Problem: convert the breeders equation into the form used in the QG of correlated traits.

$R = h^2 S$ — The breeders equation

$\Delta\bar{z}_1 = h_1^{\,2} S_1$ — substitution: $R = \Delta\bar{z}_1$

$\Delta\bar{z}_1 = \dfrac{V_{A(1)}}{V_{P(1)}} S_1$ — substitution: $h_1 = \dfrac{V_{A(1)}}{V_{P(1)}}$

$\Delta\bar{z}_1 = \dfrac{V_{A(1)}}{V_{P(1)}} \mathrm{COV}(z_1, w)$ — substitution: $S_1 = \mathrm{COV}(z_1, w)$

$\Delta\bar{z}_1 = V_{A(1)} \dfrac{\mathrm{COV}(z_1, w)}{V_{P(1)}}$ — rearrangement

$\Delta\bar{z}_1 = V_{A(1)} B(z_1, w)$ — substitution: $\dfrac{\mathrm{COV}(\bar{z}_1, w)}{V_{P(1)}} = B(z_1, w)$

$\Delta\bar{z}_1 = G_{11} B_1$ — $V_{\mathrm{A}(1)} = G_{11}$ and $B(\mathrm{z}_1,\mathrm{w}) = B_1$

$\Delta\bar{\mathbf{z}} = \mathbf{GB}$ — the standard form for correlated traits

$$\begin{bmatrix} \Delta\bar{z}_1 \\ \Delta\bar{z}_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta\bar{z}_p \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & . & . & . & G_{1p} \\ G_{21} & G_{22} & . & . & . & G_{2p} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ G_{p1} & G_{p2} & . & . & . & G_{pp} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_p \end{bmatrix}$$

the matrix/vector expansion

$\Delta\bar{z}_1 = G_{11} B_1 + G_{12} B_2$

and

$\Delta\bar{z}_2 = G_{21} B_1 + G_{22} B_2$

some concrete examples

$$B_1 = \frac{S_1 V_{P(1)} - S_2 \mathrm{COV}(z_1, z_2)}{V_{P(1)} V_{P(2)} - \mathrm{COV}(z_1, z_2)^2}$$

expansion of a partial regression coefficient

Note that if the COV=0, then

$$B_1 = \frac{S_1 V_{P(1)} - 0}{V_{P(1)} V_{P(2)} - 0} = \frac{S_1}{V_{P(1)}}$$

$$\Delta \bar{z}_1 = G_{11} B_1 = G_{11} \frac{S_1}{V_{p(1)}} = \frac{V_A}{V_{p(1)}} S = h^2 S$$