In[29]:=
```
ai =.
apop =.
R =.
Wi =.
Cd =.
```

This is Fisher's sex ratio problem. We what to show that the solution apop=1/2 is a CSS.

In[34]:=
$$Wi = \frac{R\,(1 - ai)}{Cd} + \frac{(R\,ai)\,(R\,(1 - apop))}{\frac{Cs\,Cd\,(R\,apop)}{Cs}}$$

Out[34]=
$$\frac{(1 - ai)\,R}{Cd} + \frac{ai\,(1 - apop)\,R}{apop\,Cd}$$

In[35]:=
$$Firstder = \partial_{ai}\,Wi$$

Out[35]=
$$-\frac{R}{Cd} + \frac{(1 - apop)\,R}{apop\,Cd}$$

In[37]:=
```
FullSimplify[Wi]
FullSimplify[Firstder]
```

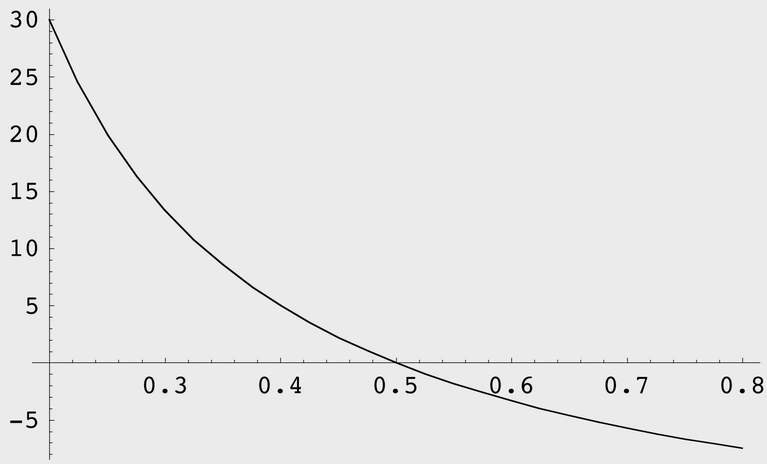
Out[37]=
$$\frac{(ai + apop - 2\,ai\,apop)\,R}{apop\,Cd}$$

Out[38]=
$$\frac{R - 2\,apop\,R}{apop\,Cd}$$

Note that, in the graph below, the first derivative is equal to zero and apop = 1/2, and that the slope is negative at that point. Hence, the solution apop=1/2 is convergence stable. That means that if the population is away from apop=1/2, mutants that are closer to 1/2 will be favored by selection, and the population will converge on the equilibrium of one half.

In[42]:=
```
R = 1;
Cd = 0.1;

Plot[Firstder, {apop, 0.2, .8}]
```



Out[44]= - Graphics -

In[20]:=
```
R =.
Cd =.
CSS = D[Firstder, apop]
```

Out[22]= $-\frac{(1-\text{apop})\ R}{\text{apop}^2\ \text{Cd}}-\frac{R}{\text{apop Cd}}$

In[26]:=
```
FullSimplify[CSS]
Reduce[CSS < 0]
```

Out[26]= $-\frac{R}{\text{apop}^2\ \text{Cd}}$

Out[27]= (R < 0 && Cd < 0 && (apop < 0 || apop > 0)) || (R > 0 && Cd > 0 && (apop < 0 || apop > 0))

Below we prove that the equilibrium is at apop=1/2.

In[28]:=
```
Reduce[Firstder == 0, apop]
```

Out[28]=
$\left(\text{apop} == \frac{1}{2} \;\&\&\; \text{Cd} \neq 0\right) \;||\; (\text{R} == 0 \;\&\&\; \text{apop Cd} \neq 0)$

In[45]:=
```
Secondder = D[Firstder, ai]
```

Out[45]= 0

Note that the second derivative (above) is zero. So the equilibrium is convergent stable, but not evolutionarily stable. Thus there will be drift at the equilibrium of 1/2, but if the population mean drifts away from 1/2, selection will tend to move the population back to one half (since the solution is convergence stable).