Extending a natural language proof theory: On ordinary comparatives

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Goals

- Provide a natural language proof theory for MacCartny’s NatLog (MacCartney (2009a,b)) that is
  - sound and complete via representation.
- Provide a semantics for ordinary comparatives
  - John is taller than Mary
that can be plausibly integrated into such a proof theory (or one like it) (see Moss (2011)).
Natural language proof

\[
\begin{align*}
\text{couch} & \equiv \text{sofa} \in \Gamma \\
\Gamma \vdash \text{couch} & \equiv \text{sofa} \quad \text{Refl} \\
\Gamma \vdash \text{sofa} & \land \text{sofa} \\
\Gamma \vdash \text{couch} & \land \text{sofa} \\
\Gamma \vdash \text{sofa} & \equiv \text{sofa} \\
\Gamma \vdash \text{sofa} & \equiv \text{sofa} \quad \land, ^1
\end{align*}
\]
Synthetic logic
Definition 1: Syntax of $S$

Let $\Phi$ be a countable set of proposition letters $p_1, \ldots, p_n$ for $n < \omega$ which I will refer to as the set of proper terms. Then,

1. If $\varphi$ is a proper term, then so is $\overline{\varphi}$;
2. If $\varphi$ and $\psi$ are proper terms, then

$$\varphi \equiv \psi, \; \varphi \square \psi, \; \varphi \preceq \psi,$$

$$\varphi \wedge \psi, \; \varphi \uparrow \psi, \; \varphi \updownarrow \psi$$

are synthetic terms.
Lexical (MacCartney) relations

- **Equivalence**
  - couch ≡ sofa
  - ‘Couch’ and ‘sofa’ are synonyms

- **Forward entailment**
  - crow □ bird
  - ‘Crow’ is a hyponym of ‘bird’

- **Reverse entailment**
  - crow ⊐ bird
  - ‘Bird’ is a hypernym of ‘crow’

- **Negation**
  - man ^ man
  - ‘Man’ and ‘non-man’ are antonyms

- **Alternation**
  - cat ^ dog
  - ‘Cat’ and ‘dog’ are alternates

- **Covers**
  - animal ^ human
  - ‘Animal’ and ‘non-human’ are covers
Definition 2: Synthetic models

Let a synthetic model \( \mathcal{M} = \langle D, \llbracket \cdot \rrbracket \rangle \), where \( D \) is a non-empty set and \( \llbracket \cdot \rrbracket \) is an interpretation function taking proper terms \( \varphi \) to their denotations in \( D \) such that

1. \( \llbracket \varphi \rrbracket = D - \llbracket \varphi \rrbracket \);
2. \( \llbracket \neg \varphi \rrbracket = \llbracket \varphi \rrbracket \);
3. \( \llbracket \varphi \rrbracket \neq \llbracket \neg \varphi \rrbracket \); and
4. \( \llbracket \varphi \rrbracket \neq \emptyset \lor \llbracket \varphi \rrbracket \neq D \) or
Definition 3: Synthetic semantics

Let \( \varphi \) and \( \psi \) be proper terms and \( R \) a MacCartney relation. Define the denotation of the synthetic term \( \varphi R \psi \), written \( \llbracket \varphi R \psi \rrbracket \), as

1. **Equivalence**
   - \( M \models \varphi \equiv \psi \iff \llbracket \varphi \rrbracket = \llbracket \psi \rrbracket \)

2. **Forward entailment**
   - \( M \models \varphi \sqsubset \psi \iff \llbracket \varphi \rrbracket \subseteq \llbracket \psi \rrbracket \)

3. **Reverse entailment**
   - \( M \models \varphi \sqsupset \psi \iff \llbracket \varphi \rrbracket \supset \llbracket \psi \rrbracket \)

4. **Negation**
   - \( M \models \varphi \land \psi \iff (\llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket = \emptyset) \land (\llbracket \varphi \rrbracket \cup \llbracket \psi \rrbracket = D) \)

5. **Alternation**
   - \( M \models \varphi \downarrow \uparrow \psi \iff (\llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket = \emptyset) \land (\llbracket \varphi \rrbracket \cup \llbracket \psi \rrbracket \neq D) \)

6. **Covers**
   - \( M \models \varphi \downarrow \rightarrow \psi \iff (\llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket \neq \emptyset) \land (\llbracket \varphi \rrbracket \cup \llbracket \psi \rrbracket = D) \)
MacCartney relations

\[
\varphi \equiv \psi \\
\varphi \sqsubset \psi \\
\varphi \sqsupset \psi \\
\varphi ^\wedge \psi \\
\varphi \downarrow \psi \\
\varphi \downarrow \psi
\]
Theorem 1: Mutual exclusivity

If $\mathcal{M}$ is a synthetic model then

$$\mathcal{M} \models \varphi R \psi \Rightarrow \mathcal{M} \nvDash \varphi S \psi$$

for $R \neq S$. 
Definition 4: Entailment

Let $\Gamma$ be a set of synthetic terms. $\Gamma \models \varphi R \psi$ just in case

$$M \models \Gamma \Rightarrow M \models \varphi R \psi$$
Definition 5: M-rules

Let $\Gamma$ be a set of synthetic formulas. Then,

$$
\frac{\Gamma \vdash \varphi R \psi \quad \Gamma \vdash \psi S \vartheta}{\Gamma \vdash \varphi T \vartheta} \quad R, S
$$

are rules of the calculus.
Sample M-rule

\[\Gamma \vdash \varphi \sqsubseteq \psi \quad \Gamma \vdash \psi \sqsubseteq \theta \]

\[\Gamma \vdash \varphi \sqsubseteq \theta \quad \sqsubseteq, \sqsubseteq\-rule\]
Definition 6: D-rules

Let $\Gamma$ be a set of synthetic terms. Then,

\[
\begin{align*}
\Gamma \vdash \varphi \equiv \varphi & \equiv_1 \\
\Gamma \vdash \psi \equiv \varphi & \equiv_2 \\
\Gamma \vdash \varphi \sqsupset \psi & \sqsupset_1 \\
\Gamma \vdash \varphi \sqsubseteq \psi & \sqsubseteq_1 \\
\Gamma \vdash \varphi \pddownarrow \psi & \pddownarrow_1 \\
\Gamma \vdash \psi \lhd \varphi & \lhd_1
\end{align*}
\]

are rules of the calculus.
**Theorem 2: Complementation**

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<thead>
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<tr>
<td>1</td>
<td>$\Gamma, \varphi \equiv \psi \vdash \varphi \uparrow \bar{\psi}$</td>
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<td>2</td>
<td>$\Gamma, \varphi \uparrow \psi \vdash \varphi \equiv \bar{\psi}$</td>
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<tr>
<td>3</td>
<td>$\Gamma \vdash \varphi \equiv \bar{\varphi}$</td>
</tr>
<tr>
<td>4</td>
<td>$\Gamma, \varphi \sqsubset \psi \vdash \bar{\psi} \sqsubset \bar{\varphi}$</td>
</tr>
<tr>
<td>5</td>
<td>$\Gamma, \varphi \sqsupset \psi \vdash \bar{\psi} \sqsupset \bar{\varphi}$</td>
</tr>
<tr>
<td>6</td>
<td>$\Gamma, \varphi \sqcap \psi \vdash \varphi \sqcap \bar{\psi}$</td>
</tr>
<tr>
<td>7</td>
<td>$\Gamma, \varphi \sqcup \psi \vdash \varphi \sqcup \bar{\psi}$</td>
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(double negation)

(contraposition)
Natural language proof

Theorem 2.1

\[ \Gamma, \varphi \equiv \psi \vdash \varphi \land \neg \psi \]

\[ \text{couch} \equiv \text{sofa} \in \Gamma \]

\[ \Gamma \vdash \text{couch} \equiv \text{sofa} \quad \text{Reflexivity} \]

\[ \Gamma \vdash \text{sofa} \land \text{sofa} \]

\[ \Gamma \vdash \text{couch} \land \text{sofa} \]

\[ \Gamma, \varphi \equiv \psi \vdash \varphi \land \neg \psi \]
Definition 7: Explosion

Let \( \Gamma \) be a set of synthetic terms. Then,

\[
\Gamma \vdash \varphi R \psi \quad \Gamma \vdash \varphi S \psi \quad \text{for } R \neq S
\]

\[
\Gamma \vdash \varphi' T \psi' \quad \text{for all } \varphi' T \psi'
\]

is a rule of the calculus.
Definition 8: Consistency

Γ is consistent if, and only if \( \Gamma \not
\varphi R\psi \) for some synthetic term \( \varphi R\psi \).
Inconsistency

\[
\begin{align*}
\text{animal} \sqsubset \text{life} &\in \Gamma \\
\Gamma \vdash \text{animal} \sqsubset \text{life} &\quad \text{Refl}
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash \text{life} \sqsupset \text{animal} &\quad \text{Exp}
\end{align*}
\]

\[
\begin{align*}
\text{animal} \sqsubset \text{human} &\in \Gamma \\
\Gamma \vdash \text{animal} \sqsubset \text{human} &\quad \text{Refl}
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash \text{life} \sqsupset \text{human} &\quad \text{Exp}
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash \psi' \quad \text{for all } \psi'
\end{align*}
\]
Representation

**Theorem 3: Completeness**

Let $\Gamma$ be a consistent of synthetic terms. Then,

$$\Gamma \vdash \varphi R \psi \iff \Gamma \models \varphi R \psi$$

- Every consistent $\Gamma$ induces an order on the set of proper terms $\Phi$;
- That ordered set can be transformed into an orthoposet;
- Every orthoposet can be represented as a system of sets (Zierler and Schlessinger (1965), Calude et al (1999) Moss (2007));
- The system of sets will function as a synthetic model.
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Ordinary comparatives
Ordinary comparatives

- **Ordinary Comparatives**
  1. John is taller than Mary

- **Negation**:
  1. John is no taller than Mary
  2. John isn’t taller than Mary

- **Coordination**:
  1. John is taller than Mary or Sue is
  2. John is taller than Mary is or Sue is

- **Quantification**:
  1. John is taller than everyone else
  2. John is taller than exactly two woman

- **Temporal expressions**:
  1. John is taller than he was yesterday
  2. John will be taller than he is today

- **Modal expressions**:
  1. John is taller than he was yesterday
  2. John will be taller than he is today
Previous analyses

1. **Degrees** (Seuren 1973)

\[ \exists d \left( \text{tall}\,(john, d) \land \neg\,(\text{tall}\,(mary, d)) \right) \]

where \( d \) is a variable ranging over degrees.

2. **Precisifications** (Kamp 1975)

\[ \exists p \left( p \models \text{tall}\,(john) \land \neg\,(p \models \text{tall}\,(mary)) \right) \]

where \( p \) is a variable ranging over precisifications of a base model \( \mathbb{M} \).

3. **Comparison classes** (Klein 1980)

\[ \exists c \left( c \models \text{tall}\,(john) \land \neg\,(c \models \text{tall}\,(mary)) \right) \]

where \( c \) is a variable ranging over comparison classes.

4. **‘Measure’ functions** (Kennedy 1997)

\[ \text{tall}\,(john) > \text{tall}\,(mary) \]

where \( \text{tall}\,(john) \) and \( \text{tall}\,(mary) \) correspond to the degrees of John’s and Mary’s (maximal) height respectively.
Derived degrees

• Authors have tried to derive ‘degrees’ as equivalence classes of more basic ontology such as individuals (Cresswell 1976; Bale 2008; van Rooij 2010; Bale 2011; van Rooij 2011).

• Natural to view entities $e$ as being an ordered set of ‘atoms’ $e = (A, \leq)$.

• Minimally extend Frege’s definition of having the same amount as to formalize Carnap’s (1967) example of arranging rods by length.

• Let $\delta$ be a function taking an individual $e$, world $w$, and time $t$, to the set of triples $(e', w', t')$ such that $e$ is order isomorphic ($\cong$), i.e., has the same height, to $e'$ at world $w'$ and time $t'$.

• $\delta$ induces a linear order $>$ over the set $I \times W \times T$, where each equivalence class of entity, world, time triples corresponds to a ‘degree’.

• This order can serve as a ‘trans-world/time measuring rod’.
Derived degrees (cont.)

\[
\left\{ (e_1, w_8, t_2), (e_2, w_1, t_2) \right\} \supset \ldots \supset \left\{ (e_1, w_3, t_1), (e_3, w_2, t_1) \right\}
\]

\[
\left\{ (e_1, w_4, t_1), (e_3, w_4, t_4) \right\}
\]

\[
\left\{ (e_2, w_3, t_9), (e_5, w_2, t_4) \right\}
\]
Current analysis

Definition 9: taller

Let $e$ and $e'$ be entities, $w$ a world, and $t$ a time. Then,

$$e \text{ taller } e' \text{ at } w \text{ and } t \iff \delta((e, w, t)) > \delta((e', w, t))$$

(which can intuitively be understood as reading, ‘$e$ has more degrees of height than $e'$ at world $w$ and time $t$’).

- Define shorter as the order dual ($<$) of taller.
- Define as tall as ($\geq$) as not taller than.
Negation

Claim: (1) ⇔ (2) ⇔ (3)

1. John is not taller than Mary
2. Mary is as tall as John
3. John is as short as Mary

\[ \neg (john \text{ taller } mary) \iff mary \text{ as tall } john \iff john \text{ as short as } mary \]

van Rooij (2008)
Entity/sentential-level coordination

**Claim:** (1) ⇔ (2)

1. John is taller than Mary (is) and Sue (is)
2. John is taller than Mary or Sue (is)

von Stechow (1984)

\[(\text{john taller } \text{mary}) \land (\text{john taller } \text{sue}) \iff \text{john taller } \max\{\text{mary, sue}\} \iff \text{john taller } (\text{mary } \sqcup \text{sue})\]
Quantification

1. John is taller than everyone (else at the current world and time)

\[ \forall x \ (x \neq john \rightarrow (\delta((john, w, t)) > \delta((x, w, t)))) \]

2. Every man is taller than some woman (at the current world and time)

\[ \forall x \ (\text{man}(x)(w)(t) \rightarrow \exists y \ (\text{woman}(y)(w)(t) \land (\delta((x, w, t)) > \delta((y, w, t))))) \]
Time and modality

1. John might be taller than Mary

\[ \exists u \ (\delta((john, u, t)) \textbf{taller} \ \delta((mary, u, t))) \]

2. John will be taller than Mary

\[ \exists s \ (\delta((john, w, s)) \textbf{taller} \ \delta((mary, w, s)) \land t < s) \]
Inference patterns

Using

- properties and concepts of (linear) orders; and
- classical rules of inference, e.g., De Morgan’s Law

we get a robust set of inferences involving comparatives.
Future work

- Develop a natural language proof theory integrating ordinary comparatives that builds on the insights of the semantics of ordinary comparatives presented here (again see Moss (2011)).
  - What orders do other adjectives induce, e.g., clever (see Kamp (1975) among others)?
  - How are those orders determined?
- Integrate ‘comparative proof theory’ with a larger natural language proof theory that allows reasoning across construction types
  - Temporal expressions (before, after, etc.)
  - Quantifiers (every, some, etc.) via
References