Inferences and Dialogues in Ludics

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Objectives of this work

Define a fundamental framework:

- that combines representations of dialogues and of reasoning

- that contains only one kind of objects
  => composition of objects logically sound

- that does not refer to formulas when there is no necessity (e.g. arguments in dialogues)
  => stay in a monotonous setting
In Logics, two notions of inference:
- Cut-elimination
- Modus Ponens

and a relation between computation/proofs and formulas:
- Typing

They may be used to model dialogues and reasoning:

<table>
<thead>
<tr>
<th>Computation</th>
<th>Logics</th>
<th>Dialogue/Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dynamics of computation</td>
<td>Cut-elimination</td>
<td>dialogue process</td>
</tr>
<tr>
<td>computation of a result typing</td>
<td>Modus Ponens</td>
<td>reasoning process</td>
</tr>
<tr>
<td>typing</td>
<td>formula</td>
<td>semantics of utterances</td>
</tr>
</tbody>
</table>
Remark that:

- A Modus Ponens between a formula and its negation induces a contradiction.
- A formula may be denoted by its set of proofs.
- A term may be used as an argument for a function even if it is incorrect as the level of types ($A \oplus B$ vs $A$).

**Motto**

- Use proofs to model dialogues and argumentation structures
- Use formulas to define/characterize utterances, ...
Interaction in dialogues

A rough attempt to focus on interaction in natural language dialogues:

associate actions to interventions

- actions are polarized: **positive** for the locutor who produces them, **negative** for the locutor who receives them
- actions/interventions of each locutor are organized in a tree-like structure: a **design**
- the dialogue is the **trace** of the interaction between two designs
Example (A. Dumas, Le Comte de Montecristo)

Dialogue between Edmond and Faria:

F  $l_1$: What was your life at this time?
E  $l_2$: I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl.
F  $l_3$: Was anyone interested in you not becoming the captain of the Pharaon?
E  $l_4$: [...] Only one man [...],
F  $l_5$: Who was he?
E  $l_6$: Danglars.
F  $l_7$: Well, tell me about that young girl ...
Example (A. Dumas, Le Comte de Montecristo)

We associate to each intervention an action: $\kappa_1, \ldots, \kappa_7$ and a justification relation between them.

- Each intervention refers to a previous one.
- Action $\kappa_1$ is initial: $F$ initiates the dialogue.
- Action $\kappa_7$ is justified by action $\kappa_2$.

$I_1 : \text{What was your life at this time?}$
$I_2 : \text{I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl.}$
$I_3 : \text{Was anyone interested in you not becoming the captain of the Pharaon?}$
$I_4 : [\ldots] \text{Only one man [\ldots],}$
$I_5 : \text{Who was he?}$
$I_6 : \text{Danglars.}$
$I_7 : \text{Well, tell me about that young girl } \ldots$
Example (A. Dumas, Le Comte de Montecristo)

The interventions polarised and organised according to both justification and chronological relations:

Point of view of Faria

Point of view of Edmond
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

<table>
<thead>
<tr>
<th>F</th>
<th>l₁ :</th>
<th>What was your life at this time?</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>l₂ :</td>
<td>I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl.</td>
</tr>
<tr>
<td>F</td>
<td>l₃ :</td>
<td>Was anyone interested in you not becoming the captain of the Pharaon?</td>
</tr>
<tr>
<td>E</td>
<td>l₄ :</td>
<td>[...] Only one man [...],</td>
</tr>
<tr>
<td>E</td>
<td>l₅ :</td>
<td>Who was he?</td>
</tr>
<tr>
<td>F</td>
<td>l₆ :</td>
<td>Danglars.</td>
</tr>
<tr>
<td>F</td>
<td>l₇ :</td>
<td>Well tell me about that young girl...</td>
</tr>
</tbody>
</table>

Faria's viewpoint

Edmond's viewpoint
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

| F | I₁ : What was your life at this time? |
| E | I₂ : I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl. |
| F | I₃ : Was anyone interested in you not becoming the captain of the Pharaon? |
| E | I₄ : [...] Only one man [...], |
| F | I₅ : Who was he? |
| E | I₆ : Danglars. |
| F | I₇ : Well tell me about that young girl... |

Faria's viewpoint

Edmond's viewpoint
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

\[ \kappa_7 \rightarrow \kappa_6 \rightarrow \kappa_5 \rightarrow \kappa_4 \rightarrow \kappa_3 \rightarrow \kappa_2 \rightarrow \kappa_1 \]

Faria's viewpoint  Edmond's viewpoint

<table>
<thead>
<tr>
<th></th>
<th>( l_1 ) :</th>
<th>What was your life at this time?</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>( l_2 ) :</td>
<td>I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl.</td>
</tr>
<tr>
<td>F</td>
<td>( l_3 ) :</td>
<td>Was anyone interested in you not becoming the captain of the Pharaon?</td>
</tr>
<tr>
<td>E</td>
<td>( l_4 ) :</td>
<td>[...] Only one man [...]</td>
</tr>
<tr>
<td>F</td>
<td>( l_5 ) :</td>
<td>Who was he?</td>
</tr>
<tr>
<td>E</td>
<td>( l_6 ) :</td>
<td>Danglars.</td>
</tr>
<tr>
<td>F</td>
<td>( l_7 ) :</td>
<td>Well tell me about that young girl...</td>
</tr>
</tbody>
</table>
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

Faria's viewpoint  Edmond's viewpoint

F

I7 : What was your life at this time?
I6 : I was ready to become captain of the *Pharaon*;
I5 : I was about to marry a beautiful young girl.

E

I3 : Was anyone interested in you not becoming the captain of the *Pharaon*?
I4 : [...] Only one man [...] ,
I2 : Who was he ?
I1 : Danglars.
I7 : Well tell me about that young girl...
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

| F |  l1 :  | What was your life at this time? |
| E |  l2 :  | I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl. |
| F |  l3 :  | Was anyone interested in you not becoming the captain of the Pharaon? |
| E |  l4 :  | [... ] Only one man [...], |
| F |  l5 :  | Who was he? |
| E |  l6 :  | Danglars. |
| F |  l7 :  | Well tell me about that young girl... |

Faria's viewpoint
Edmond's viewpoint

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Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

<p>| | | | | | |</p>
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<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>l₁ : What was your life at this time?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>l₂ : I was ready to become captain of the Pharaon; I was about to marry a beautiful young girl.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>l₃ : Was anyone interested in you not becoming the captain of the Pharaon?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>l₄ : [...] Only one man [...],</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
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<td></td>
<td></td>
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<tr>
<td>E</td>
<td>l₆ : Danglars.</td>
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</tr>
<tr>
<td>F</td>
<td>l₇ : Well tell me about that young girl...</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Faria’s viewpoint  Edmond’s viewpoint
Example (A. Dumas, Le Comte de Montecristo)

The dialogue as a step by step interaction between two designs:

- **Faria's viewpoint**

- **Edmond's viewpoint**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>l₁ : What was your life at this time?</td>
<td>E</td>
</tr>
<tr>
<td>E</td>
<td>l₂ : I was ready to become captain</td>
<td>l₃ : Was anyone interested in you</td>
</tr>
<tr>
<td></td>
<td>of the <em>Pharaon</em>;</td>
<td>not becoming the captain</td>
</tr>
<tr>
<td></td>
<td>I was about to marry a beautiful</td>
<td>of the <em>Pharaon</em>?</td>
</tr>
<tr>
<td></td>
<td>young girl.</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>l₄ : [... ] Only one man [... ],</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>l₆ : Danglars.</td>
<td>E</td>
</tr>
<tr>
<td>F</td>
<td>l₇ : Well tell me about</td>
<td></td>
</tr>
<tr>
<td></td>
<td>that young girl...</td>
<td></td>
</tr>
</tbody>
</table>

Faria's viewpoint    Edmond's viewpoint
What is a *dialog act*?

Following Landragin, we define a *dialog act* as

“The minimal unit of communication in a dialogic context”.

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What is a *dialog act*?

A dialog act is a communicational fact whose role is to allow the dialogue to progress and to determine the dialogue shape:

**Definition (Dialog Act)**

A *dialog act* $\kappa$ is:

- either a proper dialog act, i.e. a tuple $(+/-, L, I, e)$ (see next slide)

- or a particular positive dialog act, called *daimon* and noted $(\dagger, e)$, that shows the end of an interaction that went well.
What is a *dialog act*

When the dialog act is a proper one $\kappa = (+/-, L, I, e)$:

- $L$ is the **focus** of $\kappa$: the address on which the act is localized with respect to the dialogic interaction one considers,

- $I = \{L_1, \ldots\}$ is the **ramification** of $\kappa$: the openings created by the dialog act on which new dialog acts may be produced,

- $e$ is the **expression** of the dialog act, i.e. the language (or communication) fact by which the dialog act is reflected:
  - a (sequence of) proposition(s), word, prosodic element, non verbal sign.
A dialogue is represented as an interaction between two **dialog designs**, i.e. designs with dialog acts.

- Dialog designs are built incrementally from the interventions, i.e. the turns of speech of the two persons. To each intervention corresponds a set of dialog acts that completes the current design of the speaker.

- Such a set may be a unique dialog act, an alternate sequence of dialog acts or a partial design. The design completed by such a set should still be a design.

- Finally, an intervention must begin and end with a positive dialog act: hence, if not ending with a daimon, the intervention of a speaker allows the addressee to speak.
The controversy: a particular case of dialogue

Among the dialogues, controversies own some specific features:

- they are composed by means of only a few speech acts: assertions, argumentations, denegations, concessions...

- a notion of “gain” arises. It is out of purpose in most cases of dialogues. But it is essential when the dialogue is a controversy.
How give an account of these specific dialogues?

Lots of argumentation studies make use of games theory but there are two important distinctions between the argumentative frame and the model of games.

- the only relevant rule in the argumentative frame is the one relative to keeping the dialogue convergent.
- in games, each player makes an elementary move. In dialogues, one intervention may contain several speech acts.
A case study

We consider an example developed by Prakken. It is a juridic controversy between a plaintiff and a defendant.

Plaintiff claimed that defendant owes him 500 euro, because both signed a contract; himself delivered but defendant did not pay.

Defendant will give two counter arguments: 1. he does not recognize his signature, 2. he was insane during the signature of the contract.

\[
\begin{align*}
\text{I}_1 & \quad \text{Plaintiff:} \quad \text{I claim that defendant owes me 500 euro.} \\
\text{I}_2 & \quad \text{Defendant:} \quad \text{I dispute plaintiff’s claim.} \\
& \quad \ldots \quad \ldots \\
\text{J}_4 & \quad \text{Judge (deciding the dispute):} \\
& \quad \text{I am convinced by plaintiff’s evidence that defendant’s signature under the contract is authentic. Yet I cannot grant plaintiff’s claim since the fact that defendant looked normal during the negotiations is insufficient to conclude that defendant’s insanity could not be known to plaintiff: he might have known if he had checked the court’s register. Therefore I deny plaintiff his claim.}
\end{align*}
\]
The dispute in Ludics

We represent the dispute between plaintiff and defendant by an interaction between two designs, as if this controversy were a dialogue between only these two locutors.

Inside this representation, the interventions of the judge are given into account by means of some forced dialog acts for either one locutor or the other one.
The three first interventions: $I_1 \ldots I_3$

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>I claim that defendant owes me 500 euro.</td>
</tr>
<tr>
<td>$I_2$</td>
<td>I dispute plaintiff’s claim.</td>
</tr>
<tr>
<td>$I_3$</td>
<td>Defendant owes me 500 euro by $r_1$ since we conclude a valid sales contract, I delivered but defendant did not pay.</td>
</tr>
</tbody>
</table>

And next intervention:

$I_4$: I concede that plaintiff delivered and I did not pay, but I dispute that we have valid contract.

Look at speech acts modelling ...
Towards a model of the speech acts of controversy

- **To claim a thesis** This corresponds to posing a proposition on which the interlocutor may continue the dialogue by negating it or by conceding it or by asking more explanations.

  *In Ludics: \((+, L, I, e)\).*

- **To argue, to negate, to ask, to request some justification** . . .

  They are represented as a claim, by a positive proper dialog act.
The speech acts of controversy

- **To concede.** This consists in accepting one assertion that the interlocutor claimed. The dialogue will not continue on this element.

  sequence \((+ , L , \{ L' \} , e_0 ) (− , L' , \emptyset , e_1 )\)

\((+ , L , \{ L' \} , e_0 )\) expresses what is conceded, its focus is the one created by the affirmation that one concedes and its ramification is a singleton.

\((− , L' , \emptyset , e_1 )\) makes the conceded affirmation disappear: it is focused on \(L'\), i.e. the immediate previous act created, it has an empty ramification.
Winning a controversy

During a controversy, one of the locutors becomes loser when:

- It is his turn
- No addresses are available
- He is obliged to play the daïmon.

*The winner has the last word.*
The fourth intervention: \(l_4\)

\(l_4\): I concede that plaintiff delivered and I did not pay, but I dispute that we have valid contract.

We give an account of this one not by a unique dialog act but by a sequence of dialog acts:

<table>
<thead>
<tr>
<th>speech act</th>
<th>dialog act</th>
<th>expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>concession</td>
<td>(+, (L_{33}), {(L_{41}}), (e_1))</td>
<td>I concede that plaintiff delivered</td>
</tr>
<tr>
<td></td>
<td>(−, (L_{41}), (\emptyset), (e_2))</td>
<td></td>
</tr>
<tr>
<td>concession</td>
<td>(+, (L_{34}), {(L_{42}}), (e_3))</td>
<td>(I concede that) I did not pay</td>
</tr>
<tr>
<td></td>
<td>(−, (L_{42}), (\emptyset), (e_3))</td>
<td></td>
</tr>
<tr>
<td>negation</td>
<td>(+, (L_{32}), {(L_{43}}), (e_4))</td>
<td>but I dispute that we have</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a valid contract</td>
</tr>
</tbody>
</table>
until the intervention $I_4$: I concede that plaintiff delivered and I did not pay, but I dispute that we have valid contract.
The dispute continues

\[ I_5 : \text{We have a valid contract by } r_2 \]
\[ \text{since this document is a contract signed by us.} \]

\[ I_6 : \text{I dispute that this is my signature.} \]

\[ I_7 : \text{Why?} \]
The first judge’s intervention: $J_1$

$J_1 : \text{By } r_3 \text{ the party who invokes a signature under a document which is not an avidavit has the burden to prove that it is authentic when this is disputed so plaintiff must prove this is defendant’s signature.}$

- It occurs inside the dispute as an intervention of the defendant.
- It is a sequence of dialog acts: it starts with a positive one (arguing), opens three elements. The two first elements are immediately closed: it is not possible to discuss neither a law ($r_3$) nor a fact (the document is not an avidavit).
- It gives the turn to plaintiff.
The first judge’s intervention: $J_1$

$J_1$: By $r_3$ the party who invokes a signature under a document which is not an avidavit has the burden to prove that it is authentic when this is disputed so plaintiff must prove this is defendant’s signature.
The Judge’s sentence

**Before**

\[ L_{16} \vdash \ldots \vdash L'_{91} \vdash \ldots \vdash L_{51} \vdash J_{10}, J_{11}, J_{12} \vdash \pi_1 \vdash \omega \vdash L_0 \vdash \]
**Plaintiff**

\[ L_{16}, L'_{91} \vdash \ldots \vdash L_{51} \vdash \pi_1 \vdash \omega \vdash L_0 \vdash \]
**Defendant**

It is the defendant’s turn. Two addresses are available:
- \( L'_{91} \) (his signature is not valid)
- \( L_{16} \) (his illness)

**After**

\[ \vdash L_{16} \vdash \ldots \vdash L'_{91} \vdash \ldots \vdash L_{51} \vdash J_{10}, J_{11}, J_{12} \vdash \pi_1 \vdash \omega \vdash L_0 \vdash \]

I am convinced by plaintiff’s evidence that defendant’s signature under the contract is authentic. Yet I cannot grant plaintiff’s claim since the fact that defendant looked normal during the negotiations is insufficient to conclude that defendant’s insanity could not be known to plaintiff: he might have known if he had checked the court’s register. Therefore I deny plaintiff his claim.
Back to intervention $I_3$ of the plaintiff

$I_3$ : Defendant owes me 500 euro by r1 since we conclude a valid sales contract, I delivered but defendant did not pay.

- This expresses a contradiction (pay/not pay) according to plaintiff’s viewpoint.

  How can we make this explicit without yielding a logical absurdity?

- The judge overcomes this contradiction when giving the sentence.

  How can we modify plaintiff’s knowledge?
Inferential rôle of interventions

We consider a framework close to ISU (Informative states update, Asher, Ginzburg, . . .)

- **Dialogue surface**: an interaction between designs made of dialogue acts.

- **Dialogue context**: one cognitive base for each locutor containing a state of their knowledge, their entitlements, . . ., and updated after each intervention. Such a dialogue context is a set of designs.

- An intervention is *built* from elements of the cognitive base.
Back to intervention $l_3$ of the plaintiff

$l_3$ : Defendant owes me 500 euro by r1 since we conclude a valid sales contract, I delivered but defendant did not pay.

- In the modelling of the dialogue:
  \[
  \begin{array}{cccc}
  D_{r_1} & D_{val-contract} & D_{del} & D_{not-paid} \\
  \vdash & \vdash & \vdash & \vdash \\
  L_{31} & L_{32} & L_{33} & L_{34} \\
  \end{array}
  \]

\[
\vdash L_2
\]

- Plaintiff’s base includes designs $D_{r_1}$, $D_{val-contract}$, $D_{del}$ and $D_{not-paid}$, that are delocalized to build his argumentation.
Plaintiff’s designs

- Designs modelling facts he put forward and that are conceded, $\mathcal{D}_{\text{del}}$ et $\mathcal{D}_{\text{not.paid}}$, together with a design $\mathcal{D}_{\neg \text{pay}}$ interpreting the fact: ‘to pay’ is the logical negation of ‘not to pay’:

$$\mathcal{D}_{\text{del}} = \vdash L_{\text{Del}} \quad \mathcal{D}_{\text{not.paid}} = \vdash L_{\text{notPaid}} \quad \mathcal{D}_{\neg \text{pay}} = L_{\text{Pay}} \not\vdash \emptyset$$

- The design $\mathcal{D}_{\text{val-contract}}$ (the validity of a contract relies on the validity of the signature):

$$\mathcal{D}_{\text{val-contract}} = \vdash L_{\text{val-contract}}$$
Finally, there is a design corresponding to the legal rule $r_1$:

$$
\begin{align*}
\vdash L_{\text{Pay}} & \quad \emptyset \\
L_{\text{Del}} & \vdash L_{\text{Pay}} & \quad \emptyset \\
\vdash L_{\text{not-Del}, L_{\text{Pay}}} & \\
L_{\text{sign}} & \vdash L_{\text{not-Del}, L_{\text{Pay}}} & \\
\vdash L_{\text{not-sign}, L_{\text{not-Del}}, L_{\text{Pay}}} & \\
L_{\text{val-contract}} & \vdash L_{\text{not-Del}, L_{\text{Pay}}} & \\
\vdash L_{\text{not-val-contract}, L_{\text{not-Del}}, L_{\text{Pay}}} & \\
D_{r_1} = & \vdash L_{\text{Cont-and-Del}} & \vdash L_{\text{Pay}}
\end{align*}
$$
Interaction in plaintiff's base

The result of the interaction between

\[
\begin{align*}
\vdash L_{\text{Pay}} & \quad \emptyset \\
L_{\text{Del}} & \vdash L_{\text{Pay}} \\
& \vdash L_{\text{not-Del}}, L_{\text{Pay}} \\
L_{\text{sign}} & \vdash L_{\text{not-Del}}, L_{\text{Pay}} \\
& \vdash L_{\text{not-sign}}, L_{\text{not-Del}}, L_{\text{Pay}} \\
L_{\text{val-contract}} & \vdash L_{\text{not-Del}}, L_{\text{Pay}} \\
& \vdash L_{\text{not-val-contract}}, L_{\text{not-Del}}, L_{\text{Pay}} \\
\end{align*}
\]

\[D_{r_1} = L_{\text{Cont-and-Del}} \vdash L_{\text{Pay}}\]

and

\[
\begin{align*}
\vdash L_{\text{sign}} & \quad \emptyset \\
& \vdash L_{\text{not-sign}} \\
L_{\text{val-contract}} & \vdash L_{\text{not-Del}} \\
& \vdash L_{\text{not-val-contract}} \\
& \vdash L_{\text{not-Del}} \\
& \vdash L_{\text{Del}} \\
\end{align*}
\]

\[\vdash L_{\text{Cont-and-Del}} \quad \emptyset\]

gives

\[\vdash L_{\text{Pay}} \quad \emptyset\]
A “contradiction” in plaintiff’s base

Designs \( D_{\text{pay}} \) and \( D_{\neg \text{pay}} \) are in plaintiff’s base. Their interaction yields a contradiction:

\[
\begin{align*}
\emptyset & \vdash L_{\text{Pay}} \\
L_{\text{Pay}} & \vdash \emptyset
\end{align*}
\]

This design is not a formula corresponding to a logical absurdity!

The aim of the judge consists in overcoming this contradiction.
The judge overcomes the contradiction as he decides that legal rule $r_1$ cannot be applied.

- We interpret the judge’s argument as: the legal rule $r_1$ needs two premisses (and not one as plaintiff supposes).

- In other words, the validity of a contract requires two conditions (hence two sub-addresses): validity of the signature and non-insanity of those signing.
Example ctd (simplified)

Formally, plaintiff’s base is changed (by interactions)
  - either to ‘delete’ the (false) design corresponding to legal rule $r_1$ plaintiff has.
  - or to ‘refine’ design $\mathcal{D}_{val\text{-}contract}$.

‘Deletion’ and ‘refinement’ are in fact done by inferring (hence with interactions) new designs.
Specificities of our approach

With respect to the usual approaches (concerning the juridic controversies):

- We refine speech acts as sequences of (more elementary) **dialog acts**.
- Instead of considering a game between three players, the judge directly intervenes inside the interaction between two protagonists.
- Reasoning is modelled at a more primitive level than with formulas.
The notions of formal proof and of formula are simultaneously **too demanding** and **too simplistic** to account for argumentation:

- during a controversy, one can agree on the validity of a statement, without this statement to be necessarily fully determined as a formal proposition,
- one may concede the correctness of a position without necessarily having explored all its premises,
- a locutor can win a debate, simply because her addressee surrendered.
Why Ludics?

- Designs are the basic objects: Formulas and proofs are not given \textit{a priori}.
- Ludics gives a uniform framework for dealing with different levels, and for combining elements from these levels.
- Interaction is the main ingredient for modelling reasoning and dialogue.
Ludics in a nutshell

In Ludics:
- objects are abstract proof trees: the *designs*
- interaction between objects may be done to
  - test the validity of an object,
  - infer a result (similar to Modus Ponens)
- interaction between two dual designs is the core of orthogonality
- closed sets of objects help to recover formulas, hence recover logics
Ludics in a nutshell

Key points:

- No more formulas but **addresses**.
- No more logical rules but **actions**:

  \[
  \text{\(A\perp \vdash B\perp\)} \quad \text{corresponds to one positive action:} \\
  \quad (+, L, \{L_1, L_2\})
  \]

  \[
  \text{\(A\perp, B\perp \vdash A\perp, C\perp\)} \quad \text{corresponds to two negative actions:} \\
  \quad (-, L, \{L_1, L_2\}) \text{ and } (-, L, \{L_3, L_4\})
  \]
Ludics in a nutshell

- No more proofs but **designs**:

\[
\begin{align*}
\phi & \vdash \phi & \psi & \vdash \psi \\
\vdash & \phi \perp, \phi & \vdash & \psi \perp, \psi \\
\phi \otimes & \phi \perp & \psi \otimes & \psi \perp \\
\vdash & ((\phi \perp \oplus \phi) \otimes (\psi \perp \oplus \psi)) \oplus \rho
\end{align*}
\]

becomes (three notations)

\[
\begin{align*}
(+, L_{21}, L_3) & \quad (+, L_{21}, L_3) \\
(-, L_1, \{L_{21}, L_{22}\}) & \quad (-, L_1, \{L_{21}, L_{22}\}) \\
(+, L_0, \{L_1, K_1\}) & \quad L_3 & \vdash & L_{22} \\
& \quad \vdash & L_{21}, L_{22} & \quad K_3 & \vdash & K_{22} \\
& \quad L_1 & \vdash & K_{21}, K_{22} & \quad K_1 & \vdash & L_0
\end{align*}
\]

or

\[
\begin{align*}
K_3 & \quad K_5 \\
K_2 & \quad K_4 \\
K_1 & \quad K_1
\end{align*}
\]
A design of base $(L_0) \vdash L_1, \ldots, L_n$ is a set of alternate sequences of actions, satisfying a few constraints:

- In each sequence, actions are either initial, i.e. in the base, or justified by an action that precedes in the sequence.

- The set is closed by prefix, total, positive, coherent.
No more cut-elimination but interaction

An interaction occurs between designs based on dual addresses:

![Diagram of interactions between designs]
Interaction begins on the design that has a positive base. Each time a positive action $\kappa$ is visited, the travel continues on the dual (negative) action $\kappa$ in the other design (if it exists), then goes to the unique positive action after $\kappa$ in this design (bottom-up reading).
Interaction between designs

...and so on.
Interaction between designs

... and so on.
Interaction between designs

... and so on.
Interaction between designs

...and so on.
... if the negative action does not exist, the interaction fails, diverges.
... if the daimon is reached, the interaction succeeds, converges.
Two designs are **orthogonal** when their interaction succeeds.

**Separation**: A design is fully defined by its counter-designs.

A formula is retrieved as a set of designs closed by bi-orthogonality: such sets are called **behaviors**.

**Completeness**: A **good design** belongs to the interpretation of a formula iff it is the interpretation of a proof.

A formula is valid when the behavior that interprets it contains such a **good** design.