Structural-Morphism System Property: atis Homomorphismness

(Structural-morphism system properties are those properties that are part of the theory and define the mapping-relatedness of object-set components.)

Homomorphismness (general morphismness), \( \mathcal{M} =_{df} \) components that have the same connections as other components.

\[
\mathcal{M} =_{df} \mathcal{M}(\mathcal{S}_1, \mathcal{S}_2) \mid [\mathcal{S}_1(P(A_1)) \equiv \mathcal{S}_2(P(A_2))]
\]

Homomorphismness is a morphism; such that, the mapping is defined by equivalent affect-relation set predicates of each system.

Every affect relation of a system defines a homomorphism that is self-mapped. Between two systems, a homomorphism is defined by affect-relation sets that are defined by the same predicate.

HOMOMORPHISM or GENERAL MORPHISM: Functions between structures that preserve relations are called homomorphisms.

STRUCTURE: Let \( \tau \) be a signature. A signature is the collection of a set of constant symbols, and a set of n-ary relation symbols and a set of n-ary function symbols.

For ATIS, the signature is: \( V = \{ a, b, c, \ldots \}; \) \( R = \{ \{ x \}, \{ x, y \} \}; \) \( =, \subset, \forall, \exists, \iota, \hat{w}, A \}, \) where \( \{ \{ x \}, \{ x, y \} \} \in A; \) and \( F = \{ \in, \cup, \cap, \} \); and additional relations or functions as required.

A \( \tau \)-structure, \( a \), consists of an object-set, \( U \), called the universe (or domain) of \( a \), together with

- For each constant symbol \( c \in \tau \), a component, \( c^a \in U \);
- For each n-ary relation symbol \( R \in \tau \), a subset, \( R^a \subseteq U^n \); and
- For each n-ary function symbol \( f \in \tau \), a function, \( f^a: U^n \rightarrow U \).

When the context makes it clear in which structure we are working, we use the elements of \( \tau \) to stand for the corresponding constant, relation, or function. When \( \tau \) is understood, we call \( a \) a structure, instead of a \( \tau \)-structure. Also, it is common to write \( a \in A \) instead of \( a \in U \).

For ATIS, the system relations are defined by the family of affect relations, \( A \).
Therefore, a homomorphism for ATIS is a function, or mapping, between two object-sets that, by definition, have the above properties of a structure. Now let’s consider two object-sets in the same or different systems with respect to the same affect relation. Let $\Sigma$ be a fixed signature, and $\mathcal{A}$ and $\mathcal{B}$ are two structures, object-sets, for $\Sigma$. A fixed signature simply means that both systems of ATIS have the same constants, relations and functions. And, while there may be numerous functions between these affect relations, the interesting functions between them; that is, from $\mathcal{A}$ to $\mathcal{B}$ are the ones that preserve the structure; that is, preserve the affect relations with respect to each object-set. Essentially, this means that we are concerned with comparable affect relations in both systems with respect to the respective object-sets. Now, homomorphism can be more carefully defined.

A function $f: \mathcal{A} \rightarrow \mathcal{B}$ is said to be a homomorphism if and only if:

1. For every constant symbol $c$ of $\Sigma$, $f(c^\mathcal{A}) = c^\mathcal{B}$. That is, every constant in $\mathcal{A}$ is mapped to a constant in $\mathcal{B}$.
2. For every n-ary function symbol $\mathcal{F}$ of $\Sigma$,
   
   $f(\mathcal{F}(a_1, a_2, \ldots, a_n)) = \mathcal{F}(f(a_1), f(a_2), \ldots, f(a_n))$.

   That is, the function; for example, the union, $\cup$, of the components in $\mathcal{A}$ is equal to the union of the functions in $\mathcal{B}$.

3. For every n-ary relation symbol $\mathcal{R}$ of $\Sigma$,

   $\mathcal{R}(a_1, a_2, \ldots, a_n) \supseteq \mathcal{R}(f(a_1), f(a_2), \ldots, f(a_n))$.

   That is, the relation; for example, the subset relation, $\subset$, of the components in $\mathcal{A}$ imply that the corresponding functions in $\mathcal{B}$ are subsets of $\mathcal{B}$.

Therefore, a function $f: \mathcal{A} \rightarrow \mathcal{B}$ is said to be a homomorphism if and only if relations with respect to each system are preserved under the mapping.

For example, consider two object-sets consisting of teachers, $\mathcal{T}$, and students, $\mathcal{S}$. Let $f: \mathcal{T} \rightarrow \mathcal{S}$ be a function between these two object-sets. We say that $f$ is an ATIS System Homomorphism if:

- Every component of $\mathcal{T}$ is mapped to a component of $\mathcal{S}$: $f(c^\mathcal{T}) = c^\mathcal{S}$;
- Every function in $\mathcal{T}$ is preserved in $\mathcal{S}$: $f(\mathcal{F}(a_1)) = \mathcal{F}(f(a_1))$; and
- Every relation in $\mathcal{T}$ is preserved in $\mathcal{S}$: $\mathcal{R}(a_1) \supseteq \mathcal{R}(f(a_1))$. 
The following mapping represents an ATIS System Homomorphism designated by the affect relation ‘guides the learning of’. All of the above properties are preserved under this mapping. By the definition of an ATIS Structure, any mapping defined by an affect relation is an ATIS System Homomorphism.

In ATIS, relations are defined by affect relations the components of which are represented by a set of two sets as follows: \( \{\{x\}, \{x, y\}\} \). This representation designates that the affect relation is defined from ‘x’ to ‘y’. Therefore, if the above homomorphism represents the affect relation, \( \mathcal{A} \), “guides the learning of,” \( \mathcal{A}_{\mathcal{G}} \), for systems \( \mathcal{S} \) and \( \mathcal{I} \) then the affect relation, the homomorphism, is represented as:

\[
\mathcal{A}_{\mathcal{G}} = \{ \{\{t_1\}, \{t_1, s_1\}\}, \{\{t_1\}, \{t_1, s_2\}\}, \{\{t_2\}, \{t_2, s_3\}\}, \{\{t_2\}, \{t_2, s_4\}\}, \{\{t_2\}, \{t_2, s_5\}\}, \{\{t_3\}, \{t_3, s_2\}\}, \{\{t_3\}, \{t_3, s_5\}\}, \{\{t_4\}, \{t_4, s_7\}\}, \{\{t_4\}, \{t_4, s_9\}\} \}
\]

As a result of the definitions of an ATIS Structure, ATIS System Homomorphism with functions that have interesting properties can be identified by those properties just as would be in any other mathematical structure. As a result, the following morphisms are defined.
Monomorphism: A homomorphism that is an *injective function*; that is, a function that is *one-to-one*, is a monomorphism.

The following homomorphism, \( f_{\text{mono}}: \mathcal{T} \rightarrow \mathcal{S} \), defines a monomorphism:

![Diagram of monomorphism]

Epimorphism: A homomorphism that is a *surjective function*; that is, a function that is *onto*, is an epimorphism.

The following homomorphism, \( f_{\text{epi}}: \mathcal{T} \rightarrow \mathcal{S} \), defines an epimorphism:

![Diagram of epimorphism]
**Isomorphism:** A homomorphism and its inverse that are bijective functions; that is, functions that are both one-to-one and onto, is an isomorphism.

The following homomorphism, $f_{iso}: \mathcal{T} \rightarrow \mathcal{S}$, defines an isomorphism:

![Diagram of an isomorphism](image)

**Endomorphism:** A homomorphism that has a domain the same as its codomain, $f: \mathcal{A} \rightarrow \mathcal{A}$, is an endomorphism.

The following homomorphism, $f_{endo}: \mathcal{T} \rightarrow \mathcal{T}$, defines an endomorphism:

![Diagram of an endomorphism](image)
Automorphism: A homomorphism that is both an endomorphism and an isomorphism is an automorphism.

The following homomorphism, \( f_{\text{auto}} : \mathcal{T} \rightarrow \mathcal{T} \), defines an automorphism:

\[
\begin{align*}
\text{Object-Set } & \mathcal{T} & \text{Object-Set } & \mathcal{T} \\
t_1 & \quad & t_1 \\
t_2 & \quad & t_2 \\
t_3 & \quad & t_3 \\
t_4 & \quad & t_4
\end{align*}
\]

Commensalmorphism: A homomorphism that is an epimorphism between the object-set of one system and the negasystem spillage of a coterminous system.

The following homomorphism, \( f_{\text{comm}} : \mathcal{A} \rightarrow (\xi(\mathcal{B})) \), defines a commensalmorphism; where \( \xi(\mathcal{B}) = \mathcal{T} \); that is, \( \xi_{\text{spillage}} : \mathcal{B} \rightarrow \mathcal{T} \).

\[
\begin{align*}
\text{Object-Set } & \mathcal{A} & \text{Object-Set } & \mathcal{T} & \text{Object-Set } & \mathcal{B} \\
a_1 & \quad & s_1 & \quad & b_1 \\
a_2 & \quad & s_2 & \quad & b_2 \\
a_3 & \quad & s_3 & \quad & b_3 \\
a_4 & \quad & s_4 & \quad & b_4 \\
b_1 & \quad & s_5 & \quad & b_5 \\
b_2 & \quad & s_6 & \quad & b_6 \\
b_3 & \quad & s_7 & \quad & b_7 \\
b_4 & \quad & s_8 & \quad & b_8
\end{align*}
\]
**Symbiomorphism:** A homomorphism and its inverse between coterminous systems.

The following homomorphisms, $f$ and $g$, and the symbiotic-quantifier, $A_{\xi_{\text{sym}}}^\text{sym}$, define a symbiomorphism: $A_{\xi_{\text{sym}}}^\text{sym}(f: \mathcal{F} \rightarrow \mathcal{S}, g: \mathcal{S} \rightarrow \mathcal{F})(\mathcal{R}(\mathcal{I}))$; where $A_{\xi_{\text{sym}}}^\text{sym}$ is true if the values of $f$ and $g$ map into $\mathcal{R}(\mathcal{I})$. 

![Diagram of Object-Set $\mathcal{F}$ and Object-Set $\mathcal{S}$](image-url)