Identification problems in DSGE models

Fabio Canova
ICREA-UPF, CREI, AMeN and CEPR
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References


Canova, F. (2002) Validating DSGE models through VARs, CEPR working paper


Canova, F. and Sala, L. (2006) Back to square one: identification issues in DSGE models, ECB working paper


DSGE models have become the benchmark for:

- Understanding business cycles/ transmission of shocks
- Conduct policy analyses / forecasting exercises

\[
E_t[A(\theta)x_{t+1} + B(\theta)x_t + C(\theta)x_{t-1} + D(\theta)z_{t+1} + F(\theta)z_t] = 0
\]

\[
z_{t+1} = G(\theta)z_t + e_t
\]

Stationary (log-linearized) RE solution:

\[
x_t = J(\theta)x_{t-1} + K(\theta)e_t
\]

\[
z_t = G(\theta)z_{t-1} + e_t
\]

- Restricted, singular VAR(1) or state space model.
How are DSGE estimated/evaluated?

1. Limited information methods

   i. GMM

   ii. Indirect Inference: "minimum distance" estimation → matching impulse responses

   iii. SVAR (magnitude and sign restrictions (Canova (2002)).

2. Full Information methods:

   i. Maximum Likelihood

   ii. Bayesian methods

3. Business cycle accounting/calibration

Chari et. al. (2007)
**Matching impulse responses** (conditional on some shock $j$):

Model responses: $X_t^M(\theta) = C(\theta)(\ell)e^j_t$

Data responses: $X_t = \hat{W}(\ell)e^j_t$ (after shock identification).

$$\hat{\theta} = \arg\min_{\theta} g(\theta) = ||X_t - X_t^M(\theta)||_{W(T)}$$

$W(T)$ weighting matrix defining distance.

**ML:** $\hat{\theta} = \arg\max_{\theta} L(X, \theta)$

**Bayesian:**

$\tilde{\theta} = \int \theta P(\theta|X) d\theta$ or

$\tilde{\theta} = \arg\max_{\theta} L(X, \theta)P(\theta)$ (constrained maximum likelihood)
Preliminary to estimation: can we recover structural parameters?

**Identifiability:**

Mapping from objective function to the parameters well behaved

- In general need:
  - Objective function has a unique minimum 0 at $\theta = \theta_0$
  - Hessian is positive definite and has full rank
  - Curvature of objective function is "sufficient"
Difficult to verify in practice because:

A) Mapping from structural parameters to solution parameters is unknown (numerical solution)

B) Objective function is typically nonlinear function of solution parameters. Different objective functions may have different “identification power”

Standard rank and order conditions can’t be used!!!
Definitions

• i) Solution identification: can we recover structural $\theta$ from the aggregate decision rule matrices $J(\theta), K(\theta), G(\theta)$?

• ii) Objective function identification: can we recover aggregate decision rule matrices $J(\theta), K(\theta), G(\theta)$ from the objective function?

• iii) Population identification (convoluting i) and ii)): can we recover the structural parameters from the objective function in population?

• iv) Sample identification: can we recover structural parameters from the objective function, given a sample of data?
Note:

- i) and ii) can occur separately or in conjunction

- i) is due to the model specification, ii) may result from the choice of objective function

- iv) may occur even if iii) does not

- iv) the focus of much of the econometric literature. Here focus on i) and ii).

Preview:

Problems with DSGE models are in the solution/objective function mapping.
What kind of population problems may DSGE models encounter?

• Observational equivalence of models. Two models may have the same (minimized) value of the objective function at two different vector of parameters (e.g. a sticky price and a stocky wage model)

• Observational equivalence within a model. Two vectors of parameters may give the same (minimized) value of the objective function, given a model (e.g. given a sticky price model, get the same responses if Calvo parameter is 0.25 or 0.75).

• Limited Information identification. A subset of the parameters of the model can’t be identified because objective function uses only a portion of the restrictions of the solution.
• Partial/under identification within a model. A subset of the structural parameter enter in a particular functional form in the solution/ may disappear from the solution.

• Weak/asymmetric identification within a model. The population mapping is very flat or asymmetric in some dimension.

Local vs. global.

Could be due to particular objective function/occur for all objective functions.
Example 1: Observational equivalence

1) \( x_t = \frac{1}{\lambda_2+\lambda_1} E_t x_{t+1} + \frac{\lambda_1 \lambda_2}{\lambda_1+\lambda_2} x_{t-1} + v_t \) where: \( \lambda_2 \geq 1 \geq \lambda_1 \geq 0 \).

2) \( y_t = \lambda_1 y_{t-1} + w_t \)

3) \( y_t = \frac{1}{\lambda_1} E_t y_{t+1} \) where \( y_{t+1} = E_t y_{t+1} + w_t \) and \( w_t \) iid \((0, \sigma^2_w)\).

Stable RE solution of 1) \( x_t = \lambda_1 x_{t-1} + \frac{\lambda_2+\lambda_1}{\lambda_2} v_t \)

Stable RE solution of 3) is \( y_t = \lambda_1 y_{t-1} + w_t \).

If \( \sigma_w = \frac{\lambda_2+\lambda_1}{\lambda_2} \sigma_v \), three processes are indistinguishable from impulse responses.

Bayer and Farmer (2004): \( Ax_t + D E_t x_{t+1} = B_1 x_{t-1} + B_2 E_{t-1} x_t + C v_t \).

Example 2: Under-identification

\[
\begin{align*}
    y_t &= a_1 E_t y_{t+1} + a_2 (i_t - E_t \pi_{t+1}) + v_{1t} \\
    \pi_t &= a_3 E_t \pi_{t+1} + a_4 y_t + v_{2t} \\
    i_t &= a_5 E_t \pi_{t+1} + v_{3t}
\end{align*}
\]

Solution:

\[
\begin{bmatrix}
    y_t \\
    \pi_t \\
    i_t
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & a_2 \\
    a_4 & 1 & a_2a_4 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    v_{1t} \\
    v_{2t} \\
    v_{3t}
\end{bmatrix}
\]

- \(a_1, a_3, a_5\) disappear from the solution.
- Different shocks identify different parameters.
- ML and distance could have different identification properties.
Example 3: Weak and partial under-identification

$$\max \beta^t \sum_t \frac{c_t^{1-\phi}}{1-\phi}$$

$$c_t + k_{t+1} = k_t^\eta z_t + (1 - \delta)k_t$$

R.E. solution for $w_{t+1} = [k_{t+1}, c_t, y_t, z_t] = Aw_t + Be_t$

Select $\beta = 0.985, \phi = 2.0, \rho = 0.95, \eta = 0.36, \delta = 0.025, z^{ss} = 1$

Strategy: simulate data. Compute population objective function. Study its shape and features.
Figure 1: Distance surface: Basic, Subset, Matching VAR and Weighted
What causes the problems?

Law of motion of capital stock in almost invariant to:

(a) variations of $\eta$ and $\rho$ (weak identification)

(b) variations of $\beta$ and $\delta$ additive (partial under-identification)

Can we reduce problems by:

(i) Changing $W(T)$? (long horizon may have little information)

(ii) Matching VAR coefficients?

(iii) Altering the objective function?

NO
Standard solution: Problem!

Figure 2: Fixing beta
Identification and objective function

What objective function should one use? Likelihood!!

It has all the information and can be computed with Kalman filter.

What does a prior do? Can help is identification problems are due to small samples but not if due to population problems!!
Figure 3: Likelihood and Posterior

Posterior not usually updated if likelihood has no information.

With constraints, updating is possible (many constraints from the model).
Identification and solution methods

• An-Schorfheide (2005) Likelihood function better behaved if second order approximation is used. How about distance function?

\[
\max E_0 \sum_t \beta^t [\log(c_t - b\bar{c}_{t-1}) - a_t N_t]
\]

\[
c_t = y_t = z_t \bar{N}_t
\]

\(\bar{c}_t\) external habit; \(a_t\) stationary labor supply shock; \(\ln(z_t/z_{t-1}) \equiv u_t^z\) technology shock.

Linear solution (only labor supply shocks):

\[
\hat{N}_t = (b + \rho)\hat{N}_{t-1} - b\rho \hat{N}_{t-2} - (1 - b)\hat{u}_t^a
\]  \((4)\)

Sargent (1978), Kennan (1988): \(b\) and \(\rho\) are not separately identified.
Second order solution (only labor supply shocks):

\[
\hat{N}_t = b\hat{N}_{t-1} + \frac{b(b-1)}{2}\hat{N}_{t-1}^2 - (1-b)\hat{a}_t - \frac{1}{2}(-(1-b)^2 + 1-b)\hat{a}_t^2
\]

\[
\hat{a}_t = \rho\hat{a}_{t-1} + u_t^a
\]
Responses to a labor supply shock

Figure 4: Distance function: linear vs. quadratic
Identification and estimation

What if we disregard identification issues and estimate models with a finite sample?

\[
y_t = \frac{h}{1+h} y_{t-1} + \frac{1}{1+h} E_t y_{t+1} + \frac{1}{\phi} (i_t - E_t \pi_{t+1}) + v_{1t}
\]

\[
\pi_t = \frac{\omega}{1 + \omega \beta} \pi_{t-1} + \frac{\beta}{1 + \omega \beta} \pi_{t+1} + \frac{(\phi + 1.0)(1 - \zeta \beta)(1 - \zeta)}{(1 + \omega \beta) \zeta} y_t + v_{2t}
\]

\[
i_t = \lambda_r i_{t-1} + (1 - \lambda_r)(\lambda_\pi \pi_{t-1} + \lambda_y y_{t-1}) + v_{3t}
\]

\(h\): degree of habit persistence (.85)

\(\phi\): relative risk aversion (2)

\(\beta\): discount factor (.985)

\(\omega\): degree of price indexation (.25)

\(\zeta\): degree of price stickiness (.68)

\(\lambda_r, \lambda_\pi, \lambda_y\): policy parameters (.2, 1.55, 1.1)

\(v_{1t}\): AR(\(\rho_1\)) (.65); \(v_{2t}\): AR(\(\rho_2\)) (.65); \(v_{3t}\): i.i.d.
Figure 5: Distance function shape
Figure 6: Distance function and contours plots
Figure 7: Density Estimates, Monetary Shocks
Figure 8: Impulse responses, Monetary Shocks
Table 1: NK model. Matching monetary policy shocks, bias

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</table>
Wrong inference

\[ 0 = -k_{t+1} + (1 - \delta)k_t + \delta x_t \]
\[ 0 = -u_t + \psi r_t \]
\[ 0 = \frac{\eta \delta}{\bar{r}} x_t + (1 - \frac{\eta \delta}{\bar{r}})c_t - \eta k_t - (1 - \eta)N_t - \eta u_t - e_z t \]
\[ 0 = -R_t + \phi_r R_{t-1} + (1 - \phi_r)(\phi_x \pi_t + \phi_y y_t) + e_r t \]
\[ 0 = -y_t + \eta k_t + (1 - \eta)N_t + \eta u_t + e_z t \]
\[ 0 = -N_t + k_t - w_t + (1 + \psi) r_t \]
\[ 0 = Et\left[ \frac{h}{1 + h} c_{t+1} - c_t + \frac{h}{1 + h} c_{t-1} - \frac{1 - h}{(1 + h) \varphi}(R_t - \pi_{t+1}) \right] \]
\[ 0 = Et\left[ \frac{\beta}{1 + \beta} x_{t+1} - x_t + \frac{1}{1 + \beta} x_{t-1} + \frac{\chi^{-1}}{1 + \beta} q_t + \frac{\beta}{1 + \beta} e x_{t+1} - \frac{1}{1 + \beta} e x_t \right] \]
\[ 0 = Et\left[ \pi_{t+1} - R_t - q_t + \beta (1 - \delta) q_{t+1} + \beta \bar{r} r_{t+1} \right] \]
\[ 0 = Et\left[ \frac{\beta}{1 + \beta \gamma_p} \pi_{t+1} - \pi_t + \frac{\gamma_p}{1 + \beta \gamma_p} \pi_{t-1} + T_p(\eta r_t + (1 - \eta) w_t - e_z t + e p_t) \right] \]
\[ 0 = Et\left[ \frac{\beta}{1 + \beta \gamma_p} w_{t+1} - w_t + \frac{1}{1 + \beta} w_{t-1} + \frac{\beta}{1 + \beta} \pi_{t+1} - \frac{1 + \beta \gamma_w}{1 + \beta} \pi_t + \frac{\gamma_w}{1 + \beta \gamma_w} w_{t-1}(w_t - \sigma N_t - \frac{\varphi}{1 - h}(c_t - h c_{t-1}) - e w_t) \right] \]
\[ T_p \equiv \frac{(1-\beta \zeta_p)(1-\zeta_p)}{(1+\beta \gamma_p)\zeta_p} \]
\[ T_w \equiv \frac{(1-\beta \zeta_w)(1-\zeta_w)}{(1+\beta)(1+(1+\lambda_w)\sigma \lambda_w^{-1})\zeta_w} \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>( \delta )</td>
<td>depreciation rate (.0182)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>parameter (.564)</td>
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<td>( \eta )</td>
<td>share of capital (.209)</td>
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<tr>
<td>( \varphi )</td>
<td>risk aversion (3.014)</td>
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<tr>
<td>( \beta )</td>
<td>discount factor (.991)</td>
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<td>( \zeta_p )</td>
<td>price stickiness (.887)</td>
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<td>( \gamma_p )</td>
<td>price indexation (.862)</td>
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<td>( \phi_y )</td>
<td>response to ( y ) (.234)</td>
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<td>( \phi_r )</td>
<td>int. rate smoothing (.779)</td>
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<td>( \lambda_w )</td>
<td>wage markup (1.2)</td>
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<tr>
<td>( \bar{\pi} )</td>
<td>steady state ( \pi ) (1.016)</td>
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<td>( h )</td>
<td>habit persistence (.448)</td>
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<tr>
<td>( \sigma_l )</td>
<td>inverse elasticity of labor supply (2.145)</td>
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<td>( \chi^{-1} )</td>
<td>investment’s elasticity to Tobin’s q (.15)</td>
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<td>( \zeta_w )</td>
<td>wage stickiness (.62)</td>
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<td>( \gamma_w )</td>
<td>wage indexation (.221)</td>
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<tr>
<td>( \phi_\pi )</td>
<td>response to ( \pi ) (1.454)</td>
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Figure 9: Objective function: monetary and technology shocks
Figure 10: Distance surface and Contours Plots
<table>
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<tr>
<th>Case</th>
<th>0</th>
<th>0</th>
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<th>0.221</th>
<th>Obj.Fun.</th>
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<td>Baseline</td>
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<td>0.862</td>
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<td>x0 = lb + 1std</td>
<td>0.8944</td>
<td>0.8251</td>
<td>0.615</td>
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<td>x0 = lb + 2std</td>
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<td>0.6095</td>
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<td>$x_0 = \text{lb} + 2 \text{std}$</td>
<td>0.8994</td>
<td>0.234</td>
<td>0</td>
<td>0</td>
<td>3.06E-07</td>
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<td>$x_0 = \text{ub} - 1 \text{std}$</td>
<td>0.905</td>
<td>0.3494</td>
<td>0.0021</td>
<td>0</td>
<td>4.14E-07</td>
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<td>$x_0 = \text{ub} - 2 \text{std}$</td>
<td>0.9343</td>
<td>0.5409</td>
<td>0.0042</td>
<td>0</td>
<td>9.64E-07</td>
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<th>Case 6</th>
<th>$\zeta_p$</th>
<th>$\gamma_p$</th>
<th>$\zeta_w$</th>
<th>$\gamma_w$</th>
<th>Obj.Fun.</th>
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<td>$x_0 = \text{lb} + 1 \text{std}$</td>
<td>0.877</td>
<td>0.0123</td>
<td>0.0229</td>
<td>0</td>
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<td>$x_0 = \text{lb} + 2 \text{std}$</td>
<td>0.8919</td>
<td>0.0411</td>
<td>0.0003</td>
<td>0</td>
<td>4.26E-07</td>
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<tr>
<td>$x_0 = \text{ub} - 1 \text{std}$</td>
<td>0.907</td>
<td>0.2056</td>
<td>0.0003</td>
<td>0.25</td>
<td>2.46E-06</td>
</tr>
<tr>
<td>$x_0 = \text{ub} - 2 \text{std}$</td>
<td>0.8839</td>
<td>0.0499</td>
<td>0.0189</td>
<td>0</td>
<td>2.46E-06</td>
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<th>$\gamma_p$</th>
<th>$\zeta_w$</th>
<th>$\gamma_w$</th>
<th>Obj.Fun.</th>
</tr>
</thead>
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<td>$x_0 = \text{lb} + 1 \text{std}$</td>
<td>0.9056</td>
<td>0.2747</td>
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<td>$x_0 = \text{lb} + 2 \text{std}$</td>
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<td>0.2805</td>
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<td>0.3669</td>
<td>0.0003</td>
<td>0.25</td>
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<td>$x_0 = \text{ub} - 2 \text{std}$</td>
<td>0.8985</td>
<td>0.194</td>
<td>0.001</td>
<td>0.25</td>
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Figure 11: Impulse responses, Case 5.
Welfare costs different!

\[ L(\pi^2, y^2) = -0.0005 \] with true parameters

\[ L(\pi^2, y^2) = -0.0022 \] with estimated parameters
Detecting identification problems:

Ex-ante diagnostics:

- Plots/ Preliminary exploration of objective function
- Numerical derivatives of the objective function at likely parameter values
- Condition number of the Hessian (ratio largest/smallest eigenvalues)

Ex-post diagnostics:

- Erratic parameter estimates as $T$ increases
- Large or non-computable standard errors
- Crazy t-test (Choi and Phillips (1992), Stock and Wright (2003)).
Tests:

Cragg and Donald (1997): Testing rank of Hessian. Under regularity conditions: 
\[(\text{vec}(\hat{H}) - \text{vec}(H))' \Omega (\text{vec}(\hat{H}) - \text{vec}(H)) \sim \chi^2((N - L_0)(N - L_0))\]
\[N = \text{dim}(H), \; L_0 = \text{rank of } H.\]

Anderson (1984): Size of characteristic roots of Hessian. Under regularity conditions:
\[
\frac{\sum_{i=1}^{N-m} \hat{\lambda}_i}{\sum_{i=1}^{N} \hat{\lambda}_i} \xrightarrow{D} \text{Normal distribution.}
\]

Concentration Statistics: \( C_{\theta_0}(i) = \int_{j \neq i} \frac{g(\theta) - g(\theta_0)}{g(\theta_0)} d\theta \), \( i = 1, 2 \ldots \) (Stock, Wright and Yogo (2002)) = measures the global curvature of the objective function around \( \theta_0 \).
Difficult to employ: just use as a diagnostic.

Applied to last model: rank of $H = 6$; sum of 12-13 characteristics roots is smaller than 0.01 of the average root $\rightarrow$ 12-13 dimensions of weak or partial identification problems.

Which are the parameters is causing problems?

$\beta, h, \sigma_l, \delta, \eta, \psi, \gamma_p, \gamma_w, \lambda_w, \phi_\pi, \phi_y, \rho_z$.

Why? Variations of these parameters hardly affect law of motion of states!

Almost a rule: for identification need states to react changes in structural parameters.
What to do when identification problems exist?

Which type?

- If population need respecify the model.

- If objective/ limited information use likelihood.

- If small sample add information (prior or other data)

- Don´t proceed as if they do not exist.

- Careful with mixed calibration-estimation. Full calibration preferable or Bayesian calibration (Canova (1995))
Conclusions:

- Liu (1960), Sims (1980):
  - Traditional models hopelessly under-identified.
  - Identification often achieved not because we have sufficient information but because we want it to be so.
  - Proceed with reduced form models
• A destructive approach:

- Most (large scale) DSGE models are face severe identification problems.

- Models are identified not because likelihood (or part of it) is informative, but because we make it informative (via partial calibration or tight priors).

- Estimation = confirmatory analysis.

- Hard to reject models.
• A more constructive one:

(i) Try to respecify the model to get rid of problems

(ii) Evaluate numerically the mapping between structural parameters and coefficients of the decision rule. Do extensive exploratory analysis.

(iii) Find out what estimation method could work also in presence of identification problems (Stock and Wright (2000), Rosen (2005))

(iv) Work out economic reasons for identification problems with submodels or simplified versions of larger ones

(v) Be less demanding of your models. Use methodologies why employ semi-structural estimation (e.g. SVARs)