Learning Dynamics: Applications

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Some Applications

- Learning dynamics
  - Of interest in their own right
  - May permit analysis of a broader classes of problem

- Today: two applications
  - Modeling central bank communication
  - Modeling macroeconomic dynamics
Application I: Communication

- Transparency and communication emphasized in modern central banking
  - Inflation targeting
  - E.g.: Bank of England, Norges Bank

- Rational expectations logic underscores importance of systematic component of policy
  - But expectations anchored by assumption
A central bank that is inscrutable gives the markets little or no way to ground these perceptions [about monetary policy] in any underlying reality — thereby opening the door to expectational bubbles that can make the effects of its policies hard to predict.
Application

- Simple model of output gap and inflation determination

- Two key informational frictions: Friedman (1968)
  - Central bank cannot observe current state
  - Agents do not have a complete economic model of aggregates
     * Expectations not pinned down

- Explore potential benefits of communicating certain kinds of information about monetary policy regime
Communication Strategies

- Benchmark analysis: no information about policy
  - Policy represents only one of several sources of uncertainty

- Evaluate advantages of:
  - Communicating complete details of the policy rule
  - Communicating the variables upon which policy decisions are condition
  - Communicating the inflation target
Literature

- Learning and policy design

- Transparency

- Build on
  - Orphanides and Williams (2005), Hellwig (2005) and Walsh (2007)
Model Agents

- Households
- Firms
- Monetary authority
- Fiscal authority
Model Features

- Money or cashless limit
- Monopolistic competition/nominal rigidities
- Incomplete asset markets
- No capital
- Non-rational expectations
Beliefs

Under rational expectations:

1. Agents optimize given beliefs

2. The probabilities they assign to future state variables coincide with the predictions of the model

This paper retains (1) and replaces (2) with

2'. Future state variables outside agent's control are forecasted using an econometric model.
Household Problem

- Maximize

\[
\hat{E}_{t-1}^i \sum_{T=t}^{\infty} \beta^{T-t} \left[ \ln C_T^i - h_T^i \right]
\]

subject to the flow budget constraint

\[
B_t^i \leq R_{t-1} B_{t-1}^i + W_t h_t^i + P_t \Pi_t - P_t C_t^i
\]

- Consumption decisions made one period in advance

- Beliefs: a-theoretical VAR of exogenous variables

- Nests MSV REE
First order conditions

- Euler equation

\[ \hat{C}_t^i = \hat{E}_{t-1} \left[ \hat{C}_{t+1}^i - (i_t - \pi_{t+1}) \right] \]

and the intertemporal budget constraint

\[ \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \omega_{t-1}^i + \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i \]

where

\[ \omega_t^i = B_t^i / \bar{Y} \]
Optimal Consumption Rule

- Log-linear approximation provides

\[
\hat{C}_t^i = (1 - \beta) \omega_{t-1}^i + \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{Y}_T^i - \beta (i_T - \pi_{T+1}) \right]
\]

- Permanent income theory
- Efficacy of policy: depends on conditional interest rate forecasts
Firms

• Continuum of firms maximize

\[ \hat{E}_{t-1}^j \sum_{T=t}^{\infty} Q_{t,T} P_T \Pi_{j,T} \]

where

\[ \Pi_{j,t} = \frac{P_{jt}}{P_t} Y_{j,t} - \frac{W_t}{P_t} h_{jt} - \frac{\psi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 \]

– Prices set one period in advance

– Rotemberg (1982) pricing
Optimal Pricing Rule

- Log-linear approximation provides

\[ \hat{P}_t = \hat{P}_{t-1} + \hat{E}_{t-1}^i \sum_{T=t}^{\infty} \beta^{T-t} \xi (\hat{s}_T + \hat{\mu}_T) \]

where \( \xi > 0; \hat{s}_t \) marginal costs; and \( \hat{\mu}_t \) a cost push shock
• Monetary Authority

• Optimal policy problem:

\[
\min E_t \sum_{T=t}^{\infty} \left( \pi_T^2 + \lambda_x x_T^2 \right)
\]

where \( \lambda_x > 0 \) and subject to the constraints

\[
x_t = E_{t-1}x_{t+1} - E_{t-1}(i_t - \pi_{t+1} - r_t^e)
\]

\[
\pi_t = \xi E_{t-1}x_t + \beta E_{t-1}\pi_{t+1} + E_{t-1}\hat{\mu}_t
\]

• First order condition

\[
E_{t-1}\pi_t = -\frac{\lambda_x}{\xi} E_{t-1}x_t.
\]
Implementation

- Central Bank adopts the rule

\[ i_t = i_t^* + \phi \left( \hat{E}_{t-1} \pi_t + \frac{\lambda_x}{\xi} \hat{E}_{t-1} x_t \right) \]

where

\[ i_t^* = \rho_r r_t^e - 1 + \frac{\rho_\mu \lambda_x + (1 - \rho_\mu) \xi}{\xi^2 + \lambda_x (1 - \beta \rho_\mu)} \rho_\mu \hat{\mu}_{t-1} \]

- Assume: \( r_t^e \) and \( \hat{\mu}_t \) autoregressive process with eigenvalues \( 0 < \rho_r, \rho_\mu < 1 \)
Alternative Formulations

- Optimal commitment: can do with a rule of the form
  \[
  i_t = \bar{i}_t^* + \phi \hat{E}_{t-1} \pi_t
  \]
  for appropriately chosen $\bar{i}_t^*$

- Policy accounting for private learning
  - Complex: Central bank needs to know a lot
  - Gaspar, Smets and Vestin (2005), Molnar and Santoro (2005)
Fiscal Authority

- Zero debt policy
Model Summary

- Log-linearizing, aggregating and imposing market clearing at time $t$ provides

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} [(1 - \beta)x_{T+1} - (i_T - \pi_{T+1} - \hat{r}_T^e)]$$

$$\pi_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \xi (x_T + \hat{\mu}_T)$$

$$i_t = i_t^* + \phi \left( \hat{E}_{t-1} \pi_t + \frac{\lambda x}{\xi} \hat{E}_{t-1} x_t \right)$$

where $x_t = \hat{Y}_t - \hat{Y}_t^e$ and $\int_0^1 \omega_t^i = 0$. 
E-Stability

- Beliefs:

\[ z_t = a_t + b_t z_{t-1} + \varepsilon_t \]

where \( z_t = \{ x_t, \pi_t, i_t, r^c_t, \mu_t \} \)

- Convergence under least-squares learning IFF the associated ODE:

\[ \frac{d}{d\tau}(a, b) = T(a, b) - (a, b) \]

is locally stable at the REE of interest.
Benchmark Property I

- Proposition 1: Under rational expectations the model has a unique bounded equilibrium if and only if $\phi > 1$.
  
  - Taylor principle
Benchmark Property II

- Proposition 2: Under learning dynamics; no communication; and \( \phi > 1 \)

1. The rational expectations equilibrium is unstable if

\[
(1 + \phi)\xi > \phi(1 - \beta)\frac{\lambda x}{\xi} + \psi(\beta)
\]

where \( \psi(\beta) > 0 \), \( \lim_{\beta \to 1} \psi(\beta) = 0 \) and \( \lim_{\beta \to 0} \psi(\beta) = \infty \). Hence:

2. If \( \beta \to 1 \), then the REE is unstable under learning for every \( \xi \) and \( \phi \).

3. If \( \beta \to 0 \), then the REE is stable under learning for every \( \xi \) and \( \phi \).
Some Intuition

- Forecast-based instrument rules argued to be desirable
  - Batini and Haldane (1999), Levin et al (2001)

- Not here: real interest rates initially decline with inflation shock

- Aggressive inflation response destabilizing
  - Not true of output
Sourcing Instability

- Two key informational frictions
  - Central bank imperfectly observes the state of the economy
  - Households and firms have incomplete model

- What is the relative importance of these frictions?
Eliminating Policy Delays

- Suppose the central bank perfectly observe current state

- Can implement the rule

\[ i_t = i_t^* + \phi \left( \pi_t + \frac{\lambda x}{\xi} x_t \right) \]

- Proposition 3: Under learning dynamics the REE is stable if $\phi > 1$
Modeling Communication

- Communication affects the form of private agents beliefs

- For example: inflation targeting
  
  - Central bank communicates average desired outcome for inflation
  
  - Impose $a_t = 0$ and estimate
    
    $$ z_t = b_t z_{t-1} + \varepsilon_t $$

- More efficient forecast
Communication: Strategy I

- Announce the precise policy rule
  - assume perfect credibility and homogeneous beliefs

- Agents know
  - all relevant conditioning variables
  - all relevant coefficients

- Implication: do not need to independently forecast nominal interest rate path
Aggregate Demand under Communication

- Knowledge of policy rule implies

\[ x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta)x_{T+1} - (i_T^* + \phi \pi_T + \phi \frac{\lambda x}{\xi} x_T - \pi_{T+1} - \hat{r}_T^e) \right] \]

- Strategy is equivalent to announcing \( \{ E_{t-1}^C B i_T \}_{T=t}^{\infty} \)

  - E.g.: Reserve Bank of New Zealand, Norges Bank
Stability Restored

- Proposition 4: Announcing the interest rate forecast $\left\{ E_{t-1}^{CB}i_T \right\}_{T=t}^{\infty}$ or, equivalently, the policy rule

$$i_t = i^*_t + \phi \left( \hat{E}_{t-1}\pi_t + \frac{\lambda x}{\xi} \hat{E}_{t-1}x_t \right)$$

implies the REE is stable if $\phi > 1$

- Procedure has the property that forecasted path of real interest rates rise
Communication Strategy II

- Announce only the set of variables upon which interest rate decisions are conditioned

- Interpretation
  - Imperfect credibility
  - Too costly to communicate complexities of the decision process
  - Agents wish to verify announced projections
Verification of Policy

- Agents adopt a two-stage forecasting model

- Estimate the model

\[ i_t = \psi_{0,t-1} + \psi_{\pi,t-1} \hat{E}_{t-1}\pi_t + \psi_{x,t-1} \hat{E}_{t-1}x_t + e_t \]

- Then construct forecasts consistent with this model of policy
Communication Strategy II

- Proposition 5: The REE is stable if $\phi > 1$
  - Equivalent to full-information case
  - Estimation uncertainty “small”
Communication Strategy III

- Announce average desired values of the inflation rate
  - E.g.: Reserve Bank of Australia
Communication Strategy III

- Proposition 6: Assume the central bank communicates only the inflation target \( \pi^* = 0 \) and the associated values for the output gap and nominal interest rates, \( x^* = i^* = 0 \).

1. Define \( \rho = \max(\rho_u, \rho_r) \) and let \( \rho \to 1 \). Then the REE is unstable under learning if conditions of Proposition 2 hold;

2. Let \( \beta \to 1 \). Then the REE is unstable under learning if

\[
\xi (\phi + 2\rho) > 2 \frac{\phi\lambda_x}{\xi} (1 - \rho) + \tilde{\psi} (\rho),
\]

where \( \tilde{\psi} (\rho) > 0, \tilde{\psi} (1) = 0 \).
Calibration

- Let $\beta = 0.99$, $\theta = 10$, and $\phi = 2$
Figure 1: Announcing the target is not enough
The Role of Nominal Rigidities

- Higher degrees of rigidity associated with greater stability
- Prices move less, permitting agents to more easily infer future path of prices
The Value of Communication

- Underscores the importance of rational expectations logic

- Knowing the systematic component of monetary policy decisions central to expectations stabilization

- Not enough to announce objectives: one must also announce how such objectives will be achieved.
Further Insights: Dynamics of Expectations

- Does communication help even in the case of convergence?

- Example: Evolution of beliefs on natural rate coefficients are given by the ODE

\[ \dot{\omega}_1 = (J^* - I_3) \omega_1 \]

where

\[ \omega_1 = \begin{pmatrix} \omega_{xr} & \omega_{\pi r} & \omega_{ir} \end{pmatrix}' \]

- Assume initially in steady state. Perturb \( \omega_{\pi r} \) to make higher that REE value — equivalent to expectational error in forecasting inflation

  - Compare strategy one and three
Figure 2: Expectation Dynamics
Figure 3: Responding to Expected Inflation
Figure 4: The Benefits of Communication
Benefits of Communication

- Even in the case of convergence communication of systematic component of policy beneficial

- Enables an more accurate forecast of real interest paths, promoting stability
  - Orphanides and Williams (2005)
Conclusion

- Uncertainty about the state and policy have importance consequences for stabilization policy

- Communicating the systematic component of policy unambiguously improves stabilization

- Announcing an inflation target is not enough to achieve stabilization of expectations
  - one must also announce how the objective will be achieved
Application II: Macroeconomic Dynamics

- Large literature on confronting dynamics stochastic general equilibrium models with data
  - Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005)

- Useful for empirical policy evaluation and forecasting

- Outstanding issues: difficult to capture sources of persistence and volatility
  - Requires ad hoc frictions
  - Exogenous persistence mechanisms
Example: Simple Model

- Recall model of output gap and inflation determination

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1} - r_t) \]
\[ \pi_t = k x_t + \beta E_t \pi_{t+1} + u_t \]
\[ i_t = \phi_\pi \pi_t + \phi_x x_t + \varepsilon_t \]

- Poor fit of data
• Model does better with
  
  – Internal habit formation
  
  – Price indexation
  
  – Inertial policy

• For example, households maximize

\[ E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U \left( C_T^i - \eta C_T^{i-1} \right) - v \left( h_T^i \right) \right] \]
Dynamics

\[ \tilde{x}_t = E_t \tilde{x}_{t+1} - (1 - \beta \eta) \sigma (i_t - E_t \pi_{t+1} - r_t) \]

\[ \tilde{\pi}_t = \xi_p \left( \omega x_t + ((1 - \beta \eta) \sigma)^{-1} \tilde{x}_t \right) + \beta E_t \tilde{\pi}_{t+1} + u_t \]

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) (\chi_\pi \pi_t + \chi_x x_t) + \varepsilon_t \]

where

\[ \tilde{x}_t = (x_t - \eta x_{t-1}) - \eta \beta E_t (x_{t+1} - \eta x_t) \]

\[ \tilde{\pi}_t = \pi_t - \gamma \pi_{t-1} \]

- Does better
Questions

- Is this an adequate modeling strategy?

- Are there alternative simpler mechanisms?
  - Learning will generate persistence and volatility
  - And data has time variation in second order moments

- Milani (2006): investigates the relative importance of these mechanisms
Assumptions

- Exogenous disturbances

\[ r_t = \phi^r r_{t-1} + \nu^r_t \]
\[ u_t = \phi^u u_{t-1} + \nu^u_t \]

where \( 0 < \phi^r, \phi^u < 1 \) and \( \nu^r_t \sim iid\left(0, \sigma^2_r\right), \nu^u_t \sim iid\left(0, \sigma^2_u\right) \)
Expectations

- Agents estimate

\[ Z_t = a_t + b_t Z_{t-1} + c_t u_t + d_t r_t + \varepsilon_t \]

where \( Z_t = [\pi_t \ x_t \ i_t]' \)

- Forecasts

\[ E_t Z_{t+1} = a_t + b_t Z_t + c_t \phi^u u_t + d_t \phi^r r_t \]

  - Assume agents know autoregressive coefficients of exogenous shocks
Beliefs

- Beliefs updates according to

\[ \hat{\phi}_t = \hat{\phi}_{t-1} + \bar{g} R_{t-1}^{-1} X_t (Z_t - X_t' \hat{\phi}_t) \]

\[ R_t = R_{t-1} + \bar{g} (X_{t-1} X_{t-1} - R_{t-1}) \]

where

\[ \hat{\phi}_t = \begin{bmatrix} a_t' \ vec(b_t)' \ vec(c_t)' \ vec(d_t)' \end{bmatrix}' \]

\[ X_t = \begin{bmatrix} 1 & Z_{t-1} & u_t & r_t \end{bmatrix}' \]
State-space Representation

- Structure and beliefs imply representation

\[
\xi_t = A_t + F_t \xi_{t-1} + G_t w_t \\
Y_t = H \xi_t
\]

where

\[
\xi_t = \begin{bmatrix} \pi_t & x_t & i_t & u_t & r_t \end{bmatrix}'
\]

\[
w_t \sim N(0, Q)
\]

\[
H \sim \text{a selection matrix}
\]

and \(A_t, F_t\) and \(G_t\) matrices with coefficients that are convolutions of model primitives and beliefs
Bayesian Estimation

- Wish to conduct inference on the parameter vector

\[ \theta \equiv \{ \eta, \beta, \sigma, \gamma, \xi_p, \omega, \rho_i, \chi_{\pi}, \chi_{\phi}, \phi^r, \phi^u, \sigma_{\varepsilon}, \sigma_u, \sigma_r, \bar{g} \} \]

- Information about parameters summarized by posterior distribution

\[
p(\theta|Y^T) = \frac{p(Y^T|\theta) \cdot p(\theta)}{p(Y^T)}
\]

where

\[ p(Y^T|\theta) \] – likelihood function
\[ p(\theta) \] – prior
\[ Y^T \] – available data
Data

- Observables
  - Inflation
  - Output gap
  - Nominal interest rates
Estimation

- Given assumption of normal disturbances construct likelihood
  - Kalman filter

- To compute posterior distribution
  - Metropolis-Hasting MCMC
    - Geweke (1999), Schorfheide (2000)
Results

- Key parameters: habit ($\eta$) and indexation ($\gamma$)

- Estimation under rational expectations: $\eta = 0.911$ and $\gamma = 0.885$

- Estimation under learning dynamics $\eta = 0.117$ and $\gamma = 0.289$
Model Comparison

- Use posterior odds ratios to compare relative fit

\[ PO = \frac{p_{M_1}(\theta)p(Y^T|M_1)}{p_{M_2}(\theta)p(Y^T|M_2)} \]

- Data prefers model with learning dynamics \textit{and} no frictions

\[ PO = 2.6 \times 10^6 \]
Future Work

- There is much to be done
  - Optimal Policy Design
  - Macroeconomic Dynamics and Business Cycles
  - Asset Pricing
  - Social Security