Private School Optimization

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Focus of talk: A key part of my research (with co-researchers) over the past decade or so concerns competition among schools where private schools play a central role. Hence, I’ll present some of this research.

I will focus on optimization by private providers of schooling under two alternative specifications of their objectives, namely profit maximization and quality maximization.

The analysis is potentially applicable to primary, secondary, and higher education.

Private provision of education currently plays a central role in higher education. It plays some role in primary and secondary, with the potential to be increasingly important if voucher policies ever take off in the U.S. (as they have already in Chile).

Of course, there are many, many issues in provision of education and our modeling only captures some of them. I suspect you’ll find unsatisfying some elements of the modeling, probably omissions of elements that might be very important. There is clearly room for much more theoretical research (and complementary empirical research of course). I’ll note some research issues as I go along.
I will delve into a bit more detail on optimization by one school, as opposed to examining the detail of equilibrium in the market (or quasi-market) for education. But I will discuss some the elements of market equilibria.

Outline of talk:
1. Profit maximization
   - Model
   - School optimization
   - Market Equilibrium
   - Vouchers

2. Quality maximization
   - Modifications to model
   - School optimization
   - Market equilibrium

3. Analysis of Affirmative Action (as time permits) and Topics for Research
Profit Maximization: The Model

Households/students/demand-side:

The student/household’s utility function is given by:

\[ U = U(x,q,b), \] where \( x \) is numeraire consumption, \( q \) is school quality, and \( b \) is student ability.

We assume \( U(\cdot) \) is increasing, differentiable, and quasi-concave in its arguments.

We assume demand for \( q \) is normal. We assume demand for \( q \) is “weakly increasing in student ability” (ie., non-negative ability elasticity of demand for \( q \)).

Note that a special case is \( U = U(x,a(q,b)) \), where \( a(\cdot) \) is student achievement, increasing in school quality and ability.

Example: Cobb-Douglas form: \( U = x^\rho q^\gamma b^\beta \). (We’ve done numerous simulations using this form.)
We’ll discuss what determines school quality in a minute.

**Distribution of potential students:** A student (/household) is characterized by ability and household income \((y)\). We assume a continuous distribution of potential students \(F(b,y)\) on \([0,\infty)\times[0,\infty)\) with density function \(f(b,y)\).

Realistically, \((b,y)\) is likely to be positively correlated (due to genetics and household educational production) but this isn’t needed for any of the results I discuss today.

**Reservation price function:** Because we will model schools as “utility takers,” finding a student’s reservation price function for attending a school is useful. The utility-taking assumption is that a school believes it can attract a student if it provides the student with the utility the student can obtain elsewhere in equilibrium. The school’s belief will be fulfilled in equilibrium.

( Utility-taking is related to price taking as the school will take as given the student’s reservation price function. It is a concept that has been used, for example, in the club goods literature.)
A student’s reservation price function $p^R(b,y,q)$ is defined in:

$$U(y - p^R, q, b) \equiv U^A(b, y);$$

where $U^A$ is the alternative utility type $(b,y)$ can obtain in equilibrium. $U^A$ will be determined by the elements of market equilibrium, but it is exogenous to any one school.

Note that:

$$\frac{\partial p^R}{\partial q} > 0 \quad \text{(quality is a good)}$$

$$\frac{\partial^2 p^R}{\partial q \partial y} > 0 \quad \text{(normality of demand for quality)}$$

$$\frac{\partial^2 p^R}{\partial q \partial b} \geq 0 \quad \text{(non-negative demand elasticity in ability)}$$
Schools:

School quality: Assume $q = q(I, \theta)$ where $I$ is educational inputs per student and $\theta$ is mean student ability -- with $q$ increasing, differentiable, and quasi-concave in $(I, \theta)$.

The model then assumes an ability based peer effect, which is very controversial. There is a sizeable and growing empirical literature on peer effects in education. One can probably interpret this research as indicating peer effects on educational achievement are small at best. On the other hand, there is overwhelming evidence that students value good peers in schools – e.g., higher housing prices where public schools have “better” peer groups. From the perspective of positive analysis, it is appropriate to include a peer measure (or measures) in the school “quality” specification. For the purposes of normative analysis, the reasons why students care about their peers is important – this is an open question. (Our focus will be mainly on positive analysis today.)

We’ll discuss alternative specifications of school quality later.
**School costs:** Let $k$ denote the number of students in a school. We assume a cost function of the following form:

$$C(k,I) = F + V(k) + kI, \text{ where } V', V'' > 0.$$ 

In the cost function, there is a fixed cost ($F$) and a variable cost $V$ that depends just on the number of students, independent of the level of educational inputs per student ($I$). That is, there are “custodial costs” independent of quality.

The cost function implies U-shaped average cost with “efficient scale” that is independent of $I$. (Average cost at the efficient scale increases with $I$ of course.)

The cost function assumes all students are provided with same inputs; alternatively, students of different types might be split up in schools and provided with different input levels. We’ve examined this some in research on tracking (*JPubE*, Jan. 2002).
**Profit maximization problem:**

We assume schools observe \((b, y)\). (I know of some research in progress on asymmetric information alternatives.)

To maximize profits, it is obviously optimal for a school to charge \(p^R\) to any student the school wants to admit. To characterize and interpret a school’s optimization problem, it is convenient to specify a separate tuition function and admission function. The admission function, \(a(b, y)\), indicates the proportion in the population of type \((b, y)\) the student admits. One can then specify that the school charges every student, admitted or not, \(p^R(b, y, q)\). (The tuition charged to non-admitted students is irrelevant, and, again, it is optimal to charge admitted students \(p^R\).) (Alternatively, the school could let the pricing function do all the work – set a price to non-admitted students above their reservation price.)

To be clear, the school expects all admitted students to matriculate – since it charges them their reservation price.
Letting $S$ denote the support of $(b,y)$, one can then write the school’s problem:

$$\max_{\theta, I, k, \alpha(b,y)} \iint_{S} p^R(b,y,q(\theta,I))\alpha(b,y)f(b,y) \text{d}b \text{d}y - F - V(k) - kI$$

subject to $\alpha(b,y) \in [0,1]$ for all $(b,y) \in S$;

$$\theta = \frac{1}{k} \iint_{S} b \alpha(b,y)f(b,y) \text{d}b \text{d}y;$$

and $k = \iint_{S} \alpha(b,y)f(b,y) \text{d}b \text{d}y$.

Write out the Langrangian function and find the first-order conditions. They simplify to a condition describing optimal admission and a condition describing optimal expenditure on inputs (keeping in mind students are charged their reservation prices).
Admission criterion:

\[
\alpha(b,y) \begin{cases} 
= 1 & \text{if } \frac{p^R(b,y,q)}{\alpha} \leq EMC(b) \forall (b,y) \\
= 0 & \text{otherwise}
\end{cases}
\]

where:

\[
EMC(b) \equiv V' + I + \frac{q_\theta}{q_I} \cdot (\theta - b).
\]

Interpretation: Admit all (no) students whose reservation price is higher (lower) than their “effective marginal cost (EMC).” EMC equals the sum of the resource cost \((V'+I)\) and cost of the student’s ability externality. The cost of the ability externality equals the change in \(\theta\) from admitting a type b student \(((b-\theta)/k)\) multiplied by the resource cost of maintaining quality \((-kq_\theta/q_I)\).

Note that the ability externality “cost” is negative for students that increase the peer measure and thus school quality, namely students with \(b > \theta\). For sufficiently high ability students EMC is negative (which in some cases can lead to student fellowships).
The lower effective cost of higher-ability students will of course increase their presence in high quality schools. How this is manifested will depend on the competitive environment. We’ll examine some examples in a minute.

**Condition on Optimal I:**

\[ \int \int_{s} \frac{\partial p^R(b,y,q(\theta,I))}{\partial I} \alpha(b,y)f(b,y)dbdy = k. \]

This is just the Samuelsonian condition for optimal provision of a congested public good.

It is optimal for the school to increase educational inputs so long as the marginal value to the student body (the LHS of above) equals the marginal cost ($k$). This is because the school fully price discriminates and thus charges students for school quality increases.
Market Equilibrium

Issues: How many private schools? What alternative is there to attending a private school?

For application to primary or secondary schools, we assume there exists a public sector. The simplest case assumes a homogeneous public school alternative, i.e., everyone has access to the same quality public school alternative. The public alternative is free and the quality of the public alternative is given by $q(\theta_p, I_p)$, where $\theta_p$ is the mean ability of students that do not attend a private school and $I_p$ is public expenditure per student, perhaps determined by voting over a tax rate. (One might make the quality function in the public sector different, reflecting a productivity difference.)

A more complicated alternative specification would have differences in public schools due to multiple districts in the local economy and/or neighborhood schools. Such a model would then require housing markets. See Epple and Romano (“Neighborhood Schools, Choice, and the Distribution of Educational Benefits” in *The Economics of School Choice*, 2003) and a stream of papers by Tom Nechyba including in the same volume. A defense of the simpler specification with
homogeneous public schools is that this might be approximated under public school choice.

**Case with Just One Private School:**

See Figure 1. We assume the parameters are such that a private school can cover its costs.

The public school alternative pins down $U^A(b,y)$ for all types.

Students along the boundary of the $(b,y)$ plane that partitions students into the public and private school have $p^R(b,y,q) = \text{EMC}(b)$. Students to the right of the boundary have $p^R > \text{EMC}$ and attend the private school and students to the left have $p^R < \text{EMC}$ and are not admitted to the private school. (The $b$-axis intercept of the boundary locus has $\text{EMC}(b) = 0$; students with no income and this level of $b$ or a higher level go to the private school for nothing.)
The private school is, of course, of higher quality than the public school. Under weak conditions it will have a higher $\theta$; e.g., a non-negative correlation between $b$ and $y$. Inputs in the private school may or may not be higher (but would be in a realistically calibrated model). At least one of $\theta$ and $I$ must be higher in the private school.

There is a mixing of relatively lower-ability higher-income students and higher-ability lower-income students in the private school. This is because higher income students are willing to pay more for quality and buy there way in, while higher ability students are desired for their peer effect. Supporting this, the pricing is such that there is a cross subsidization from the former to the latter students, e.g., think of the case where the private school breaks even in equilibrium.
Figure 1: Partition of Students with One Private School
Free Entry Equilibrium: Assume the same public alternative but that there is free entry into the private sector. Suppose further that the parameters are such that more than one private school would arise in equilibrium.

There is an existence of equilibrium problem, but ignore it for the moment (it may not be a problem).

A fundamental property of an equilibrium is that private schools must be quality differentiated.

Informally, the argument in this: Suppose that there are two clone private schools. Each student would pay exactly EMC(b) at each school, because each student would have the other school, priced at cost, as an alternative. Then either school could increase profits by admitting and expelling students so as to become higher ability and richer – or lower ability and poorer. If the school becomes higher ability and richer (see Figure 2), then the school increases profits because if can increase tuition being higher quality and with richer students who are willing to pay “substantially” for quality increases. If the school alternatively becomes lower ability and poorer, then it has to decrease tuition but it gets rid of higher-ability students from whom tuition is low to start with. In either case, essentially the school is better honing its
student body to their demands for quality. (Formal proof is in Epple and Romano, *AER*, 1996 with inputs held constant in all schools, and in Epple and Romano, “Educational Vouchers and Cream Skimming,” with variable inputs.)
Figure 2: Increasing Profits by Getting Richer and Higher Ability Student Body
An equilibrium partition will look as shown in the upper left panel of the next figure, where there are five private schools. In this example, public schools spend $7,000 per student. Private schools are assumed to be a little more efficient (ed achievement is 4\% higher with same peer group and 2\% less expenditure, based on some Hoxby estimates).

Properties of Equilibrium:

Strict quality hierarchy.

$U^A(b,y)$ and thus reservation price at the attended school is determined by one of the “adjacent” alternative schools. Students on the boundaries pay exactly EMC at their attended (private) school since they are indifferent between the adjacent alternatives, while students in the interior of admission spaces pay tuition above EMC. But for realistic parameterization tuitions in all but the top school are very close to EMC because there is a close alternative (that would admit the student at EMC). In spite of the differentiation, things are very competitive (of course given our parameterization but it is attempted to be realistic).
Again, there is a cross subsidization in schools. Obviously, the rich and higher ability attend relative better schools in equilibrium. That higher ability students attend better schools is cost driven, not demand driven – in the example, the utility specification is Cobb Douglas implying 0 ability elasticity of demand for quality.
The number of schools is such that they all earn 0 profits – free entry equilibrium. In fact, though, this does not hold precisely; and this relates to the existence-of-equilibrium problem. The parameters would need to be “just right” for each school to earn 0 profits. Put differently, equilibrium fails to exist generically. So we examine approximate equilibria where we let profits differ some from 0.

The existence problem is a version of an integer problem (as schools have fixed cost). Note, however, that eliminating the fixed cost would lead to an infinite number of differentiated schools, another version of an existence problem.

It is notable that in this example some (high ability and poor) students in every school pay negative tuition – since their EMC is negative at their best alternative school.

Last, income stratification along the school quality hierarchy must hold and ability stratification is very likely to hold.
Private-School Vouchers:

An important and interesting policy issue concerns the effects of government financed vouchers.

Issues: How is the voucher designed? For whom? Any strings attached? How financed?

Simplest case: Non-targeted, flat-rate, no strings attached. Any student that attends a private school gets a voucher of value \( v \), finance with an income tax.

To analyze: Let \( y_t \) denote after tax income. Substitute \( y_t \) for \( y \) and add \( v \) to \( y_t \) in the utility function for students that attend a private school.

Effects on school optimum:

Everything is the same where income is now \( y_t + v \) and \( p^R \) is now evaluated at \( p^R(b,y_t+v,q) \). E.g., to be admitted it must be that \( p^R(b,y_t+v,q) \geq \text{EMC}(b) \).
The effects on market equilibrium depend crucially on whether there is free entry of not.

**Effects on Case of Monopoly Private School:**

In this case, because the alternative to the private school will still be the public school (which will change a bit depending on the level of the voucher), for not too big a voucher $p^R(b,y_t+v,q) \approx p^R(b,y) + v$; and the private school will largely capture the voucher in increased profits.

More students will be admitted because, then, the admission criterion will be approximated by $p^R(b,y) \geq EMC(b) - v$. The boundary separating the public and private sector students will shift in some. The public and private school will get worse, but the students who end up in the private school will get higher quality education although there utility might not be higher.

Intuitively, there isn’t a competitive effect here.

Note, too, that inputs won’t much be affected in the private school – the Samuelsonian condition will still be satisfied.
With free entry, things are different:

First, a competitive effect will lead the voucher to be largely passed along to private school students in reduced tuition. Because many students will have access to another school at effective marginal cost, schools will be forced to pass along the voucher in lower tuition in equilibrium. (The alternative utility function will change to reflect the voucher received if another private school is attended.)

The voucher will induce entry of new schools, more so, of course, the higher the voucher. The other panels in the previous figure show what happens in the calibrated example.

There is “cream skimming” in the sense that private schools form by attracting students from the public sector that are relatively high ability and high income. Hence, the public sector will lose relatively good peers and deteriorate. The variation in school quality obviously increases substantially.
With the voucher of $4,520, 65% of students are in private schools and 53.4% are in lower quality schools (including everyone still in the public school). There are also 2 “bottom feeder” private schools that form that are worse than the public school. They kick back money to students (everyone gets a fellowship). Welfare effects are different (see next example).

With a $7,000 voucher the public sector is eliminated. 57.8% are in lower quality schools. But 85% have higher utility. For example, some relatively high ability students who were in the public sector with no voucher end up in weaker private schools but they get fellowships.

Note that increasing the voucher would not lead schools to spend more and more on inputs, rather it would lead schools to lower tuition more and more (since the Samuelsonian conditions will continue to be satisfied).
Two voucher design issues:

1. The case above allowed schools to do anything with tuition so, for example, bottom-feeder schools arose. Rules on tuition relative to the voucher might be set and I’ll discuss an example in a moment.

2. A voucher might be targeted to some student types. Again, I’ll discuss some in a moment.

**Tuition restriction:** A simple restriction like the school must accept exactly the voucher for tuition is arguably undesirable since it might prevent a good school from charging lower ability students more and then spending more on inputs (and very high ability students could not get fellowships).

A more flexible rule that would prevent bottom-feeder schools would require schools to spend at least as much per student as the voucher. A school couldn’t give a kick-back on average.
This would add the following constraint to the private school’s problem:

\[ F + V(k) + kI \geq \int_S v\alpha(b, y)f(b, y)dbdy. \]

The admission criterion is the same, but with:

\[
EMC(b) = \Omega \cdot [V' + I + \frac{q_\theta}{q_I} (\theta - b)] + (1 - \Omega) v;
\]

where:

\[ \Omega \in [0, 1) \text{ with } \Omega \equiv [k - \int_S \frac{\partial p^R}{\partial I} \alpha f dbdy] / k. \]

**Interpretation:** EMC is now a weighted average of EMC with no constraint and the voucher. For not too high a voucher and high-spending schools, \( \Omega = 0 \) and nothing changes. For high vouchers and not (otherwise) so high-spending schools, \( \Omega \) will become positive. It will have the effect of making high ability students relatively less attractive because the school can no longer substitute their ability in increasing quality for reduced expenditure on inputs. For such schools the Samuelsonian condition will no longer be satisfied. As the voucher increases, more spending will result. We are currently examining this case quantitatively.
Targeted Vouchers:

The general form in our model would be $v(b,y)$. There are many issues.

In “Ed Vouchers and Cream Skimming,” Epple and I examine the following voucher design issue. Suppose that one would like to use a voucher to inject competition into provision of schooling but not cause cream skimming. That is, one desires to create schools that serve a cross section of students as does the initial public sector in the model but do so more efficiently.

We show that a voucher of the form $v(b) = EMC(b)$ combined with a requirement that schools accept the voucher for tuition could accomplish this. (There are other forms that due the same.) The basic idea is that the system must create incentives to admit all student types. I refer you to the paper for details.
Quality Maximization

What schools try to accomplish is an open question. The profit maximization assumption is natural for economists, but may not really tell the right story. Another objective that we have studied is school quality maximization. We have applied this empirically to the study of competition among institutions of higher education (e.g., Epple, Romano, Sieg, *Econometrica* July 2006).

Basic Model:

Assume the demand side is the same as in the profit maximization model.

Regarding schools, there are two differences:

1. They maximize \( q(\theta,I) \) subject to a profit constraint.

2. We allow an exogenous additional source or revenues, in addition to tuition revenues.
The idea in assumption 2 is to permit voluntary donations and government subsidies, having in mind colleges (although such happens as well in private primary and secondary schools).

So a school’s problem is:

$$\max_{I,k,\theta,\alpha(b,y)} q(\theta,I)$$

s.t. $\int_S p^R(b,y,q(\theta,I))\alpha(b,y)f(b,y)dbdy + M \geq F + V(k) + kI$

$$\theta = [\int_S b\alpha(b,y)f(b,y)dbdy]/k$$

$$k = \int_S \alpha(b,y)f(b,y)dbdy$$

$$\alpha(b,y) \in [0,1] \forall (b,y) \in S.$$
The optimal admission criterion is exactly the same as under profit maximization!

\[ \alpha(b,y) \begin{cases} = 1 \\ \in [0,1] \\ = 0 \end{cases} \text{ if } \begin{cases} p^R(b,y,q) > \text{EMC}(b) \forall (b,y) \\ \leq \text{EMC}(b) \forall (b,y) \end{cases} \]

where:

\[ \text{EMC}(b) \equiv V' + I + \frac{q_\theta}{q_I} \cdot (\theta - b). \]

EMC is the effective marginal cost of maintaining quality when student of ability b is admitted so the optimal admission criterion is the same.
The condition determining optimal inputs is different, however. Let \( R \) denote tuition revenues.

\[
\frac{\partial R}{\partial I} - k = - \frac{q_i}{\eta} < 0; \text{ where } \eta \text{ is the (positive) multiplier on the profit constraint.}
\]

The condition indicates that inputs exceed the Samuelsonian level (which satisfies \( \frac{\partial R}{\partial I} - k = 0 \)). This is because the school values increasing \( I \) both because it increases quality per se and because it permits higher tuition and relaxes the profit constraint.

The tighter is the profit constraint, i.e., the higher is \( \eta \), the closer will optimal \( I \) be to the Samuelsonian level. In turn, competition among colleges for students will tighten the profit constraint: Students prefer lower \( I \) and lower tuition. Hence, the endowment revenues, differentiation of schools, and related market power are important in determining the extent to which \( I \) differs from the Samuelsonian level.
Aside on Research Topic: Examining other objectives of schools is of interest. A criticism of the quality objective is that it does not place weight on the number of students educated. We’ve examined some a specification with maximization of aggregate achievement of students. There is certainly room for research here.

Market Equilibrium:

Assume there are \( N \) schools with \( M_1 < M_2 < \ldots < M_N \).

Assume the alternative is free (or a low given price) and of given quality. The alternative might be a state school or no school (quality then corresponds to high-school educational quality). Assume, further, that the parameters are such that the \( N \) private schools are present in equilibrium.
Properties of Equilibrium:

A strict hierarchy of private schools results that follows the endowment hierarchy.

A school with a higher endowment must have a higher quality in equilibrium simply because it can use the endowment to spend more on inputs and/or provide greater subsidies to high ability students. Note, though, that absent differences in endowments, a strict hierarchy of qualities would need to arise in equilibrium by an argument analogous to that under profit maximization.

The endowment hierarchy “helps” overcome the existence problem.

Regarding the partition of students into schools and the “outside option” it looks just like that under profit maximization. See Figure 4.

And the pattern of prices is analogous. These results are because the admission criterion and optimal pricing are the same as under profit maximization.

Again, the difference is that inputs will be at higher levels (with “general equilibrium” effects relative to profit maximization).
Figure 4: Partition of Students with Quality Maximization
Extension to Price Caps:

An unrealistic prediction of the latter model with regard to a private college equilibrium is that any student can buy his way into any school and then some very rich pay exorbitant tuition.

If one introduces exogenous price caps that rise along the school hierarchy, then this leads to ability minima in the colleges and a subset of students that pay the price cap.

See Figure 5.

But we don’t have a good theory explaining the price caps.
Figure 5: Schools with Price Caps (Two Schools)
Diversity Preference:

Colleges, especially highly ranked ones, have demonstrated a preference for diversity of their student bodies.

Diversity preference regards race/ethnicity, socioeconomics, and other student characteristics (e.g., region in U.S.).

In several papers, we have extended the quality maximization model to include preference for student diversity. I will relate the ideas in the context of a two-race specification with preference for racial diversity.

Assume races, \( r \), are \( r \in \{w, nw\} \), \( nw \) is the under-represented race in a college \( i \), \( \Gamma \) is the proportion \( nw \) in the population of potential students, and \( \Gamma_i \) is the proportion of \( nw \) in the college with \( \Gamma_i < \Gamma \).

The college quality index is assumed to be of the form: \( q = q(\theta, I, \Gamma_i/\Gamma) \). \( q \) is increasing in \( \Gamma_i/\Gamma \) so long as \( \Gamma_i < \Gamma \). The idea is that proportional representation is desired all else constant.
Regarding the set of potential students, assume that \( f(b|y) \) is the same for the two races, but the income distribution of \( w \)’s first-order stochastically dominates that of \( nw \)’s. Consequently, the race conditioned distributions \( f^r(b,y) \) are such that for the quality index that doesn’t place weight on diversity, \( nw \)’s would in fact be under-represented. (This will hold as well with diversity valued but less so.)

Consider the college’s problem assuming that it can legally practice affirmative action, which means it can have admission policies that depend on race as well as tuition policies that depend on race. We assume that the outside option (e.g., state college alternative) is the same across races conditional on \( (b,y) \), which may be a strong assumption.

Issue: Does everyone have the same college quality index, i.e., how do potential students feel about diversity? Perhaps they share a preference for diversity, perhaps they are indifferent, perhaps they prefer to be with their own race. (Realistically, potential students probably vary with respect to this.) In Epple, Romano, Sieg (AER May 2002) we examine several specifications. Here I discuss the case having everyone with the same, diversity-dependent, quality index.
One can write down the quality maximization problem analogously as above. Note that there is a separate admission function for the races, \( \alpha^r(b,y) \), \( r \in \{w,nw\} \); and that the reservation price functions differ by race in general since multiple schools might be practicing affirmative action. (It remains optimal to charge any admitted student their reservation price.)

Let’s just focus on the solution to the problem.
Admission Policy:

\[
a_i(r,b,y) \begin{cases}
=1 & \text{if } \alpha \in [0,1] \\
=0 & \text{if } \alpha > \text{EMC}_i(r,b,y)
\end{cases}
\]

\[
\text{EMC}_i(nw,b,y) \equiv V' + I_i + \frac{q_0}{q_t}(\theta_i - b) + \frac{q_{ri}}{q_t}(\Gamma_i - 1);
\]

\[
\text{EMC}_i(w,b,y) \equiv V' + I_i + \frac{q_0}{q_t}(\theta_i - b) + \frac{q_{ri}}{q_t}\Gamma_i.
\]

The admission policy is analogous, but with reservation price race dependent and a racial diversity term in EMC that differs by race. For nw, it is the negative of the change in the proportion of nw’s, multiplied by the cost of substituting inputs to maintain quality. Since \( \Gamma_i < 1 \), this element of cost for a nw is negative. The structure of the term is the same for admission of a w, but positive.

(Inputs satisfy the same condition as in the simpler quality max problem.)
In market equilibrium, affirmative action is manifest in that for the same \((b,y)\):

1. nw’s have weakly higher utility if attending college;

2. nw’s pay weakly lower tuition if attending the same college as a w with the same \((b,y)\);

3. nw’s attend a weakly better college (if the shadow value of improving diversity rises along the quality hierarchy).

Note, however, that these results are conditional on the same \((b,y)\). Since nw’s have lower income and thus lower b on average, these results imply that only a subset of nw’s are better off then comparable w’s.

The next figure shows the effect on admission spaces in a calibrated model that examines the practice of affirmative action.
FIGURE 2

Boundary Loci for Nonwhites and Whites when Affirmative Action Permitted
The reason, incidentally, the boundary loci are curves in this example (while have been straight lines in other examples) is because this model also has income diversity as an element of the quality index. (Along a boundary, when income drops, ability doesn’t have to rise by as much to keep a student in the better school.)

In “Diversity, Profiling, and Affirmative Action in Higher Education,” Epple, Romano, and Sieg also examine the consequences of a ban on affirmative action, assuming schools must practice race-blind policies but they will use \((b, y)\) strategically to still try to increase representation of nw’s.

The next figure shows the effect on admission spaces which are now the same for w’s and nw’s. Essentially, schools bias admission toward relatively low scoring but relatively affluent students who are relatively likely to be nw in our calibrated model.

The table shows that banning affirmative action would have dire consequences for attendance of nw’s.
FIGURE 4

Boundary Loci when Affirmative Action Proscribed and

Contours of Constant Density of Nonwhites Relative to Whites

Income

120000
110000
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0

SAT

400
800
1200

25%
20%
15%
10%
5%
### TABLE 2

Affirmative Action Permitted

<table>
<thead>
<tr>
<th>College</th>
<th>k white</th>
<th>nonwhite</th>
<th>total</th>
<th>sp SAT</th>
<th>inputs</th>
<th>White</th>
<th>Nonwhite</th>
<th>sp income</th>
<th>% nonwhite</th>
<th>sp race</th>
<th>Quality</th>
<th>Average Tuitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-college</td>
<td>71.4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.4%</td>
<td>977</td>
<td>810</td>
<td>943</td>
<td>$ 892</td>
<td>$ 3,825</td>
<td>$ 33,333</td>
<td>$ 29,223</td>
<td>-0.033</td>
<td>14.2%</td>
<td>$ 1,108</td>
<td>1.565</td>
</tr>
<tr>
<td>2</td>
<td>6.2%</td>
<td>995</td>
<td>829</td>
<td>962</td>
<td>$ 941</td>
<td>$ 4,107</td>
<td>$ 39,559</td>
<td>$ 33,666</td>
<td>-0.030</td>
<td>14.4%</td>
<td>$ 1,175</td>
<td>1.570</td>
</tr>
<tr>
<td>3</td>
<td>6.0%</td>
<td>1019</td>
<td>855</td>
<td>986</td>
<td>$ 1,020</td>
<td>$ 4,562</td>
<td>$ 48,569</td>
<td>$ 40,117</td>
<td>-0.027</td>
<td>14.5%</td>
<td>$ 1,292</td>
<td>1.579</td>
</tr>
<tr>
<td>4</td>
<td>5.6%</td>
<td>1061</td>
<td>900</td>
<td>1029</td>
<td>$ 1,210</td>
<td>$ 5,644</td>
<td>$ 64,046</td>
<td>$ 51,005</td>
<td>-0.026</td>
<td>14.7%</td>
<td>$ 1,583</td>
<td>1.599</td>
</tr>
<tr>
<td>5</td>
<td>3.8%</td>
<td>1186</td>
<td>1052</td>
<td>1159</td>
<td>$ 2,087</td>
<td>$ 11,005</td>
<td>$ 94,403</td>
<td>$ 66,740</td>
<td>-0.034</td>
<td>15.5%</td>
<td>$ 2,915</td>
<td>1.681</td>
</tr>
</tbody>
</table>

Affirmative Action Prohibited

<table>
<thead>
<tr>
<th>College</th>
<th>k white</th>
<th>nonwhite</th>
<th>total</th>
<th>sp SAT</th>
<th>inputs</th>
<th>incomes</th>
<th>sp income</th>
<th>% nonwhite</th>
<th>sp race</th>
<th>Quality</th>
<th>Average Tuitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-college</td>
<td>71.4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.4%</td>
<td>974</td>
<td>870</td>
<td>953</td>
<td>$ 874</td>
<td>$ 3,771</td>
<td>$ 32,171</td>
<td>$ 35,084</td>
<td>-0.033</td>
<td>11.9%</td>
<td>$ 1,305</td>
</tr>
<tr>
<td>2</td>
<td>6.2%</td>
<td>989</td>
<td>859</td>
<td>963</td>
<td>$ 929</td>
<td>$ 4,075</td>
<td>$ 38,041</td>
<td>$ 41,772</td>
<td>-0.030</td>
<td>11.2%</td>
<td>$ 1,498</td>
</tr>
<tr>
<td>3</td>
<td>6.0%</td>
<td>1010</td>
<td>866</td>
<td>981</td>
<td>$ 1,020</td>
<td>$ 4,572</td>
<td>$ 46,671</td>
<td>$ 51,324</td>
<td>-0.028</td>
<td>9.9%</td>
<td>$ 1,901</td>
</tr>
<tr>
<td>4</td>
<td>5.6%</td>
<td>1044</td>
<td>895</td>
<td>1014</td>
<td>$ 1,233</td>
<td>$ 5,739</td>
<td>$ 61,934</td>
<td>$ 68,522</td>
<td>-0.026</td>
<td>7.8%</td>
<td>$ 3,021</td>
</tr>
<tr>
<td>5</td>
<td>3.8%</td>
<td>1161</td>
<td>888</td>
<td>1107</td>
<td>$ 2,177</td>
<td>$ 11,372</td>
<td>$ 92,967</td>
<td>$ 100,028</td>
<td>-0.035</td>
<td>5.0%</td>
<td>$ 9,326</td>
</tr>
</tbody>
</table>

Definitions

- sp SAT: Shadow price per 200 SAT points
- sp income: Shadow price on income
- sp race: Shadow price on racial composition
I mentioned that we have also examined preference for socio-economic diversity; the quality index increases as income of the student body better reflects that of the population.

Note that this can help to explain the prevalence of need-based aid.
Concluding Remarks:

Many things to study; here are a few (not to say there has been no work on):

1. Non-Quality Differentiation of Schools
   ● Religious Schools (Maria Ferrerya)
   ● Varied Curriculum (Sinan Sarpca)

2. Competition Among Schools In Richer Settings
   ● Differentiated Public Schools
   ● Regulations on Schools (e.g., curriculum rules that public schools must satisfy; admission rules that state colleges must satisfy)
   ● Charter Schools
   ● Geography Playing More of a Role

3. Alternative Objective Functions of Ed Providers

4. Alternative Affirmative Action Regulations
   ● X-percent Rules
5. Imperfect Information

6. Multi-Stage Education