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P1.6 SPECTRAL DEPENDENCE OF THE CORRECTION FOR PATH-SMOOTHING BY 3D ANEMOMETERS

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1. INTRODUCTION

The use of long-term eddy covariance measurements to accurately assess net ecosystem carbon exchange is critically affected by small biases in the measurement system, particularly during nighttime periods. It is correspondingly important to accurately correct measured fluxes for the effect of limitations of the measuring system (e.g. Moore, 1986; Massman, 2000).

One of the corrections which is applied is that for the smoothing due to pathlength of the sonic anemometer - any method of spectral correction which involves estimating the instrumental response and integrating over a spectrum uses this correctional curve. There are two problems with this correction as commonly applied: (a) the description of this correction is one which has been integrated over the so-called "Kaimal" spectra (Kaimal *et al*. 1972) which may not be appropriate to the chosen experimental site; (b.) in many 3D sonic anemometers the transducer heads are not aligned with the axes on which the velocity components are being measured. The current paper separates the description of the instrument's geometry from that of the turbulence, allowing the examination of both questions.

If $T(n)$ is the system response function as a function of frequency n, and $S(n)$ is the corresponding co-spectrum being measured, then an amount ΔF of the actual covariance F will be lost, where

$$F = (F - \Delta F) . \frac{\int S(n) dn}{\int T(n) S(n) dn} \tag{1}$$

and the $S(n)$ used are commonly the universal curves established during the Great Plains experiments. The smoothing due to the finite path length of the anemometer is one component of the system's response $T(n)$. $T(n)$ is the product of all such applicable response functions.

Kaimal *et al* (1968) (henceworth KWH) calculated the sensor correction for finite path-length where the path is parallel to the component measured as

$$T_i(k_j, \mathbf{p}) = \frac{\int_{-\infty}^{+\infty} \frac{\sin^2(\mathbf{k}\cdot\mathbf{p})}{(\mathbf{k}\cdot\mathbf{p})^2} \cdot \Phi_{ij}(\mathbf{k})\, dk_2 dk_3}{\int_{-\infty}^{+\infty} \Phi ij(\mathbf{k})\, dk_2 dk_3} \tag{2}$$

where $\mathbf{k}$ is the wave number vector (k_1, k_2, k_3), $\mathbf{p}$ is the path vector and Φ_{ij} is the spectral density tensor. The equation is generally used in an approximate form for the vertical component, developed by Moore (1986),

$$T_W(x) = \frac{4}{x}\left(1 + \frac{e^{-x}}{2} - \frac{3(1-e^{-x})}{2x}\right) \tag{3}$$

where $x = 2\cdot\pi\cdot n\cdot p/U$, p being the magnitude of $\mathbf{p}$ and U being the windspeed. While (3) is appropriate to the one-component sonic system that Moore (1986) refers to, the non-orthogonal geometry of typical three-dimensional anemometers render this approximation generally invalid.

The sensitivity of the form of the path correction to the spectral shape is investigated, and the correction is found to be relatively insensitive to the exact spectral form.

The effect of the misalignment between axes and velocity components is calculated for the Kaimal spectra, and the errors are found to be significant at high frequencies.

2. BASIC PRINCIPLE

The major difficulty in this problem is that of reconciling the two reference frames, those of the instrument and of the turbulence which it is exposed to. Since turbulence data are reduced to a common frame to allow comparisons among measurements to be made, and since the power spectra are measured in this frame, all quantities must be converted to the turbulence frame.

The orthogonal velocity components U_i which are used for describing wind fields are actually reconstructed from the windspeeds V_j measured along the three non-orthogonal transducer paths of the anemometer. Therefore the effect of path smoothing must be calculated for these non-orthogonal paths. Since the path smoothing can be expected to affect higher frequencies more than low frequencies, the magnitude of path smoothing is dependent upon the relative abundances of different spectral frequencies, that is, the power spectrum: this information is generally available only for orthogonal axes, and so the spectra parallel to the non-orthogonal axes must first be calculated from those for orthogonal axes.

The two ways in which the instrument geometry affects the path smoothing are through the path length and the separation of the centers of the sensor paths. If the centers of the sensor paths are not collocated (i.e., the instrument has low rotational symmetry) then the orientation of the instrument relative to the wind can have a large effect upon the path smoothing.

Following the arguments in KWH, the smoothed velocity components W_{kp} may be expressed as Fourier-Stieltjes integrals

$$W_{kp} = \int_{-\infty}^{+\infty} e^{i\mathbf{k}\cdot\mathbf{x}_o} \cdot e^{i\mathbf{k}\cdot(\mathbf{x}_p - \mathbf{x}_o)} \frac{\sin(\mathbf{k}\cdot\mathbf{l}_p/2)}{(\mathbf{k}\cdot\mathbf{l}_p/2)} \cdot dU_k(\mathbf{k}) \tag{4}$$

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where W_{kp} is the k^{th} component of the velocity vector as smoothed over the p^{th} sensor path $\mathbf{l}_p$, $\mathbf{x_o}$ is the center of the anemometer array, $\mathbf{x}_p$ is the center of the path along which the velocity is smoothed. The dU_k are random functions with orthogonal increments

$$\overline{dU_i(\mathbf{k})dU_j^*(\mathbf{k}')} = \Phi_{ij}(\mathbf{k})d\mathbf{k} \qquad ,\mathbf{k}'=\mathbf{k}$$
$$= 0 \qquad ,\mathbf{k}'\neq\mathbf{k} \qquad (5)$$

Φ_{ij} the spectral density tensor can, assuming isotropy, be written

$$\Phi_{ij}(\mathbf{k}) = \frac{E(k)}{4\pi k^4}(k^2\delta_{ij} - k_i k_j) \qquad (6)$$

where k is the magnitude of $\mathbf{k}$, and $E(k)$, the three-dimensional spectrum, can be written for an inertial subrange

$$E(k) \propto k^{-5/3} \qquad (7)$$

A quick test of the sensitivity of the correction to the actual form of the spectrum can be made by altering the equation for $E(k)$.

In developing these equations for a fully three-dimensional system (KWH's results are for a Kaijo-Denki sonic with one vertical transducer path and two non-orthogonal horizontal paths) the description of the geometry of the instrument (path center separation, path length and orientation to the wind) was separated out from the integration. This allows the integration to be tested for a known geometry, specifically, the sonic which KWH used, and then with no changes other than to the arrays detailing the geometry, the investigation of other geometries.

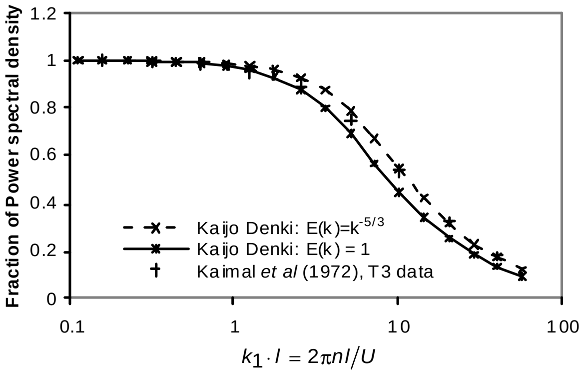

Fig 1. Comparison of data from Kaimal *et al*. (1972) with those obtained by integrating the full 3-D equations for the same geometry. Third trace demonstrates the low sensitivity of the integrated transfer function to an extreme alteration to the spectral shape.

3. RESULTS

The results are in three main parts – to show that the reworked equations can reproduce the proven integrations of KWH, to investigate the effect of using Moore's equation for 3D sonics, and to investigate the sensitivity of the correction to the form of the spectrum.

Figure 1 shows the transfer function for the vertical component, T3, from KWH together with the same transfer function produced by (4)-(7), both using the normal inertial subrange function $E(k) \sim k^{-5/3}$. The close agreement between the two sets of curves is encouraging. Figure 1 also shows the results of setting $E(k)$=1. This is an extreme alteration to the spectral form, but there is negligible influence upon the integrated transfer function. The implication is that the transfer function value is dominated by large scales, though it is a function of k_1 only it has been integrated over both k_2 and k_3.

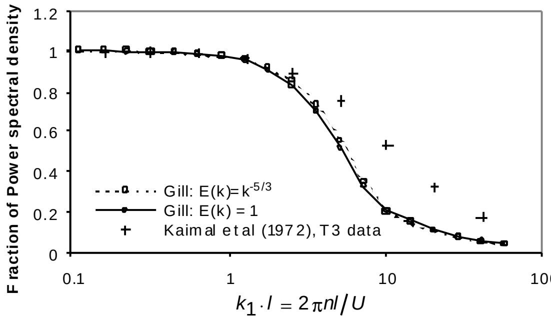

Fig 2. Comparison of data from Kaimal et al. (1972) with those obtained for Gill sonic anemometer. Third trace demonstrates low sensitivity of transfer function to spectral shape.

Figure 2 shows the same T3 data from KWH together with calculated values for the Gill sonic, a model for which the transducer paths are not parallel to the vertical axis. There is a distinctly different shape to the transfer function for frequencies $k_1 l = (2\pi \cdot n \cdot l/U) > 1$.

4. CONCLUSION

The most important factor in calculation of the spectral transfer function for the vertical wind velocity component by a sonic anemometer is not the shape of the spectrum, to which the calculation is relatively insensitive, but the geometry of the sensor, and in particular the fact that the transducer paths are not aligned with the component to be measured.

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