Problem: convert the breeder's equation into the form used in the QG of correlated traits.

$R = h^2 S$ — The breeder's equation

$\Delta \bar{z}_1 = h_1{}^2 S_1$ — substitution: $R = \Delta \bar{z}_1$, where the 1 stands for trait 1.

$\Delta \bar{z}_1 = \dfrac{V_{A(1)}}{V_{P(1)}} S_1$ — substitution: $h_1 = \dfrac{V_{A(1)}}{V_{P(1)}}$

$\Delta \bar{z}_1 = \dfrac{V_{A(1)}}{V_{P(1)}} \mathrm{COV}(w, z_1)$ — substitution: $S_1 = \mathrm{COV}(w, z_1)$

$\Delta \bar{z}_1 = V_{A(1)} \dfrac{\mathrm{COV}(w, z_1)}{V_{P(1)}}$ — re-arrangement.

$\Delta \bar{z}_1 = V_{A(1)} B(w, z_1)$ — substitution: $\dfrac{\mathrm{COV}(w, z_1)}{V_{P(1)}} = B(w, z_1)$ the selection gradient.

$\Delta \bar{z}_1 = G_{11} B_1$ — $V_{A(1)} = G_{11}$ and $B(\mathrm{w}, \mathrm{z}_1) = B_1$

$\Delta \bar{\mathbf{z}} = \mathbf{GB}$ — the standard form for correlated traits, where **G** is the additive genetic variance/covariance matrix, and **B** is the vector of selection gradients

$$\begin{bmatrix} \Delta \bar{z}_1 \\ \Delta \bar{z}_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta \bar{z}_p \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdot & \cdot & \cdot & G_{1p} \\ G_{21} & G_{22} & \cdot & \cdot & \cdot & G_{2p} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ G_{p1} & G_{p2} & \cdot & \cdot & \cdot & G_{pp} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_p \end{bmatrix}$$

the matrix/vector expansion

Thus, for two traits, we get

$\Delta \bar{z}_1 = G_{11} B_1 + G_{12} B_2$

and

$\Delta \bar{z}_2 = G_{21} B_1 + G_{22} B_2$

where G_{11} = the additive genetic variance for trait 1, and G_{22} = the additive genetic variance for trait 2. And: $G_{12} = G_{21}$ = Additive genetic covariance between traits 1 and 2.

Now, the selection gradients here are not simple regression coefficients as in the single-trait case. The selection gradients are coefficients in a multiple regression.

$$B_1 = \frac{S_1 V_{P(1)} - S_2 \mathrm{COV}(z_1, z_2)}{V_{P(1)} V_{P(2)} - \mathrm{COV}(z_1, z_2)^2}$$

expansion of a partial regression coefficient

Note that if the COV=0, then

$$B_1 = \frac{S_1 V_{P(1)} - 0}{V_{P(1)} V_{P(2)} - 0} = \frac{S_1}{V_{P(1)}}$$

$$\Delta \bar{z}_1 = G_{11} B_1 = G_{11} \frac{S_1}{V_{p(1)}} = \frac{V_A}{V_{p(1)}} S = h^2 S$$

But if the covariance is not equal to zero, selection on trait 2 affects the selection gradient for trait 1.

Here is the main point. Selection on a trait can affect other traits. This could affect the magnitude and even the direction of change.

See article on web by Steve Arnold.