

ai = .
ares = .
astar = .
R = .
Wi = .
Cd = .
Noffspring = .

Variables are defined as in class, except that Noffspring is the number of offspring produced by each females. As you will see, this is a constant, and drops out of the solution.

This is Fisher's sex allocation problem. We want to show that the solution $ares=1/2$ is a CSS.

$$W_i = \text{Noffspring} \left(\frac{R(1 - a_i)}{C_d} + \frac{R(a_i)}{C_s} \left(\frac{(1 - a_{res}) / C_d}{(a_{res}) / C_s} \right) \right)$$

$$\text{Noffspring} \left(\frac{(1 - a_i) R}{C_d} + \frac{a_i (1 - a_{res}) R}{a_{res} C_d} \right)$$

$$\text{Firstder} = D[W_i, a_i]$$

$$\text{Noffspring} \left(-\frac{R}{C_d} + \frac{(1 - a_{res}) R}{a_{res} C_d} \right)$$

Now we want to determine the effect of ares on the firstderivative in order to evaluate convergence stability. Note that the first derivative is equal to zero at $ares = 1/2$. Also note that the slope is negative at that point. Hence, the solution $ares=1/2$ is convergence stable. That means that if the population is away from $ares=1/2$, mutants that are closer to $1/2$ will be favored by selection, and the population will converge on the equilibrium of one half.

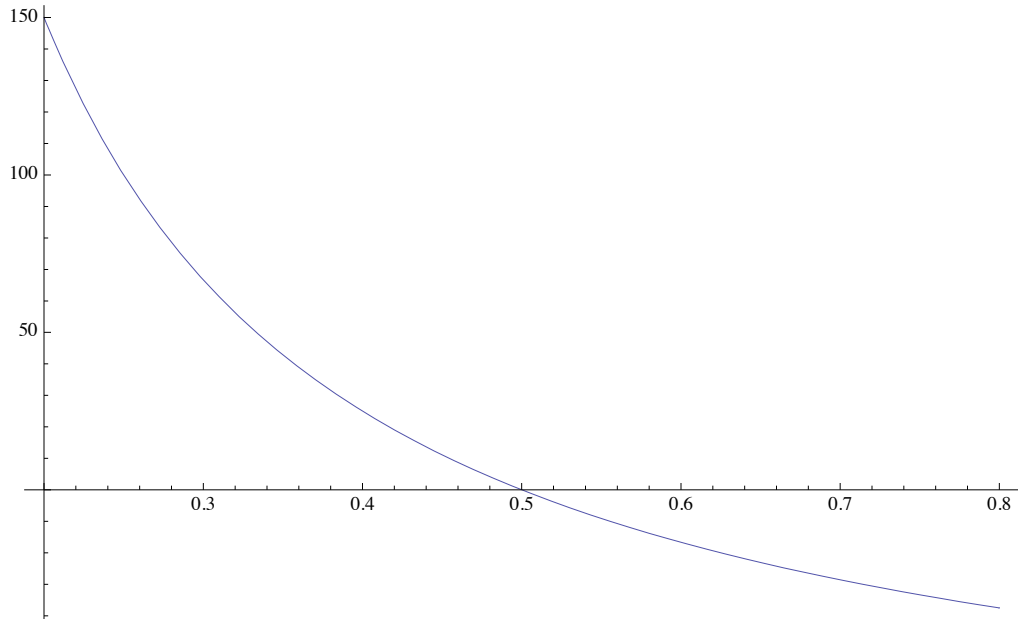
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Noffspring = 5;
R = 1;
Cd = 1 / 10;
Firstder

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Plot[Firstder, {ares, 0.2, 0.8}]
```

$$5 \left(-10 + \frac{10 (1 - \text{ares})}{\text{ares}} \right)$$



```

Noffspring =.
R =.
Cd =.
Cs =.
CSS = D[Firstder, ares]

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$$\text{Noffspring} \left(-\frac{(1 - \text{ares}) R}{\text{ares}^2 \text{Cd}} - \frac{R}{\text{ares} \text{Cd}} \right)$$

Below we prove that the equilibrium is at $\text{ares}=1/2$.

```
Reduce[Firstder == 0, ares]
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$$\left(\text{ares} == \frac{1}{2} \ \&\& \ \text{Cd} \neq 0 \right) \ || \ (\text{Noffspring} == 0 \ \&\& \ \text{ares} \ \text{Cd} \neq 0) \ || \ (\text{R} == 0 \ \&\& \ \text{ares} \ \text{Cd} \neq 0)$$

Now we want to determine whether $\text{ai}=1/2$ is an ESS.

```
Secondder = D[Firstder, ai]
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0
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Note that the second derivative (above) is zero. So the equilibrium is convergent stable, but not evolutionarily stable. Thus there will be drift at the equilibrium of $1/2$, but if the population mean drifts away from $1/2$, selection will tend to move the population back to one half (since the solution is convergence stable).