Here we are interested in there trade-off between the size and number of offspring. We want to know the size of offspring that maximizes individual fitness.

DEFINING THE VARIABLES...

q = fraction of resouces allocated to each offspring.

qmin = the fraction of resources required to produce and offspring

that has a suvival probability greater than zero.

x = exponent that controls the shape of expected fitness a function of q.

No = number of offspring, which is equal to 1/q

Wo = fitness of individual offspring.

In[1]:=

```
q=.
qmin=.
x=.
No=.
Wo=.
```

FIRST WE DEFINE THE Number OF OFFSRPING, No

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[6]:=

```
No = 1/q
```

Out[6]=

$$\frac{1}{q}$$

NEXT WE DEFINE THE FITNESS OF OFFSRPING, Wo

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[7]:=
```
Wo = (q - qmin)^x
```

Out[7]= $(q - qmin)^x$

THE FITNESS OF THE PARENT IS THE SIZE (QUALITY) OF OFFSPRING

TIMES THE NUMBER OF OFFSPRING. HENCE WE GET,

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[8]:=
```
Wp = No*Wo
```

Out[8]= $\frac{(q - qmin)^x}{q}$

WE WANT TO KNOW WHERE PARENTAL FITNESS IS AT A LOCAL MAXIMUM. SO WE TAKE THE FIRST PARTIAL DERIVATIVE OF PARENTAL FITNESS WITH RESPECT TO ALLOCATION TO OFFSPRING,

AND SET IT EQUAL TO ZERO. FIRST WE TAKE THE DERIVATIVE...

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[9]:=
```
Firstder = D[Wp, q]
```

Out[9]= $-\frac{(q - qmin)^x}{q^2} + \frac{(q - qmin)^{-1+x}\, x}{q}$

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[10]:=
```
Simplify [Firstder]
```

Out[10]=
$$\frac{(q - qmin)^{-1+x} \, (qmin + q \, (-1 + x))}{q^2}$$

Now we set the first derivative equal to zero, giving:
((q - qmin)^(-1 + x)*(-q + qmin + q*x))/q^2 = 0.
Multiply both sides by q^2, and we get:
(q - qmin)^(-1 + x)*(-q + qmin + q*x) = 0.

Let the left-hand side of the above equality be "LHS"

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[11]:=
```
LHS = ((q - qmin)^(-1 + x)*(-q + qmin + q*x))
```

Out[11]=
$$(q - qmin)^{-1+x} \, (-q + qmin + q \, x)$$

Set LHS = 0, and solve for q:

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[12]:=
```
Solve [LHS == 0, q]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[12]= $\{\{q \to 0^{\frac{1}{-1+x}} + qmin\}, \{q \to -\frac{qmin}{-1+x}\}\}$

Thus at the ESS,

$$q = q^* = \frac{qmin}{1-x}$$

Now for the second derivative test...

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[13]:=
```
Secondder = D[Firstder, q]
```

Out[13]= $\frac{2\ (q - qmin)^x}{q^3} - \frac{2\ (q - qmin)^{-1+x}\ x}{q^2} + \frac{(q - qmin)^{-2+x}\ (-1 + x)\ x}{q}$

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[14]:=
```
Simplify [Secondder]
```

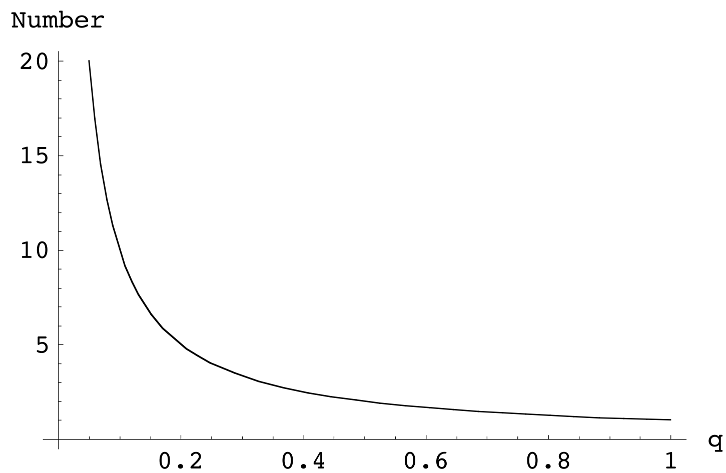
Out[14]=
$$\frac{(q - qmin)^{-2+x}\ (2\ qmin^2 + 2\ q\ qmin\ (-2 + x) + q^2\ (2 - 3\ x + x^2))}{q^3}$$

Whoa, the second derivative is a bit gnarly; lets try the graphical approach.

First plot number of offspring as a function of investment in individual offspring.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[15]:=
```
Plot [No, {q, 0.05, 1}, AxesLabel->{"q", "Number"}]
```



Out[15]=
```
- Graphics -
```

Yes, the number of offspring decreases as the investment in individual offpring increases.

Now set x equal to some value less than one, say 0.2, and qmin = 0.1

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[16]:=
```
x= 0.5
qmin = 0.1
```

Out[16]=
```
0.5
```

Out[17]=
```
0.1
```

Solve for the ESS, given these parameters

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[18]:=
```
ESS = qmin/(1 - x)
```

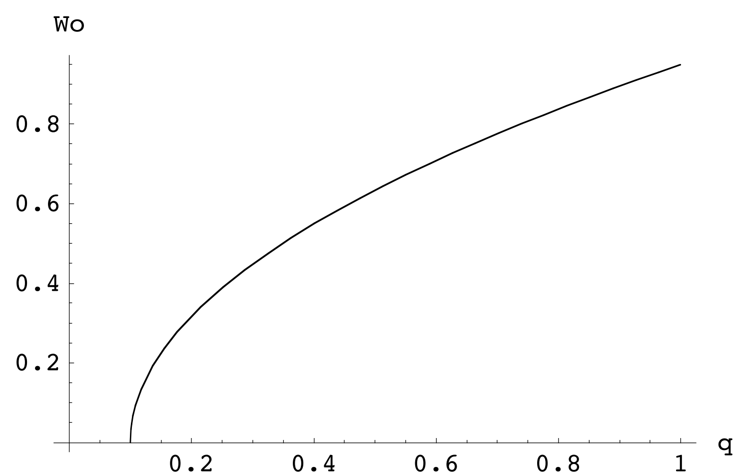
Out[18]=
```
0.2
```

Hence the ESS is at q = 0.125. Remember this value!

Now, plot offspring fitness as a function of individual allocation to offspring, q.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[19]:=
```
Plot [Wo, {q, qmin, 1.0}, AxesLabel->{"q", "Wo"}]
```



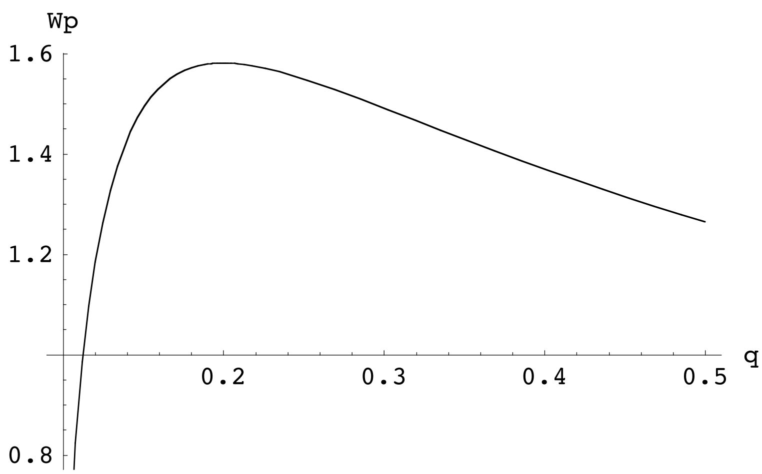
Out[19]=
```
- Graphics -
```

Yes. Diminishing returns!

Finally, plot parental fitness as a function of individual allocation to offspring, q.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[20]:=
```
Plot [Wp, {q, qmin, 0.5}, AxesLabel->{"q", "Wp"}]
```



Out[20]=
```
- Graphics -
```

Note that the ESS is at 0.125, as expected. And note that q = 0.125 is a local maximum.

Below, you can change the values for x and qmin. Just highlight the question marks, and type in a new value.

Or you can save the file under a different name, and change anything you would like to change.

In[24]:=
```
x= .25
qmin = .2
```

Out[24]=
```
0.25
```

Out[25]=
```
0.2
```

Solve for the ESS, given these parameters

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

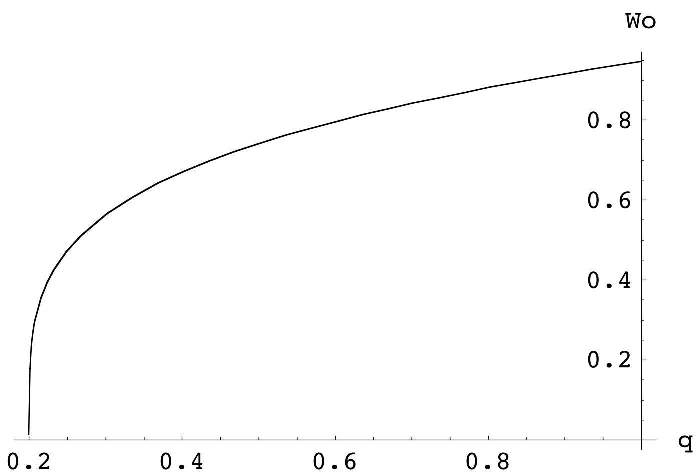
In[26]:=
```
ESS = qmin/(1 - x)
```

Out[26]=
```
0.266667
```

Now, plot offspring fitness as a function of individual allocation to off-spring, q.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[27]:=
```
Plot [Wo, {q, qmin, 1.0}, AxesLabel->{"q", "Wo"}]
```



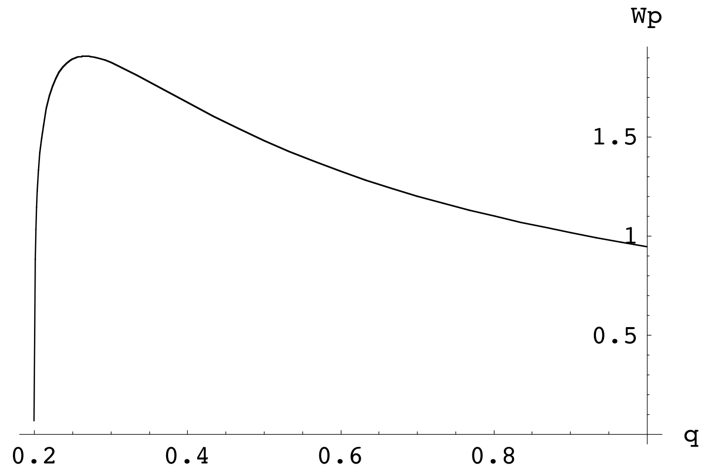
Out[27]=
```
- Graphics -
```

Are there diminishing returns ?

Finally, plot parental fitness as a function of individual allocation to offspring, q.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

In[28]:=
```
Plot [Wp, {q, qmin, 1.0}, AxesLabel->{"q", "Wp"}]
```



Out[28]=
```
- Graphics -
```

Is there a hump-shaped curve?

Doe the top of the curve correspond to the ESS?