Abstract This paper studies the information transmission and the effect of ambiguous information and transaction cost on trading volume. We consider a market with risk-averse informed and uninformed investors with CARA utility function and the supply of the risky asset is random. In this model, all investors have ambiguous beliefs about the probability distribution of the risky asset payoff before the signal was announced. After the signal is arrival, the speculator, informed investor, receives a private information signal about the realization of the risky asset payoff such that the speculator know precisely the conditional distribution of the payoff.
1 Introduction

This paper studies the information transmission and the effect of ambiguous information and transaction cost on trading volume and asset price in asset markets when investors receive public information. There are various information in financial markets. Investors want to minimize their risk when facing uncertainty. The ambiguous information is a kind of uncertainty in financial market.

We consider a market with risk-averse informed investors, uninformed arbitrageurs and random supply of a single risky asset, which is a modification from Vives (1995a,b) with unambiguous information and Ozsoylev and Werner (2009). And

We assume that all investors have ambiguous beliefs about the probability distribution of the risky asset payoff before the signal was announced. After the signal is arrival, the speculator, informed investor, receives a private information signal about the realization of the risky asset payoff such that the speculator know precisely the conditional distribution of the payoff. However, the risky asset payoff still remains uncertain. The uninformed arbitrageurs do not observe the signal such that the uninformed arbitrageur still has ambiguous belief about the distribution of the risky asset.

We use a different ambiguous information environment from Epstein and Schneider (2008). In Epstein and Schneider (2008) they assume that investors’ prior beliefs is unambiguous, but their posterior beliefs become ambiguous after receiving information signals. But, we assume that investors have ambiguous prior beliefs, after receiving the signal, the ambiguity disappears for informed trader, speculator. But, uninformed investors still have ambiguous information.

There are many papers study the effects of ambiguous uncertainty and ambiguity aversion in financial markets. Cao, Wang and Zhang (2005) build a model of asset markets with heterogeneous ambiguity. Their model showed that, in equilibrium, ambiguity aversion may lead to limited participation in asset market trading. Using a similar model, Easley and OHara (2009) focus on the role of regulation in mitigating the effects of nonparticipation
induced by ambiguity aversion. Epstein and Schneider (2008) studies asset prices in dynamic markets with ambiguous information signals, but there is no information transmission in their model.

2 The model

In this section, we present a model of asset market with ambiguous information and asymmetric information. There are one risk-averse informed speculator, one risk-averse uninformed arbitrageur and one noise trader. The risk-averse speculator and arbitrageur have a CARA utility function $-e^{\rho \omega}$, where $\omega$ is their end-of-period random wealth, $\rho > 0$ is the absolute risk aversion.

There are two assets in this market: risky asset and risk-free asset. The price of risk-free asset is normalized to 1. The price of the risky asset is $p$. The payoff of the risky asset is $\tilde{v}$ which is the sum two random variables $\tilde{\theta}$, and $\tilde{\epsilon}$.

$$\tilde{v} = \tilde{\theta} + \tilde{\epsilon}$$

where, $\tilde{\theta}$ and $\tilde{\epsilon}$ are two independent random normal distribution with mean $\mu$ and 0, variance $\sigma_{\theta}^2$ and $\sigma_{\epsilon}^2$, respectively. We use $\tilde{\epsilon}$ to capture the idiosyncratic shock.

This model has the feature of ambiguous information and information asymmetric which is showing on the observation of the realization of $\tilde{\theta}$. We assume that speculator can observe the realization $\theta$ of $\tilde{\theta}$ but the arbitrageur cannot. And the arbitrageur has ambiguous beliefs about the distribution of $\tilde{\theta}$. In more specific, the arbitrageur thinks the distribution of $\tilde{\theta}$ is a normal distribution $\mathcal{P}$, with mean $\mu \in [\mu, \bar{\mu}]$ and variances $\sigma_{\epsilon}^2 \in [\sigma_{\theta}^2, \sigma_{\bar{\theta}}^2]$. Also, $\mathcal{P}$ is independent of $\tilde{\epsilon}$. We use the interval $[\mu, \bar{\mu}], [\sigma_{\theta}^2, \sigma_{\bar{\theta}}^2]$ to reflect investor’s subjective aversion to ambiguity.

For the supply of risky asset $L$, we assume that $L \sim N(0, \sigma_L^2)$. $L$ is independent of $\tilde{\theta}$ and $\tilde{\epsilon}$. As Ozsoylev and Werner(2009) states, randomness in $L$ can be thought as resulting from
trade by noise traders and serves as an additional source of uncertainty and prevents asset
prices from fully revealing agents’ information.

The speculator’s random wealth $\omega = \omega_s + x(\bar{v} - p)$ is the sum of speculator’s initial wealth $\omega_s$ plus the gain from purchasing $x$ shares of risky assets, where $\bar{v}, p$ are the payoff and the price of the risky asset respectively. After observing the realization $\theta$ of $\tilde{\theta}$, the speculator’s information set is $I_s = \{\tilde{\theta} = \theta\}$

We also assume that there exists a per share transaction cost, $c$. Hence, the speculator’s portfolio is

$$
\max_x E[-e^{-\rho(\omega_s + x(\bar{v} - p - c))}\mid I_s].
$$

(2)

Since $[\bar{v}\mid I_s]$ distributes as normal, the speculator’s maximization problem can be rewrite as follow:

$$
\max_x (\rho(\omega_s + xE[(\bar{v} - p - c)\mid I_s]) - \frac{1}{2}\rho^2 x^2 \text{var}[\bar{v}\mid I_s]).
$$

(3)

The speculator’s demand of the risky asset, $x_s$ is solution of (3) which is as follow

$$
x_s(I_s, p) = \frac{E[\bar{v}\mid I_s] - p - c}{\rho \text{var}[\bar{v}\mid I_s]}.
$$

(4)

The arbitrageur maxmin their expected utility with risk-averse CARA utility function and the set of priors $\mathcal{P}$ since the arbitrageur are ambiguous about the mean and variance of an asset. By using maxmin expected utility, the arbitrageur considers the worst-case in the possible portfolio choice. The maxmin expected utility exhibits aversion to ambiguity and are axiomatized by Gilboa and Schmeidler (1989).

Now, we want to find the arbitrageur’s optimal demand for the risky asset. The degree of the ambiguous information can be measured by $\sigma_\theta^2 - \sigma_{\tilde{\theta}}^2$, and the larger the interval, $[\sigma_\theta^2, \sigma_{\tilde{\theta}}^2]$, the information are more ambiguous such that the arbitrageur is more averse to ambiguity. The arbitrageur’s maxmin expected utility of random wealth resulting from purchasing $x$
shares of risky asset is

$$\min_{\pi \in \mathcal{P}} E[-e^{-\rho(\omega_a + x(\tilde{v} - p - c))} | I_a].$$ \hspace{1cm} (5)$$

Equation (5) means that the arbitrageur is averse to ambiguous information, where $E_\pi$ stands for the expectation under belief $\pi \in \mathcal{P}$ and $I_a$ is the arbitrageur’s information set. Thus, the arbitrageur’s portfolio choice given $I_a$ becomes

$$\max_{x} \min_{\pi \in \mathcal{P}} E[-e^{-\rho(\omega_a + x(\tilde{v} - p - c))} | I_a].$$ \hspace{1cm} (6)$$

Hence, the set of solution $x_a(I_a, p)$ is (not completed)

$$x_a(I_a, p) =$$ \hspace{1cm} (7)

3 Rational Expectation Equilibrium

In rational expectations equilibrium, the speculator knows the real value $\theta$ of $\tilde{\theta}$. Hence, the speculator’s information set $I_s^* = \{\tilde{\theta} = \theta\}$. The arbitrageur doesn’t the realization of $\theta$, the arbitrageur only can extra information about $\theta$ from equilibrium price. Hence, the speculator’s information set $I_a^* = \{P(\tilde{\theta}, L) = p\}$. Where $L$ is the realization of the randomness asset supply $\tilde{L}$.

Therefore, in equilibrium we we would know an equilibrium price function $P(\theta, L)$ and equilibrium demand functions $X_s(\theta, L)$ and $X_a(\theta, L)$ and $p = P(\theta, L)$ which are

$$X_s(\theta, L) = x_s(I_s^*; p); X_a(\theta, L) \in x_a(I_a^*; p),$$ \hspace{1cm} (8)$$

where $X_s(\theta, L)$ is the optimal portfolio demand for the speculator, and $X_a(\theta, L)$ is the optimal portfolio demand for the arbitrageur. $I_s^*, i = [s, a]$ is the speculator’s and the arbitrageur’s information set under rational expectation respectively.
And the market clearing condition holds.

\[ X_a(\theta, L) = L - X_s(\theta, L); \]  

(9)

as Ozsoylev and Werner(2009), we know that for almost every realizations \( \theta \) and \( L \) of \( \tilde{\theta} \) and \( \tilde{L} \), there exists equilibrium demand function for the speculator and the arbitrageur.

How does the arbitrageur extra information from trading behavior? Assume that the arbitrageur could observe order flow in the end of transaction. Let \( f = \{ \tilde{L} - x_s(I^*_s; p) \} \) denote the order flow such that information will reveal through order flow. In equilibrium, given \( p = P(\theta, L) \) and \( f = \{ L - x_s(I^*_s; p) \} \), we know that information revealed by order flow and information revealed by price is equal. We will use this observation to derive the equilibrium demand function and the equilibrium price.

We can derive the speculator’s equilibrium demand function at information \( I^*_s = \{ \theta \} \) from (4):

\[ x_s(I^*_s; p) = \frac{\theta - p - c}{\rho \sigma^2} \]  

(10)

Hence, substitute the \( x_s(I^*_s; p) \) in to order flow \( f \), we could have the information revealed by order flow,

\[ \tilde{L} - \frac{\tilde{\theta} - p - c}{\rho \sigma^2} = f. \]

We can rewrite the information set as \( \{ \rho \sigma^2 \tilde{L} - \tilde{\theta} = a \} \), where \( a = \rho \sigma^2 f - p - c \) is a parameter which is known by the arbitrageur. Thus, the arbitrageur’s information set is \( I^*_a = \{ \rho \sigma^2 L - \theta \} \). It means that the arbitrageur’s information set is ambiguous.

Given the arbitrageur’s information set, \( I^*_a \), we can know the arbitrageur’s conditional expectation of asset payoff \( \tilde{v} \) is \( E_\pi[\tilde{v}|I^*_a] \) under a probability distribution \( \pi \) from the set of multi priors \( \mathcal{P} \) is:

\[ E_\pi[\tilde{v}|I^*_a] = E_\pi[\tilde{v} | \rho \sigma^2 L - \theta] \]
\[
E_{\pi}[\tilde{\theta}] + \frac{\text{cov}_{\pi}(\tilde{\theta}, \rho \sigma^2 \tilde{L} - \tilde{\theta})}{\text{Var}_{\pi}(\rho \sigma^2 \tilde{L} - \tilde{\theta})} (\rho \sigma^2 \tilde{L} - \theta - E_{\pi}[\rho \sigma^2 \tilde{L} - \theta])
\]
\[
= \mu_{\pi} + \frac{\sigma^2_{\pi}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \mu_{\pi}),
\]
(11)

where \( \mu_{\pi} = [\underline{\mu}, \bar{\mu}] \) is the mean of distribution \( \pi \) on \( \tilde{\theta} \) and \( \sigma_{\pi} = [\sigma^2_{\theta}, \sigma^2_{\theta}] \) is the variance of distribution \( \pi \) on \( \tilde{\theta} \).

Hence, we can derive \( \min_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] \) and \( \max_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] \) from above equation.

If \( \theta - \rho \sigma^2 \tilde{L} - \underline{\mu} < 0 \), then

\[
\min_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \mu + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \underline{\mu}),
\]
(12)

\[
\max_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \bar{\mu} + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \bar{\mu}).
\]
(13)

If \( \underline{\mu} \leq \theta - \rho \sigma^2 \tilde{L} \leq \bar{\mu} \), then

\[
\min_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \mu + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \underline{\mu}),
\]
(14)

\[
\max_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \bar{\mu} + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \bar{\mu}).
\]
(15)

If \( \theta - \rho \sigma^2 \tilde{L} > \bar{\mu} \), then

\[
\min_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \mu + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \underline{\mu}),
\]
(16)

\[
\max_{\mu \in \mathcal{P}} E_{\pi}[\tilde{v}|I^*_a] = \bar{\mu} + \frac{\sigma^2_{\theta}}{\sigma^2 + \sigma^2_{L}\rho^2\sigma^4_{\epsilon}} (\theta - \rho \sigma^2 \tilde{L} - \bar{\mu}).
\]
(17)

4 Effects of Ambiguity on Trading Volume

In this section, we want to study the effect of ambiguity information on total trading volume. We define the total trading volume as a function of information signal, \( \theta \) and the
realization of asset supply \( L \).

\[
V(\theta, L) = |X_a(\theta, L)| + |X_s(\theta, L)| + |L|, \tag{18}
\]

where, \( |X_a(\theta, L)| \) is the trading volume of the arbitrageur, \( |X_s(\theta, L)| \) is the trading volume of the speculator and \( L \) is the volume of supply.

## 5 Conclusion and Future Works

For future works, we could add a risk-neutral liquidity trader with linear utility and we can study the interaction between risk-averse informed trader and risk-neutral liquidity trader and we also can study the interaction between risk and ambiguity. Furthermore, we may allow informed traders have different risk parameter, therefore, we may have fruitful results. Besides, we could include more risky assets into our model and allow correlation among risky assets. The random return of other risky assets could serve additional resource of uncertainty. Moreover, we could model the effect of public signal of more risky asset on less risky asset. I think those extensions could provide some interesting results and could capture more financial market activities.

## References


Ozsoylev, Han N. and Werner, Jan (2009), Liquidity and Asset Prices in Rational Expectations Equilibrium with Ambiguous Information, working paper.
