Evaluating Macroprudential Policy with Financial Friction DSGE Model

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Abstract

In general, macroprudential policy refers to a set of regulatory policy imposed mainly on financial institutions, for macroeconomic purposes. In this paper, I aim to provide a DSGE framework to assess issues regarding macroprudential policy. Based on New Keynesian setup, I embedded financial accelerator mechanism by Bernanke et al. (1999) in both business and household lending contract and designed bank capital functioning as a buffer stock. Then I evaluate the effectiveness of various macroprudential policy rules given different shocks, using a policy evaluation measure in terms of inflation and output volatility. It turns out the target capital ratio reacting to output deviation performs the best, as it can reduce the volatility of inflation and output in most cases. LTV rule on household lending is in general not effective, as credit shifts away to the business sector.

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1 Introduction

The financial crisis that hit the U.S. and European economy in late 2000s changed the overall perspectives on how we should design macroeconomic model and policy, from various angles. One among them is the emphasis put on financial friction and the macroprudential policy. Economists who used to describe the economy with frictionless financial market now come to conceive more seriously that the financial friction can be an important factor in the economy. Accordingly, the role of credit, leverage, systemic risk, and policy instruments which can effectively control those variables are now being studied more actively. Blanchard et al. (2010) address this new challenge for policymakers in a straightforward way. While it becomes evident that policymakers should gain more control of leverage and systemic risk, monetary policy alone cannot achieve both price and the financial market stability. Nor there is any significant additional benefit from reacting monetary policy aggressively to asset price movement, as mentioned in Bernanke and Gertler (2001), because financial variables have large volatility and monetary policy is too blunt tool to control them. Therefore it is necessary for us to find additional set of policy instruments to achieve our dual objectives, price stability and financial market stability. They claim that the macroprudential financial regulation combined with the monetary policy, when well-coordinated, can provide us tools to pursue those dual objectives.

In general, macroprudential policy refers to a set of regulatory policy imposed mainly on financial institutions, for macroeconomic purposes. The objectives and instruments of macroprudential policy are well summarized in a paper from BIS (2010a). According to this, there are two, which are not mutually exclusive, objectives of macroprudential policy. The first is to strengthen the financial system’s resilience against adverse shocks in the economy, and the second is to actively limit the buildup of financial systemic risk. Clearly, these goals suggest countercyclical nature of macroprudential policy. Also it is distinguished from the crisis management policy, since macroprudential policy is preventive in its orientation. Macroprudential policy instruments include wide range of financial regulation measures, from balance sheet regulation such as bank capital requirement and provisioning, liquidity requirement and FX position limit, to lending contract regulation such as loan-to-value (LTV) ratio and debt service-to-income (DTI) ratio. In securities market, margin/haircut limit to control borrowers’ leverage can be also seen as macroprudential policy tools.
Some of those measures are already in operation in a lot of countries, but there are still a lot of issues about the implementation of macroprudential policy that requires further study. The first is about the transmission mechanism. We need to be able to explain the channels those macroprudential prudential policy can influence not only nominal but also real variables in the economy. This is why we need financial friction models to evaluate macroprudential policy, because they are essentially models about the role of credit and leverage in the economy, which are often the target object of the macroprudential policy. Second, the comparison between rules vs. discretion in policy operation needs to be discussed. In most countries financial regulation instruments mentioned above are conducted largely by policymaker’s discretion, except for Spain’s dynamic provisioning rule requiring banks to set aside provisions during phases of rapid credit expansion according to a formula. However, it has been argued that the rule-based approach can perform better\textsuperscript{1}, because it can better anchor agents’ expectation about future regulatory stance, as well as it lessens the regulator’s burden to justify every regulatory action. Also it can help to reduce regulator’s incentive not to tighten the regulation based on “this time is different” belief. Third, there is signal extraction problem for policymakers, because identifying the buildup of the systemic risk is not a simple task. For example, there could be only market-specific turbulence, or even worse, counter signal from different segments of the financial market. Determining which information should be taken into consideration to which degree can present a big challenge for policymakers. This signal extraction problem directly brings up another question of to which variable macroprudential policy should react to. Fourth, there is an issue of policy coordination with the monetary policy or other macroeconomic policy. It is not uncommon that the economy enters a state where monetary and macroprudential policy objectives diverges from each other. In fact we experienced it in global housing price and credit boom in mid-2000s, when those variables indicated the buildup of systemic risk but the monetary policy was kept accommodative because of the low inflation environment. Finally, there is a possibility of regulatory arbitrage, as the credit can shift from heavily regulated market to other markets where there is less regulation. It can generate unexpected side effects, often resulting in offsetting the effectiveness of the regulatory policy.

In this paper, I aim to provide a DSGE framework to assess many, albeit not all, of those questions and to suggest both qualitative and quantitative results. I embedded financial accelerator mechanism by Bernanke et al. (1999) in both business and household lending

\textsuperscript{1}See BIS (2010b)
contract, and will evaluate the effectiveness of macroprudential policy given different policy rules and different shocks.

Researches about macroprudential policy began to develop only quite recently. Some researches emphasized the complementary role of macroprudential policy to the monetary policy. Borio and Shim (2007) brought up discussions about macro dimension of prudential policy and its supportive role for monetary policy as a built-in stabilizer. N’Diaye (2009) also explored how countercyclical prudential regulation can support monetary policy, in a reduced-form monetary model with contingent claims analysis. Researches closest to this paper are those which are based on DSGE framework with financial friction, although main research questions and model setups vary. Angeloni and Faia (2009) featured optimal credit allocation problem in the banking sector within a New Keynesian model, and showed how countercyclical and procyclical bank capital requirement ratio influences monetary policy performance. Kannan et al. (2009) discusses stabilizing effects of countercyclical macroprudential policy in a model setup where durable goods are used as collateral by the borrowing household. Angelini et al. (2010) has the framework that is the closest to this paper, as it uses the model setup by Gerali et al. (2010), where we can find saving/borrowing household and deposit/saving bank distinction. They also suggested the same set of policy instruments as in this paper, bank capital requirement ratio and LTV regulation. However, core financial friction and bank capital dynamics differ, and so is LTV regulation in this paper as it is designed to be specific only to household lending. They found the optimal combination of monetary and macroprudential policy rule given policy loss function, and casted another interesting question of how different policy objectives of macroprudential policymaker and monetary policymaker can create a cooperation problem.

As I mentioned earlier, financial friction is crucial in explaining the transmission mechanism of macroprudential policy. Two of the most frequently referred financial friction models are Bernanke-Gertler-Gilchrist (BGG) financial accelerator model and liquidity constraint model by Kiyotaki and Moore (1997). They focus on borrower’s balance sheet, and show how borrower’s leverage condition generates significant effects in economy. On the other hand, Holmstrom and Tirole (1997) cast “double-decker” contract problem that both borrower and lender are subject to moral hazard, which give rise to the importance of financial intermediary’s capital. The financial intermediary’s balance sheet is also emphasized in Adrian and Shin (2010), in terms of Value-at-Risk (VaR) constraint. The financial friction in this paper
is close to Zhang (2009), which refines BGG financial accelerator mechanism by introducing risk sharing banking sector and bank capital which functions as a buffer stock to unexpected realization of aggregate return on capital.

The rest of the paper is composed of five sections. In section 2, I will explain the model in detail. Parameter calibration and functional form assumption will be discussed in section 3. Dynamics from the baseline model will be described in section 4, and the effectiveness of macroprudential policy will be discussed in section 5. Section 6 concludes.

2 Model

The model in this paper is based on BGG financial accelerator mechanism in New Keynesian framework. In addition, I distinguish between saving household and borrowing household, so that there are dual credit markets (business lending and household lending). Housing goods are not only in the utility function but also used as collateral for the borrowing household. The banking sector consists of deposit bank and lending bank, linked with each other by an interbank money market. Monetary policy determines saving household’s deposit rate, and macroprudential policy affects bank lending rates by imposing regulation on the banking sector. Figure 1 summarizes the basic framework of the model.

2.1 Household

There exist two types of households, saving (s) and borrowing (b) household. They are distinguished by time preference parameter ($\beta$), as borrowing household is less patient about future consumption than saving household ($\beta_b < \beta$). This distinction by time preference parameter is often used in credit friction model, as in Iacoviello (2005). Because of different time preference, saving household will always save and borrowing household will always borrow in steady state and its neighborhood. I assume that both types have the same population, as if there are one saving member and one borrowing member in a single household. It is also assumed that agents do not move between two groups.
2.1.1 Saving Household

Saving household (denoted by $s$) with future discount rate $\beta$ maximizes

$$
\max_{C_t, H_t, N_t, I_{t,s}^H, B_t, D_t, e_t} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{t,s} + \left(1 - \gamma \epsilon_t^\gamma \right) \log H_{t,s} - \frac{(N_{t,s})^{1+\phi}}{1+\phi} \right] \right\}
$$

subject to

$$
C_{t,s} + P_t^H I_{t,s}^H + \frac{B_t}{P_t} + D_t + e_t + T_{t,s} \leq R_{t-1}^N \frac{B_{t-1}}{P_t} + R_{t-1}^D D_{t-1} + R_{t-1}^e (1 - \phi_t) e_{t-1} + w_t N_{t,s}
$$

where $C$ and $H$ denote consumption of consumption goods and housing goods, and $N$ is labor supply. $\epsilon^\gamma$ is preference shock on housing goods, so that a positive $\epsilon^\gamma$ shock can cause housing demand to drop. In the budget constraint, $P^H$ is the relative price of housing goods in terms of final consumption goods and $I^H$ is the investment on housing goods. Saving household can invest on asset portfolio that consists of risk-free nominal asset $B$, real bank
deposit $D$ and real bank equity $e$. $R^N$ is nominal gross return on $B$, $R^D$ and $R^e$ is real gross return on $D$ and $e$, respectively. Finally $T$ is lump-sum tax and $\phi$ is default probability of the bank equity.

Saving household chooses $C_{t,s}, H_{t,s}, N_{t,s}, I^H_{t,s}, B_t, D_t, e_t$ given prices and rate of return on each asset. We can setup Lagrangian as:

$$\mathcal{L} = E_{0} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{t,s} + \left(1 - \gamma \epsilon_t^c \right) \log H_{t,s} - \frac{(N_{t,s})^{1+\phi}}{1+\phi} \right] + \lambda_t(R^N_{t-1} \frac{B_{t-1}}{P_t}) + R^D_{t-1} D_{t-1} + R^e_{t-1} (1 - \phi_t) e_{t-1} + w_t N_{t,s} \right. \right.$$  

$$ - C_{t,s} - P^H_t I^H_{t,s} - \frac{B_t}{P_t} - D_t - e_t - T_{t,s}) + \mu_t ((1 - \delta_H) H_{t-1,s} + I^H_{t,s} - H_{t,s})]$$

where Lagrange multiplier $\lambda$ is on household budget constraint and $\mu$ is on the law of motion for housing goods stock.

FOCs are shown as below:

$$C_{t,s} : \frac{\gamma}{C_{t,s}} = \lambda_t$$  

$$N_{t,s} : (N_{t,s})^\phi = \lambda_t w_t \Rightarrow (N_{t,s})^\phi = \frac{\gamma}{C_{t,s}} w_t$$  

$$D_t : \lambda_t = \beta E_t \lambda_{t+1} R^D_t \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} R^D_t$$  

$$B_t : \frac{\lambda_t}{P_t} = \beta E_t \lambda_{t+1} \frac{R^N_{t+1}}{P_{t+1}} \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} \frac{R^N_t P_t}{R^N_{t+1} P_{t+1}}$$  

$$e_t : \lambda_t = \beta E_t \lambda_{t+1} R^e_t (1 - \phi_{t+1}) \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} R^e_t (1 - \phi_{t+1})$$  

$$I^H_{t,s} : \lambda_t P^H_t = \mu_t$$  

$$H_{t,s} : \mu_t = \frac{1 - \gamma \epsilon^c_t}{H_{t,s}} + \beta E_t \mu_{t+1} (1 - \delta_H)$$

Combining (4) and (5) gives us labor supply decision. (6) and (7) are Euler equation with real deposit and nominal asset, and combining them gives us Fisher equation. Here nominal and real assets are substitutes, and the rest of the paper I will assume the equilibrium quantity of nominal asset is zero. In other words, all outstanding deposit, lending and
borrowing will be in real terms. (8) shows that the agent will demand higher return for bank equity because of the possibility of default, and that premium will go up as the probability of default goes up. (9) and (10) show that the shadow price of housing goods this period is given by the sum of this period’s marginal utility from housing goods and discounted value of next period’s expected shadow price.

2.1.2 Borrowing Household

Borrowing household (denoted by \( b \)) with future discount rate \( \beta_b \) maximizes

\[
\max_{C_{tb}, H_{tb}, N_{tb}, I_{tb}^H, L^H} E_o \left\{ \sum_{t=0}^{\infty} \beta_b^t \left[ \gamma \log C_{tb} + (1 - \gamma \epsilon_t) \log H_{tb} - \frac{(N_{tb})^{1+\varphi}}{1+\varphi} \right] \right\}
\]

subject to

\[
C_{tb} + P_t^H I_{tb}^H + R_{LH}^t L_{tb}^H + T_{bt} \leq w_t N_{tb} + L_{tb}^H
\]

where \( L^H \) is household loan, \( R_{LH}^t \) is the rate of return on household loan and all other terms are defined similarly as the saving household problem.

First order conditions for borrowing household will be similar to those of saving household, except there is one for intertemporal borrowing decision rather than saving:

\[
\frac{1}{C_{tb}} = \beta_b E_t(\frac{1}{C_{t+1,b}} R_{LH}^t).
\]

In steady state, borrowing household has to provide more labor and consume less, to pay interest for its debt.

2.2 Entrepreneurs

Entrepreneurs produce intermediate goods using capital, labor, entrepreneurs’ labor \( (N_{t,e}) \) and bankers’ labor \( (N_{t,f}) \). Production technology includes entrepreneurs’ and bankers’ labor to guarantee that those labor income can make their net worth nonzero in steady state. However, their contribution in aggregate output is negligible. Entrepreneurs choose the
quantity of the capital next period, financed by their net worth and borrowing from the banking sector. Loan contract between entrepreneurs and the bank will be discussed in detail later.

\[ Y_t = A_t(K_{t-1}^{\alpha_k})(N_t^{\alpha_n})(N_{e,t}^{\alpha_{ne}})(N_{f,t}^{\alpha_{nf}}) \]

and

\[ \alpha_k + \alpha_n + \alpha_{ne} + \alpha_{nf} = 1. \]

Let the value of marginal product of capital \( z_t \) be

\[ z_t = mc_t \cdot \frac{Y_t}{K_{t-1}} \tag{14} \]

where \( mc \) is the real marginal cost. Then the gross return from one unit of capital is defined by:

\[ R^K_t = \frac{z_t + (1 - \delta)q_t}{q_{t-1}}. \tag{15} \]

\( q \) is the price of capital in terms of consumption goods, so the return on capital reflects price change \((q_t/q_{t-1})\) of capital as well as return from \( z_t \).

Firm’s optimization also gives us labor demand for each labor supplier as

\[ w_t = mc_t \cdot \frac{Y_t}{N_t} \tag{16} \]

\[ w_{t,e} = mc_t \cdot \frac{Y_t}{N_{t,e}} \tag{17} \]

\[ w_{t,f} = mc_t \cdot \frac{Y_t}{N_{t,f}} \tag{18} \]

Entrepreneur’s net worth, denoted by \( W_t \), is the retained earning and similar concept with owner’s capital in balance sheet. Using aggregated terms, we have

\[ W_t = vV_t + w_{t,e} \tag{19} \]

where \( v \) is entrepreneurs’ survival rate and \( V \) is the return from each period’s project net of borrowing cost, defined by
\[ V_t = \int_{\bar{\omega}^b_{t-1}}^{\infty} \omega R^K_q t_{t-1} K_{t-1} f(\omega) d\omega - (1 - F(\bar{\omega}^b_{t-1}))(R^{LB}_{t-1} L^{LB}_{t-1}). \]  

(20)

Here \( \omega \) is idiosyncratic shock that hits each entrepreneur and \( \bar{\omega}^b \) is the default threshold which will be defined below. \( R^{LB} \) denotes the gross return for business lending and \( L^{B} \) is the amount of lending. The first term in right hand side is the gross payoff for entrepreneurs when they do not default, and the second term is the debt repayment obligation to the bank. If a entrepreneur defaults, \((\omega < \bar{\omega}^b)\), it will end up with nothing, and the remaining value of the project will be under control of the bank. Each period \( 1 - \upsilon \) proportion of entrepreneurs are assumed to be thrown out of business, in order to prevent entrepreneurs from accumulating too much net wealth so that they don’t need any borrowing. Those entrepreneurs who find themselves out of business will just consume \((1 - \upsilon)V\).

### 2.3 Capital Producer

At the beginning of each period, the capital producer purchases \( I_t \) amounts of consumption goods at price one, and turns them into the same amount of new capital. Transformation cost arises during the process, and at the end of the period it resells new capital to entrepreneurs at price \( q_t \). The law of motion for capital stock is given as

\[ K_t = I_t + (1 - \delta)K_{t-1} \]  

(21)

and \( q_t \), the price of capital in terms of consumption goods, can be obtained from optimization problem of the capital producer.

\[ \max_I (q_t - 1)I_t - f(\frac{I_t}{I_{t-1}})I_t \]  

(22)

with \( f(1) = f'(1) = 0, f''(1) = \chi_K \). Functional form of \( f \) will be discussed in the next section.

FOC can be written as

\[ q_t = 1 + f(\frac{I_t}{I_{t-1}}) + f'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}}. \]  

(23)
2.4 Financial Contract and Banking Sector: BGG Mechanism

The basic framework of financial contracts in this paper is based on BGG model. In a literature about credit implication in macro business cycle model, either BGG or Kiyotaki-Moore (KM) liquidity constraint friction has been most frequently applied. This is also the case in works about macroprudential policy, as Kannan et al. (2009) used BGG model and Angelini et al. (2010) used KM model. The financial contract here resembles that of Zhang (2009) the most, which refines BGG model. There are two main advantages of choosing Zhang’s model over standard BGG or KM model. First, while KM model does not allow the default of borrower (in the first place, one cannot borrow the amount exceeding what is backed by his collateral value), Zhang’s model allows agent to default when idiosyncratic shock falls below the default threshold level. Second, it has a mechanism which bank capital absorbs unexpected shock in aggregate variables. By distinguishing between ex-ante and ex-post default threshold, the forecast error in next period’s return on capital or housing price creates a discrepancy between expected default rate and actual default rate. The bank has to absorb this discrepancy by accumulating or writing off bank capital, hence the bank capital functions as a buffer stock. This mechanism is not found in BGG model, where the financial intermediary does not share any default risk. Based on this framework, I introduced another financial accelerator mechanism in household lending contract, and also added deposit bank-lending bank distinction so that there is an interbank money market between them.

2.4.1 Financial Contract: Business Loan

The size of business loan for an individual entrepreneur is defined by the difference between the size of investment project and the entrepreneur’s net worth:

\[ L_{t}^{B,i} = q_{t} K_{i}^{t} - W_{i}^{t}. \]  
(24)

Define \( \bar{\omega}_{i,a}^{t} \) as ex-ante threshold idiosyncratic shock level which determines whether an entrepreneur defaults or not. Then there is a relationship between gross loan repayment value and the project value such as:

\[ R_{t}^{LB,i} L_{t}^{B,i} = \bar{\omega}_{i,a}^{t} E_{t} R_{t+1}^{K} q_{t} K_{i}^{t}. \]  
(25)
The expected return for the entrepreneur net of debt repayment is:

$$\int_{\omega_i}^{\infty} \omega E_t R_{t+1}^K q_t K_i f(\omega) d\omega - (1 - F(\omega)) R_t^{LB,i} L_t^{B,i}. \quad (26)$$

The entrepreneur should guarantee the same expected return to the bank as its funding cost:

$$(1 - F(\omega)) R_t^{LB,i} L_t^{B,i} + (1 - \mu) \int_{0}^{\infty} \omega E_t R_{t+1}^K q_t K_i f(\omega) d\omega = R_t^f(q_t K_i - W_t^i). \quad (27)$$

$\mu$ represents the monitoring cost for the bank in case of default, arising due to ‘costly state verification’ problem between the entrepreneur and the bank, first introduced by Townsend (1979). Since the bank has disadvantage of verifying the borrower’s states, when it takes to manage entrepreneur’s remaining project in case of default, $\mu$ fraction of monitoring cost arises. This monitoring cost is the core friction in BGG model.

The entrepreneur maximizes (26) subject to (27). Solving this problem and aggregating, with a little derivation, gives rise to the leverage-external finance premium relationship that drives BGG financial accelerator mechanism:

$$E_t R_{t+1}^K = S(\frac{q_t K_t}{W_t}, \epsilon_t^f) R_t^f \quad (28)$$

where $S$ is increasing and convex function. Define $lev_t \equiv q_t K_t / W_t$. $\epsilon^f$ is riskiness shock reflecting overall financial market sentiment, and is affected by factors such as the variance of $\omega$.

External finance premium, $(E_t R_{t+1}^K / R_t^f)$, is increasing in debt-net worth ratio, or leverage ratio $(lev_t)$. This is the reason why the mechanism is called financial accelerator, because if a positive shock which improves net worth of the entrepreneur is realized, with better balance sheet condition she can further increase investment with lower external finance premium.

From (25), gross rate of return for business loan can be written as

$$R_t^{LB} = \frac{\omega_{i} E_t R_{t+1}^K q_t K_t}{L_t^{B,i}}. \quad (29)$$
In period \( t+1 \), given loan rate, after \( R_{t+1}^K \) is realized ex-post threshold productivity \( \bar{w}_t^b \) is defined as

\[
\bar{w}_t^b = \frac{R_{t+1}^{LB} L_t^B}{R_t^{K} q_t K_t} = \bar{w}_t^a E_t R_{t+1}^K \frac{R_t^K}{R_{t+1}}.
\]  

(30)

2.4.2 Financial Contract : Household Loan

Similar to BGG mechanism applied to business sector, we can derive a mechanism that the household lending rate is increasing in loan-to-value (LTV) ratio \( (ltv_t = L_t^H / P_t^H H_{t,b}) \) of the borrowing household.

\[
R_{t}^{LH} = G \left( \frac{L_t^H}{P_t^H H_{t,b}}, \epsilon_t^H \right) R_{i}^f.
\]  

(31)

Suppose idiosyncratic housing price shock hits each borrowing household at the beginning of each period. The \( i \)th borrowing household will default if the value of idiosyncratic price shock \( \omega_{H,i}^t \) is less than some threshold level \( \bar{\omega}_{H,i}^t \). Then the household lending contract can be written as

\[
R_{t}^{LH} L_{t}^{H,i} = \omega_{t}^{H,i,a} E_t P_{t+1}^H H_{t,b}^i.
\]  

(32)

Here \( E_t P_{t+1}^H H_{t,b}^i \) is the expected value of the collateral that the borrowing household has. Noting the bank will ask the same expected return from household lending as its opportunity cost, we have

\[
R_{t}^f L_{t}^{H,i} = (1 - F(\omega_{t}^{H,i,a})) R_{t}^{LH} L_{t}^{H,i} + (1 - \mu) \int_0^{\omega_{t}^{H,i,a}} \omega E_t P_{t+1}^H H_{t,b}^i f(\omega) d\omega.
\]  

(33)

I assume the bank will set \( R_{t}^{LH} \) and \( \omega_{t}^{H,i,a} \) to satisfy (33), given \( L_{t}^{H,i}, H_{t,b}^i, P_t^H \) and \( E_t P_{t+1}^H \). Substituting (32) to (33) and assuming \( \omega_{t}^{H,i,a} \) is in a region where the expected revenue of the bank is increasing in \( \omega_{t}^{H,i,a} \), it is possible to show that \( \omega_{t}^{H,i,a} \) and \( R_{t}^{LH} \) is increasing in \( L_{t}^{H,i} / P_t^H H_{t,b}^i \). Since every borrowing household is homogenous, it is possible to aggregate them and deduce a financial accelerator relationship for household lending (31). Again ex-post default threshold price shock for household, \( \bar{\omega}_t^{H,b} \) is defined by

\[
\bar{\omega}_t^{H,b} = \frac{R_{t}^{LH} L_t^H}{P_{t+1}^H H_{t,b}^i}.
\]  

(34)
2.4.3 Banking Sector

There are two types of banks, deposit and lending bank, in banking sector. Deposit bank receives deposit from household, and simply intermediates it to interbank market. Hence deposit bank does not accumulate profit, nor issues equity capital. On the other hand, lending bank has two funding channels, interbank borrowing and equity capital financing, and its asset consists of business and household lending. Deposit rate $R^{D}$ is determined by the nominal rate set by central bank, and interbank rate ($R^{I}$) is determined by adding regulatory markup to deposit rate. This markup captures the restriction imposed on banking sector by the regulatory action, which is a function of capital requirement ratio and actual capital ratio of the lending bank.

\[
R^{I}_{t} = f(R^{D}_{t}, \bar{\kappa}_{t} - \kappa_{t}).
\]  

(35)

Here $\kappa$ is the actual capital ratio (capital/asset) of lending bank, and $\bar{\kappa}$ is capital requirement ratio set by financial regulator, which will be discussed below. $R^{I}_{t}$ is increasing in $\bar{\kappa}_{t} - \kappa_{t}$, meaning it will be higher when lending bank’s capital ratio is below the required level. When bank capital ratio is below the required level, the financial regulator will enforce corrective measure affecting adversely its reputation, or constraining its managerial decision. This in turn worsen the lending bank’s funding cost, as higher markup is charged on the lending bank.

Lending bank capital, $e_{t}$, evolves as below:

\[
e_{t} = (1 - \phi_{t})e_{t-1} + R^{LB}_{t-1}L^{B}_{t-1}(1 - F(\bar{\omega}^{b}_{t-1})) + (1 - \mu) \int_{0}^{\bar{\omega}^{b}_{t-1}} \omega R^{K}_{t}q_{t-1}K_{t-1}f(\omega) d\omega + R^{LB}_{t-1}L^{H}_{t-1}(1 - F(\bar{\omega}^{H,b}_{t-1})) + (1 - \mu) \int_{0}^{\bar{\omega}^{H,b}_{t-1}} \omega P^{H}_{t}H_{t-1,b}f(\omega) d\omega - R^{I}_{t-1}(L^{B}_{t-1} + L^{H}_{t-1}) + w_{t-1,f}.
\]

(36)

It can be also written as
\[ e_t = (1 - \phi_t)e_{t-1} + w_{t-1,f} \]  \hspace{1cm} (37)

\[ + R_{t-1}^{LB} L_{t-1}^B (F(\bar{\omega}_{t-1}^a) - F(\bar{\omega}_{t-1}^b)) + (1 - \mu) \int_{\bar{\omega}_{t-1}^a}^{\bar{\omega}_{t-1}^b} \omega R_t^K q_{t-1} K_{t-1} f(\omega) d\omega \]

\[ + R_{t-1}^{LH} L_{t-1}^H (F(\bar{\omega}_{t-1}^H,a) - F(\bar{\omega}_{t-1}^H,b)) + (1 - \mu) \int_{\bar{\omega}_{t-1}^H,a}^{\bar{\omega}_{t-1}^H,b} \omega F_t^H H_{t-1,b} f(\omega) d\omega. \]

Equation (37) tells us that unexpected realization of return on capital or housing price can increase or decrease banking capital, as the difference between ex-ante and ex-post default threshold (\( \bar{\omega}^a \) and \( \bar{\omega}^b \)) arises because of forecast errors in return on capital or housing price. When realized value of \( R^K \) or \( P^H \) is lower than its expected value, we have \( \bar{\omega}^b > \bar{\omega}^a \) (or \( \bar{\omega}^H,b > \bar{\omega}^H,a \)) and hence \( F(\bar{\omega}^a) - F(\bar{\omega}^b) < 0 \) (or \( F(\bar{\omega}^H,a) - F(\bar{\omega}^H,b) < 0 \)).

Total lending is the sum of business and household lending. \( (L_t = L_t^B + L_t^H) \). Now define bank capital ratio, \( \kappa \) as the ratio of bank capital to total lending.

\[ \kappa_t = \frac{e_t}{L_t}. \]  \hspace{1cm} (38)

The asset side of the lending bank must be balanced with its liability and equity side:

\[ (D_t + e_t) = L_t. \]  \hspace{1cm} (39)

The default rate of the lending bank is decreasing function of bank capital ratio and subject to a default shock.

\[ \phi_t = F(\kappa_{t-1}, \epsilon_t^\phi), F_\kappa < 0. \]  \hspace{1cm} (40)

Finally, lending bank’s funding rate is weighted average of equity capital return rate and interbank borrowing rate:

\[ R_t^f = \kappa_t R_t^e + (1 - \kappa_t) R_t^f. \]  \hspace{1cm} (41)

Therefore, the lending bank has to pay two different costs for carrying smaller capital.
ratio, one from interbank borrowing rate, and the other from equity financing cost. (Re-
member from household optimization investors demand higher bank equity return when they
perceive higher probability of default).

2.5 Retail Goods Producers

Each retail goods producer \((i)\) purchases intermediate goods, and turns into retail good \(Y_t(i)\) in monopolistic competitive market. Total final usable goods \(Y_t\) are the following composite of retail goods:

\[
Y_t = \left[ \int_0^1 Y_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \quad (42)
\]

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \quad (43)
\]

Only \((1-\theta)\) fraction of retail goods producers are allowed to change price, \(\text{à la Calvo (1983).}\)

They solve following maximization problem:

\[
\max_{P_t} E_t \left[ \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} \Omega_{t+k}(i) Y_{t+k}^*/P_{t+k} \right] \quad (44)
\]

where \(\Lambda_{t,k} = \beta^k C_t C_{t+k}\), and nominal profit \(\Omega = P^* - MC\), \(MC\) being nominal marginal cost, subject to the demand function

\[
Y_{t+k}^*(i) = \left( \frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}. \quad (45)
\]

By substituting demand constraint into the objective function and differentiate with respect to \(P_t^*\), the first order condition for the retail goods producer can be shown as

\[
P_t^*(i) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} MC_{t+k}(i) Y_{t+k}(i)/P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i)/P_{t+k}}. \quad (46)
\]

Then the aggregate price will be determined as:

\[
P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \quad (47)
\]

Log-linearizing and combining optimality condition and aggregate price dynamics, as-
Assuming steady state inflation $\pi$ is 1, we can obtain New Keynesian Philips curve expression.

$$\beta E_t \pi_{t+1} = \pi_t - (1 - \beta \theta) \frac{1 - \theta}{\theta} \hat{m}C_t. \quad (48)$$

### 2.6 Exogenous Processes and Market Clearing

Assume productivity shock, financial riskiness shock, housing preference shock all follow stationary AR(1) process in log-linearized form:

$$\hat{A}_t = \rho A \hat{A}_{t-1} + \hat{\epsilon}^A_t \quad (49)$$

$$\hat{\epsilon}^f_t = \rho_f \hat{\epsilon}^f_{t-1} + \hat{\xi}^f_t \quad (50)$$

$$\hat{\epsilon}^\gamma_t = \rho_\gamma \hat{\epsilon}^\gamma_{t-1} + \hat{\xi}^\gamma_t \quad (51)$$

Housing stock is determined by following law of motion:

$$H_{t,j} = (1 - \delta_H)H_{t-1,j} + I^H_{t,j}, \quad j = s, b. \quad (52)$$

We can get aggregate housing investment demand by adding housing investments from both types of households.

$$I^H_{t,s} + I^H_{t,b} = I^H_t(D). \quad (53)$$

For simplicity, assume housing investment supply is given exogenously, following AR(1) process in log-linearized terms.

$$\tilde{I}^H_t(S) = \rho_H \tilde{I}^H_{t-1}(S) + \hat{\epsilon}^H_t. \quad (54)$$

In equilibrium, $I^H_t(D) = I^H_t(S)$.

Besides, we have market clearing conditions for goods and labor market. Note in aggregate resource constraint we have terms for monitoring cost and regulatory markup.
\[ Y_t = C_{t,s} + C_{t,b} + C_t^e + C_t^f + q_t I_t + I_t^H + \]
\[ + \mu \int_0^{\omega_{t-1}} \omega R_t^K q_{t-1} K_{t-1} dF(\omega) + \mu \int_0^{\omega_{t-1}^H} P_t^H H_{t-1,b} dF(\omega) + \varsigma_t. \] (55)

\[(C_t^e = (1 - \gamma)V_t, \quad C_t^f = \phi_t e_{t-1}, \text{ and } \varsigma_t \text{ denotes the sum of regulatory markups imposed by capital requirement and LTV regulation)}\]

\[ N_{t,s} + N_{t,b} = N_t(S), \quad N_t(S) = N_t(D). \] (56)

### 2.7 Monetary and Fiscal Policy

Monetary policy is set to follow standard Taylor rule in log-linearized form.

\[ \hat{R}_t^N = \rho_r \hat{R}_{t-1}^N + (1 - \rho_r)[\phi_{\pi} \hat{\pi}_t + \phi_Y \hat{Y}_t] + \epsilon_t. \] (57)

Regarding fiscal policy, government is restricted to play very simple role in this model. It collects lump-sum tax from both types of households and just spends the same amount away each period.

\[ G_t = T_{s,t} + T_{b,t} (T_{s,t} = T_{b,t}). \] (58)

### 2.8 Macroprudential Policy

The regulatory authority sets target capital requirement ratio \( \kappa_t \) and target Loan-to-Value ratio \( ltv_t \), according to rules systemically reacting to observable macro variables such as output, inflation, credit growth or housing price.
2.8.1 Capital Requirement

As mentioned in banking sector, lending bank has to pay higher borrowing cost in interbank borrowing market when its capital ratio is below target capital requirement ratio. Target capital requirement ratio is set by financial regulator using a simple rule:

\[
\kappa_t = \zeta_{\kappa}(Y_t, L_t, P^H_t).
\]  

(59)

The set of variables which the macroprudential policy reacts to are chosen with practical considerations, among those which policymakers are the most likely to concern about. For simplicity, throughout the paper inflation variable will be excluded from this set, with the assumption that the policy mix will be operated in a way that the monetary policy alone can install the price stability.

2.8.2 Loan-to-Value Ratio

Recall (31), \( R_{t}^{LH} = G(\hat{ltv}_t, \epsilon^L_t) \hat{R}^L_t \), with \( G'(\cdot) > 0 \) and \( G''(\cdot) > 0 \). We can construct a mechanism that the borrowing household not only has to bear higher borrowing cost for its higher indebtedness, but also has to pay regulatory penalty for taking higher LTV ratio than the target LTV ratio set by financial regulator. Given the functional form which will be shown in the next section, we can rewrite equation (31) in log-linearized form as

\[
\hat{R}_{t}^{LH} = \hat{R}^L_t + \psi^{LH}\hat{ltv}_t + \psi^{LTV}(\hat{ltv}_t - \bar{ltv}_t) + \epsilon^{H}_t
\]  

(60)

where \( \psi^{LH}, \psi^{LTV} > 0 \). The third term on the right hand side represents the regulatory markup in household lending.

Again, target LTV ratio is set using a simple rule:

\[
\bar{ltv}_t = \zeta_{ltv}(Y_t, L_t, P^H_t).
\]  

(61)
3 Functional Forms and Calibration

3.1 Functional Forms

In capital producer’s optimization problem, the adjustment cost function is assumed to be a simple quadratic form:

\[ f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\chi_k}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2. \] (62)

We suppose the functional form of external finance premium for business lending as:

\[ E_t R_{t+1}^K = R_t f' \left( \frac{q_t K_t}{W_t} \right)^{\nu_a} \epsilon_t^f. \] (63)

The functional form for household lending premium is assumed to be

\[ R_{t+1}^{LH} = R_t f' (1 + \ell t v_t)^{\nu_a} \epsilon_t^{fH} + \nu^H \exp[\nu_c^H (\ell t v_t - \overline{\ell t v_t})] \] (64)

where the second term on the right hand side is the expression for regulatory penalty (markup) for taking higher LTV than the target LTV.

Interbank rate is set by the deposit rate plus regulatory markup applied to lower level of capital ratio than the target capital ratio. We assume a functional form for this relationship as

\[ R_{t+1}^I = R_t^D + \nu^I \exp[\nu^I (\kappa_t - \kappa_t)]. \] (65)

Default probability of the bank equity capital is decreasing in capital ratio of lending bank. I assume logistic function for this probability,

\[ \phi_t = F(\kappa_{t-1}, \epsilon_t^\phi) = \frac{\exp[\nu^\phi - \nu^\phi \kappa_{t-1}]}{1 + \exp[\nu^\phi - \nu^\phi \kappa_{t-1}]} \cdot \epsilon_t^\phi. \] (66)
3.2 Calibration

Future discount factors are chosen to be 0.99 for saving household and 0.982 for borrowing household. This implies steady state spread between household deposit rate and household borrowing rate is about 340bps. $\gamma$, which determines the weight of consumption goods and housing goods in utility function, is calibrated as 0.843, and inverse Frisch elasticity of labor $\varphi$ is assumed to be 1. The steady-state ratio of saving household’s consumption/borrowing household’s consumption is set to 0.65/0.35, which implies that given parameter values of utility function, saving household’s labor/borrowing household’s labor ratio should be 0.35/0.65 in steady state. The rate of depreciation for housing goods ($\delta^H$) is chosen to be 0.02, implying that it takes about 12.5 years for housing goods to completely depreciate.

In production sector, the share of capital in Cobb-Douglas production function $\alpha$ is chosen to be 0.34. The rate of depreciation for capital ($\delta$) is 0.025, and the parameter for capital adjustment cost $\chi_K$ is set at 1.5. Calvo parameter $\theta$, which indicates the fraction of retail goods producers who can reset its sales price each period, is 0.25, implying on average a firm can reset the price once a year. The parameter for retailer’s degree of monopolistic power, $(\epsilon)$, is chosen so that steady state real markup is 1.1. Parameters for financial contract and banking sector are calibrated so that they imply a certain spread level between various kinds of interest rates in steady state. The steady state interest rate spreads (in annual terms) are shown in the table below. Those parameters also imply that in steady state the probability of default for entrepreneurs and household is 3.78% and 3.54%, respectively. The probability of default of the lending bank capital is set at 1.8%. Entrepreneurs’ capital-net worth ratio (K/W) in steady state is set at 2, and entrepreneur’s survival rate, $\nu$ is set at (1-0.272). They both are the same as in BGG’s original calibration. For macroprudential variable, steady state bank capital ratio $\kappa$ is set at 0.12, and LTV ratio is set at 0.70. Besides, autoregressive coefficients for exogenous processes were chosen to be $\rho_A = 0.9, \rho_H = 0.9, \rho_f = 0.8$ and $\rho_g = 0.9$. Finally, it is assumed that the financial riskiness shock in the household lending, $\hat{\epsilon}_f^H$, is assumed to have a correlation coefficient of 0.7 with the financial riskiness shock in the business lending, $(\hat{\epsilon}_f^B)$.

The table below displays calibrated steady state interest rates and variable ratios.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Deposit rate</td>
<td>1.0410</td>
</tr>
<tr>
<td>$R^I$</td>
<td>Interbank borrowing rate</td>
<td>$R + 0.002$</td>
</tr>
<tr>
<td>$R^{LH}$</td>
<td>Household borrowing rate</td>
<td>$R + 0.0343$</td>
</tr>
<tr>
<td>$R^K$</td>
<td>Return of capital</td>
<td>$R + 0.0664$</td>
</tr>
<tr>
<td>$R^{LB}$</td>
<td>Entrepreneur borrowing rate</td>
<td>$R + 0.0449$</td>
</tr>
<tr>
<td>$R^e$</td>
<td>Lending bank equity capital return</td>
<td>$R + 0.0785$</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>Consumption-output ratio</td>
<td>0.612</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>Investment-output ratio</td>
<td>0.152</td>
</tr>
<tr>
<td>$I^H/Y$</td>
<td>Housing investment-output ratio</td>
<td>0.048</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Government spending-output ratio</td>
<td>0.186</td>
</tr>
<tr>
<td>$K/W$</td>
<td>Entrepreneur’s capital-net worth ratio</td>
<td>2</td>
</tr>
<tr>
<td>$L^B/L$</td>
<td>Business lending-total lending ratio</td>
<td>0.636</td>
</tr>
<tr>
<td>$L^H/Y$</td>
<td>Household debt-output ratio</td>
<td>0.435</td>
</tr>
</tbody>
</table>

Note: all values are in real, annualized terms
4 Dynamics of the Model

In this section, I will discuss how variables in the model react to different kinds of shocks, assuming there is no macroprudential policy instrument. This model with monetary policy only will also serve as a benchmark when I discuss the effectiveness of macroprudential policy in the next section. For this baseline model, the Taylor rule parameters are chosen as $\phi_\pi = 1.5$, $\phi_Y = 0.3$ and $\rho_r = 0.8$. There are four shocks which are taken into considerations, monetary shock $\epsilon^r$, productivity shock $\epsilon^A$, financial riskiness shock $\epsilon^f$ and housing demand shock $\epsilon^H$. To analyze the model dynamics, I will present impulse response functions given one unit realization of each shock at time zero in figure 2-5.

Figure 2 shows impulse response functions given expansionary monetary shock, that is, one unit reduction in nominal interest rate by the central bank. As in usual New Keynesian model we can observe rise in output and inflation, as well as investment and consumption. We have greater increase in borrowing household’s consumption than saving household’s, as the former is more benefitted because of a reduction in debt-service burden. In fact saving household’s consumption drops after a few period and and stays negative until about period 20, and we can see there is indeed redistribution of wealth from saving household to borrowing household. What is counterintuitive among those impulse response functions is the credit dynamics. Empirical evidences suggest that the credit should increase given lower interest rate. However, in this model the bank asset increases initially, but falls immediately and stays negative for a quite while. Both components of bank asset, business and household lending, show the similar pattern. There are three possible explanations for this behavior. First, lower interest rate provides saving household less incentive to save, thus the less becomes the supply of credit. Second, while entrepreneur’s net worth improves following an expansionary monetary shock, investment does not increase as much for external financing to be necessary, because of the adjustment cost in investment. Indeed, if the adjustment cost parameter $\chi_K$ is set at zero, we can observe that the business lending increases. Finally, regarding the household lending, lower interest rate makes borrowing household less burdensome of its debt service. It has similar effect with providing borrowing household additional wealth, and some of them are used to repay the remaining debt. Because the credit decreases, leverage ratio of the entrepreneur and LTV ratio of borrowing household both fall, and the bank capital ratio rises.
Figure 3 displays impulse response functions given positive productivity shock. As usual, we can observe rise in output, consumption and investment, and fall in inflation. Housing price rises, because in this model housing supply is given exogenously and housing price is completely determined by its demand. Back to the household optimization problem, it can be understood that the housing demand must increase when the consumption increases and the marginal utility of consumption goods goes down. Contrast to the expansionary monetary shock, positive productivity shock increases both business and household lending. It results entrepreneur’s leverage ratio and household’s LTV ratio to rise and the bank capital ratio to fall, leaving the economy more leveraged in overall.

Financial riskiness shock is basically a shock to the spread between bank funding rate and bank lending rate. It influences financial accelerator relationship for both entrepreneur and household, as are shown in equation (28) and (31). Impulse response functions given positive financial riskiness shock, which increases the spread by one unit, are shown in figure 4. It is observed that output, investment decreases, and inflation rises following the shock. The bank asset decreases as both business lending and household lending get reduced, and the capital ratio rises, displaying more risk averse bank balance sheet structure against an adverse shock, just as what we can observe in periods of credit crunch in reality. Interestingly, consumption increases for a while and starts to fall eventually. The initial increase of consumption is mainly due to that of saving household, which is related with large and immediate reduction of credit demand (bank lending). Because this reduction in credit demand dominates the credit market in early periods, saving household who feels more difficult to save will rather choose to spend away now. However, eventually consumption should decrease as output, labor income and investment are kept below its steady state.

Figure 5 shows impulse response functions given one unit housing preference shock. The shock in housing preference is given so that it can cause housing price to fall. Impulse responses of output, inflation, investment or consumption are rather small, all ranging between ±0.05. We can observe that the decrease in housing preference induces housing price to fall, raising household’s LTV ratio and household lending rate, leading household lending to fall. However, business lending increases, albeit slightly, indicating that there is a substitution in bank lending portfolio from household lending to business lending. Reduced household lending improves the capital ratio of the bank, provides more favorable funding condition for the bank, and the bank can utilize this by expanding credit to the other sector. As a
result, business lending and investment rises initially. However, output and inflation falls as the decrease in borrower’s consumption outweighs.

5 The Effectiveness of Macroprudential Policy

As we have seen in section 2.8, macroprudential policy is assumed to be simple rules on bank capital requirement ratio or LTV ratio. They are designed to resemble the Taylor rule in its functional forms, that is:

$$\hat{\kappa}_t = \rho \hat{\kappa}_{t-1} + \phi_{\kappa} \hat{Y}_t + \phi_{L} \hat{L}_t + \phi_{P} \hat{P}_t$$  \hspace{1cm} (67)

and

$$\hat{lt}_t = \rho \hat{lt}_{t-1} + \phi_{\kappa} \hat{Y}_t + \phi_{L} \hat{L}_t + \phi_{P} \hat{P}_t$$  \hspace{1cm} (68)

For quantitative evaluation of the effectiveness of the macroprudential policy, we need welfare analysis on how agents’ lifetime utility changes given different combination of monetary and macroprudential policy. It requires perturbation solution method using higher order approximation of the utility function and the equilibrium condition of the economy. I will leave it for future research, instead I will suggest an alternative policy evaluation measure which can reflect the volatility of business cycle in simple way. Also qualitative features that come along with macroprudential policy will be explained.

Define the policy evaluation measure, $\sigma$, as the squared sum of impulses response functions during 40 periods (10 years) after the shock, in log-linearized terms.

$$\sigma_X = \sum_{i=0}^{40} \beta^i (\partial \hat{X}_{t+i}/\partial \epsilon_t)^2.$$  \hspace{1cm} (69)

By examining this measure for inflation and output, it is possible to figure out at least relatively the size of fluctuation in the business cycle. Those two variables are chosen because in basic New Keynesian setup the objective for policymaker is given as to minimize
loss function which is some linear combination of the inflation and output gap. Hereby I will evaluate \( \sigma_\pi \) and \( \sigma_Y \) given various macroprudential policy rules and different types of shocks. The set of shocks will be the same as the one in the previous section.

Table 2-5 displays \( \sigma_\pi \) and \( \sigma_Y \) for different macroprudential policy parameters. For each policy reaction parameter three different values are assigned, and those values are chosen by considering the volatility of variables that the policy reacts to. For example, in general housing price is more volatile than output, hence I assign smaller values for policy reaction coefficients on housing price. While varying values of one parameter, other parameters are assumed to be held at zero, except I allow for two mixed macroprudential policy, policy mix (a) \((\phi_Y^c = 2.5, \phi_L^c = 1)\) and policy mix (b) \((\phi_Y^c = 2.5, \phi_L^c = 1, \phi_{PH}^{ltv} = 0.1)\). In table 2, monetary policy is given as the same Taylor rule as in the baseline model, \( \phi_\pi = 1.5, \phi_Y = 0.3 \). Table 3 shows the policy gain (or loss) from macroprudential policy, as numbers indicate the decrement (or increment) of \( \sigma_\pi \) and \( \sigma_Y \) compared with the case where there is no macroprudential policy. Therefore, when there is reduction of \( \sigma_\pi \) or \( \sigma_Y \) with macroprudential policy, negative numbers will appear.

In general, we can observe that target capital requirement ratio rule reacting to output is effective in reducing the volatility of business cycle. In particular, when the response is stronger, \((\phi_Y^c = 5 \text{ or } 10)\) this rule can reduce \( \sigma_\pi \) and \( \sigma_Y \) given any kinds of shocks. When target capital ratio rule reactions to credit growth, \((\phi_L^c > 0)\), it can reduce \( \sigma_\pi \) for any kinds of shocks, but in terms of \( \sigma_Y \) it does not perform well when given shock is originated from housing demand. When housing price falls by a housing preference shock, household credit decreases, and due to the bank portfolio substitution there is an increase in business credit. However, because the former effect is more dominant, we observe a fall in total credit, and the macroprudential policy reacting to total credit will lower target capital requirement ratio. It results in further expansion in business credit, which in turn makes investment and output more volatile. Nevertheless, this rule is the best performing against financial riskiness shock \( \epsilon_f \), as it shows the greatest reduction in both \( \sigma_Y \) and \( \sigma_\pi \). The capital ratio rule reacting to housing price does not perform well, as it reduces \( \sigma_\pi \) and \( \sigma_Y \) given only productivity and financial riskiness shock. On the other hand, when given other types of shocks, policy loss from using this rule is much greater than other rules. It is possible to see that when policymaker is uncertain about the origination of the shock, capital ratio rule reacting to housing price is not the safest bet. Meanwhile, our first policy mix (a), mild reaction to both
output and credit \((\phi_Y^\kappa = 2.5, \phi_L^\kappa = 1)\), gives us policy gain given any types of shocks.

On the other hand, LTV regulation does not perform well with any types of rule specification. It gives us policy gain only given some specific shocks, and the policy loss given other types of shocks is rather large. Note LTV regulation is household lending-specific, thus this result can be interpreted that the universal regulation such as capital requirement ratio is better-performing tool than the market specific regulation such as LTV. With LTV regulation, although figures will not be presented here, it is possible to observe the substitution effect in bank portfolio corresponding to degree of regulation imposed on each credit market, as the credit in strongly regulated market just shifts away to less regulated market. This can produce undesirable consequences to policymakers, because they lose some of their ability to control aggregate credit. By the way, the best performance among those LTV rule is found from mild reaction to housing price \((\phi_{ltv}^P = 0.1)\) case, which is similar to the way current LTV regulation is operated in some countries. In this case, both \(\sigma_\pi\) and \(\sigma_Y\) are reduced given housing preference shock, and the magnitude of policy loss given other kinds of shocks is smaller than other rules. Policy mix (b), obtained by combining this mild LTV rule with policy mix (a), can give us reduction of both \(\sigma_\pi\) and \(\sigma_Y\) given any types of shocks. Moreover, the magnitude of reduction given housing preference shock becomes the largest. Therefore, it is conceivable that LTV regulation can still play a supportive role when it accompanies capital requirement policy, especially against housing market-oriented shock.

In order to see the qualitative consequences of introducing macroprudential policy, in figure 6-9, I compare impulse response functions from the baseline model and from the model with policy mix (b) type macroprudential policy. Impulse response functions from the baseline model are drawn using real lines, and those from macroprudential policy model are drawn using dashed lines. Figure 6 shows that there is little difference between two models when given shock is the monetary policy shock. In figure 7, when positive productivity shock hits the economy, it is more evident that the existence of the countercyclical macroprudential policy is influencing the economy’s dynamics. Because of the countercyclical operation of the macroprudential policy, bank capital ratio, which falls in the baseline case, now goes up and LTV, which goes up in the baseline case, falls. For the bank capital ratio to rise, both household and business credit should be repressed, and so become investment and output. We also can observe smaller volatility in inflation compared to the baseline case. However, we cannot regard these results as an evidence of welfare-improvement, as the consumption volatility at the earlier periods gets larger. This occurs because the macroprudential policy
represses credit, and it leads the saving household unable to save as much as the baseline case. Housing price is also more volatile, as housing demand dynamics resembles consumption dynamics. In figure 8, when there occurs adverse shock in financial market condition, the macroprudential policy can be helpful to stabilize the downturn by easing the bank capital requirement ratio and LTV. The reduction in both household and business credit becomes smaller, and it dampens the reduction in investment, output and rise in inflation. Consumption and housing price also show more stable dynamics. In figure 9, impulse response functions given housing demand shock are displayed. Housing price is still falling, but the reduction in household credit is much smaller now by the easing of the macroprudential policy. In result, we have less volatile business cycle, as the movements of investment, output, inflation and consumption are dampened.

What would happen to those results if monetary policy behavior is different? In order to answer this question, I repeated the same experiment given larger inflation coefficient \( \phi_\pi = 2.0 \), and zero output coefficient \( \phi_Y = 0 \). In other words, this monetary policy regime puts more weight in price stabilization, responding strictly only to inflation and more aggressively. Table 4 and 5 summarize the result. Again, numbers in table 5 show the increment (or decrement) of \( \sigma_\pi \) and \( \sigma_Y \) compared with the case where there is only monetary policy. We can observe similar pattern with the baseline monetary policy case, except that we cannot find any macroprudential policy which can reduce \( \sigma_\pi \) when given technology shock. On the other hand, the reduction of \( \sigma_Y \) becomes greater almost uniformly given any parametrization or given any type of shock, when our policy instrument is capital requirement ratio or policy mix (a) or (b). This result suggests that there can be a coordination problem between monetary and macroprudential policy, and optimal macroprudential policy might be different given different monetary policy rules. Therefore, it would be an interesting experiment to find the optimal combination of monetary and macroprudential policy, as well as optimal macroprudential policy given a particular monetary policy.

6 Conclusion

Analyzing the effects of macroprudential policy requires a model which can explain how credit and leverage interact with other real variables. I lay out a model economy where credit market is composed of household and business credit, bank capital functions as a buffer stock and macroprudential policy can affect bank capital ratio and LTV ratio in household lending.
Based on this model, I conducted several experiments to see which macroprudential policy rule can help stabilize the economy, using a policy evaluation measure in terms of inflation and output volatility given different shocks. It turns out the target capital ratio responding to output deviation performs the best, as it can reduce the volatility of inflation and output in most cases. Only one exception is the case when monetary policy is strictly disinflationary and given shock is the productivity shock, as there is no macroprudential policy that can reduce inflation volatility. This result suggests there can be a coordination problem between monetary and macroprudential policy. Target capital ratio responding to lending can also help stabilizing the economy, especially in case the shock is originated from the financial sector. LTV rule is in general not effective, since it is a regulation only pertaining to household lending and bank can just shift its lending to the entrepreneurs. Nevertheless, mild LTV rule reacting to housing price combined with capital ratio rule can be effective against housing demand shock. However, as mentioned in the previous section, it should be noted that the precise evaluation of welfare gain or loss from macroprudential policy requires higher order approximation of the utility function and the equilibrium condition, and I will leave it for future work. One last thing to think about is the possible conflict between macroprudential objective and microprudential objective, as macroprudential policy imposes another constraint in financial intermediary’s optimization behavior. Although it automatically keeps financial institutions from too much leveraged and increases protection against systemic risk, it may also worsen bank profitability because of that additional constraint. It would be an interesting subject studying how serious the latter effect would be to individual institutions’ stability.
References


Paolo Angelini, Stefano Neri, and Fabio Panetta. Monetary and macroprudential policies. Manuscript, Banca d’Italia, September 2010.


Appendix. 1. Figures and Tables

Figure 2: Impulse Response Functions: Expansionary Monetary Shock
Figure 3: Impulse Response Functions: Productivity Shock
Figure 4: Impulse Response Functions: Financial Riskiness Shock
Figure 5: Impulse Response Functions: Housing Preference Shock
Figure 6: The Effects of Macroprudential Policy: Expansionary Monetary Shock

Note: dashed lines show impulse response functions from the model with macroprudential policy, and real line show those from the baseline model.
Figure 7: The Effects of Macroprudential Policy: Productivity Shock

Note: dashed lines show impulse response functions from the model with macroprudential policy, and real line show those from the baseline model.
Figure 8: The Effects of Macroprudential Policy : Financial Riskiness Shock

Note: dashed lines show impulse response functions from the model with macroprudential policy, and real line show those from the baseline model.
Figure 9: The Effects of Macroprudential Policy: Housing Preference Shock

Note: dashed lines show impulse response functions from the model with macroprudential policy, and real line show those from the baseline model.
Table 2: The Effectiveness of Macroprudential Policy, with Baseline Monetary Policy: Policy Evaluation Measure

<table>
<thead>
<tr>
<th>Policy Evaluation Measure</th>
<th>$\sigma_Y$</th>
<th>$\sigma_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon^r$</td>
<td>$\epsilon^A$</td>
</tr>
<tr>
<td>Types of Shock</td>
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<tr>
<td>No Macroprudential</td>
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Note: $\sigma_Y = \sum_{i=0}^{40} \beta^i (\partial \hat{Y}_{t+i}/\partial \epsilon_t)^2$, $\sigma_\pi = \sum_{i=0}^{40} \beta^i (\partial \hat{\pi}_{t+i}/\partial \epsilon_t)^2$.

Taylor rule parameters chosen as $\rho_r = 0.8$, $\phi_\pi = 1.5$, $\phi_Y = 0.3$.

AR coefficients in macroprudential policy: $\rho_\kappa = 0.95$, $\rho_{ltv} = 0.8$. 


Table 3: The Effectiveness of Macroprudential Policy, with Baseline Monetary Policy: Gains

<table>
<thead>
<tr>
<th>Policy Evaluation Measure</th>
<th>Types of Shock</th>
<th>$\sigma_Y$</th>
<th>$\sigma_\pi$</th>
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</thead>
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</table>

Note: Numbers are differences from the baseline measure when macroprudential policy is nonexistent. Taylor rule parameters chosen as $\rho_r = 0.8, \phi_r = 1.5, \phi_Y = 0.3$. Non-positive values are highlighted. AR coefficients in macroprudential policy: $\rho_\kappa = 0.95, \rho_{ltv} = 0.8$. 
Table 4: The Effectiveness of Macroprudential Policy, with Hard Monetary Policy: Policy Evaluation Measure

<table>
<thead>
<tr>
<th>Policy Evaluation Measure</th>
<th>$\sigma_Y$</th>
<th>$\sigma_\pi$</th>
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<tbody>
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<td>Types of Shock</td>
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Note: $\sigma_Y = \sum_{i=0}^{40} \beta^i (\partial \hat{Y}_{t+i}/\partial \epsilon_t)^2$, $\sigma_\pi = \sum_{i=0}^{40} \beta^i (\partial \hat{\pi}_{t+i}/\partial \epsilon_t)^2$.

Taylor rule parameters chosen as $\rho_r = 0.8, \phi_\pi = 2, \phi_Y = 0$.

AR coefficients in macroprudential policy: $\rho_\kappa = 0.95, \rho_{ltv} = 0.8$. 


Table 5: The Effectiveness of Macroprudential Policy, with Hard Monetary Policy: Gains

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<tr>
<th>Policy Evaluation Measure</th>
<th>( \sigma_Y )</th>
<th>( \sigma_\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>( \epsilon^\lambda )</td>
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<tr>
<td>Types of Shock</td>
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<td>0.0000</td>
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<tr>
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</tr>
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<td>-1.0305</td>
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</tbody>
</table>

Note: Numbers are differences from the baseline measure when macroprudential policy is nonexistent. Taylor rule parameters chosen as \( \rho_r = 0.8, \phi_\pi = 2.0, \phi_Y = 0 \). Non-positive values are highlighted. AR coefficients in macroprudential policy: \( \rho_\kappa = 0.95, \rho_{ltv} = 0.8 \).
Appendix. 2. Log-linearizing Around Steady State

1. Household and Housing Goods Market

\[ \varphi \hat{N}_{t,j} = \hat{w}_t - \hat{C}_{t,j} \quad (70) \]

\[ \hat{C}_{t,s} = E_t(\hat{C}_{t+1,s} - \hat{R}_t^N + \hat{\pi}_{t+1}) \quad (71) \]

\[ \hat{C}_{t,s} = E_t(\hat{C}_{t+1,s} - \hat{R}_t^D) \quad (72) \]

\[ \hat{C}_{t,s} = E_t(\hat{C}_{t+1,s} - \hat{R}_t^L + \frac{\phi}{1-\phi} \hat{\phi}_{t+1}) \quad (73) \]

\[ \hat{C}_{t,b} = E_t(\hat{C}_{t+1,b} - \hat{R}_t^{LH}) \quad (74) \]

\[ \gamma \frac{P^H}{C_j}(\hat{P}^H_{t,j} - \hat{C}_{t,j}) = -(1 - \gamma) \hat{H}_{t,j} - \frac{\gamma}{H_j} \hat{I}^g_j + \beta \gamma (1 - \delta^H) \frac{P^H}{C_j} E_t[\hat{P}^H_{t+1,j} - \hat{C}_{t+1,j}] \quad (75) \]

\[ \hat{H}_{t,j} = (1 - \delta^H) \hat{H}_{t-1,j} + \delta^H \hat{I}^H_{t,j} \quad (76) \]

\[ \frac{I^H_s}{I^H} \hat{I}^H_{t,s} + (1 - \frac{I^H_s}{I^H}) \hat{I}^H_{t,b} = \hat{I}^H_t \quad (77) \]

2. Entrepreneur and Capital Goods Producer

\[ \hat{Y}_t = \hat{A}_t + \alpha_k \hat{K}_{t-1} + \alpha_n \hat{N}_t + \alpha_{ne} \hat{N}_t e_t + \alpha_{nf} \hat{N}_t f \quad (78) \]

\[ \hat{z}_t = \hat{m}c_t + \hat{Y}_t - \hat{K}_{t-1} \quad (79) \]

\[ \hat{w}_t = \hat{m}c_t + \hat{Y}_t - \hat{N}_t \quad (80) \]

\[ \hat{w}_{t,e} = \hat{m}c_t + \hat{Y}_t - \hat{N}_{t,e} \quad (81) \]
\[ \hat{w}_{t,f} = \hat{m}c_t + \hat{Y}_t - \hat{N}_{t,f} \]  
\[ R^K \hat{R}^K_t = z(\hat{z}_t - \hat{q}_t) + (1 - \delta)(\hat{q}_t - \hat{q}_{t-1}) \]  
\[ \hat{W}_t = v \left[ \frac{K}{W} (R^K \hat{R}^K_t - R^f \hat{R}^f_{t-1}) + R^f (\hat{R}^f_{t-1} + \hat{W}_{t-1}) \right] + \eta_t^W \]  
(\eta_t collects terms with second order importance)

\[ \hat{q}_t = \chi_K (\hat{I}_t - \hat{I}_{t-1}) \]  

3. Financial Contract

\[ \frac{L^B}{W} \hat{L}_t^B = \frac{K}{W} (\hat{q}_t + \hat{K}_t) - \hat{W}_t \]  
\[ l\hat{c}_v_t = \hat{q}_t + \hat{K}_t - \hat{W}_t \]  
\[ E_t \hat{R}^K_{t+1} = \hat{R}^f_t + \nu^B l\hat{c}_v_t + \hat{\varepsilon}_t^f \]  
\[ l\hat{v}_t = \hat{L}_t^H - \hat{P}_t^H - \hat{H}_{t,b} \]

\[ R^{LH} \hat{R}^{LH}_t = R^f (1 + ltv)\nu^H [\hat{R}^f_t + \hat{\varepsilon}^H_t] + \nu^H R^f (1 + ltv)\nu^H - 1 ltv l\hat{v}_t + \nu^H \nu^H (ltv)(l\hat{v}_t - l\hat{v}_t) \]  
\[ \hat{L}_t = \frac{L^B}{L} \hat{L}_t^B + (1 - \frac{L^B}{L}) \hat{L}_t^H \]  
\[ \hat{\kappa}_t = \hat{\varepsilon}_t - \hat{\kappa}_t \]  
\[ \hat{L}_t = (1 - \kappa) \hat{D}_t + \kappa \hat{\varepsilon}_t \]  
\[ R^d \hat{R}^d_t = R^d \hat{R}^d_t + \nu^d \nu^d (\hat{\kappa}_t - \hat{\kappa}_t) \]
\[ R^f \hat{R}^f_t = \kappa R^e(\hat{R}^e_t) + (1 - \kappa)R^t \hat{R}^t_t + \kappa(R^e - R^t)\hat{k}_t \] (95)

\[
\hat{e}_t = (1 - \phi)\hat{e}_{t-1} - \phi \hat{\phi}_t + \mu \frac{R^{L_B} L^B}{e} \tilde{\omega}^d f(\tilde{\omega}^d)(R^K_t - E_{t-1} R^K_t) \\
+ \mu \frac{R^{L_H} L^H}{e} \tilde{\omega}^a f(\tilde{\omega}^a)(P^H_t - E_{t-1} P^H_t) + \eta_t^e
\] (96)

\[
\hat{\phi}_t = F'(\kappa)\hat{k}_{t-1} + \epsilon_t^\phi
\] (97)

4. Retail Sector

\[
\sum_{k=0}^{\infty} (\beta \theta)^k \hat{P}_t^* = \sum_{k=0}^{\infty} (\beta \theta)^k \hat{M}C_{t+k} \Rightarrow \hat{P}_t^* = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \hat{M}C_{t+k}
\] (98)

\[
\hat{P}_t = \beta \hat{P}_{t-1} + (1 - \theta)\hat{P}_t^* \Rightarrow E_t \hat{\pi}_{t+1} = \theta \hat{\pi}_t + (1 - \theta)(E_t \hat{P}_{t+1}^* - \hat{P}_t^*)
\] (100)

\[
E_t \hat{\pi}_{t+1} = \theta \hat{\pi}_t + (1 - \theta)(E_t \hat{P}_{t+1}^* - \hat{P}_t^*)
\] (101)

\[
E_t \hat{P}_{t+1}^* - \hat{P}_t^* = (1 - \beta \theta)E_t \hat{P}_{t+1}^* - (1 - \beta \theta)[\hat{m}c_t + \hat{P}_t]
\] (102)

\[
E_t \hat{\pi}_{t+1} = \theta \hat{\pi}_t - (1 - \beta \theta)(1 - \theta)\hat{m}c_t - (1 - \beta \theta)(1 - \theta)\hat{P}_t + (1 - \beta \theta)(E_t \hat{P}_{t+1} - \theta \hat{P}_t)
\] (103)

\[
\beta E_t \hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{m}c_t
\] (104)

5. Market Clearing Conditions

\[
\hat{C}_t^\epsilon = \hat{V}_t
\] (105)
\[ \hat{C}_t^f = \hat{\phi}_{t-1} + \hat{e}_{t-1} \]  

\[ Y\hat{Y}_t = C_s \hat{C}_{t,s} + C_b \hat{C}_{t,b} + \bar{I}(\hat{I}_t + \hat{q}_t) + I^H(\hat{I}_t^H) + \eta_Y \]  

6. Monetary Policy

\[ \hat{R}_t^N = \rho_r \hat{R}_{t-1}^N + \rho_{x\pi} \hat{\pi}_t + \rho_{Y\hat{Y}_t} \hat{Y}_t + \hat{e}_t^r \]