Housing Demand Shock and the Implication of Monetary Policy - A Small Open Economy Case. *

Shu-Ling Wang†

April, 15, 2011

Abstract

The paper develops a two-agent small-open economy DSGE model, to address the issue about how does the overall macro-economy respond to the fundamental housing demand shock, how does the current account has changed conjunction to the shock, and what is the monetary policy transmission to the real economy.

Key words: Housing demand shock, housing investment, small open economy, policy experiments.

*This is the first version of my 3rd-year paper. I could like to thank Prof. Edward Buffie, Gerhard Glomm, Todd Walker, Eric Leeper and Bulent Guler at Indiana University for many helpful comments and suggestion. All the errors are mine.

†Indiana University, Bloomington. Email: sw41@indiana.edu
I Introduction

According to IMF (2009) housing study, real estate prices in some Asian economies are rebounding quickly. Housing prices and transactions volumes recovered in the second half of 2009 from their 2008 downturn in eastern Asian economies such as Mainland China, Taiwan, Hong Kong, Singapore, and South Korean. The housing prices in major metropolitan areas exceeded their 2008 peaks. Form the following housing index trend figure, we can see the real housing price in those economies steadily increase over a long period.

Three reasons result in this trend. First, the unprecedented low policy rate conjunct to the U.S financial crisis from 2007 to 2009 decreases the borrowing cost of housing investment. Second, housing-related tax initiatives in late 2008 to revive domestic real estate markets also strengthen this trend. Third, which is also the main point in this paper, capital inflows due to U.S financial crises have brought an influx of buyers from other countries, especially those who aim at the housing market as an investment subjective. Take Taiwan an an example, a huge number of housing investors have participated in Taiwanese housing market transaction especially in luxury apartment, causing the housing boom housing inflation. Those investors mostly come from Mainland China after the ECFA\(^1\), which enforces the run-up of the housing price.

\(^1\)ECFA, is an trade negotiation between ROC (Taiwan) and PRC (Mainland China) that aims to reduce tariffs and commercial barriers is effective.
Whether the surge in housing price is owing to over-optimism psychology, which is so-called non-fundamental shock that induces housing price bubble and thus threatens the financial soundness, or the run-up of price purely comes from fundamental shock, has not arrived consensus since it is hard to recognize whether bubble has formed. However, the central banks have been alert to the the prevailing upward trend in housing prices since the housing price fluctuation will affect the overall macroeconomic stability due to a large portion of income spent on housing expenditure on one hand, and the housing value affects the credit capacity and thus affect the overall consumption as well as inflation rate, on the other hand.

In this paper, I work on a small open economy model with flexible exchange rate regime, which confronts the fundamental housing demand shock from the international investors. The main focus is to investigate how does the shock affect the overall economy and how do the different policies stabilize the economy. The main differences between the existing model from the recent literature are twofold. First, comparing to the international literature, traditional literature focuses on the shock of international interest rate or the export shock. However, in this paper, the demand shock comes from the foreign investors’ demand shock in domestic housing who only care about maximizing the present value of the wealth but neither care the consumption nor care about how much housing service they enjoy since they do not occupy it. To best of my knowledge, no literature addresses this issue. Second, comparing to the closed economy housing literature, there are also two agents in this model (domestic households and the foreigner), but only the households enjoys housing services as well as consumption, the foreigner just determines how much to invest to maximize wealth with a housing demand shock. Due to this setting, foreigner’s behaviour may bring heavy impact on the overall economy, which is worth of noting.

The main question in this paper are three. First, how does the overall economy respond to the housing investment demand shock and what is the transmission mechanism. Second, whether the shock worsens or benefits the current account. Third, how do the different policies stabilize the overall economy. The following are three policy experiments implemented by the government, which will provide some insights to this model and policy analysis.

1 The central bank only targets current composite goods inflation. In this experiment, we can compare aggressive Taylor rule with non-aggressive one.

2 The central bank not only targets composite goods inflation as well as target the the change of nominal exchange rate.

Since the interest rate parity condition implies that the larger differential between the domestic and foreign nominal interest rate will build expectation of currency depreciation, the domestic currency is going to appreciate today. That self-fulling effects may hurt the high-exporting dependency economy and thus the central bank also targets exchange rate if necessary. However, we should notice that the change of exchange rate is implicitly incorporated into the composite goods inflation in this model\(^2\), but we can ask whether putting more weights on targeting nominal exchange rate will bring more satisfactory effects. In addition, we can also play with the policy experiments of targeting non-traded goods price inflation to see different results.

\(^2\)This is also the characteristics when we work on the non-trade and traded model.
3 The central bank only targets composite goods price inflation in the Taylor rule, and the fiscal authority levies housing investment taxation to mitigate the demand shock. Here, I assume the fiscal authority cannot distinguish whether the housing investor is a foreigner or the domestic households\(^3\). In order to help control the inflation, the fiscal policy levies investment taxation to both agents.

Three points are worth of being mentioned. First, in practice, many central banks target CPI, which includes the housing rents as well as the consumption goods price. However, in this existing model, I leave the housing aside rent expenditure for simplicity. That is the reason why the inflation defined by this paper is not the CPI, but just the composite goods inflation. Second, a great body of housing boom-bust literature recently addresses this issue from the "credit channel" point of view, emphasizing the importance of the credit capacity is tied to the asset value. However, since the loan-to-value ratio is relatively low in Asian small open economy, I will not address this issue in this paper. Third, in the policy experiments, since I assume the housing producer is like a aggregator which produce house with the bundle of traded and non-traded goods, and thus the price of real housing is just the combination between those two prices. Thus, when we target the composite price inflation, it also targets the real housing price in some sense.

II Literature Review

Since the U.S financial crisis from 2007, the housing mortgage problem has become popular in the literature. As the data reveals, housing expenditure is crucial in wealth and thus the value affects the business cycle a lot. The consensus of how the housing affects the macro-economy through two channels according to Boivin et al(2010) - neoclassical and non-neoclassical channel. Neoclassical channel refers to that central bank adjust short-term nominal interest rate to affect the real interest rate and thus affects the business/residential investment through user's costs facets, while affects consumption through intertemporal substitution effects or through life-cycle wealth effects\(^4\), and affects the trade through exchange rate since the loosen interest rates policy lowers the domestic assets relative to foreign assets, and thus the domestic currency will depreciate, which will stimulate the export and thus increase the aggregate demand.

As for the non-neoclassical channel, it involves with the financial liberalization a lot and thus relates to the asymmetric information problem. If we focus on households facet, the main channel which monetary policy conveys is through balance-sheet channel. When the households face some credits constrain which is tied to the housing value (since borrowing is secured by the housing value), the run-up of housing price will affect the borrowing ability and also increase their consumption, which will further affect the inflation and the underlying variables, which is so-called boom - bust cycle. Under international framework, if the households face the credit constrain which involves with the foreign-currency denominated assets, then not only the domestic monetary policy will affect the asset value and thus affects the borrowing capacity of the households, but the fluctuation in exchange rate

\(^3\)We can imagine that the domestic households can invest housing in favour of the foreign investor.

\(^4\)However, wealth effect plays an important role in modelling effects, but plays a secondary role to real interest rate channels from empirical work
can also determine the credit capacity.

Knowing how the monetary policy affects the macro economy, we turn our focus on the housing-related literature, to investigate the link monetary policy to housing demand shock. Prevailing literature mostly follows Iacoviello (2005), focusing on how the financial accelerator mechanism works under a DSGE framework with financial frictions, and ask how the monetary policy affects the real economy.\(^5\) Those models start with two types of agents with different discount rates, an entrepreneur who face credit constraint, and retailers under monopolistic competition. They conclude that the increase in housing demand will increases consumption, output, and inflation rate through the credit channel. Moreover, it implements policy experiments and conclude that Taylor rule functions well whereas targeting the asset price directly gains little benefit. The later work such as Iacoviello and Neri (2009) and the Kannan et al (2009) borrow basic elements from BGG (1999) and Iacoviello (2005) when addressing macro-prudential policy issue. They conclude the central bank ought to target credit directly, since it will serve as a role to counteract the loosen lending conditions when the asset price goes up, to stabilize the macro economy.

It is not hard to find that most of the recent housing literature confines to the closed economy framework. To some extent, we should admit that open economies will face more challenging problems since exchange rate channel has provided another channel to affect the real economy and thus interest rate policy should be used with scrutiny. As an extended work for housing literature with credit friction, Funke and Paets (2010) studies Hong Kong’s housing market; they argue the housing preference shocks can largely explain boom-bust cycle owning to financial liberalization such as higher LTV ratio. One different point of view comes from Aspachs-Braconsy and Rabanalz (2010). In this paper, the authors compare different shocks in Spain, and concludes that the labour markets rigidities provide stronger amplification than that from financial friction. They also show that the non-durable sector suffers from large contraction when the central bank attempts to stabilize the housing prices and thus it remains dispute whether the boom-bust cycle can be avoided by adopting monetary policy. Another popular issue under discussion relates to housing (durable goods) and current account imbalances. Punzi (2006) concludes that higher financial liberalization help households to relax the credit constraint and thus generate the boom-bust cycle, also worsens current account. Also, some papers make effort on explaining stylized facts. Buffie and Atolia (2009) is an example, which introduces durable goods consumption to explain the boom-bust cycle as well as the stylized facts during the ERBS program in the small open economy under crawling-pegged regime. They find that adding durable goods consumption can explain the stylized facts qualitatively and quantitatively under weak credibility assumption.

As for the policy issue, even since Taylor (1993) seminar paper, many countries started to adopt inflation-targeting (IT) policy by adjusting nominal interest rates. This kind of policy has three main features: credibility, and timely, and thus it is highly-accepted policy rule. However, except for the inflation targeting, the so-called augmented Taylor Rule (ATR) is also popular in practice. For example, targeting exchange rate fluctuation might be a choice for the central bank. The reason for implementation of augmented Taylor rule lie in, in practice, the central bank targets CPI inflation, it may not be sufficient to stabilize the exchange rate since the larger difference between domestic

---

\(^5\)The credit view is used under general equilibrium model by Bernanke and Gertler (1989), but they modelled the financial accelerator mechanism in firm’s level, which enhances the amplitude of busyness cycle.
and international nominal interest rate will bring large capital inflows or outflows, which affects the overall stability, and hence there exists a trade-off to open economy when the central bank implements Taylor rule. Especially in many export-dependency small open economy, the central bank also makes effort on stabilizing the exchange rate fluctuation, no matter the fluctuation resources come from the world-trend fluctuation of from the economy. When it comes to targeting asset price, most of the theoretical work oppose central bank to intervene housing price except that is can recognize the price bubble. However, when the housing service enters to agents’ utility function, central bank may target asset prices since the agent is eager to smooth housing consumption, which is one of the policy experiment in Iacoviello (2005).

III Model

I will lay out a DSGE model under small open economy framework which is composed of a representative household, perfectly monopolistic firms which is normalized to one, fiscal and monetary authority.

Demand Side

III.1 Domestic Households

\[
\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \kappa \frac{S_{t+1}^{1-\sigma}}{1-\sigma} - \chi \left( \frac{L_N^t + L_T^t}{1+\nu} \right) \right)
\]

subject to the nominal budget constraint:

\[
P_t C_t + (1 + \tau)Q_t I_t + R_t(S_{t+1} - H_{t+1}) + P_t T_t + i_{t-1} B_t + e_t B_t^* t_{t-1}^*
\]

\[
= B_{t+1} + e_t B_{t+1}^* - P_t \left( \frac{e_t B_{t+1}^*}{P_t} \right)^2 + W_N^t L_N^t + W_T^t T_T^t + \Pi_N^t + \Pi_T^t
\]

where \( S_{t+1} \) is the flow of housing service which provides utility to the households, we assume the housing stock convert to housing service one-to-one with free cost. \( H_{t+1} \) is the housing stock owned by the domestic households at time \( t \) while \( I_t \) is his/her housing investment. \( Q_t \) and \( P_t \) stand for the prices of housing investment and composite consumption goods, respectively. In this model, households can rent housing in the market with the nominal rent \( R_t \) when the agent is eager to enjoy more housing service than the owned-occupying amount. \( \tau \) is the tax rate levies on the housing investment; I assume \( \tau = 0 \) in the baseline model. If \( S_{t+1} = H_{t+1} \), the model turns out to be the special case as recent literature has modelled, which excludes the rent market. Households provide labour to both the traded and non-trade sectors in a perfectly competitive labour market with nominal wage \( W_N^t, W_T^t \), and the total supply is \( L_t \). \( \Pi_T^t \) and \( \Pi_N^t \) denote the nominal profit from the traded and non-traded goods sectors, which will rebate to the domestic households in lump-sum terms since the firms are owned by the domestic agent.
In addition, \( C_t \) denotes the composite goods which is composed of a bundle of traded and non-traded goods, which satisfies the following relationship:

\[
C_t = \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} (c_N)^\gamma (c_T)^{1-\gamma}
\]  \hspace{1cm} (3)

where \( c_N \) and \( c_T \) denote the non-traded and traded goods. The composite price level is defined as the geometric weighted average between the price of non-trade goods \( p_t \) as well as the exchange rate \( e_t \):

\[
P_t = e_t^\gamma p_t^{1-\gamma}
\]  \hspace{1cm} (4)

where \( \gamma \) denotes the expenditure share spent on traded goods out of the total consumption expenditure. We can regard this optimization problem as a two-stage problem. In the first stage, the household minimizes the total expenditure \( E_t \) on \( C_t \) subject to \( P_tC_t = p_t c_N + e_t c_T \); in the second stage, the household allocate expenditure to both traded and non-traded goods, taking the price as given. As for the financial assets for the domestic households, the agent either hold nominal domestic bonds \( B_t \) or nominal foreign bonds \( B_t^* \) with gross risk-free return \( i_t \) and \( i_t^* \). To avoid the unit-root problem, I introduce a transaction cost for holding foreign bonds according to Uribe (2002) owing to technique concern.

Now, rewrite the nominal budget constraint in terms of composite consumption goods, then the real budget constraints reads:

\[
C_t + (1 + \tau) q_t I_t + r_t (S_{t+1} - H_{t+1}) + T_t + \frac{i_{t-1} B_t}{P_t} + \frac{e_t B_t^*}{P_t} - \frac{e_t B_t^{*2}}{P_t} = \frac{B_{t+1}}{P_t} + \frac{e_t B_t^{*2}}{P_t} - \frac{\psi}{2} \left( \frac{e_t B_{t+1}}{P_t} \right)^2 + \frac{w_t L_t^N}{P_t} + \frac{w_t^T L_t^T}{P_t} + \frac{\pi_t^N}{P_t} + \frac{\pi_t^T}{P_t}
\]  \hspace{1cm} (5)

where \( q_t \) is the real price of housing investment, \( r_t \) is the real rent, and \( w_t, w_t^T, \pi_t^N, \pi_t^T \) denotes the real wage and real profits in terms of composite consumption goods from each sector.

The law of motion for housing investment is:

\[
H_{t+1} = H_t (1 - \delta) + I_t - S \left( \frac{I_t}{I_{t-1}} \right) I_{t-1}
\]  \hspace{1cm} (6)

where \( S \left( \frac{I_t}{I_{t-1}} \right) I_{t-1} \) denotes the convex adjustment cost function. Put differently, a unit of housing investment cannot convert to one increment of housing stock since some adjustment cost will incur. Also, the size of adjustment cost has a negative relationship with how much of investment in the last period of time. The reason why to assume this type the adjustment costs are twofold. On the one hand, we should prevent from ”bang-bang solution”; on the other hand, we hope to display the hump-shaped investment trend, which is supported by the empirical data. Also, assume \( S() = S'(()) = 0 \) under steady state and \( S''() > 0 \).

\[^6\text{In this paper, the composite price level is not the CPI, since CPI incorporates the price of housing services} R_t. \text{However, I exclude the price for housing services, and thus } P_t \text{ is not CPI.}\]

\[^7\text{For the derivation of aggregate price level, please see the appendix for details.}\]
The choice variable for the households are composite goods $C_t$, housing services $S_{t+1}$, housing stock $H_{t+1}$, housing investment $I_t$, labour supply $L^N_t$ and $L^T_t$ to non-traded and traded sector, as well as the domestic or foreign bonds holdings $B_{t+1}$ and $B^*_t+1$. The necessary first-order conditions for domestic households, with Lagrangian multiplier $\lambda_t$ associated with the flow of the budget constraint(5) and $\mu_t$ associated with the law of motion for housing investment, are:

\[ S_{t+1} : \quad C^{-\sigma}_t r_t = \kappa S^{-\sigma}_{t+1} \]  
\[ H_{t+1} : \quad C^{-\sigma}_t r_t = \mu_t - \beta (1 - \delta) E_t \mu_{t+1} \]  
\[ I_t : \quad -C^{-\sigma}_t q_t (1 + \tau) + \mu_t = \mu_t S'(I_t \frac{I_{t+1}}{I_t}) + \beta E_t \mu_{t+1} S(I_{t+1}) - \beta E_t \mu_{t+1} S'(I_{t+1} \frac{I_{t+1}}{I_t}) \]  
\[ L^T_t : \quad \chi L^T_t = C^{-\sigma}_t w^T_t \]  
\[ L^N_t : \quad \chi L^N_t = C^{-\sigma}_t w^N_t \]  
\[ B_{t+1} : \quad C^{-\sigma}_t = \beta i_t E_t \frac{C^{-\sigma}_{t+1}}{\pi_{t+1}} \]  
\[ B^*_t+1 : \quad C^{-\sigma}_t (1 - \psi B^*_t(\frac{\epsilon_t}{P_t})) = \beta i^*_t E_t C^{-\sigma}_{t+1} \frac{\epsilon_{t+1}}{\pi_{t+1}} \]  

where $\epsilon_{t+1} = \frac{\epsilon_{t+1}}{\epsilon_t}$, denoting the gross exchange rate.

We can regard it (9) from Tobin’s q theory. Since housing is owned by the households, the households will determine whether to increase or decrease investment according to (9). $q_t$ is the real price for housing investment, $\mu_t$ is the shadow price of housing stock. The net gain of increase housing investment today can separate into two. First, the net asset value in terms of marginal utility of consumption; second, the increase in housing investment today will decrease the adjustment cost tomorrow, and thus this gain will appear in the first-order condition in a discounted term.

In addition, (12) is the standard consumption Euler’s equation. Combining (12) and (13), we can get the modified interest rate parity condition\(^8\).

\(^8\)Here, I call it as modified interest rate parity condition because I introduce an extra transaction costs when holding the foreign bonds. If there is no transaction cost, (12) + (13) reduces to the standard interest rate parity condition.
III.2 Foreign Investor

For a wealthy foreign investor, I assume this investor owns the housing $H_{t+1}'$ in this economy but does not occupy it. The only thing this investor does is to maximize his present value of wealth in this small open economy by participating housing transactions. The discount rate is $\beta'$, and there is an AR(1) shock as the housing demand shock incurred to foreigner’s discount factor such that the foreign investor increases housing investment, which can be regarded as the preference shock. I assume $\hat{j}_t = \rho \hat{j}_{t-1} + \eta_t$, where $\eta_t$ is iid and mean zero. Moreover, those investors are free from credit friction since they are self-financed.

$$\max E_t \sum_{t=0}^{\infty} \beta'^t j_t (r_t H_{t+1}' - (1 + \tau)q_t(I_t'))$$

(14)

Also, the low of motion for the foreigner is identical to that for domestic households:

$$H_{t+1}' = H_t'(1 - \delta) + I_t' - S(I_t'/I_{t-1})I_{t-1}'$$

(15)

, where $H_{t+1}'$ denotes the housing stock owned by the foreign investor, $I_t'$ denotes foreigner’s housing investment, and $z_t$ is the lagrangian multiplier associated with the law of motion of investment $I_t'$.

The above-equation implies that this foreign agent maximizes the present value of wealth by renting the house to the domestic households with the real return $r_t$ as well as the housing value subject to the law of motion with a quadratic adjustment cost. The same, $\tau$ denotes the housing investment taxation; when $\tau = 0$, it means fiscal authority does not help control the inflation.

The first-order condition is:

$$H_{t+1}': \quad j_t r_t = j_t z_t - \beta' (1 - \delta) E_t j_{t+1} z_{t+1}$$

(16)

$$I_t: \quad j_t ((-q_t(1 + \tau) + z_t) = j_t z_t S(\frac{I_t'}{I_{t-1}'}) + \beta' E_t \hat{z}_{t+1} S(I_{t+1}'/ I_t') j_{t+1} - \beta' E_t \hat{z}_{t+1} S(I_{t+1}'/ I_t') j_{t+1}$$

(17)
Supply Side

III.3 Non-traded sector

Assume there is a representative final goods producer act as a aggregate who chooses how many inputs it will use from the intermediate firms and the amount it plans to produce, implementing the following Dixit-Stigliz constant return to scale technology. The optimization problem for the final goods producer chooses output $y_t^N$ as well as the input $y_t, i$ it will use as:

$$\max p_t y_t - \int_{i=0}^{1} p_t(i) y_t(i) di$$

$$\left[\int_{0}^{1} y_t(i) \frac{i-1}{i} \right]^{\frac{-1}{i-1}} \geq y_t^N$$

Then we can get the relationship between the input and output as\(^9\):

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\epsilon} y_t$$

As for the intermediate firms, there are infinite (continuum) number of monopolistic competitive firms which produce differential goods $y_t^N$, and the number of firms is normalized to one. Moreover, I assume the labour market is perfectly competitive and thus all the firms pay the same wage to the labour provided from the households. Since the intermediate firms are monopolistic competitive, firms have market power to set the product’s price. In addition, I introduce the price-setting friction according to Calvo-type setup only in the non-traded sector. Moreover, since for the intermediate firms, output is demand-driven, intermediate firms should take the aggregate demand as given and decide the amount to produce.

The production function for each intermediate firm is:

$$y_{t,i}^N = A_t l_{t,i}$$

where $A_t$ is the economy-wide technology parameter, and

$$L_t = \int_{i=0}^{1} l_{i} di$$

For each intermediate firm $i$, the objective reads:

$$\max \Pi_{t}^N = E_t \sum_{k=0}^{\infty} (\beta \theta)^k \frac{N_{t+k} y_{t+k}}{\lambda_t} (p_{i,t} - MC_{t+k}) y_{i,t+k}$$

\(^9\)For derivation details, please see the appendix.
where $MC_t$ is the real nominal marginal cost in terms of non-traded goods. Assume that the equilibrium for the intermediate firms is symmetric and thus

$$y_t^N = A_t L_t^N$$

From firm’s cost-minimize problem, real marginal cost in terms of non-traded goods $MC_t = \frac{W_t^N}{\bar{p}_t A_t}$, and $\theta$ stands for the probability that the firms remain their current prices. $\beta^\lambda_{t+k}$ stands for domestic agents’ nominal discount factor between period $k$ and $t+k$ since the non-traded firms are owned by the households.

The optimal behaviour from the intermediate and the final goods firm give us the following domestic New-Kaynesian Phillips Curve (NKPC, henceforth) equation in log-linearized form:

$$\dot{\pi}_t^* = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} MC_t + \beta E_t \pi_{t+1}^*$$

where $\pi_t^* = \dot{p}_t - p_{t-1}^*$ is the gross inflation rate in non-traded sector, and I assume $\pi^* = 1$ under the steady state. If we exploit the relationship among $P_t, p_t$, and $e_t$ in (4), $\pi_t^*$ can be expressed in terms of $\pi_t$ and $\epsilon_t$. Then, (25) can be rewritten as

$$\frac{\dot{\pi}_t}{1 - \gamma} - \frac{\gamma \epsilon_t}{1 - \gamma} = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \left( w_t^N + \frac{\gamma}{1 - \gamma} \left( \frac{\dot{e}_t}{P_t} - \dot{A}_t \right) \right) + \beta E_t \left( \frac{\pi_t}{1 - \gamma} - \frac{\gamma \epsilon_{t+1}}{1 - \gamma} \right)$$

III.4 Traded Sector

For traded sector, since it’s perfectly competitive and I assume the price of traded-goods is normalized to unity in dollars and thus the traded-goods price is just the nominal exchange rate $e_t$. The production function is:

$$y_t^T = A_t L_t^T$$

and thus the objective function is:

$$\max \Pi_t^T = e_t y_t^T - W_t^T L_t^T$$

The first-order condition is:

$$L_t^T : e_t A_t = W_t^T$$
III.5 Housing Construction sector

Assume housing construction sector is also perfectly competitive. According to Buffie and Atolia (2009) working paper, for the housing investment (durable goods), we can assume the housing investment per unit is produced by one unit of traded goods and $a_1$ units of non-traded goods. According to Burstein et al. (2001), non-traded durables can be regarded as distribution costs associated with bringing imported durables to the retail market.\(^\text{10}\) Therefore, the nominal housing investment price is:

$$Q = e_t + p_t a_1 \quad (30)$$

Rewrite (30) as the real term, we get:

$$q_t = \frac{e_t}{P_t} + \frac{p_t}{P_t} a_1 \quad (31)$$

III.6 Government

Since 1990s, Taylor rule has been prevailing in many central banks. In practice, most of the central banks stabilize CPI inflation, but also some central banks target exchange rate change, which is so-called augmented Taylor rule (ATR). However, whether the asset (housing) asset should be incorporated directly into Taylor rule remains an open question as it is mentioned before.

But in the existing model, I leave aside the academic debate for a while, since I assume the housing construction sector act as a aggregator which packages non-traded and traded goods into housing investment goods, the price is indeed a combination between non-trade and traded goods prices. Thus, when the central bank targets the composite consumption price level, it implicitly targets the housing price as well. But trying to put different weights on each target to see different effects will be another idea to work on. Also, we can make some artificial prices index such as non-traded goods price level and see the different combinations between targeting non-traded goods price level as well as the asset price. Also, I will see whether the cooperation between fiscal and monetary policy is able to bring more satisfactory effects.

Monetary Policy I

The central bank adopts simple Taylor rule which simply targets composite goods inflation, which is denoted as $\pi_t$. I set $\alpha_\pi$ equals to 1.5 or 3.

$$\hat{i}_t = \alpha_\pi \hat{\pi}_t \quad (32)$$

\(^{10}\)Please see Buffie and Atolia (2009)\(^\text{”Exchange-Rate-Based Stabilization, Durables Consumption, and the Stylized Facts”}\)
Monetary Policy II

The central bank implements augmented Taylor rule, which also targets the exchange rate.

\[ \hat{i}_t = \alpha_\pi \hat{\pi}_t + \alpha_\epsilon \hat{\epsilon}_t \] (33)

Fiscal Policy I/II

The fiscal authority only remains its budget constraint balanced:

\[ G_t = T_t \] (34)

where \( G_t \) and \( T_t \) stands for exogenous real government spending and lump-sum taxation. Lump-sum tax is always passively to meet fiscal adjustment under different monetary policy rule. I also assume government does not hold bonds for simplicity.

Monetary Policy III

\[ \hat{i}_t = \alpha_\pi \hat{\pi}_t \] (35)

In the third policy experiment, I assume that the central bank targets the simple Taylor rule, but the fiscal policy levies housing investment taxation to both agents.

Fiscal Policy III

\[ G_t = T_t + q_t (I_t + \dot{I}_t) \tau \] (36)
Closing the Model

We also need several conditions to close the model:

Domestic bond market:

\[ B_t = 0 \]  

(37)

The total housing stock:

\[ BH_{t+1} = H_{t+1} + H'_{t+1} \]  

(38)

The total housing service is enjoyed by the domestic household:

\[ S_{t+1} = H_{t+1} + H'_{t+1} \]  

(39)

Labour Market:

\[ L_t = L^N_t + L^T_t \]  

(40)

Non-traded goods market clearing condition:

\[ c^N_t + a_1(I_t + I'_t) = y^N_t \]  

(41)

Balance of payments (assume \( \tau = 0 \) here.\(^{11}\):

\[ \gamma C_t + \frac{e_t}{P_t} I_t + r_t H'_{t+1} + \frac{e_t B^*_t I'_{t-1}}{P_t} = \frac{e_t}{P_t} y^T_t + \frac{p_t}{P_t} a_1 I'_t + \frac{e_t B^*_t}{P_t} - \frac{\psi}{2} \left( \frac{e_t B^*_t}{P_t} \right)^2 \]  

(42)

The current account equations is defined as the change in net foreign assets under the floating exchange rate system.

Finally, we also need law of motion for each investment:

\[ H_{t+1} = H_t(1 - \delta) + I_t - S(\frac{I_t}{I_{t-1}})I_{t-1} \]  

(43)

\[ H'_{t+1} = H'_t(1 - \delta) + I'_t - S(\frac{I'_t}{I'_{t-1}})I'_{t-1} \]  

(44)

\(^{11}\)Please see the attachment for details.
Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor of the domestic households</td>
<td>$\beta$</td>
<td>0.98</td>
</tr>
<tr>
<td>Subjective discount factor of the foreign investor</td>
<td>$\beta'$</td>
<td>0.98</td>
</tr>
<tr>
<td>Baseline inverse Frisch labour elasticity</td>
<td>$\frac{1}{\nu}$</td>
<td>1</td>
</tr>
<tr>
<td>Consumption substitution between non-traded and traded goods</td>
<td>$\sigma$</td>
<td>1</td>
</tr>
<tr>
<td>Share of expenditure in housing services</td>
<td>$\kappa$</td>
<td>0.25</td>
</tr>
<tr>
<td>Scale parameter conjunct to labour supply when $\tau = 0$</td>
<td>$\chi$</td>
<td>3.14</td>
</tr>
<tr>
<td>Adjustment cost parameter in foreign bonds</td>
<td>$\psi$</td>
<td>0.019</td>
</tr>
<tr>
<td>Depreciation rate in housing stock</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Adjustment costs in housing investment</td>
<td>$S''()$</td>
<td>0.25</td>
</tr>
<tr>
<td>Probability of resetting price in non-traded sector</td>
<td>$1 - \theta$</td>
<td>0.3</td>
</tr>
<tr>
<td>Elasticity between differentiate goods in non-trade sector</td>
<td>$\epsilon$</td>
<td>6</td>
</tr>
<tr>
<td>Share of non-traded goods in composite goods</td>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Steady State Value

<table>
<thead>
<tr>
<th>Description</th>
<th>Steady state</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic gross nominal interest rate</td>
<td>$i$</td>
<td>1.02</td>
</tr>
<tr>
<td>Foreign gross nominal interest rate</td>
<td>$i^*$</td>
<td>1.0125</td>
</tr>
<tr>
<td>Foreign Bond level</td>
<td>$B^*$</td>
<td></td>
</tr>
<tr>
<td>Consumption share when $\tau = 0$</td>
<td>$c/Y$</td>
<td>0.65</td>
</tr>
<tr>
<td>Labour share in non-traded sector</td>
<td>$\frac{L_N}{L}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Labour share in traded-sector</td>
<td>$\frac{L_T}{L}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Steady state value of the technology parameter</td>
<td>$A$</td>
<td>1</td>
</tr>
<tr>
<td>Steady state value of the AR(1)shock</td>
<td>$j$</td>
<td>1</td>
</tr>
<tr>
<td>Housing stock owned by the domestic households</td>
<td>$\frac{H}{HH}$</td>
<td>0.9</td>
</tr>
<tr>
<td>Housing stock owned by the foreign investors</td>
<td>$\frac{H'}{HH}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Results

Still working on the results. Please see the last page for impulse response function (IRF) to the underlying variables.
Reference

1 Aspachs-Braconsy and Rabanalz (2010), "The effects of housing prices and monetary policy in a currency union."


4 Funke and Paets (2010), ”What can an open-economy DSGE model tell us about Hong Kong’s housing market?” Bank of Finland Discussion Papers, 19.


8 Posen (2009), Finding the right tool for dealing with asset price booms. Speech to the MPR monetary policy and the economy conference, London.

Appendix

1 Steady State

We have the following equation to get the steady state values for all the variables:

\[ C^{-\sigma} r = \kappa B H^{-\sigma} \quad (45) \]

\[ r = (1 - \beta(1 - \delta))q(1 + \tau) \quad (46) \]

\[ \chi L^\nu = C^{-\sigma} w^T \quad (47) \]

\[ w^T = w^N \quad (48) \]

\[ \frac{1}{\beta} = i \quad (49) \]

\[ r = (1 - \beta'(1 - \delta))q(1 + \tau) \quad (50) \]

\[ y^T = L^T \quad (51) \]

\[ y^N = L^N \quad (52) \]

\[ \frac{e_t}{P_t} = w^T \quad (53) \]

\[ w^N = \frac{\epsilon - 1}{\epsilon - \frac{p_t}{P_t}} \quad (54) \]

\[ q = \frac{e_t}{P_t} + a_1 \frac{p_t}{P_t} \quad (55) \]

\[ 1 - \frac{\psi B^*_{t+1} e_t}{P_t} = \beta i^*_t \quad (56) \]

\[ \gamma C + r H' + \frac{e}{P} I + \frac{e}{P} (B^* i^* - 1) = \frac{e}{P} y^T + \frac{p}{P} a_1 I' - \frac{\psi}{2} \left( \frac{e B^*}{P} \right)^2 \quad (57) \]
2 Composite Price Level
Suppose in the first stage, the households choose how much they want to spend on traded and non-traded goods, subject to the composite goods technology. The optimization problem is:

$$\min E_t = p_t c_t^N + e_t c_t^T = P_t C_t$$ (58)

$$C_t = k(c_t^T)^\gamma (c_t^N)^{1-\gamma}$$ (59)

where $k$ is the normalized parameter.

After manipulating the algebra, we can get the relationship among composite price level, price of non-traded and traded goods is:

$$P_t = (p_t)^{-\gamma}(e_t)^\gamma$$ (60)

3 New Keynesian Phillips Curve

$$\max E_t \sum_{j=0}^\infty (\beta \theta)^j \frac{\lambda'_{t+j}}{\lambda_t} (p_t^* y_{t+j}(i) - p_{t+j} TC y_{t+j}(i))$$ (61)

where $TC_{t+j}$ denotes the marginal cost in terms of non-traded goods.

$$s.t \ y_{t+j}(i) = \left(\frac{p_t^*}{p_{t+j}}\right)^\epsilon y_{t+j}$$ (62)

The first-order condition of choosing $p_t^*$ turns out to be:

$$E_t \sum_{j=0}^\infty (\beta \theta)^j \frac{\lambda'_{t+j}}{\lambda_t} [y_{t+j}(i) - p_t^* \epsilon + \epsilon p_{t+j} MC_{t+j}]$$ (63)

After rearranging it, we get

$$\frac{p_t^*}{p_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^\infty (\beta \theta)^j \frac{\lambda'_{t+j}}{\lambda_t} \left(\frac{p_t^*}{p_{t+j}}\right)^{-\epsilon} y_{t+j} MC_{t+j}}{E_t \sum_{j=0}^\infty (\beta \theta)^j \frac{\lambda'_{t+j}}{\lambda_t} \epsilon^{t+j}} = \frac{\epsilon}{\epsilon - 1} \frac{K_t}{F_t}$$ (64)

After log-linearized (62), we get:

$$K_t = y_t MC_t + \beta \theta E_t \sum_{j=0}^\infty (\beta \theta)^j \frac{\lambda'_{t+j}}{\lambda_t} \pi_t^* \epsilon^{t+1} K_{t+1}$$ (65)
$$F_t = y_t + \beta \theta E_t \sum_{j=0}^{\infty} (\beta \theta)^j \frac{\lambda_t^j}{\lambda_t} \pi_{t+1}^{\pi^*}$$ \hspace{1cm} (66)

Then we can get standard NKPC in non-traded goods sector in a log linearized form:

$$\hat{\pi}_t^* = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} MC_t + \beta E_t \hat{\pi}_t^{\pi^*}$$ \hspace{1cm} (67)

Now, we should rewrite (65) as the function of composite consumption goods inflation as well as the gross nominal exchange rate, we make use of (4) such that:

$$\left(\frac{\hat{P}_t}{p_t}\right) = \frac{\gamma}{1 - \gamma} \hat{e}_t$$ \hspace{1cm} (68)

and

$$\pi_t^* = \frac{\hat{\pi}_t}{1 - \gamma} - \frac{\gamma \hat{e}_t}{1 - \gamma}$$ \hspace{1cm} (69)

Moreover, we also need to convert \(MC_t\) (which in terms of the non-traded goods) to the one that is in terms of the composite consumption goods, making use of:

$$MC_t = \frac{W_t}{p_t} = \frac{W_t \cdot P_t}{P_t \cdot p_t}$$ \hspace{1cm} (70)

and hence we get real marginal cost in terms of \(C_t\) as:

$$m\hat{c}_t = \hat{w}_t + \frac{\gamma}{1 - \gamma} \left(\frac{\hat{e}_t}{P_t}\right) - \hat{A}_t$$ \hspace{1cm} (71)

Therefore, from (65)-(69) and (4), the NKPC reduces to:

$$\frac{\hat{\pi}_t}{1 - \gamma} - \frac{\gamma \hat{e}_t}{1 - \gamma} = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} (\hat{w}_t + \frac{\gamma}{1 - \gamma} (\frac{\hat{e}_t}{P_t}) - \hat{A}_t) + \beta E_t (\frac{\hat{\pi}_{t+1}^{\pi^*}}{1 - \gamma} - \frac{\gamma \hat{e}_{t+1}}{1 - \gamma})$$ \hspace{1cm} (72)

Lastly, under steady state, the real marginal cost \(MC_t = \frac{e - 1}{e}\), we should also convert \(MC\) into \(mc\) which is in term of the \(C_t\) as:

$$\frac{W}{P} = \frac{\epsilon - 1}{\epsilon} = \frac{W \cdot P}{P \cdot p} = mc \cdot \frac{P}{p}.$$ \hspace{1cm} (73)

4 Balance of Payments

$$e_t c_t^T + p_t c_t^N + Q_t I_t + R_t (BH_t + H_t) + e_t B_t^{*} = \epsilon_t y_t^T + p_t y_t^N + e_t B_t^{*} - P_t \frac{\psi}{2} \left(\frac{e_t B_t^{*}}{P_t}\right)^2$$ \hspace{1cm} (74)

$$e_t c_t^T + p_t c_t^N + (e_t + a_1 p_t) I_t + R_t (BH_t + H_t) + e_t B_t^{*} = \epsilon_t y_t^T + p_t (c_t^N + a_1 (I_t + I')) + e_t B_t^{*} - P_t \frac{\psi}{2} \left(\frac{e_t B_t^{*}}{P_t}\right)^2$$ \hspace{1cm} (75)
Hence, BOP reduces to:

\[
e^T_t c_t + e^T_t I_t + R_t (BH_{t+1} - H_{t+1}) + \frac{e^T_t B_t^* i^*_{t-1}}{P_t} = e^T_t y^*_t + p_t a_1 I'_t + \frac{e^T_t B^*_{t+1}}{P_t} - \frac{\psi}{2} \left( \frac{e^T_t B^*_{t+1}}{P_t} \right)^2
\]  

(76)

Rewrite as the real term:

\[
\gamma C_t + \frac{e^T_t}{P_t} I_t + r_t H'_{t+1} + \frac{e^T_t B^*_{t-1}}{P_t} = e^T_t y^*_t + p_t a_1 I'_t + \frac{e^T_t B^*_{t+1}}{P_t} - \frac{\psi}{2} \left( \frac{e^T_t B^*_{t+1}}{P_t} \right)^2
\]  

(77)

5 Log-Linearize system

We need to use the Taylor first-approximation method to linearize all the equation to get the solution as well as the impulse response function.

\[
- \sigma \dot{C}_t + \dot{r}_t = \sigma BH'_{t+1}
\]  

(78)

\[
rr_t - \sigma \dot{C}_t = q \dot{\mu}_t - q \beta (1 - \delta) E_t \mu_{t+1}
\]  

(79)

\[
- \sigma \dot{C}_t - \dot{q}_t + \tilde{m} \mu_t = S''(\hat{I}_t - \hat{I}_{t-1}) + \beta S''(\hat{I}_{t+1} - \hat{I}_t)
\]  

(80)

\[
\nu \dot{L}_t + \sigma \dot{C}_t = w^T_t
\]  

(81)

\[
- \sigma \dot{C}_t = - \sigma E_t C_{t+1} + \hat{i}_t - \pi_{t+1}
\]  

(82)

\[
- \sigma (1 - \psi B^* \frac{e}{P}) \dot{C}_t - \psi B^* \frac{e}{P} B^*_{t+1} + \psi^* B^* \frac{e}{P} \frac{\dot{e}_t}{P_t}
\]

\[
= \beta i^* E_t (\epsilon_{t+1} - \pi_{t+1}) - \sigma \beta i^* E_t C_{t+1} + \beta i^* \dot{i}_t
\]

(83)

\[
(1 - \beta') \dot{r}_t - \beta' \dot{r}_t = \dot{z}_t - \beta' E_t \dot{z}_{t+1} - \beta' E_t \dot{j}_{t+1}
\]

(84)

\[
- \dot{q}_t + \dot{z}_t = S''(\hat{I}_t - \hat{I}_{t-1}) + \beta' S''(\hat{I}_{t+1} - \hat{I}_t)
\]  

(85)
\[
\frac{\hat{\pi}_t}{1 - \gamma} - \gamma \hat{\epsilon}_t = \frac{(1 - \beta\theta)(1 - \theta)}{\theta} (\hat{w}_t + \gamma \frac{\hat{\epsilon}_t}{P_t}) - \hat{A}_t + \beta E_t \frac{\hat{\pi}_t}{1 - \gamma} - \gamma \hat{\epsilon}_{t+1}
\]

(86)

\[
\hat{A}_t + \hat{L}_t^T = \hat{y}_t^T
\]

(87)

\[
\hat{A}_t + \hat{L}_t^N = \hat{y}_t^N
\]

(88)

\[
\frac{\hat{\epsilon}_t}{P_t} + \hat{A}_t = \hat{w}_t^T
\]

(89)

\[
\hat{q}_t q = \left( \frac{e}{P} + \left( \frac{e}{P} \right)^{\gamma - 1} \right) \frac{a_1 \gamma}{\gamma - 1} \frac{\hat{\epsilon}_t}{P_t}
\]

(90)

\[
i_t = \alpha \hat{\pi}_t
\]

(91)

\[
(1 - \gamma) C \hat{C}_t + a_1 (I \hat{I}_t + I' \hat{I}'_t) \frac{\hat{P}_t}{P_t} = y^N (\hat{y}_t^N)
\]

(92)

\[
\hat{L}_t = \frac{L_N}{L} \hat{L}_t^N + \frac{L_T}{L} \hat{L}_t^T
\]

(93)

\[
B \hat{H}_{t+1} = \frac{H}{BH} \hat{H}_{t+1} + \frac{H'}{BH} \hat{H}_{t+1}
\]

(94)

\[
\gamma C_t + \frac{e}{P} \hat{I}_t + \frac{e}{P} \frac{\hat{\epsilon}_t}{P_t} + r H' \hat{I}_t + r H' \hat{H}_{t+1} + \frac{e}{P} B^* \hat{\epsilon}_{t+1} + \frac{e}{P} \hat{\epsilon}_t = \frac{e}{P} \hat{\epsilon}_t + \frac{e}{P} \hat{\epsilon}_t - \gamma \frac{\hat{e}_t}{P_t} - \gamma \frac{\hat{e}_t}{P_t}
\]

(95)

\[
\hat{j}_t = \rho \hat{j}_{t-1} + \eta_t
\]

(96)
\[ \hat{A}_t = \rho \hat{A}_{t-1} + \eta_t \]  \hspace{1cm} (97)

\[ \hat{\imath}_t^* = \rho \hat{\imath}_{t-1}^* + \eta_t \]  \hspace{1cm} (98)

\[ \frac{\dot{\pi}_t}{P_t} - \frac{\dot{\pi}_{t-1}}{P_{t-1}} = \pi_t - \dot{\epsilon}_t \]  \hspace{1cm} (99)

Also, we need to define two more state variable for \( I_{t-1} \) and \( I'_{t-1} \) to captures their effects on housing investment.

\[ X_{t+1}^* = \hat{I}_t \]  \hspace{1cm} (100)

\[ Y_{t+1}^* = \hat{I}'_t \]  \hspace{1cm} (101)

and we have two more law of motion to both housing investment

\[ HH_{t+1}^* = (1 - \delta) HH_t + I \hat{I}_t \]  \hspace{1cm} (102)

\[ H' H'_{t+1}^* = (1 - \delta) H' H'_t + I' \hat{I}_t \]  \hspace{1cm} (103)
Figure 2: Policy (I), $\alpha_\pi = 1.5$

Figure 3: Policy (I), $\alpha_\pi = 3$