GOVERNMENT SPENDING MULTIPLIER IN A MODEL WITH FINANCIAL FRICITION

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Abstract. During the recent financial crisis, huge fiscal stimulus packages were initiated and bold actions were taken by monetary authorities in many countries. Inspired by these, I explore the impact of fiscal stimulus when I take financial friction and liquidity trap into account. In particular, I investigate the fiscal multiplier in a dynamic stochastic general equilibrium model which is featured with financial accelerator mechanism and zero lower bound for nominal interest rate. And I find that the presence of financial friction (financial accelerator mechanism following from BGG (1999)) leads to an increase in fiscal multiplier. While this increase is mild, the binding zero nominal interest rate environment induces a significant increase in the impact of fiscal expansion.

1. Introduction

The 2008-2009 financial crisis has witnessed the meltdowns of both financial market and housing market: the collapse of housing bubble has dramatically damaged the health conditions of many large financial institutions, and the household has also suffered from real estate losses which results in numerous foreclosures and prolonged vacancies. In 2009, the Obama administration signed the American Recovery and Reinvestment Plan entitling a $787 billion stimulus package. The US Fed also conducted quantitative easing policy which tremendously enlarged the Fed’s balance sheet, and also the policy rate has been set to near zero which they claim that the policy rate will remain zero for foreseeable future. Although the endeavors made by the US governments, the US economy is still enjoying a slow recovery with a high unemployment rate and low inflation environment. In this paper, in light of this financial crisis and near zero policy rate set by monetary authority, I investigate the fiscal multiplier in a DSGE model which is featured with financial accelerator mechanism and zero lower bound for nominal interest rate.

The paper is constructed as follows. Section 2 lays out the baseline model, the solution method is include in section 3. Section 4 presents the model dynamics. Section 5 provides the calculations of fiscal multipliers in variants of the baseline model. And last section contains some concluding remarks.

2. The Model

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This draft is preliminary. I thank Eric Leeper and Todd Walker for many suggestions and comments. Also I thank Manuel Gonzalez-Astudillo and Hyunduk Suh for helpful conversations. All errors are mine. Department of Economics, Indiana University Bloomington, gzuo@indiana.edu.
The model I use here is a conventional New Keynesian model featured with Calvo style nominal rigidity. On top of this, entrepreneurial sector is added into the model to demand the credit which incorporates the financial accelerator mechanism following from BGG (1999). Moreover, zero lower bound (ZLB) for nominal interest rate is also considered for setting monetary policy.

2.1. **Households.** The economy is inhabited by a representative household who provides the labor endowments and have life-time utility which is given by:

\[
E_0 = \beta t \rho t \left[ \frac{C_t \gamma (1-H_t)^{1-\gamma}}{1-\sigma} \right]^{1-\sigma} - \frac{1}{1-\sigma}
\]  

And here \(E_0\) is the conditional expectation operator, \(C_t\) and \(H_t\) denote time \(t\) consumption and labor supply respectively. We assume that \(\sigma > 0\) and \(\gamma \in (0, 1)\). And here \(\rho_t\) is the preference shock which is motivated by the changing desires of households on savings. The stochastic process for \(\hat{\rho}_t\) (throughout the paper, \(\hat{x}\) stands for the log deviation from its steady state value of some variable \(x\)) is assume to be AR(1) process:

\[
\hat{\rho}_t = \rho \hat{\rho}_{t-1} + \epsilon_{\rho,t}
\]  

The representative household enters period \(t\) with \(D_{t-1} - 1\) units of real deposits which earn risk-free net real interest rate \(R_{n,t-1}\), \(B_{t-1}\) units of nominal government bonds which earn risk-free net nominal interest rate \(R_{t-1}\). During period \(t\), this representative household supplies labor to the entrepreneur and receive labor payment \(W_tH_t\) where \(W_t\) is the real wage in the economy. Furthermore, the household receive dividend payments \(\Pi_t\) from retail firms and pay lump sum tax \(T_t\) to government. Household allocates the funds to private consumption \(C_t\), nominal government bond and real deposit \(D_t\). Hence the budget constraint in real term is:

\[
C_t + D_t + \frac{B_t}{P_t} + T_t = W_tH_t + (1 + R_{t-1})D_{t-1} + \frac{B_{t-1}(1 + R_{n,t-1})}{P_t} + \Pi_t
\]  

2.2. **Entrepreneurs.** Following from BGG (1999) entrepreneurs are introduced to demand the credit in order to purchase capital in each period. And capital is used in combination with hired labor from workers to produce wholesale output in the subsequent period. In order to preclude the possibility that the entrepreneurial sector will ultimately accumulate enough net worth to fully self-finance their own projects. We assume that each entrepreneur has a probability \(v_t\) of surviving to the next period. And we assume \(\hat{v}_t\) follows from AR(1) stochastic process:

\[
\hat{v}_t = \rho_v \hat{v}_{t-1} + \epsilon_{v,t}
\]  

2.2.1. **Optimal Financial Contract.** There is a continuum of risk neutral entrepreneurs indexed by \(j \in [0, 1]\). At time \(t\), the entrepreneur \(j\) who manages firm \(j\) purchases capital \(K_j^t\) for use at time \(t + 1\) at price \(Q_t\). Moreover, each entrepreneur \(j\) suffers from an idiosyncratic shock \(\omega_{j,t+1}\) to the return on the capital. Hence the ex post gross return on capital bought at time \(t\) for entrepreneur \(j\) is \(\omega_{j,t+1} R_{k,t+1}^j\) where \(R_{k,t+1}^j\) is the ex post aggregate return to capital (i.e., the gross return averaged across firms). Followed from BGG (1999), we assume that the random variable \(\omega^j\) is i.i.d. across time and across firms with a continuous and differentiable
c.d.f. \( F(\omega^j) \) over a non-negative support and \( \mathbb{E}\omega^j = 1 \). We also impose the restriction on the corresponding hazard rate \( h(\omega^j) \):

\[
\frac{\partial (\omega^j h(\omega^j))}{\partial \omega^j} > 0
\]

(2.5)

where \( h(\omega) \equiv \frac{f(\omega)}{1-F(\omega)} \). This regularity condition is a relatively weak restriction that is satisfied by most conventional distribution including for example the log-normal.

At the end of period \( t \), entrepreneur \( j \) uses the available real net worth \( N^j_t \) and borrows an amount \( B^j_t \) from financial intermediary to finance the capital purchase \( Q_tK^j_t \) which is used for the production next period. Hence,

\[
B^j_t = Q_tK^j_t - N^j_t
\]

(2.6)

The financial intermediary uses household’s deposit which pays risk-free net real interest rate \( R^n_t \) to finance the capital purchases of entrepreneurs. And the optimal financial contract may be characterized by a gross non-default loan rate \( Z^j_t \) and a threshold value of the idiosyncratic shock \( \omega_{t+1} \) such that for values of the idiosyncratic shock greater than or equal to \( \omega_{t+1} \), the entrepreneur is able to repay the loan at the contractual rate \( Z^j_t \). We have,

\[
\omega_{t+1}\mathbb{E}_t R^k_t Q_tK^j_t = Z^j_t B^j_t
\]

(2.7)

Therefore, when \( \omega_{t+1} \geq \omega_{t+1} \), under the optimal contract the entrepreneur repays the financial intermediary the promised amount \( Z^j_t B^j_t \) and keeps the difference which is equal to \( \omega_{t+1} \mathbb{E}_t R^k_t Q_tK^j_t - Z^j_t B^j_t \). When \( \omega_{t+1} < \omega_{t+1} \), the entrepreneur cannot pay the contractual return and thus declares default. In this case, following from the CSV (costly state verification) in Townsend (1979), the financial intermediary needs to pay a monitoring cost in order to observe the borrower’s realized return. And for simplicity, we assume that the financial intermediary’s net receipts are \((1 - \mu)\omega_{t+1} \mathbb{E}_t R^k_t Q_tK^j_t \). The defaulting entrepreneur receives nothing.

We need to characterize this optimal financial contract more explicitly. Denote \( \Gamma(\omega_{t+1}) \) as the expected gross share of profits going to the lender and \( \mu G(\omega_{t+1}) \) as the expected monitoring costs, where

\[
\Gamma(\omega_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega f(\omega)d\omega + \omega_{t+1} \int_{\omega_{t+1}}^{\infty} f(\omega)d\omega
\]

(2.8)

and

\[
\mu G(\omega_{t+1}) = \mu \int_0^{\omega_{t+1}} \omega f(\omega)d\omega
\]

(2.9)

Then the problem that the financial intermediary faces is to maximize the expected profit of entrepreneur subject to the zero profit condition, or more explicitly

\[
\max_{K^j_t,\omega_{t+1}} (1 - \Gamma(\omega_{t+1})) \mathbb{E}_t R^k_t Q_tK^j_t
\]

(2.10)

s.to

\[
(\Gamma(\omega_{t+1}) - \mu G(\omega_{t+1})) \mathbb{E}_t R^k_t Q_tK^j_t = (1 + R_t)(Q_tK^j_t - N^j_t)
\]

(2.11)
And therefore we can get optimal level of capital that the financial intermediary lends to the entrepreneur $j$:

$$Q_tK^j_t = \psi(\mathbb{E}_t \frac{R^k_{t+1}}{1 + R_t})N^j_t$$  \quad (2.12)

with $\psi(1) = 1$ and $\psi'(\cdot) > 0$. The above equation is a key relationship in the model. It describes the critical link between the risk premium $\mathbb{E}_t \frac{R^k_{t+1}}{1 + R_t}$ and the leverage $\frac{Q_tK^j_t}{N^j_t}$ the firm takes. Everything else being equal, the less leverage the firm takes, the less risk premium the financial intermediary charges for the lending. Or the entrepreneurs depend more on their internal funds ($N^j_t$), the less risk premium the financial intermediary would charge for the external funds. Moreover, the above equation characterizes a linear relationship between the capital and the entrepreneur’s net worth. And therefore we can easily aggregate across all the entrepreneurs to get the economy wide supply of capital:

$$\mathbb{E}_t \frac{R^k_{t+1}}{1 + R_t} = s \left( \frac{N_t}{Q_tK_t} \right) \quad \text{where} \quad s'(\cdot) < 0$$  \quad (2.13)

2.2.2. Entrepreneur in General Equilibrium. Entrepreneurs purchase capital in each period $t-1$ for use in the subsequent period $t$. Capital $K_{t-1}$ is used in combination with hired labor $H_t$ from household and the labor $H^e_t$ from themselves to produce wholesale output $Y_t$ which will be sold to the retailer. We assume that the entrepreneurs are perfectly competitive and production for them is constant returns to scale and hence it allows us to treat entrepreneurs in the economy as a whole and write the production function as an aggregate relationship. We specify the production function at time $t$ as follows:

$$Y_t = A_tK^\alpha_{t-1} \left( H^\Omega_t \left( H^e_t \right)^{1-\Omega} \right)^{1-\alpha}$$  \quad (2.14)

In order to solve the problem for the entrepreneurs as a whole, we follow two steps which we first solve cost minimization problem to determine the optimal allocation between capital and labor under certain level of output and then we solve the profit maximization problem to determine the price that entrepreneurs charge the wholesale output.

First, for entrepreneurs as a whole, they take the wage rates $W_t$ and $W^e_t$ for labors from household $H_t$ and entrepreneur themselves $H^e_t$ as given, then the cost minimization problem is given as follows:

$$\min_{\{K_{t-1}, H_t, H^e_t\}} TC(Y_t) = W_tH_t + W^e_tH^e_t + R^k_tQ_{t-1}K_{t-1} - (1 - \delta)Q_tK_{t-1} - (2.15)$$

subject to

$$A_tK^\alpha_{t-1} \left( H^\Omega_t \left( H^e_t \right)^{1-\Omega} \right)^{1-\alpha} \geq Y_t$$  \quad (2.16)
Let $\lambda$ be the Lagrangian multiplier associated with the constraints (2.2.2) and the first order conditions are given:

\[ H_t : \quad W_t = \lambda(1 - \alpha)\frac{Y_t}{H_t} \]  
\[ H^e_t : \quad W^e_t = \lambda(1 - \alpha)(1 - \Omega)\frac{Y_t}{H^e_t} \]  
\[ K_{t-1} : \quad Q_{t-1}R^k_t = \lambda\alpha\frac{Y_t}{K_{t-1}} + (1 - \delta)Q_t \]  

Some manipulations give us that:

\[ \lambda = \frac{1}{A_t} \left( \frac{Q_{t-1}R^k_t}{\alpha} - (1 - \delta)Q_t \right)^\alpha \left( \frac{1}{1 - \alpha} \left( \frac{W_t}{\Omega} \right)^\Omega \left( \frac{W^e_t}{1 - \Omega} \right)^{1 - \Omega} \right)^{1 - \alpha} \]  

And moreover, we can get that $TC(Y_t) = \lambda Y_t$. Therefore we have $MC_t = \lambda$. Second, for the profit maximization problem, we need to set the price of wholesale output to be $MC_t$ since the entrepreneurs in this economy are perfectly competitive.

Moreover, we assume that the entrepreneurs supply their unit labor endowment inelastically to the wholesale production. And hence we can characterize the evolution of the entrepreneurs’ net worth. Let $V_t$ be entrepreneurial equity or the wealth accumulated by entrepreneurs from operating firms. Recall that only a fraction $\nu$ of the entrepreneurs will survive to the next period, hence aggregate entrepreneurial net worth at the end of period $t$, $N_t$, is given by:

\[ N_t = \nu_t V_t + W^e_t \]  

where

\[ V_t = R^k_t Q_t K_{t-1} - \left( (1 + R_{t-1})(Q_{t-1}K_{t-1} - N_{t-1}) + \mu \int_0^{\omega t} \omega f(\omega)d\omega R^k_t Q_{t-1} K_{t-1} \right) \]  

2.3. Capital Producer. At the end of period $t$, after production has taken place, a competitive capital producer buys the existing undepreciated capital stock $(1 - \delta)Q_tK_{t-1}$, combine it with a proportion of final goods denoting as aggregate investment $I_t$ and produces the new stock of capital $K_t$ to be used in period $t+1$. We assume that the capital producer faces investment adjustment costs denoted by $S(I_t/I_{t-1})$ where $S(\cdot)$ is an increasing and concave function and $S(1) = S'(1) = 0$. Then the profit maximization problem for capital producer is given as follows:

\[ \max_{\{I_t, K_{t-1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t A_{0,t}(Q_tK_t - I_t - (1 - \delta)Q_tK_{t-1}) \]  

s.to

\[ K_t = (1 - \delta)K_{t-1} + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \]
2.4. **Final Good Producers.** The final goods, $Y^f_t$, used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by $z \in [0, 1]$ via a typical Dixit-Stiglitz aggregator which is given as follows:

$$Y^f_t = \left( \int_0^1 Y_t(z)^{e/(e-1)} dz \right)^{\varepsilon/(\varepsilon-1)}$$

(2.25)

The maximization problem for final good producers yield a demand function for type $z$ intermediate goods:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{\varepsilon} Y^f_t$$

(2.26)

And the corresponding price index:

$$P_t = \left( \int_0^1 P_t(z)^{1-\varepsilon} dz \right)^{1/(1-\varepsilon)}$$

(2.27)

2.5. **The Retailer Firms.** In order to introduce sticky price into the economy, the model allows for monopolistically competitive retail firms which are a continuum of firms living in the unit interval. They purchase the wholesale goods from entrepreneurs and diversify them at no cost. Let $Y_t(z)$ be the quantity of output sold by retailer $z \in (0, 1)$ and let $P_t(z)$ be the nominal price.

To introduce price inertia, we assume that the retailer is free to change its price in a given period only with probability $\theta$ following Calvo (1983). Let $P^*_t(z)$ denote the price set by retailer $z$ who is able to change prices at $t$ and $Y^*_t(z)$ be the corresponding demand curve. Hence subjecting to the demand of its goods given by equation (33), retailer $z$ chooses his price to maximize expected discounted profits:

$$E_t \sum_{j=0}^{\infty} (\beta \theta)^j \Lambda_{t+j} \left( P^*_t(z)Y_{t+j}(z) - P_{t+j}MC_{t+j}Y_{t+j}(z) \right)$$

(2.28)

And from here we can get Phillips curve:

$$\pi_t = \beta E_{t+1} \pi_{t+1} + \frac{(1-\beta \theta)(1-\theta)}{\theta} MC_t$$

(2.29)

2.6. **Monetary Policy.** We assume that monetary policy follows the rule:

$$R_{t+1} = \max (Z_{t+1}, 0)$$

(2.30)

where

$$Z_{t+1} = \frac{1}{\beta} (1 + \pi_t)^{\phi_1(1-\rho_N)} \left( \frac{Y_t}{Y} \right)^{\phi_2(1-\rho_N)} \left( \beta(1 + R_t) \right)^{\rho\mu} \exp (\beta \epsilon_{Rn.t+1}) - 1$$

Throughout the paper, a variable without time subscript denotes its steady state value. Here $Y$ denotes the steady state level of output. And we assume that $\phi_1 > 1$ and $\phi_2 \in (0, 1)$.

According to the above rule for monetary policy, monetary authority follows a Taylor rule as long as the implied nominal interest rate is non-negative. Whenever the Taylor rule implies a negative nominal interest rate, the monetary authority simply sets the nominal interest rate to be zero.
2.7. Fiscal Policy. Each period the government buys a fraction of the final goods \(G_t\), issues bonds \(B_t\) and collects lump-sum taxes \(T_t\) to retire the bonds issued last period \(B_{t-1}\), then the budget constraint in real term is given by:

\[
G_t + \frac{B_t - 1(1 + R_{t-1})}{P_t} = \frac{B_t}{P_t} + T_t \tag{2.31}
\]

And for the government spending, as long as the zero bound on the nominal interest rate is not binding it evolves according to:

\[
G_{t+1} = G_t e^{\eta_{t+1}} \tag{2.32}
\]

where \(\eta_{t+1}\) is an i.i.d shock with zero mean.

2.8. Aggregate Resource Constraint. Final output may be either transformed into consumption goods, invested, consumed by the government or used up in monitoring costs. More specifically, the aggregate resource constraint in this economy is given by:

\[
Y^f_t = C_t + C^e_t + I_t + G_t + \mu \int_0^{\omega_t} \omega f(\omega) d\omega R^t_t Q_{t-1} K_{t-1} \tag{2.33}
\]

Here \(C^e_t = (1 - \upsilon_t) V_t\) is entrepreneurial consumption from those who exit the entrepreneur sector each period. And \(\mu \int_0^{\omega_t} \omega f(\omega) d\omega R^t_t Q_{t-1} K_{t-1}\) reflects aggregate monitoring costs.

3. Calibration and Model Solution

3.1. Calibration. I calibrate the model using quarterly US data. \(\beta\) is 0.99 which implies 4% annual risk free nominal interest rate. \(\delta\) is set to be 0.025 to ensure a 10% annual capital depreciation rate. Calvo pricing parameter is set to be 0.85. \(\alpha\) is set to be 0.35 which implies approximately 65% of the total output will be wage income. The utility curvature parameter is calibrated to be 2 and the complementarity parameter between consumption and leisure \(\gamma\) is set to be 0.29 as in Christiano et. al. (2010). Following from BGG (1999), we assume a 1% share of total income accruing to entrepreneur’s income and accordingly we have \(\Omega(1 - \alpha)\) to be 0.64. To ensure a 1.1 price markup in monopolistic competitive production sector, and this implies a quarterly capital to output ratio to be 7.93 which is more or less consistent with the US data. Moreover, this leads to investment output ratio to be 20%. And government spending to output ratio is set to be 20%. Again following from BGG (1999), the annualized business failure rate, \(F(\omega_s)\), is 3%. The idiosyncratic productivity shock to entrepreneur \(\omega_t\) is assumed to have log normal distribution such that \(\ln \omega_t\) follows \(N(-\sigma^2/2, \sigma^2)\) and hence we have \(E(\omega_t) = 1\). Moreover, the standard deviation \(\sigma\) for entrepreneurial idiosyncratic shock is assumed to be 0.28 which implies the steady state threshold idiosyncratic shock to be 0.526. And the steady state capital to net worth ratio is set to be 2, which is equivalent to have a leverage ratio of 0.5. The fraction of realized payoffs lost in bankruptcy, \(\mu\), is set to be 0.12. These parameters imply a 58.4% consumption output ratio which is more or less consistent with US data. As for the monetary policy rule, I follow from Smet and Wouters (2003) to set nominal interest rate smoothing coefficient to be 0.928 and reaction coefficients to inflation and output gap to be 1.668 and 0.144 respectively.\(^1\)

\(^1\)The full specification of parameters can be found in the appendix.
3.2. **Model Solution.** In order to mimic this financial crisis where we have been experiencing the near zero nominal interest rate, I assume that at time 0 the economy is hit by some shocks which makes nominal interest rate bind for periods $[t_1, t_2]$. Moreover, following from Christiano (2010) I assume that although agents would experience near zero nominal interest environment during period $[t_1, t_2]$, they perceive that from $t_2$ onward nominal interest will never hit zero lower bound and therefore always follow Taylor rule. Therefore, the model can be solved piecewisely. The solution for the period $t_2$ onward can be obtained by using undetermined coefficient method which is described in Uhlig (1997). And the evolution path for the time period from 0 to $t_2$ can be derived using backward induction. The detailed solution method can be found in the appendix.

4. **Model Dynamics**

In this section, I consider the model without zero lower bound for nominal interest rate and hence monetary authority is always able to manipulate the nominal interest rate following Taylor rule. The impulse responses of macro variables to various structure shocks are explored. And the model here can offer sensible macro implications.

4.1. **Responses to Technology Shock.** Figure 1 shows the impulse responses of macro variables to a positive productivity shock. Due to the rise in productivity, marginal cost falls on impact. As monetary policy does not response strongly enough to offset this rise, the inflation falls gradually. Output, consumption and investment rise while employment falls. And this fall in employment is consistent with estimated impulse responses of identified productivity shocks in the US.

4.2. **Responses to Preference Shock.** Figure 2 displays the impulse responses of macro variables to a positive preference shock. This changes the households’ inter-temporal choice between consumption and saving, they becomes more impatient and hence are more willing to consume. While output and consumption are increasing significantly, the significant crowd-out effect on investment is present. In order to meet the increase demand in consumption, there is an increase in employment. Moreover, this type of demand shock causes inflation to rise.

4.3. **Responses to Monetary Policy Shock.** Figure 3 reports the impact of a negative monetary policy shock. The nominal interest rate decreases on impact and leads real interest rate to decrease. This encourages households to investment more and also leads to a significant increase in entrepreneur’s net worth. And due to the increase in entrepreneur’s net worth, credit frictions is weakened and therefore total output picks up. This leads to the increase in labor employment, real wage and consumption.

4.4. **Responses to Government Spending Shock.** Figure 4 illustrates the impact of a positive government spending shock. Although output increases on impact, the strong crowd-out effects for both household’s consumption and investment are present. Due to the increase in government spending, the inflation increases since the increase in demand capacity raises marginal cost. Moreover, the monetary authority raises the nominal interest rate to fight the upward pressure in inflation and this drives up the real interest rate. One
more thing needed to be mentioned is that the rise in government spending alleviates the risk premium in credit market. This is because the banking sector is well functional in this model and the increase in demand capacity initiates more projects and therefore drives up the net worth of the whole entrepreneur sector.

5. Fiscal Multipliers in Variants of Baseline Model

In this section, the fiscal multipliers are investigated in variants of the baseline model. Let me denote the baseline model as NK+FA+ZLB, or the conventional new Keynesian model featured with financial accelerator mechanism\(^2\) and zero lower bound for nominal interest rate. More specifically, beside the baseline model two other models, conventional new Keynesian model (NK) and new Keynesian model featured with financial accelerator (NK+FA), will be explored here. In order to make the comparison of fiscal multipliers across difference models meaningful, I use present value fiscal multiplier which is defined as the cumulative change in discounted output over the cumulative change in discounted fiscal expenditures. And over some time horizon \(t\), present value fiscal multiplier is given by:

\[
\text{Present Value Fiscal Multiplier}(t) = \frac{\sum_{k=0}^{t} (\prod_{s=0}^{k} (1 + R_s)^{-1}) \Delta Y_k}{\sum_{k=0}^{t} (\prod_{s=0}^{k} (1 + R_s)^{-1}) \Delta G_k}
\]

5.1. Present Value Fiscal Multipliers. Present value fiscal multiplier summarizes the overall effect of fiscal expansion which I believe to be a more appropriate measure. And moreover when \(t = 0\), the present value fiscal multiplier collapses to impact fiscal multiplier. Figure 5 plots the present value fiscal multipliers for variants of the model. For all three models, the calculation of fiscal multiplier is based on a 2% increase in government consumption at time 0\(^3\). In the NK model with neither financial accelerator mechanism nor zero lower bound for nominal interest rate, the fiscal multiplier is 0.72 on impact and over a long horizon the present value fiscal multiplier is around 0.4. The NK+FA model has a higher impact fiscal multiplier, 0.82, and also a higher, 0.53, present value fiscal multiplier over a long horizon. While the increase in fiscal multiplier due to financial friction is mild, the increase due to the presence of zero lower bound for nominal interest rate is significant. In the NK+FA+ZLB model, the impact fiscal multiplier is much larger, 1.78 as well as a much larger, 1.4, present value fiscal multiplier over a long horizon. In the subsequent sections, the roles of financial accelerator mechanism and zero lower bound for nominal interest rate in determining the size of fiscal multipliers are investigated.

5.2. The Role of Financial Friction. Figure 6 shows the impact of government spending under two different scenarios. The solid line shows the impulse responses of macro variables to government spending shock in the NK+BGG model where financial accelerator mechanism are present. Whereas the dash line shows the responses in the NK model without financial accelerator mechanism. The discrepancy lies in the financial sector. Different from the

\(^2\)The financial accelerator mechanism here follows from Bernanke, Gertler and Gilchrist (1999).

\(^3\)This follows from Christiano (2010) who argues that although the huge fiscal stimulus plan enacted in February 2009, total government consumption rose by only 2%. The two factors are (1): a substantial part of the stimulus plan involved an increase in transfers to households; (2): there was a large fall in state and local purchases that offset a substantial part of the increase in federal government purchases.
model with financial accelerator, the NK model cannot alleviate the risk premium as the NK+FA model does. Hence the supply of capital will be cut back more than in the NK+FA model, and therefore the investment is crowded out more severely in the NK model. On the other hand, the sluggish in the credit market lead to less increase in real interest rate which makes the crowd-out effect of government spending on household’s consumption to be milder. Altogether, although the presence of financial accelerator mechanism does differ the impact of fiscal expansion, the difference is mild.

5.3. The Role of ZLB for Nominal Interest Rate. The presence of financial accelerator mechanism contributes a mild increase in government spending, but zero lower bound for nominal interest rate leads to a significant increase in the fiscal multiplier which is in line with Christiano (2010). In order to examine the role of ZLB in determining the size of fiscal multiplier, I introduce a negative preference shock in two different models: one is NK+FA model where the nominal interest rate is not subject to zero lower bound and therefore is free to vary following Taylor rule, the other is NK+FA+ZLB model where nominal interest rate is subject to zero lower bound. Following Christiano (2010) the discount factor is subject to a 50% decrease to have a 11 quarters binding periods for nominal interest rate in NK+GA+ZLB model\(^4\). Figure 7 reports the impulse responses of two models to this negative preference shock. As you can see from the figure, the differences are dramatic. The negative shock to discount factor induce people to save more which drives down nominal interest rate, but in NK+FA+ZLB model the nominal interest rate cannot fall below the zero lower bound and therefore real interest rate goes up even higher. This raises the funding cost for banking sector and therefore more entrepreneurs are excluded from operating their projects. Hence net worth in entrepreneurial sector goes down sharply and therefore drives up the risk premium sharply. In equilibrium output needs to suffer a big loss in order to bring the increased desire to save in line with the lack of demand in capital. Due to the large friction introduced by zero lower bound for nominal interest rate and effect of fiscal expansion (increase the demand putting more entrepreneurs back into operating their projects), the larger fiscal multiplier is expected.

6. Conclusion and Future Directions

Inspired by the 2008-2009 financial turmoil and the huge fiscal expansion packages initiated by many fiscal authorities over the world during that time, I investigate the fiscal multiplier in variants of models trying to figure out the effect of fiscal stimulus, or how large is actually the fiscal multiplier. Three models are considered here, NK model (conventional new Keynesian model), NK+FA model (NK model featuring with financial accelerator mechanism) and NK+FA+ZLB model (zero lower bound for nominal interest rate is incorporated into the model). In the NK model, fiscal multiplier is 0.72 on impact and the present value fiscal multiplier over a long horizon is around 0.4. The NK+FA model instead has a higher impact fiscal multiplier which is 0.82 as well as a higher present value fiscal multiplier, 0.53. Because of the effect of fiscal stimulus on reducing the risk premium, fiscal stimulus is slightly more efficient in the model featuring with financial accelerator mechanism. Due to the fact that

\(^4\)The justification is given by figure 8 showing the personal savings rate in the US. The figure shows that during the financial crisis the savings rate rises sharply from roughly 2% in 2007 to a level around 5.5%.
we are in this near zero nominal interest rate environment\(^5\), changing the policy rate is no longer an available policy tool for monetary authority. I investigate the fiscal multiplier in a model with binding zero nominal interest rate when the model economy is hit by a severe preference shock. Consistent with Christiano (2010), I find that the ZLB for nominal interest rate leads to a significant increase in the effect of fiscal expansion. According to my calculation, if fiscal expansion is timely\(^6\), the impact fiscal multiplier is 1.78 and a 1.4 present value fiscal multiplier over a long horizon.

The above results are well expected, but throughout the analysis several deficiencies need to be pointed out. First, the large fiscal multiplier in this model (NK+FA+ZLB), or the big impact of fiscal expansion on the real economy, hinges on a well functional banking sector. The increase in demand capacity due to the fiscal expansion will immediately initiate more entrepreneurs’ projects and therefore increase the net worth of entrepreneur sector to alleviate the risk premium. However, during the financial crisis we witnessed a dysfunctional banking sector who raised their lending standard\(^7\) and reduced their supply of credits. Therefore, it might be misleading to consider the effect of fiscal expansion in a model with a well functional banking sector. Second, the models here use lump sum taxes to finance government spendings which introduces no distortion on private agents’ choices. However, in reality the ways of financing government spending do provide some distortions and therefore in order to explore the impact of fiscal expansion, I need to model fiscal policy more seriously. Third, zero lower bound for nominal interest rate always presents and hence it is not guaranteed that near zero environment for nominal interest rate will not reappear. However, the model here is solved under the assumption which is against this point. The agents in this model economy perceive the binding zero nominal interest rate to be temporary and do not worry about the reappearance of this environment. Fourth, the fiscal multipliers I calculated here only have qualitative implications. In order to obtain quantitatively acceptable estimations, I plan to implement Bayesian technique to achieve this.

Therefore, for my future research, I want to make the following modifications: (1) to incorporate a seriously modelled banking sector which might be dysfunctional during the financial crisis; (2) to model carefully the different ways to finance fiscal expenditures; (3) to modify the assumption that the binding zero nominal interest rate is just temporary and resolve the model; (4) to obtain fiscal multipliers which has quantitative implications.

---

\(^5\)Figure 9 shows the historical plot of federal funds rate.

\(^6\)I mean here the fiscal expansion comes in line with exactly the time when a negative preference shock hits the model economy.

\(^7\)It can be seen in “Senior Loan Officer Opinion Survey on Bank Lending Practices”. And the survey data can be found in Federal Reserve Board’s website.
7. APPENDIX

7.1. **Calibrated Parameters.** The calibrated parameters are provided as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Complementarity parameter between consumption and leisure</td>
<td>0.29</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Consumption curvature parameter</td>
<td>2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share of production</td>
<td>0.35</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Degree of price stickiness</td>
<td>0.85</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Proportion of household labor in aggregate labor</td>
<td>0.9846</td>
</tr>
<tr>
<td>( R )</td>
<td>Net risk-free interest rate</td>
<td>0.0101</td>
</tr>
<tr>
<td>( R^k )</td>
<td>Gross return on capital</td>
<td>1.0151</td>
</tr>
<tr>
<td>( H )</td>
<td>Labor in the economy</td>
<td>0.32</td>
</tr>
<tr>
<td>( S''(1) )</td>
<td>Investment adjustment cost</td>
<td>2.48</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Entrepreneur surviving rate</td>
<td>0.9728</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Bankruptcy monitoring cost</td>
<td>0.12</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Variance of log idiosyncratic shock</td>
<td>0.28</td>
</tr>
<tr>
<td>( F(\pi) )</td>
<td>Quarterly business failure rate</td>
<td>3%( \times )4</td>
</tr>
<tr>
<td>( C/Y )</td>
<td>Household consumption share</td>
<td>0.6</td>
</tr>
<tr>
<td>( C_e/Y )</td>
<td>Entrepreneur consumption share</td>
<td>0.01</td>
</tr>
<tr>
<td>( G/Y )</td>
<td>Government spending share</td>
<td>0.2</td>
</tr>
<tr>
<td>( K/N )</td>
<td>Capital to net worth ratio</td>
<td>2</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>Elasticity of nominal interest rate to inflation</td>
<td>1.668</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>Elasticity of nominal interest rate to output gap</td>
<td>0.144</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>Persistence in nominal interest rate setting</td>
<td>0.928</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Persistence in government spending</td>
<td>0.945</td>
</tr>
<tr>
<td>( \rho_\theta )</td>
<td>Persistence in preference shock</td>
<td>0.7</td>
</tr>
<tr>
<td>( \rho_\nu )</td>
<td>Persistence in entrepreneurs’ surviving rate</td>
<td>0.8</td>
</tr>
</tbody>
</table>

7.2. **Solution method of the model with a zero lower bound (ZLB) constraint.** To mimic the financial crisis, we assume that at time zero the economy is hit by two shocks: (1) A temporary 10% decrease in the surviving rate of entrepreneur \( \nu_t \); (2) A temporary 70% decrease in the preference factor \( \varrho_t \). And these shocks have the nominal interest rate bind during time period \([t_1, t_2]\).

Use Uhlig (1997)’s notation, the model we are dealing with can be written as:

\[
Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0 \tag{7.1}
\]

\(^8\)The reasons why we use two shocks is that (1) it’s motivated by the fact that we do see the decrease of the net worth of entrepreneur in the economy during the crisis; (2) in order to let the nominal interest rate hit the zero lower bound, only one shock is needed to be unreasonable large.
\[ \mathbb{E}_t (Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t) = 0 \quad (7.2) \]
\[ z_{t+1} = Nz_t + \epsilon_{t+1}, \quad \text{with} \quad \mathbb{E}_t \epsilon_{t+1} = 0 \quad (7.3) \]

Then according to the different scenarios we have, we can divide the time span into three pieces and solve the model piece-wisely. The general strategy we follow here is to get the solution for the model from time \( t_2 + 1 \) onward and then use backward induction to solve the model recursively to get the solution from time 0 to time \( t_2 \).

7.2.1. **When \( t \geq t_1 + 1 \):** Within this period, the Taylor rule applies and zero lower bound (ZLB) for nominal interest rate does not bind, hence the model can be solved using the methods of undetermined coefficients as in Uhlig (1997). And the solution is given by:
\[ x_t = Px_{t-1} + Qz_t \quad (7.4) \]
\[ y_t = Rx_{t-1} + Sz_t \quad (7.5) \]

7.2.2. **When \( t_1 \leq t \leq t_2 \):** Within this period ZLB binds for nominal interest rate and Taylor rule no longer applies, the monetary authority simply set the net nominal interest rate to be zero. Then equilibrium conditions within this period is given as follows:
\[ Ax_t + Bx_{t-1} + Cy_t + Dz_t = U \quad (7.6) \]
\[ \mathbb{E}_t (Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t) = 0 \quad (7.7) \]
where all the elements in \( U \) is zero except for the column corresponding with the Taylor rule for nominal interest setting and in our case this element should be \( -R \) (\( R = 1/\beta - 1 \) is the steady state value for nominal interest rate). And moreover, the \( B, C \) and \( D \) matrices should be modified in some way so as to have \( R^n_t = 0 \) during this time period. Let’s assume the solution is given in the following form:
\[ x_t = P_t x_{t-1} + Q_t z_t + V_t \quad (7.8) \]
\[ y_t = R_t x_{t-1} + S_t z_t + W_t \quad (7.9) \]

Then use (7.8) and (7.9) to substitute \( x_{t+1}, y_{t+1} \) in equation (7.7) and use (7.3) to get rid of the expectation, we have the following relations:
\[
\begin{bmatrix}
    FP_{t+1} + G + J R_{t+1} & K \\
    A & C
\end{bmatrix}
\begin{bmatrix}
    x_t \\
    y_t
\end{bmatrix}
= 
\begin{bmatrix}
    -H & -(F Q_{t+1} + J S_{t+1} + L) N - M \\
    -B & -D
\end{bmatrix}
\begin{bmatrix}
    x_{t-1} \\
    z_t
\end{bmatrix}
+ 
\begin{bmatrix}
    -F V_{t+1} - J W_{t+1} \\
    U
\end{bmatrix}
\quad (7.10)
\]

Compare the solution in (7.8) and (7.9) with (7.10), we can get recursive formula for the solution coefficients \( P_t, Q_t, R_t, S_t, V_t \) and \( W_t \):
\[
\begin{bmatrix}
    P_t & Q_t \\
    R_t & S_t
\end{bmatrix}
= 
\begin{bmatrix}
    FP_{t+1} + G + J R_{t+1} & K \\
    A & C
\end{bmatrix}^{-1}
\begin{bmatrix}
    -H & -(F Q_{t+1} + J S_{t+1} + L) N - M \\
    -B & -D
\end{bmatrix}
\begin{bmatrix}
    x_{t-1} \\
    z_t
\end{bmatrix}
+ 
\begin{bmatrix}
    -F V_{t+1} - J W_{t+1} \\
    U
\end{bmatrix}
\quad (7.11)
\]
and
\[
\begin{bmatrix}
    V_t \\
    W_t
\end{bmatrix}
= 
\begin{bmatrix}
    FP_{t+1} + G + J R_{t+1} & K \\
    A & C
\end{bmatrix}^{-1}
\begin{bmatrix}
    -F V_{t+1} - J W_{t+1} \\
    U
\end{bmatrix}
\quad (7.12)
\]
7.2.3. When $0 \leq t \leq t_1 - 1$: Within this time period the ZLB for nominal interest rate does not bind and Taylor rule applies. In particular, the equilibrium condition within this period is given as follows:

\[ Ax_t + B x_{t-1} + C y_t + D z_t = 0 \]  
\[ \mathbb{E}_t (F x_{t+1} + G x_t + H x_{t-1} + J y_{t+1} + K y_t + L z_{t+1} + M z_t) = 0 \]

And suppose the solution has the following form within this period,

\[ x_t = P_t x_{t-1} + Q_t z_t + V_t \]  
\[ y_t = R_t x_{t-1} + S_t z_t + W_t \]

Then same as in last section, we plug in the solution of $x_{t+1}$, $y_{t+1}$ as indicated in equation (7.15) and (7.16) into (7.14) and use (7.3) to get rid of the expectation. Therefore we can get the following equilibrium conditions:

\[
\begin{bmatrix}
FP_{t+1} + G + JR_{t+1} & K \\
A & C
\end{bmatrix}
\begin{bmatrix}
x_t \\
y_t
\end{bmatrix}
= 
\begin{bmatrix}
-H & -(FQ_{t+1} + JS_{t+1} + L)N - M \\
-B & -D
\end{bmatrix}
\begin{bmatrix}
x_{t-1} \\
z_t
\end{bmatrix}
+ 
\begin{bmatrix}
-FV_{t+1} - JW_{t+1} \\
0
\end{bmatrix}
\]

(7.17)

Compare (7.17) with equation (7.15) and (7.16), we can get the recursive formula for the solution coefficients:

\[
\begin{bmatrix}
P_t \\
R_t \\
S_t
\end{bmatrix}
= 
\begin{bmatrix}
FP_{t+1} + G + JR_{t+1} & K \\
A & C
\end{bmatrix}^{-1}
\begin{bmatrix}
-H & -(FQ_{t+1} + JS_{t+1} + L)N - M \\
-B & -D
\end{bmatrix}
\begin{bmatrix}
P_t \\
R_t \\
S_t
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
V_t \\
W_t
\end{bmatrix}
= 
\begin{bmatrix}
FP_{t+1} + G + JR_{t+1} & K \\
A & C
\end{bmatrix}^{-1}
\begin{bmatrix}
-FV_{t+1} - JW_{t+1} \\
0
\end{bmatrix}
\]

(7.18)

(7.19)

7.3. **Optimal Financial contract between entrepreneur and financial intermediaries.** The optimal financial contract may be characterized by a gross non-default loan rate $Z^j_t$ and a threshold value of the idiosyncratic shock $\overline{w}_{t+1}$ such that for values of the idiosyncratic shock greater than or equal to $\overline{w}_{t+1}$, the entrepreneur is able to repay the loan at the contractual rate $Z^j_t$, or more precisely, the financial intermediaries get $\overline{w}_{t+1} R^k_{t+1} Q_t K^j_t = Z^j_t B^j_t$, and entrepreneur $j$ keep the remaining return. When the idiosyncratic shock is less than $\overline{w}_{t+1}$, the entrepreneur $j$ cannot pay back the loan and hence declare bankruptcy. The financial intermediaries take over the project and in order to observe the realized return, they need to pay $\mu$ fraction of the return as monitoring cost.

7.3.1. **Derivation of optimal financial contract.** The returns for financial intermediaries and entrepreneur $j$ are summarized as follows:

\[
\text{Return of Financial Intemediaries} = \begin{cases} 
\overline{w}_{t+1} \mathbb{E}_t R^k_{t+1} Q_t K^j_t & \text{when } \omega_{t+1} \geq \overline{w}_{t+1} \\
(1 - \mu) \omega^j_{t+1} \mathbb{E}_t R^k_{t+1} Q_t K^j_t & \text{when } \omega_{t+1} < \overline{w}_{t+1}
\end{cases}
\]

(7.20)

and

\[
\text{Return of Entrepreneur } j = \begin{cases} 
(\omega_{t+1} - \overline{w}_{t+1}) \mathbb{E}_t R^k_{t+1} Q_t K^j_t & \text{when } \omega_{t+1} \geq \overline{w}_{t+1} \\
0 & \text{when } \omega_{t+1} < \overline{w}_{t+1}
\end{cases}
\]

(7.21)
Then the return of the project going to the financial intermediaries should be \( \Gamma(\omega_{t+1}) - \mu G(\omega_{t+1}) \mathbb{E}_t R_{t+1}^k Q_t K_t^j \) whereas the return going to entrepreneur \( j \) is \( (1 - \Gamma(\omega_{t+1})) \mathbb{E}_t R_{t+1}^k Q_t K_t^j \), and \( \Gamma(\omega_{t+1}), \mu G(\omega_{t+1}) \) are given as follows:

\[
\Gamma(\omega_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega f(\omega) d\omega + \omega_{t+1} \int_{\omega_{t+1}}^{\infty} f(\omega) d\omega
\]

and

\[
\mu G(\omega_{t+1}) = \mu \int_0^{\omega_{t+1}} \omega f(\omega) d\omega
\]

In order to get the optimal supply of the loan, financial intermediaries tend to maximize the return for entrepreneur \( j \). The problem becomes:

\[
\max_{k_t, \omega_{t+1}} \left( 1 - \Gamma(\omega_{t+1}) \right) \mathbb{E}_t R_{t+1}^k Q_t K_t^j
\]

s.t

\[
\left( \Gamma(\omega_{t+1}) - \mu G(\omega_{t+1}) \right) \mathbb{E}_t R_{t+1}^k Q_t K_t^j = (1 + R_t)(Q_t K_t^j - N_t^j)
\]

Introduce the risk premium \( s_t = \mathbb{E}_t R_{t+1}^k / (1 + R_t) \) and leverage ratio \( k_t = Q_t K_t^j / N_t^j \) and the above problem can be rewritten as:

\[
\max_{k_t, \omega_{t+1}} \left( 1 - \Gamma(\omega_{t+1}) \right) s_t k_t
\]

s.t

\[
\left( \Gamma(\omega_{t+1}) - \mu G(\omega_{t+1}) \right) s_t k_t = k_t - 1
\]

Let \( \lambda \) be the Lagrangian multiplier associated with budget constraint and therefore the first order conditions are:

\[
\omega_{t+1}: \quad \Gamma'(\omega_{t+1}) - \lambda (\Gamma(\omega_{t+1}) - \mu G(\omega_{t+1})) = 0
\]

\[
k_t: \quad [1 - \Gamma(\omega_{t+1}) + \lambda (\Gamma(\omega_{t+1}) - \mu G(\omega_{t+1}))] s_t - \lambda = 0
\]

\[
\lambda: \quad (\Gamma(\omega_{t+1}) - \mu G(\omega_{t+1})) s_t k_t = k_t - 1
\]

We assume the hazard rate \( h(\omega) \equiv \frac{f(\omega)}{1 - F(\omega)} \) has the property that \( \omega h(\omega) \) is increasing in \( \omega \), or

\[
\frac{\partial(\omega h(\omega))}{\partial \omega} > 0
\]

Hence there exists a global maximum point \( \omega^* \) for \( \Gamma(\omega) - \mu G(\omega) \) such that

\[
\Gamma'(\omega) - \mu G'(\omega) = (1 - F(\omega))(1 - \mu \omega h(\omega)) \leq 0 \quad \text{for} \quad \omega \leq \omega^*
\]

Moreover, we have the following implication:

\[
\Gamma' \omega G''(\omega) - \Gamma''(\omega) G'(\omega) = \frac{\omega h(\omega)}{d\omega} (1 - F(\omega))^2 > 0 \quad \text{for all} \quad \omega
\]

According to (7.29), we know that the share of return going to financial intermediaries, \( \Gamma(\omega) - \mu G(\omega) \), is increasing on \( (0, \omega^*) \) and decreasing on \( (\omega^*, \infty) \). And we restrict ourself to the case that \( 0 < \omega_{t+1} < \omega^* \) where we can have interior solution. And we would argue below that a sufficient condition to guarantee an interior solution is given by:

\[
s_t < \frac{1}{\Gamma(\omega^*) - \mu G(\omega^*)} \equiv s^*
\]
Now we are able to characterize the solution. From (7.26) we can get \( \lambda \) as a function of \( \omega t+1 \):

\[
\lambda(\omega t+1) = \frac{\Gamma'(\omega t+1)}{\Gamma'(\omega t+1) - \mu G'(\omega t+1)}
\]

(7.32)

We can determine that \( \lambda \) is increasing in \( \omega t+1 \in (0, \omega^*) \) using equation (7.29),

\[
\lambda'(\omega t+1) = \frac{\mu \left[ \Gamma''(\omega t+1) G''(\omega t+1) - \Gamma''(\omega t+1) G'(\omega t+1) \right]}{[\Gamma'(\omega t+1) - \mu G'(\omega t+1)]^2} > 0
\]

(7.33)

Taking limit of \( \lambda(\omega t+1) \) we obtain:

\[
\lim_{\omega t+1 \to 0} \lambda(\omega t+1) = 1, \quad \lim_{\omega t+1 \to \omega^*} \lambda(\omega t+1) = +\infty
\]

(7.34)

From (7.27) we can get:

\[
s_t = \rho(\omega t+1)
\]

(7.35)

where,

\[
\rho(\omega) \equiv \frac{\lambda(\omega)}{1 - \Gamma(\omega) + \lambda(\Gamma(\omega) - \mu G(\omega))}
\]

And hence the first order derivative for \( \rho \) has the following property, for \( \omega t+1 \in (0, \omega^*) \):

\[
\rho'(\omega t+1) = \rho(\omega t+1) \frac{\lambda(\omega t+1)}{\lambda(\omega t+1)} \left( \frac{1 - \Gamma(\omega t+1)}{1 - \Gamma(\omega t+1) + \lambda(\Gamma(\omega t+1) - \mu G(\omega t+1))} \right) > 0
\]

(7.36)

Then by using (7.34) we can get the following limits for \( \rho \):

\[
\lim_{\omega t+1 \to 0} \rho(\omega t+1) = 1, \quad \lim_{\omega t+1 \to \omega^*} \rho(\omega t+1) = \frac{1}{\Gamma(\omega^*) - \mu G(\omega^*)} \equiv s^*
\]

(7.37)

And from here, it validates our claim of the sufficient condition for the interior solution stated in equation (7.31). Due to the monotonicity of \( \rho(\cdot) \) function, we can inverse equation (7.35) to get:

\[
\omega t+1 = \omega(s_t), \text{ where } \omega'(s_t) > 0 \text{ for } s \in (1, s^*)
\]

(7.38)

And from first order condition (7.28), we get for a given cut-off \( \omega \in (0, \omega^*) \) it implies a unique leverage ratio which is given by:

\[
k_t = \Psi(\omega t+1)
\]

(7.39)

where,

\[
\Psi(\omega) \equiv 1 + \frac{\lambda(\Gamma(\omega) - \mu G(\omega))}{1 - \Gamma(\omega)}
\]

and the one-to-one mapping from \( \omega t+1 \) to \( k_t \) is justified by:

\[
\Psi'(\omega) = \frac{\lambda'(\omega)}{\lambda(\omega)} (\Psi(\omega) - 1) + \frac{\Gamma'(\omega)}{1 - \Gamma(\omega)} \Psi(\omega) > 0 \text{ for } \omega t+1 \in (0, \omega^*)
\]

(7.40)

Lastly, combining equation (7.38) with equation (7.39) we have the optimal supply of loan for entrepreneur:

\[
k_t = \psi(s_t)
\]

(7.41)

where,

\[
\psi'(s_t) > 0 \text{ for } s_t \in (1, s^*)
\]
7.3.2. Distribution assumption on idiosyncratic shock $\omega$. Following from BGG (1999), we assume that $\omega$ is log-normally distributed and in order to ensure $E(\omega) = 1$, we assume:

$$\ln(\omega) \sim N \left( -\frac{\sigma^2}{2}, \sigma^2 \right)$$  \hspace{1cm} (7.42)

And under these assumptions, we obtain:

$$\Gamma(\omega) = \Phi(z - \sigma) + \omega [1 - \Phi(z)] \hspace{1cm} (7.43)$$

and

$$\Gamma(\omega) - \mu G(\omega) = (1 - \mu) \Phi(z - \sigma) + \omega [1 - \Phi(z)] \hspace{1cm} (7.44)$$

where $z \equiv (\ln(\omega) + 0.5\sigma^2) / \sigma$

7.4. Equilibrium Conditions. The following section is characterizing equilibrium conditions and log-linearized equilibrium conditions.

7.4.1. Equilibrium system. The representative household maximizes the life-time utility subject to the budget constraint. And hence we can get the following equilibrium conditions.

Intra-temporal labor choice equation:

$$(1 - \gamma) C_t = \gamma W_t (1 - H_t) \hspace{1cm} (7.45)$$

Consumption Euler equation:

$$E_t \beta \frac{\varrho_{t+1}}{\varrho_t} \left( 1 + \frac{R^n_t}{1 + \pi_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right)^{\gamma - \gamma \sigma - 1} \left( \frac{1 - H_{t+1}}{1 - H_t} \right)^{(1-\gamma)(1-\sigma)} = 1 \hspace{1cm} (7.46)$$

Fisher equation:

$$1 + R_t = E_t \left( \frac{1 + R^n_t}{1 + \pi_{t+1}} \right) \hspace{1cm} (7.47)$$

The supply of capital:

$$E_t \frac{R^k_{t+1}}{1 + R_t} = s \left( \frac{N_t}{Q_t K_t} \right) \text{ where } s'(\cdot) < 0 \hspace{1cm} (7.48)$$

The demand of labor from household:

$$W_t = MC_t (1 - \alpha) \Omega \frac{Y_t}{H_t} \hspace{1cm} (7.49)$$

The demand of labor from entrepreneur:

$$W^e_t = MC_t (1 - \alpha) (1 - \Omega) \frac{Y_t}{H^e_t} \hspace{1cm} (7.50)$$

The demand of capital:

$$Q_{t-1} R^k_t = MC_t \alpha \frac{Y_t}{K_{t-1}} + (1 - \delta) Q_t \hspace{1cm} (7.51)$$

Marginal cost:

$$MC_t = \frac{1}{A_t} \left( \frac{Q_{t-1} R^k_t - (1 - \delta) Q_t}{\alpha} \right)^{\alpha} \left( \frac{1}{1 - \alpha} \left( \frac{W_t}{\Omega} \right)^{\Omega} \left( \frac{W^e_t}{1 - \Omega} \right)^{1-\Omega} \right)^{1-\alpha} \hspace{1cm} (7.52)$$
The evolution of capital:
\[ K_t = I_t + (1 - \delta)K_{t-1} - S \left( \frac{I_t}{I_{t-1}} \right) I_t \]  \hfill (7.53)

The evolution of entrepreneurs’ net worth:
\[ N_t = v_t V_t + W_t^e \]  \hfill (7.54)

where
\[ V_t = R_t^k Q_{t-1} K_{t-1} - \left( (1 + R_{t-1})L_{t-1} + \mu \int_{0}^{\omega_t} \omega f(\omega) d\omega R_t^k Q_{t-1} K_{t-1} \right), \]  \hfill (7.55)

\[ L_t = Q_t K_t - N_t \]  \hfill (7.56)

The consumption of entrepreneurs:
\[ C_t^e = (1 - v_t)V_t \]  \hfill (7.57)

Zero profit condition for financial intermediaries:
\[ [(1 - F(\omega_t))\omega_t + (1 - \mu) \int_{0}^{\omega_t} \omega f(\omega) d\omega] R_t^k Q_{t-1} K_{t-1} = (1 + R_{t-1})L_{t-1} \]  \hfill (7.58)

The price of capital:
\[ \mathbb{E}_t \left\{ \Lambda_{0,t} \left[ \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \right) \frac{I_t}{I_{t-1}} \right] Q_t - 1 \right\} + \beta \Lambda_{0,t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}^2}{I_t^2} Q_t \right\} = 0 \]  \hfill (7.59)

Phillips curve:
\[ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{M} C_t \]  \hfill (7.60)

Aggregate resource constraint:
\[ C_t + C_t^e + I_t + G_t + \mu \int_{0}^{\omega_t} \omega f(\omega) d\omega R_t^k Q_{t-1} K_{t-1} = Y_t \]  \hfill (7.61)

Taylor Rule:
\[ R_t^n - R = \rho_R (R_{t-1}^n - R) + \frac{1 - \rho_R}{\beta} (\phi_1 \pi_t + \phi_2 \hat{Y}_t) + \epsilon_{Rn,t} \]  \hfill (7.62)

Exogenous process for government spending:
\[ \hat{G}_{t+1} = \rho \hat{G}_t + \epsilon_{G,t+1} \]  \hfill (7.63)

Exogenous process for preference shock:
\[ \hat{\varrho}_{t+1} = \rho_\varrho \hat{\varrho}_t + \epsilon_{\varrho,t+1} \]  \hfill (7.64)

Exogenous process for entrepreneurs’ surviving rate:
\[ \hat{v}_{t+1} = \rho_v \hat{v}_t + \epsilon_{v,t+1} \]  \hfill (7.65)
7.4.2. Log-linearized equilibrium system.

\[ \dot{\hat{k}}_t = \delta \hat{i}_t + (1 - \delta)\hat{k}_{t-1} \]  
\[ \dot{\hat{q}}_t = S''(1)\mathbb{E}_t[-\hat{i}_{t-1} + (1 + \beta)\hat{i}_t - \beta\hat{i}_{t+1}] \]  
\[ \mathbb{E}_t \left[ \dot{\hat{q}}_{t+1} - \dot{\hat{q}}_t + \beta (R^n_t - R) - \pi_{t+1} + (\gamma - \gamma \sigma - 1)(\hat{C}_{t+1} - \hat{C}_t) \right] \]  
\[ -(1 - \gamma)(1 - \sigma) \frac{H}{1 - H} (\dot{\hat{H}}_{t+1} - \dot{\hat{H}}_t) = 0 \]  
\[ R_t - R = R^n_t - R - \frac{1}{\beta} \mathbb{E}_t \pi_{t+1} \]  
\[ \tilde{W}_t = \hat{C}_t + \frac{H}{1 - H} \dot{\hat{H}}_t \]  
\[ \mathbb{E}_t \tilde{R}^k_{t+1} - \beta (R_t - R) = \frac{S'(N/QK)N/QK}{S(N/QK)} (\hat{\pi}_t - \dot{\hat{q}}_t - \dot{\hat{k}}_t) \]  
\[ \hat{\dot{\hat{R}}}_t = (1 - \epsilon)(\hat{\dot{Y}}_t + \hat{M} \dot{C}_t - \dot{\hat{k}}_{t-1}) + \frac{1 - \delta}{R^k} \dot{\hat{q}}_t - \dot{\hat{q}}_{t-1} \]

\[ \hat{M} \dot{C}_t = \frac{R^k}{R^k - (1 - \delta)} \alpha (\hat{R}^k_t + \dot{\hat{q}}_{t-1}) - \frac{1 - \delta}{R^k - (1 - \delta)} \alpha \dot{\hat{q}}_t - \dot{\hat{A}}_t \]  
\[ + (1 - \alpha) \Omega \hat{W}_t + (1 - \alpha)(1 - \Omega) \hat{W}_t^e \]  
\[ \hat{\pi}_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{M} \dot{C}_t \]  
\[ \hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{C^e}{Y} \hat{C}_t^e + \frac{I}{Y} \dot{\hat{H}}_t + \frac{G}{Y} \dot{\hat{C}}_t + \frac{\mu R^k K \varpi^2 f(\varpi)}{Y} \hat{\dot{\omega}}_t \]  
\[ + \frac{\mu R^k K}{Y} \int_0^{\varpi} \omega f(\omega) d\omega (\hat{R}^k_t + \dot{\hat{q}}_{t-1} + \dot{\hat{k}}_{t-1}) \]  
\[ \hat{\dot{C}}_t^e = \hat{V}_t - \frac{\gamma}{1 - \gamma} \dot{\hat{v}}_t \]  
\[ \hat{\dot{\hat{\pi}}}_t = \frac{\nu V}{N} \dot{\hat{v}}_t + \frac{\nu V}{N} \dot{\hat{V}}_t + \frac{W^e}{N} \dot{\hat{W}}_t^e \]  
\[ \dot{\hat{V}}_t = \left( \frac{R^k K}{V} - \text{coeff2} \right) (\hat{R}^k_t + \dot{\hat{q}}_{t-1} + \dot{\hat{k}}_{t-1}) - \text{coeff1} \varpi_t - (1 + R) \frac{L}{V} \hat{L}_{t-1} - \frac{L}{V} (R_{t-1} - R) \]  

where,

\[ \text{coeff1} \equiv \mu \frac{R^k K}{V} \varpi^2 f(\varpi) \]  
\[ \text{coeff2} \equiv \mu \int_0^{\varpi} \omega f(\omega) d\omega \frac{R^k K}{V} \]
\[ [(1 - F(\bar{\omega}))(\bar{\omega} - \mu \omega^2 f(\bar{\omega})) R^k \hat{\omega}_t = \text{coeff3}(\hat{R}^k_t) + \dot{q}_{t-1} + \dot{k}_{t-1}) + (1 + R) \frac{L}{K} \hat{L}_{t-1} + \frac{L}{K}(R_{t-1} - R) \] 

(7.81)

where,

\[ \text{coeff3} = - \left[ (1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega \right] R^k \]

\[ \hat{L}_t = \frac{QK}{L}(\dot{q}_t + \dot{k}_t) - \frac{N}{L}\dot{N}_t \]

\[ R^n_t - R = \rho_R(R^n_{t-1} - R) + \frac{1 - \rho_R}{\beta}(\phi_1 \pi_t + \phi_2 \dot{Y}_t) + \epsilon_{Rn,t} \]

(7.82)

\[ \hat{G}_{t+1} = \rho \hat{G}_t + \epsilon_{G,t+1} \]

(7.84)

\[ \hat{\theta}_{t+1} = \rho_b \hat{\theta}_t + \epsilon_{\theta,t+1} \]

(7.85)

\[ \hat{v}_{t+1} = \rho_v \hat{v}_t + \epsilon_{v,t+1} \]

(7.86)

\[ \hat{A}_{t+1} = \rho_A \hat{A}_t + \epsilon_{A,t+1} \]

(7.87)
Figure 1. Positive Technology Shock
Figure 2. Positive Preference Shock

![Graph showing the relationship between net worth, consumption, marginal cost, inflation, investment, lending, labor, entrep wage, wage, capital return, threshold ogm, price of capital, risk premium, output, asset, and capital rate.]
Figure 3. Negative Monetary Policy Shock
Figure 4. Positive Government Spending Shock
Figure 5. Present Value Fiscal Multiplier
FIGURE 6. The Role of Financial Friction
Figure 7. The Role of ZLB for Nominal Interest Rate
Figure 8. Personal Savings Rate in US

Figure 9. Federal Funds Rate in US