Insider Rates vs Outsider Rates in Bank Lending

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ABSTRACT:

When an inside lender has private information about a firm, there is an information asymmetry among lenders. The effect of this asymmetry on the relative rates for firms borrowing from an insider or outsider is not well established. In the Sharpe (1990) model, an inside lender with private information competes against an outside lender with public information. In a correction to the model, von Thadden (2004) shows that there is no pure-strategy equilibrium due to the “winner’s curse.” Because the outsider must compensate for the winner’s curse to break even, one might expect outsider rates to be higher than insider rates. However, due to the winner’s curse, the expected interest rates for each firm type are higher at the inside lender. In this paper, I show analytically that the inside rate can be either higher or lower depending on the parameter space. This result is limited because it depends somewhat on an assumption about which lender wins in the event of a tie bid. Under the assumption that the lenders split the firms in a tie bid, an analytical solution shows that the inside (outside) rate is higher for low (high) quality borrower pools. A numerical analysis further investigates the parameter space where the outside rate is higher than the inside rate. The results show that the magnitude of the difference between the outside and inside rate is greatest for borrower pools in which the probability of success for a good firm is significantly higher than the probability of success for the average firm.

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1. Introduction

Sometimes banks have different amounts of information about a firm when competing for a loan. How does this private information affect the competition among lenders? Where should we expect the interest rates to be higher? This question is important for comparing the informed bidding of insider banks with the uninformed bidding of outsider banks. Intuition may suggest that the expected interest rates will be higher at an outside lender due to adverse selection. However, this paper shows that this is not always the case in a standard framework of insider vs. outsider lending. Depending on borrower characteristics, the expected interest rates at an inside lender may be higher or lower than the expected interest rates at an outside lender.

The standard theoretical framework used in this paper is the Sharpe (1990) model of inside and outside banks. In the Sharpe model, a bank learns private information about a firm through the process of lending. This bank becomes the “inside” lender in subsequent bidding on a new loan for the firm, whereas all other lenders without private information are the “outside” lenders. In the second period of the model, when banks compete for the firm’s loan, the inside lender has an informational advantage over all the outside lenders. A correction to the Sharpe paper by von Thadden (2004) shows that there is no pure-strategy equilibrium. Both the inside and outside bank offer rates from mixed strategies and the firm chooses the lowest offer.1

In this model of inside vs. outside lending, the outsider faces a clear winner’s curse problem. This problem is due to the adverse selection present in the probability of the outside bank winning the bid. If both banks were equally informed, each bank would win the same share of bad firms. However, in this model, the outside lender has an informational disadvantage. Even though the outsider chooses a mixed strategy so that it has some possibility of winning a good firm, the informational disadvantage increases the probability of the outsider winning a bad firm. I confirm analytically that the outside

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1 This is a common-value first-price closed-bid auction. There are two bidders, one with private information and one with public information. Although Sharpe allows for the outside bank to receive a weaker version of the insider’s signal, I analyze the case where the outside bank has only public information.
bank wins a greater proportion of bad firms relative to the proportion of bad firms in the total pool of firms.

The winner’s curse would seem to imply that the expected interest rate at the outside bank is higher. If the outside bank wins more bad firms and bad firms pay a higher rate, the comparison of insider vs. outsider rates seems clear. Von Thadden (2004) states that the prediction of the model is a higher interest rate at the outsider. However, this paper shows that this conclusion is premature. Although the outside bank wins a greater proportion of bad firms, the expected interest rate for each firm type is higher at the inside bank. Combining the probabilities of winning and the expected interest rates for each firm reveals a surprising result. This paper shows analytically that the expected interest rate at the inside bank is sometimes higher than the expected interest rate at the outside bank.

This paper also points out that the result of the model for expected interest rates is dependent on the assumption that tie bids go to the insider. This assumption would appear to be unimportant; however, the bidding functions of both firms have a point mass at the highest rate. Therefore, whenever the offers are both at the highest rate, the loan goes to the inside lender. This is an important factor in causing the inside rate to be higher than the outside rate. Under an alternative assumption that the outside bank wins in a tie, the inside rate is always lower.

Under the assumption that the lenders split the firms in a tie, the predictions of the model largely depend on the probability of success of a good firm and the probability of success for the average firm. This paper shows analytically that the inside rate is higher than the outside rate in low-quality borrower pools and the outside rate is higher than the inside rate for high-quality borrower pools. In the parameter space where the outside rate is higher, the paper also investigates the magnitude of the difference between the outside and inside rate. The numerical results show that the difference is largest for a borrower pool in which the probability of success of a good firm is large relative to the probability of success of the average firm.

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2 Von Thadden incorrectly states that “Consistent with the prediction of the present analysis, [Degryse and Van Cayseele (2000)] conclude that ‘some firms occasionally switch to an outside bank (MAIN=0), and the outside bank charges a higher interest rate, because it takes into account a winner’s curse effect.’” (p12)
The remainder of the paper is laid out as follows. Section 2 provides a brief overview of the model. Section 3 provides the theoretical predictions of the model for the probabilities of each lender winning and the expected inside and outside rates conditional on firm type. Section 4 analyzes the complete expected interest rates analytically and discusses the sensitivity of the results to the assumption about tie bids. Under a split tie-bid assumption, Section 5 first analyzes analytically the sign of the difference between the inside and outside rate, and then, analyzes numerically the magnitude of the difference between the inside and outside rate when the outside rate is higher. Section 6 concludes.

2. Model

In the Sharpe model, there are two qualities of firms, \( q = L, H \), with probabilities of project success \( p_L < p_H \). The proportion of high quality firms is \( \theta \). For the entire pool of borrowers, the expected probability of default is \( p = \theta p_H + (1 - \theta) p_L \). Banks all have this same public information when bidding for firm loans in period 1. After the bidding process in the first period, one bank makes a loan to the firm and becomes the “inside” bank. This inside bank receives a private signal \( \gamma \) about the quality of the firm, based on the firm’s performance on the first loan. If the firm succeeds on the first loan, the inside bank receives a signal of \( S \), and if the firm fails on the first loan, the inside bank receives a signal of \( F \). This private information received by the inside bank allows the inside bank to distinguish between “good” firms (an \( S \) signal) and “bad” firms (an \( F \) signal). The signals are correlated with the underlying quality of the firm, such that the probability of success conditional on these signals follows the order of \( p(S) > p > p(F) \). This results in the corresponding interest rate ordering of \( r_S < r_p < r_F \). The “outside” banks are the banks that did not lend to the firm and did not receive the private signal. In the second stage of the model, the bidding is between the inside bank and an outside bank. The inside bank conditions its bid on its private signal, while the outside bank only has public information.
Von Thadden (2004) shows that the inside bank and outside bank both choose a mixed strategy for their bids. My analysis is based on Proposition 2 (p17) of von Thadden (2004):

**Proposition 2.** “The inside bank’s equilibrium strategy is to offer \( r(F) = r_{p} \) with certainty and is an atomless distribution on \([r_{p}, r_{f}]\) for \( \gamma = S \), with density

\[
h^{S}_{i}(r) = \frac{p(S)(1 + r_{p}) - (1 + r)}{(p(S)(1 + r) - (1 + r))}.
\]

The outside bank’s equilibrium strategy has a point mass of \( 1 - p(S) \) at \( r = r_{e} \) and an atomless distribution on \([r_{p}, r_{f}]\) with density \( h_{o}(r) = p(S)h^{S}_{i}(r) \).”

The inside bank has different bidding functions for each firm type. For S firms, the inside bank bids from \( h^{S}_{i} \), a distribution of rates \( r \in [r_{p}, r_{f}] \). Note that the possible offer rates from \( h^{S}_{i} \) are all greater than \( r_{s} \). Because the good signal about the firm is not public information, the firm pays more than \( r_{s} \). Good firms will never receive the fully competitive rate of \( r_{s} \). For F firms, the inside bank bids \( r_{f} \), which is the fully competitive rate. The inside lender cannot extract rents from the worse of the two types.

The outside bank has the same bidding function for all firms. The outside bank does not know the firm types, so it must offer the same distribution of rates to all firms. However, the outside bank does know that the inside bank has private information. In equilibrium, the outside bank’s optimal bidding is a mixed strategy. This prevents the inside bank from undercutting the outside bank on all good firms. The outside bank bids from \( h^{S}_{i} \) with probability \( p(S) \) and bids \( r_{f} \) with probability \( 1 - p(S) \).

The important part of this model is the competition between the insider and outsider. Because the inside bank has different bidding functions for the two firm types, the competition can be separated by firm type. Consider first the S firms. The inside bank always makes an offer from \( h^{S}_{i} \) to an S firm, whereas the outside bank makes an offer from \( h^{S}_{i} \) with a probability \( p(S) \) or an offer of \( r_{f} \) with probability \( 1 - p(S) \). If the
outside bank makes an offer from $h^S_i$, then this is a competitive offer. The inside bank will only win the bid with probability $1/2$ and the outside bank will win with probability $1/2$. However, if the outside bank offers $r_F$, then this is not a competitive offer and the inside bank always wins.

Now consider the F firms. The inside bank always makes an offer of $r_F$ to an F firm. Again, the outside bank makes an offer from $h^S_i$ with a probability $p(S)$ or an offer of $r_F$ with probability $1 - p(S)$. If the outside bank makes an offer from $h^S_i$, then this is a negative profit offer. The outside bank always wins in this case. Such bidding is profit maximizing, because the outside bank must accept the possibility of some of these losses in order to be competitive for the good firms. Von Thadden shows that the outside bank makes zero profit in expectation. The positive profits earned on good firms are balanced out by the negative profits earned on bad firms. If the outside bank offers $r_F$, then this is a competitive offer. The banks make the same offer in this case, and, by assumption, the bad firms stay with the inside bank.

### 3.1. Probability of Winning

Proposition 2 establishes the bidding functions of both the inside and outside bank. The inside bank has two different bidding functions for each firm type, while the outside bank has only one bidding function. The contribution of this paper is to analyze the competition between these bidding functions. This section analyzes the probabilities of a particular lender winning a bid.

The probabilities of winning can first be analyzed for each of the two firm types. For S firms, if the outside bank makes an offer from $h^S_i$, the banks split the firms in expectation, but if the outside bank makes an offer of $r_F$, the inside bank always wins. This implies the following probabilities of winning for the inside and outside bank, conditional on the firm being an S firm:
\[
\text{Prob}_i(win \mid S) = p(S)\left(\frac{1}{2}\right) + (1 - p(S)) \\
\text{Prob}_o(win \mid S) = p(S)\left(\frac{1}{2}\right).
\]

It is already clear that the inside bank will win more of the \( S \) firms than the outside bank. This is the first step toward identifying the winner’s curse.

For \( F \) firms, if the outside bank makes an offer from \( h^S_i \), then the outside bank always wins the bid (at a loss). Sharpe assumes that if the banks make the same offer, then firms stay with the insider. Therefore, if the outside bank offers \( r_F \), then the inside bank always wins. This implies the following probabilities of winning for the inside and outside bank, conditional on the firm being an \( F \) firm:

\[
\text{Prob}_i(win \mid F) = 1 - p(S) \\
\text{Prob}_o(win \mid F) = p(S).
\]

Basically, the insider wins if the outsider does not bid from \( h^S_i \) and the outsider wins if it does bid from \( h^S_i \). Note that the outsider may or may not win more of the \( F \) firms, depending on the level of \( p(S) \).

These probabilities can now be combined by using the probabilities of each firm type. The probabilities of each firm type depend simply on the probabilities of succeeding or failing in repaying the first period loan. A firm repays the first period loan with a probability of \( p \) and fails to repay the loan with a probability of \( 1 - p \). Therefore, a firm is the \( S \) type with probability \( p \) and an \( F \) type with probability \( 1 - p \).

We now have both the probabilities of winning conditional on firm type as well as the probabilities of each firm type. These can be combined to specify the probabilities of each lender winning without conditioning on firm type:
The winner’s curse implies that the outside bank wins more than its fair share of $F$ firms. The composition of $S$ and $F$ firms at the insider and outsider conditional on winning can be compared with the proportion of $S$ to $F$ firms in the total firm pool. The proportion of $S$ to $F$ firms in the total firm pool is $\frac{p}{1-p}$. If the insider and outsider split the firms evenly, then this would be the proportion of $S$ to $F$ firms at both the insider and outsider. But this is not the case.
**Corollary 1:**

The winner’s curse faced by the outside bank affects the composition of firm types at the insider and outsider:

\[
\frac{\text{Prob}_o(\text{win } S)}{\text{Prob}_o(\text{win } F)} < \frac{\text{Prob}(S)}{\text{Prob}(F)} < \frac{\text{Prob}_i(\text{win } S)}{\text{Prob}_i(\text{win } F)}.
\]

The insider wins a greater proportion of S firms than are in the pool and the outsider wins a greater proportion of F firms than are in the pool.

**Proof:**

The proportion of good firms to bad firms in the total firm pool is 
\[
\frac{\text{Prob}(S)}{\text{Prob}(F)} = \frac{p}{1-p}.
\]

For the outside bank, the probability of winning an S firm relative to the probability of winning an F firm is
\[
\frac{\text{Prob}_o(\text{win } S)}{\text{Prob}_o(\text{win } F)} = \frac{p}{1-p} \left( \frac{p(S)(1/2)}{p(S)} \right) = \frac{p}{1-p}.
\]

For an inside bank, the probability of winning an S firm relative to the probability of winning an F firm is
\[
\frac{\text{Prob}_i(\text{win } S)}{\text{Prob}_i(\text{win } F)} = \frac{p}{1-p} \left( \frac{p(S)(1/2) + (1-p)(S)}{1-p(S)} \right) > \frac{p}{1-p}.
\]

Corollary 1 shows that the outside bank wins a greater proportion of F firms than the inside bank. This winner’s curse is what would seem to imply that the expected interest rates at the outside bank are higher than the expected interest rates at the inside bank. The assumption behind this implication is that F firms are charged a higher rate than S firms. More F firms and a high rate for F firms would lead to higher interest rates at the outside bank. However, this is not necessarily the right conclusion. In order to compare the inside and outside rates, the analysis must go beyond the probabilities of winning to also consider the expected interest rates conditional on firm type. The expected interest rates for the firm types differ across the two banks, which complicates the problem. The interesting finding of the next section is that the interest rates may undo the apparent implication of the winner’s curse.
3.2. Expected Interest Rates

This section establishes the expected interest rate for each firm type at each lender. The expected interest rate for a given firm depends on the firm type and how many of the banks bid for the firm from $h_i^S$. The analysis in this section shows that the expected interest rate for each firm type is lower at the outside bank. This is a surprising result, given that the outside bank faces the winner’s curse problem.

If the firm quality signal were public information, then each firm would receive the same offer from each bank and the offer would perfectly match the firm’s credit risk. An $S$ firm would receive identical offers of $r_s = \frac{1 + F}{p(S)} - 1$ and an $F$ firm would receive identical offers of $r_F = \frac{1 + F}{p(F)} - 1$. In the model where the firm quality signal is not public information, the inside and outside bids are no longer necessarily identical nor are they necessarily perfectly matched to the firm’s credit risk. We will see that $S$ firms always pay a rate above $r_s$, which is to be expected when the signal is private. The lowest draw from $h_i^S$ is $r_p$, which shows that the $S$ firm will pay a minimum of the pooling rate for all firms. Surprisingly, we will see that $F$ firms sometimes pay a rate below $r_F$, thereby benefiting from the signal being private. However, the most important conclusion of this section is that the expected interest rates for both $S$ and $F$ firms are higher at the inside bank.

For a given firm in the private information model, there are three possible expected interest rates. The expected interest rate is determined by the number of banks bidding from $h_i^S$ for the firm. There are three possible expected rates, because there are three possible combinations of the number of banks bidding from $h_i^S$: both banks bid from $h_i^S$, one bank bids from $h_i^S$, or neither bank bids from $h_i^S$. If both banks bid from $h_i^S$, then the firm receives the minimum of two draws from the distribution, which is the “low rate.” If only one bank bids from $h_i^S$, then the firm receives a single draw from the
distribution, which is the “medium rate.” Finally, if neither bank bids from $h_i^S$, then the firm receives only $r_F$, which is the “high rate.” The three rates are shown below, followed by an explanation of which firm types receive which rate in expectation.

If both banks bid from $h_i^S$, the expected interest rate is

$$E(r \mid \text{two bidders}) = \int_{r_F}^{r_L} r \left[ 2(1 - H_i^S(r))h_i^S(r) \right] dr = r_L.$$  \hspace{1cm} \text{(Low Rate)}$$

This expected interest rate is only possible for $S$ firms. We know that the inside bank always bids for $S$ firms. If the outside bank also bids from $h_i^S$, then there are two competitive bids made for the loan. The expected rate for two bidders is based on the minimum of two draws from the $h_i^S$ distribution.

If only one bank bids from $h_i^S$, the expected interest rate is

$$E(r \mid \text{one bidder}) = \int_{r_F}^{r_L} r h_i^S(r) \, dr = r_M.$$  \hspace{1cm} \text{(Medium Rate)}$$

This rate can apply to either an $S$ firm or an $F$ firm. If the firm is an $S$ firm, then the insider always bids, but the outsider does not always bid. If the outside bank does not bid, this is the expected rate for an $S$ firm. If the firm is an $F$ firm, the inside bank does not bid, but sometimes the outside bank bids. This is the expected rate for an $F$ firm that receives an offer from the outsider.

If neither bank bids from $h_i^S$, then the expected interest rate is

$$E(r \mid \text{no bidder}) = r_F = r_H.$$  \hspace{1cm} \text{(High Rate)}$$

This expected interest rate is only possible for $F$ firms. The inside bank never bids from $h_i^S$ for $F$ firms, so the only possibility of an offer from $h_i^S$ is from the outside bank. These are the $F$ firms that do not get a bid from $h_i^S$ from the outside bank either.

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3 This is the general equation for the expected value of the minimum draw from two identical distributions.
It is clear that the expected interest rates can be ranked as \( r_S < r_L < r_M < r_H = r_F \). It is interesting to note that both banks can extract rents from the \( S \) firm, but neither bank can extract rents from the \( F \) firm. Using these rates, we can establish the relative rates at the inside lender, unconditional on firm type. But first, I will show the expected interest rate for each firm type at the inside and outside lender. The expected interest rates at the inside and outside lender are simply weighted averages of \( r_L, r_M, \) and \( r_H \), with the weights determined by the probability of the insider or outsider winning the bid. The surprising result is that the expected interest rate at the inside bank is higher than the outside bank, when we condition on firm type.

**Expected Interest Rates Conditional on Firm Type**

An \( S \) firm will always pay an expected interest rate of \( r_L \) or \( r_M \). Although these rates are lower than \( r_H \), note that they are greater than \( r_S \), the fully competitive rate for an \( S \) firm. This premium above \( r_S \) is the rent extracted from the \( S \) firm. Because the good signal is not public, the good firm cannot expect to pay the good-firm rate. The inside bank is able to extract rents from the good firm, because the inside bank has private information about the firm. Interestingly, the outside bank is also able to extract some rent from an \( S \) firm, when it succeeds in underbidding the insider. Because of the mixed strategies equilibrium, the outside bank sometimes wins the \( S \) firm at an expected interest rate of \( r_L \), which is greater than \( r_S \).

The \( S \) firms that borrow from the insider sometimes get a low rate in expectation and sometimes get a medium rate. They get a medium rate from the outsider when the outside bank does not make them an offer from \( h_t^S \). This is where the inside bank is able to extract the greatest rents. The outside bank, on the other hand, only wins an \( S \) firm when it bids from \( h_t^S \) and its draw from \( h_t^S \) is lower than the inside bank’s draw. Therefore, the inside bank only wins \( S \) firms at the low expected interest rate.
An $F$ firm will always pay an expected interest rate of $r_M$ or $r_H$. $F$ firms always pay the fair rate $r_H$ if they borrow from the insider, because this is all that the insider offers them. However, the outside bank does not know a firm’s type. Therefore, the outsider sometimes offers an $F$ firm a rate from $h_i^S$ and wins the firm at an expected rate of $r_M$. The outsider loses money on these transactions, but it must make bids from a competitive distribution in order to sometimes also win $S$ firms. Under the assumption that the insider always wins in a tie, the outside bank only wins $F$ firms when it makes an offer from $h_i^S$.

**Corollary 2:**
The winner’s curse faced by the outside bank affects the expected interest rate for each firm type at the insider and outsider:

\[ E_o(r \mid S) < E_i(r \mid S) \quad \text{and} \quad E_o(r \mid F) < E_i(r \mid F). \]

The expected interest rate for an $S$ firm at the insider is higher than the expected interest rate for an $S$ firm at the outsider. This is also true for $F$ firms.

**Proof:**
The expected interest rates at the inside and outside bank, conditional on the firm being an $S$ firm, are:

\[
E_i(r \mid S) = \frac{p(S)\left(\frac{1}{2}\right) r_L + (1 - p(S)) r_M}{p(S)\left(\frac{1}{2}\right) + (1 - p(S))} = \alpha r_L + (1 - \alpha) r_M
\]

\[
E_o(r \mid S) = r_L,
\]

where $\alpha = p(S)/(2 - p(S))$. 

The expected rate for an $S$ firm at the insider is a weighted combination of the low and medium rates, whereas $S$ firms that borrow from the outsider always get a low rate in expectation. Therefore, the expected rate for an $S$ firm at the insider is clearly higher than the expected rate for an $S$ firm at the outsider. It is important to note that this difference comes from the ability of the inside bank to lend to an $S$ firm at the medium rate. This additional ability to extract rent leads to a higher interest rate for $S$ firms at the inside bank.

The expected interest rates at the inside and outside bank, conditional on the firm being an $F$ firm, are:

\[ E_i(r \mid F) = r_{ff} \]
\[ E_o(r \mid F) = r_{mf}. \]

The expected rate for an $F$ firm at the insider is clearly higher than the expected rate for an $F$ firm at the outsider. This result derives from a very different reason than the same result for $S$ firms. Although the inside bank is able to extract greater rents from $S$ firms than the outside bank, the inside bank always offers an $F$ firm its fully competitive rate. The expected interest rate for an $F$ firm at the outside bank is lower, because the outside bank sometimes makes an offer to an $F$ firm below its fully competitive rate. Although the outside bank loses money ex post, this strategy is profit maximizing, because there is positive probability ex ante of the firm being an $S$ firm. (end of proof).

It was established in the previous section that the outside bank lends to a greater proportion of $F$ firms relative to the inside bank. This is due to the winner’s curse, which seems to indicate that the expected rates at the inside bank are lower. However, I have also shown that each firm type pays a higher rate when borrowing from the insider, which is also due to the winner’s curse. This complicates the apparent intuition of this model. Therefore, the winner’s curse does not necessarily imply that the expected interest rates at the inside bank are lower than the expected interest rates at the outside bank. The probabilities of winning and the expected rates must be combined to analyze the complete picture.
4.1. Probabilities and Interest Rates

Having established the probabilities of winning a bid and the expected rates for each bidding outcome, it is important to identify the overall expected interest rates at the inside and outside bank, without conditioning on firm type. The expected rates at the insider and outsider depend on the probabilities of each lender winning a bid and the interest rate on the bid. Combining the probabilities with the weights yields the complete picture of the winner’s curse and the interest rate differentials for each firm type. The expected interest rates at the inside and outside bank are:

\[
E_i(r) = \frac{p \left[ p(S) \left( \frac{1}{2} \right) r_L + (1 - p(S)) r_M \right] + (1 - p) \left[ (1 - p(S)) r_H \right]}{1 - p(S) \left( 1 - \frac{p}{2} \right)}
\]

\[
E_o(r) = \frac{p \left[ p(S) \left( \frac{1}{2} \right) r_L \right] + (1 - p) \left[ p(S) r_M \right]}{p(S) \left( 1 - \frac{p}{2} \right)}.
\]

The denominators in each equation are the total probabilities of the inside bank and outside bank winning the bid, respectively. These probabilities are the simplified expressions of Prob\(_i\)(win) and Prob\(_o\)(win), already shown above. The numerators reflect both the probabilities of each bank winning a bid as well as the expected interest rate of a loan conditional on winning. As before, the p and 1-p are the probabilities of a firm being an S firm or F firm, respectively. The terms in the brackets are the expected rates for each firm type (the conditional expected interest rates without weighting by the probability of winning the firm type).
The equations for $E_i(r)$ and $E_o(r)$ show the low rate, medium rate, and high rate contracts all together, along with the probability of each bank winning at each of these rates. The expected interest rate at the inside and outside bank will depend on the relative weighting of these three rates. The key to comparing the inside and outside bank rates is to begin with a comparison of which bank wins more of each of the expected interest rate contracts.

The insider and outsider expect to win the same number of low rate contracts. The low expected rate is only possible when both banks make an offer from $h^S_i$. When the outsider makes an offer from the $h^S_i$, then each bank has equal chance of winning the contract.

The insider expects to win a lesser number of medium rate contracts ($p(1 - p(S)) < (1 - p)p(S)$). Note that the firms receiving these contracts are very different. It is only $S$ firms that borrow from the inside bank at the medium rate and it is only $F$ firms that borrow from the outside bank at the medium rate. The inside bank makes a profit on the $S$ firms and the outside bank takes a loss on the $F$ firms.

The insider expects to win a greater number of high rate contracts. Sharpe and von Thadden both assume that firms which have identical offers will borrow from the insider. Therefore, firms that are offered the high rate by both lenders will borrow at the high rate from the insider.

The expected interest rates of the insider and outsider are quite complex, so to compare the expected rates, it is more straightforward to consider the following simplification of the probabilities:

$$\text{Prob}_i(\text{win}) = pp(S)\left(\frac{1}{2}\right) + p(1 - p(S)) + (1 - p)(1 - p(S))$$

$$\text{Prob}_o(\text{win}) = pp(S)\left(\frac{1}{2}\right) + (1 - p)p(S).$$

The probability terms are ordered to match the three expected interest rates $r_L$, $r_m$, and $r_H$. The first term in each equation is the probability of winning a low-rate contract, the second term in each equation is the probability of winning a medium-rate contract, and
the third term in the inside equation is the probability of winning a high-rate contract. It is clear in this form that, compared to the outsider, the insider wins the same number of low rate contracts, fewer medium rate contracts, and more high rate contracts.

It is difficult to compare the expected interest rates at the insider and outsider without using the actual sizes of \( r_L, r_M, \) and \( r_H \). The inside bank wins fewer of the medium rate contracts, but more of the high rate contracts; therefore, the difference between the inside and outside rate will generally depend on more than just the ranking of \( r_L, r_M, \) and \( r_H \). However, there is a special case, which can be solved analytically with just the ranking.

**Corollary 3a:**
The expected interest rate at the inside bank is *higher* than the expected interest rate at the outside bank when the inside bank wins at least as many bids as the outside bank (in expectation).

**Proof:**
In terms of the parameter space, this is the case when \( p(1 - p(S)) + (1 - p)(1 - p(S)) \geq (1 - p)p(S) \). This expression simplifies to the case where \( p(S) \leq 1/(2 - p) \). An example of this would be \( p = 0.5 \) and \( p(S) \leq 0.66 \). This case is unique, because it implies that the proportion of low-rate loans at the inside bank is equal to or less than the proportion of low-rate loans at the outside bank. Under this condition, we only need to consider the relative expected rates on the remaining medium and high rate contracts. The remaining loans at the inside bank include high-rate loans, whereas the remaining loans at the outside bank are only medium-rate loans. Therefore, in this case, the expected rate at the insider is *higher* than the expected rate at the outsider.

Corollary 3a shows a special case in which it is clear analytically that \( E_i(r) > E_o(r) \). In other words, the expected inside rate is higher than the expected outside rate. This contradicts the apparent intuition of the winner’s curse. Even though
the outside bank wins a greater proportion of bad firms, the outside bank earns less interest on both good firms and bad firms. Therefore, in this case, the winner’s curse does not imply a higher expected rate at the outsider.

The model can also be solved analytically in the case where the firm types converge in probability of success. This is the case where the insider and outsider rates converge.

**Corollary 3b:**
The expected interest rate at the inside bank is the same as the expected interest rate at the outside bank when \( p_H - p_L \to 0 \) or \( \theta \to 0 \) or 1.

Proof:
In this case, all the firms are identical in the limit, so all information is public. By definition, when firms approach an identical probability of success, then \( p(S) - p \to 0 \). The interest rate bidding distributions collapse to a single rate and all firms borrow at that rate. The two banks split the loans.4

Lastly, the model can be solved for an analytical case where the firm types completely diverge in probability of success.

**Corollary 3c:**
The expected interest rate at the inside bank is lower than the expected interest rate at the outside bank when \( p_H \to 1 \), \( p_L \to 0 \), and \( \theta \) does not approach 0 or 1.

Proof:
In this part of the parameter space, \( p(S) \to 1 \) and \( p \to \theta \). This part of the parameter space is unique, because the outsider is bidding from \( h_i^S \) with probability approaching 1, but the firm types are still unique. As always, the insider bids from \( h_i^S \)

---

4 Again, this result depends on the assumption about who wins in a tie. Technically, if the inside bank always wins in a tie, then the insider would win all bids in this case.
for good firms and bids \( r_f \) for bad firms. In this case, the insider never wins a bad firm and it never wins a good firm at the medium rate (because the outsider is bidding from \( h_1 \)). Therefore, the expected rate at the insider is \( r_L \). The outsider wins half the good firms at \( r_L \) and all of the bad firms at \( r_M \). Therefore, the expected outside rate is

\[
E_o(r) = \frac{\theta \left[ \left( \frac{1}{2} \right) r_L \right] + (1-\theta) [r_M]}{(1-\theta/2)}.
\]

At these limits, we also know that \( r_p \to \frac{1+r}{\theta} \) and \( r_f \to \infty \). This implies that \( h_1 \) does not collapse to a single point and \( r_M - r_L \) does not approach zero. Therefore, it is clear in this case that the inside rate is lower.

Corollaries 3a-c show analytically that the inside rate can be higher or lower than the outside rate in different parts of the parameter space. These results show that there is not a universal prediction of the Sharpe model for insider vs. outsider interest rates. The prediction of the model depends on the parameter space, or, in other words, the prediction depends on the characteristics of the borrower pool.\(^5\)

4.2. An Important Assumption

It still remains to interpret the result and understand why the inside rate is generally higher in this benchmark model. It is clear from the analytical result that the allocation of \( r_L, r_M \), and \( r_H \) is a primary determinant of the difference between the inside and outside rates. Even without knowing the magnitudes of these three rates, we have shown that the inside rate is higher whenever the inside bank wins at least as many loans as the outsider. It is immediately apparent from the probabilities of winning that high rate loans are only made by the inside bank. This obviously increases the expected rate at

\(^5\) Appendix A shows numerical results for the inside rate vs outside rate under the original assumption that the inside bank wins all of the high rate contracts.
the insider. Yet the reason for these high rate loans appearing at the inside bank is entirely exogenous to the model. One of the seemingly unimportant assumptions of the Sharpe (1990) model is that the inside bank wins the loan in the event of a tie bid. Altering this assumption can reverse the comparison of inside vs. outside rates entirely.

There are two reasons why the tie-bid assumption is usually innocuous, but these reasons do not apply in the current analysis. First of all, this assumption would not affect the results of a model with continuous bidding functions, because the probability of two offers being identical from a continuous distribution is zero. However, in the Sharpe model (with the von Thadden correction) the bidding functions are non-continuous. Both banks have a positive probability mass at the high rate, $r_f$. A second reason why this assumption would not seem to matter is that the high rate loans are zero-profit loans. Winning or losing one of these loans does not affect the expected profit of either bank, so it does not affect their bidding strategies. Again, this reasoning does not apply here, because we are comparing relative expected rates at the inside and outside bank, not their relative profits. When comparing the relative expected rates, the assumption about which bank gets the high rate loans does affect the results.

The previous analytical result of Corollary 3a is based on the assumption that the inside bank makes all the high rate loans. Notably, this result is driven by the fact that all the high rate loans are at the inside bank. This clearly indicates that the assumption is important. If we were to make the opposite assumption, then all the high rate loans would be allocated to the outside bank. This implies that the outside bank wins more medium rate contracts and more high rate contracts than the inside bank. Therefore, under this opposite assumption, the analytical solution is reversed and it applies to the entire parameter space. The expected interest rate at the inside bank is always lower than the expected interest rate at the outside bank.

The two extreme assumptions about tie bids lead to opposite conclusions. The original assumption is that the inside bank wins all tying bids. Analytically, this assumption was shown to lead to a case in which the expected interest rate at the inside bank is higher than the expected interest rate at the outside bank. The opposite assumption is that the outside bank wins all tying bids. Analytically, this assumption was shown to lead to the result that the expected interest rate at the inside bank is always
lower than the expected interest rate at the outside bank. These results indicate that the result of the model for the specific question of interest rates is not independent of a seemingly unimportant assumption.6

5. Predictions Under the Split-Tie Assumption

This section proceeds under the assumption that the inside and outside bank split the loans in the event of a tie bid. Under this neutral assumption, the inside bank and outside bank would lend to an equal number of high rate firms. Interestingly, this assumption also leads to a mixed result for the difference between inside and outside rates, even though it shifts some of the high rate contracts into the outside bank loan portfolio. In some parts of the parameter space, the inside rate is higher than the outside rate and, in other parts of the parameter space, the outside rate is higher than the inside rate. The sign of this difference and the size of this difference depends on the characteristics of the borrower pool.

Under the split-tie assumption, the only difference between the inside and outside bank is the number of medium rate firms. Both banks win the same number of low rate contracts and high rate contracts in expectation. The expected probabilities of winning the bid are the following:

\[
\text{Prob}(\text{win})_i = pp(S)\left(\frac{1}{2}\right) + p(1-p(S)) + (1-p)(1-p(S))\left(\frac{1}{2}\right)
\]

\[
\text{Prob}(\text{win})_o = pp(S)\left(\frac{1}{2}\right) + (1-p)p(S) + (1-p)(1-p(S))\left(\frac{1}{2}\right).
\]

6 Another alternative would be to treat the “high rate” offer instead as a “no offer.” This is done in Rajan (1992). In this case, only the low rate and medium rate contracts would be observed. The insider and outsider win the same number of low rate contracts, but the outsider wins more medium rate contracts. Therefore, under this assumption, the expected rate at the outside lender would be higher.
By definition of \( p \) and \( p(S) \), it follows that \( p(1 - p(S)) < (1 - p)p(S) \), so the outside firm always wins more of the bids. More specifically, the outside bank wins more of the bids in total, because it wins more of the medium rate bids.

This implies that the proportion of both low rate contracts and high rate contracts are lower at the outside bank. In this case, it is not clear whether the expected inside rate or the expected outside rate is greater. The answer depends on whether the medium rate is less or more than the weighted average of the low and high rate. If it is more than the weighted average, then the outside rate is higher. If it is less than the weighted average, then the inside rate is higher. Under the split-tie assumption, the expected interest rates at the inside and outside bank are:

\[
E_i(r) = p \left[ p(S) \left( \frac{1}{2} r_L + (1 - p(S)) r_M \right) + (1 - p) \left( (1 - p(S)) \left( \frac{1}{2} r_H \right) \right) \right] \frac{1}{1 + \frac{1}{2} p - \frac{1}{2} p(S)}.
\]

\[
E_o(r) = p \left[ p(S) \left( \frac{1}{2} r_L \right) + (1 - p) \left( p(S)r_M + (1 - p(S)) \left( \frac{1}{2} r_H \right) \right) \right] \frac{1}{1 + \frac{1}{2} p - \frac{1}{2} p(S)}.
\]

The results for the split-tie assumption appear to be similar to the results with the original assumption. Analytically, the same result occurs in this case when \( p_H \to 1 \) and \( p_L \to 0 \). This is not surprising, because the probability of a high rate loan goes to zero under these limits. Therefore, the result showing that the expected outside rate is higher in this case is independent of the assumption about tie bids.

The important conclusion for this analysis is the difference between the inside and outside rate. The equations above can be combined to identify the difference between the inside and outside rate:

\[
E_i(r) - E_o(r) = 2 \left( \frac{p(S) - p}{1 - (p(S) - p)^2} \right) \left( p(1 - p(S))(r_H - r_M) - pp(S)(r_M - r_L) \right)
\]
Proof:
Let
\[
\alpha = pp(S)\left(\frac{1}{2}\right)
\]
\[
\beta = -pp(S)
\]
\[
\delta = (1 - p)(1 - p(S))\left(\frac{1}{2}\right).
\]

Then
\[
E_i(r) - E_o(r) = \frac{(\alpha + \beta + \delta + p(S))(\alpha r_L + \beta r_M + \delta r_H + pr_m) - (\alpha + \beta + \delta + p)(\alpha r_L + \beta r_M + \delta r_H + p(S)r_m)}{(\alpha + \beta + \delta + p(S))(\alpha + \beta + \delta + p)} \left[\frac{[(\alpha + \beta + \delta + p(S)) - (\alpha + \beta + \delta + p)](\alpha r_L + \beta r_M + \delta r_H)}{(\alpha + \beta + \delta + p(S))(\alpha + \beta + \delta + p)} + \frac{[(\alpha + \beta + \delta + p(S)) - p - (\alpha + \beta + \delta + p)p(S)]r_M}{(\alpha + \beta + \delta + p(S))(\alpha + \beta + \delta + p)} \right]
\]
\[
= \frac{(p(S) - p)(\alpha r_L + \beta r_M + \delta r_H) + (\alpha + \beta + \delta)(p - p(S))r_M}{(\alpha + \beta + \delta + p(S))(\alpha + \beta + \delta + p)} \left[\frac{p(S) - p}{(\alpha + \beta + \delta + p(S))(\alpha + \beta + \delta + p)} \right] \left(\delta(r_H - r_m) - \alpha(r_m - r_L)\right)
\]

With some minor algebraic manipulations, this expression can be reduced to the equation shown above.

The sign of the difference between the inside and outside rate depends on the sign of \((1 - p)(1 - p(S))(r_H - r_m) - pp(S)(r_m - r_L)\). This difference reflects the relative weightings of the low and high rate contracts in the inside and outside bank portfolios. If \((1 - p)(1 - p(S))\) is high, then the proportion of high rate contracts is large in both banks’ portfolios. The outside bank wins more medium rate contracts, so the outside rate is pushed lower than the inside rate, making \(E_i(r) - E_o(r)\) positive. On the other hand, if \(pp(S)\) is high, then the proportion of low rate contracts is large in both banks’ portfolios.
The outside bank wins more medium rate contracts, so the outside rate is pushed higher than the inside rate, making \( E_i(r) - E_o(r) \) negative.

The multiplicative factor \( \frac{p(S) - p}{1 - (p(S) - p)^2} \) stems from the difference in insider and outsider probabilities of winning. The outside bank wins with probability \( \frac{1}{2} - \frac{1}{2} p + \frac{1}{2} p(S) \) and the inside bank wins with probability \( \frac{1}{2} + \frac{1}{2} p - \frac{1}{2} p(S) \). Therefore, the difference between the outside probability of winning and the inside probability of winning is \( p(S) - p \). And the difference is precisely in the medium rate loans, \( r_M \). The outside rate is greater than the inside rate when \( r_M \) is higher or lower than the average of \( r_L \) and \( r_H \), weighted by the proportion of each contract. The factor \( p(S) - p \) expands the difference in the proportion of \( r_M \) contracts at the outsider vs. insider, thereby expanding the difference between the inside and outside rate, whichever is higher.

The Sign of the Inside Rate vs. Outside Rate Difference

The expected sign of the difference between inside rates and outside rates can be formalized into the following proposition:

**Proposition A**

\[
E_i(r) > E_o(r) \iff \frac{pp(S)}{(1 - p)(1 - p(S))} < \frac{r_H - r_M}{r_M - r_L} \text{ and } \frac{pp(S)}{(1 - p)(1 - p(S))} > \frac{r_H - r_M}{r_M - r_L}.
\]

Proof:
This proposition follows directly from the equation for the difference between the inside and outside rate. \( E_i(r) - E_o(r) > 0 \) iff \((1-p)(1-p(S))(r_H - r_M) > pp(S)(r_M - r_L)\). Proposition A is simply a rearranging of this inequality.

It is difficult to show analytically where this threshold is crossed, therefore Figure 1 illustrates the results numerically. The three panels in Figure 1 (a-c) show the results for three different levels of \( \theta \). Because \( p_L < p_H \) by definition, the parameter space in \((p_H, p_L)\) is triangular. In each panel, the light area is the region in \((p_H, p_L)\) space where \( E_i(r) > E_o(r) \), and the dark area is the region in \((p_H, p_L)\) space where \( E_i(r) < E_o(r) \).

The numerical results indicate that the changes in \( \frac{pp(S)}{(1-p)(1-p(S))} \) are the primary determinant of how the inside rate and outside rate change over the parameter space. The changes in \( \frac{r_H - r_M}{r_M - r_L} \) over the parameter space are relatively small compared to the changes in \( \frac{pp(S)}{(1-p)(1-p(S))} \). The following analysis of the observed difference in the inside and outside rate shows that the results are consistent with the changes in \( p \) and \( p(S) \) over the parameter space.

Before proceeding, it is important to restate the definitions of \( p \) and \( p(S) \), so as to understand their relationship to \( p_H \) and \( p_L \):

\[
p = \theta p_H + (1-\theta)p_L
\]

\[
p(S) = \frac{\theta p_H^2 + (1-\theta)p_L^2}{p}.
\]

It is clear that \( p \) and \( p(S) \) are low when \( p_H \) and \( p_L \) are both low and \( p \) and \( p(S) \) are high when \( p_H \) and \( p_L \) are both high. In the corner where \( p_H \) is high and \( p_L \) is low, \( p(S) \) is high, with almost no dependence on \( \theta \), whereas \( p \) depends heavily on \( \theta \). When \( \theta \) is high, \( p \) is high in this corner, and when \( \theta \) is low, \( p \) is low in this corner.
This is because a high $\theta$ weights $p$ toward $p_H$ and a low $\theta$ weights $p$ toward $p_L$. These mappings of $(p_H, p_L, \theta)$ space into $(p, p(S))$ space facilitate the interpretation of the results.

If $\frac{r_H - r_M}{r_M - r_L}$ is considered to be relatively constant, it is clear from Proposition A that the inside rate is higher than the outside rate when $p$ and $p(S)$ are sufficiently low and the outside rate is higher than the inside rate when $p$ and $p(S)$ are sufficiently high. The analysis of the inside rate vs the outside rate in Proposition A by $p$ and $p(S)$ corresponds with the numerical results shown in Figure 1.

It is first apparent that the inside rate is higher than the outside rate when $p_H$ and $p_L$ are both low. This corresponds to low values of $p$ and $p(S)$. Recall that $p$ is the proportion of good firms in the borrower pool. Therefore, this is an area where the proportion of good firms is low. Also, $p(S)$ is low, which means that the outsider bids from $h_i^S$ with a low probability. The combination of these factors implies that this is an area in the parameter space where there is a high proportion of $r_H$ loans made by both the inside and outside bank. The outside bank makes more $r_M$ loans than the inside bank, therefore the inside bank rate is higher.

On the other side of the parameter space, the outside rate is higher than the inside rate when $p_H$ and $p_L$ are both high. In all three panels, as $p$ and $p(S)$ both approach 1 (which occurs when $p_H \to 1$ and $p_L \to 1$), the outside rate is higher than the inside rate, independent of $\theta$. At this limit, $\frac{pp(S)}{(1-p)(1-p(S))}$ approaches infinity, whereas the ratio $\frac{r_H - r_M}{r_M - r_L}$ does not. Although $r_M - r_L$ approaches zero at the limit, $r_H - r_M$ also approaches zero at the limit; therefore, the ratio of rate differentials is bounded. This is why the outside rate is always higher in this corner.

This situation depicts the case where there is a high proportion of good firms and the outside bank is bidding from $h_i^S$ with a high probability. In this part of the parameter
space, there is therefore a high proportion of $r_L$ loans made by both the inside and outside bank. The outside bank makes more $r_M$ loans than the inside bank, therefore the outside bank rate is higher.

The panels also indicate that the threshold depends on $\theta$. This is especially true in the corner of $(p_H, p_L)$ space where $p_H$ is high and $p_L$ is low. In this corner, when $\theta$ is low, the inside rate is higher than the outside rate, and when $\theta$ is high, the outside rate is higher than the inside rate. This result makes sense in light of the values of $p$ and $p(S)$, given these parameter values. When $p_H$ is high and $p_L$ is low, the value of $p$ depends heavily on the value of $\theta$. In this corner, when $\theta$ is low, $p$ is low, and when $\theta$ is high, $p$ is high. In other words, the proportion of good firms, $p$, largely corresponds to the value of $\theta$ in this corner. When $\theta$ is low, the proportion of good firms is low, and the inside bank has a larger proportion of $r_H$ loans. When $\theta$ is high, the proportion of good firms is high, and the inside bank has a larger proportion of $r_L$ loans. Therefore, the inside rate is higher than the outside rate when $\theta$ is low, but the outside rate is higher than the inside rate when $\theta$ is high.\footnote{The nonlinearity in the threshold, which is most apparent in the second panel, can be partly explained by the nonlinearity of $p(S)$ in $p_L$.}

These results show that the outside rate is not always higher than the inside rate, as might seem the case due to the winner’s curse. The outside rate is higher than the inside rate when there is a high proportion of good firms in the borrower pool and the outside bank bids aggressively. The inside bank wins mostly good firms at the low rate and the outside bank rate is driven up by the outside bank mistakenly offering some bad firms a rate below $r_F$. On the other hand, the inside rate is higher than the outside bank when there is a low proportion of good firms in the borrower pool and the outside bank does not bid aggressively. In this case, the inside bank rate is high because it lends mostly to bad firms at the high rate. The outside rate is reduced below the inside rate, because the outside bank mistakenly makes low rate offers to some bad firms.

In summary, the results based on $p$ and $p(S)$ indicate that the outside rate is higher than the inside rate when the quality of the borrower pool is high. The difference
between the inside and outside rate is driven primarily by the higher proportion of medium rate contracts at the outside bank, which is driven by the winner’s curse. If the two banks have mostly high rate contracts, then the additional medium rate contracts at the outside bank push the outside rate below the inside rate. On the other hand, if the two banks have mostly low rate contracts, then the additional medium rate contracts at the outside bank push the outside rate above the inside rate. Therefore, the comparison of the inside and outside rates depends primarily on the whether the borrow pool is high quality (mostly low rate contracts) or low quality (mostly high rate contracts). The theory predicts that the inside rate is higher in low quality borrower pools and the outside rate is higher in high quality borrower pools.

The Size of the Inside Rate vs. Outside Rate Difference

This section focuses on the magnitude of the difference between the outside rate and the inside rate when the outside rate is higher. The previous results for analyzing the sign of $E_i(r) - E_o(r)$ show that the ordinal difference between the rates depends primarily on $\frac{pp(S)}{(1-p)(1-p(S))}$. If $p$ and $p(S)$ are sufficiently low, then the inside rate is higher than the outside rate and, if $p$ and $p(S)$ are sufficiently high, the outside rate is higher than the inside rate. However, empirical results from Degryse and Van Cayseele (2000) and Black (2006) indicate that the average outside rate is higher than the inside rate in an empirical specification over a broad sample of firms. This empirical finding is consistent with the Sharpe model for the specific case of a borrower pool with a high $p$ and $p(S)$. However, as $p$ and $p(S)$ change over the parameter space, the size of the gap between the inside and outside rate also changes. If the outside rate tends to be higher for a large pool of firms, how does the magnitude of this difference differ across different subsamples of the borrower pool? Within the area of the parameter space where the outside rate is higher than the inside rate ($E_o(r) > E_i(r)$), this section shows how the size of the difference between the two rates depends on the parameters of the model.
As discussed above, the expression for $E_o(r) - E_i(r)$ (order reversed for this section) includes a multiplicative factor of $\frac{p(S) - p}{1 - (p(S) - p)^2}$, which magnifies or diminishes the weighted difference between the interest rate contracts. The following analysis of the magnitude of $E_o(r) - E_i(r)$ shows that the cardinal difference depends primarily on this multiplicative factor. In other words, the size of the difference between the outside and inside rate depends primarily on $p(S) - p$. Although this result is difficult to show analytically, the numerical results in this section indicate the following proposition:

**Proposition B**
If $E_o(r) > E_i(r)$, then $E_o(r) - E_i(r)$ is large if $p(S) - p$ is large.

The difference between $p(S)$ and $p$ is large when $p_H$ is high and $p_L$ is low. Therefore, the following illustration of Proposition B proceeds from an analysis of the parameters $p_H$, $p_L$, and $\theta$.

The goal of this analysis is to show where the Sharpe theory predicts the greatest difference between the outside and inside rate when the outside rate is higher. It is apparent from Figure 1 that $E_o(r) > E_i(r)$ if and only if $p_H$ is high. The outside rate is always higher than the inside rate if both $p_H$ and $p_L$ are high. However, the outside rate can also be higher than the inside rate if $p_L$ is low, as long as $\theta$ is sufficiently large. Therefore, there are two primary cases to consider in $(p_H, p_L)$ space where $E_o(r) > E_i(r)$: the case where both $p_H$ and $p_L$ are high, and the case where $p_H$ is high and $p_L$ is low. These two cases provide insight into the magnitude of the difference between the outside and inside rate when the outside rate is higher than the inside rate. In accordance with Proposition B, the results for these two cases are driven primarily by the difference in the value of $p(S) - p$ for these two cases.

The first case is the corner where $p_H$ and $p_L$ are both high. Although this is where the outside rate is always higher, it is also where the difference between the
outside and inside rate is the smallest. As discussed above, the outside rate is always higher than the inside rate in this corner because \( \frac{pp(S)}{(1 - p)(1 - p(S))} \) approaches infinity, whereas the \( \frac{r_H - r_M}{r_M - r_L} \) is bounded. This would seem to make the difference between the two rates very large; however, the multiplicative factor \( \frac{p(S) - p}{1 - (p(S) - p)^2} \) approaches zero in the corner where \( p_H \) and \( p_L \) are both high. Therefore, the difference is very small in this corner (on the order of \( 10^{-4} \)).

In the second case, \( p_H \) is high and \( p_L \) is low. As discussed earlier, the outside rate is higher than the inside rate in this corner when \( \theta \) is sufficiently high. A high value of \( \theta \) implies a high value of \( p \), which means that both banks make a high proportion of low contract loans. However, \( p(S) - p \) is always greatest in this corner of \( (p_L, p_H) \) space, regardless of \( \theta \). It follows that the multiplicative factor \( \frac{p(S) - p}{1 - (p(S) - p)^2} \) takes its largest value in this corner.

The intuition from these two cases is confirmed in Figure 2, which illustrates the numerical results for the cardinal difference between the outside and inside rates. The three panels of Figure 2 correspond to the three values of \( \theta \) (0.1, 0.5, and 0.9) used in Figure 1. The x-axis is the value of \( p_L \) ranging from 0 to 0.9 and the value of \( p_H \) varies across the three lines in each panel. For each line, \( p_H \) takes on a value of either 0.99, 0.95, or 0.90. The large values of \( p_H \) are used due to the observation from Figure 1 that a high value of \( p_H \) (at least 0.7) is a necessary condition for the outside rate to be higher than the inside rate. The y-axis in each panel is the difference between the outside rate and inside rate \( (E_o(r) - E_i(r)) \) for the given values of \( p_H \), \( p_L \), and \( \theta \).

Figure 2a shows that the inside rate is higher than the outside rate for low values of \( p_L \) when \( \theta = 0.1 \). It is not apparent in the figure as shown, but the lines cross the x-axis around the value of \( p_L \approx 0.7 \). This result corresponds to the result in Figure 1, showing that the outside rate is higher than the inside rate for \( \theta = 0.1 \), if \( p_L \) is
sufficiently large. The important result from Figure 2a is the magnitude of this difference. When \( p_H \) and \( p_L \) are both high, the difference \( E_s(r) - E_i(r) \) is very small.

Figure 2b shows that the outside rate is higher than the inside rate when \( \theta = 0.5 \) as long as \( p_H \) is sufficiently high. For a value of \( p_H = 0.99 \) and \( p_L = 0.1 \), the difference between the inside and outside rate is about 70 basis points. The difference is smaller when \( p_H = 0.95 \) and the difference is negative for \( p_H = 0.90 \). This result corresponds to the illustration in Figure 1b, which shows that the difference between the inside and outside rates flips just above \( p_H = 0.90 \). The main result shown in Figure 2b is that the magnitude of the difference is large when \( p_L \) is low and small when \( p_L \) is high. This corresponds to \( p(S) - p \) being large when \( p_L \) is low and small when \( p_L \) is high.

In Figure 2c, where \( \theta = 0.9 \), the difference between the outside and inside rate is positive for the entire range of \( p_L \), at each of the three levels of \( p_H \). This same result can be seen from Figure 1c, in which each of the three levels of \( p_H \) is entirely in the dark region. Most importantly, Figure 2c shows that the magnitude of the difference between the outside and inside rate is significantly positive for a larger range of \( p_L \). This result corresponds to a borrower pool consisting of mostly good firms with a high probability of repayment and a few bad firms with a low probability of repayment. In this case, the difference between the outside and inside rate is gradually decreasing in \( p_L \).

Both Figure 2b and Figure 2c illustrate Proposition B. If the outside rate is higher than the inside rate, it is clear that the difference between the outside and inside rate is greatest when \( p_H \) is high and \( p_L \) is low. These values of \( p_H \) and \( p_L \) correspond to the parameter space where \( p(S) - p \) is the greatest. This result provides additional insight into the expected difference between the outside and inside rate. According to the Sharpe theory, the difference should be significant when \( p_H \) is high and \( p_L \) is low and insignificant when \( p_H \) and \( p_L \) are both high.

The magnitude of the difference between the outside and inside rate can be seen relative to the overall interest rates in Figure 3a-c. Again, the three panels of Figure 3 correspond to the three values of \( \theta \) (0.1, 0.5, and 0.9) used in Figure 1. In this figure,
each panel shows $E_r(r)$ and $E_o(r)$ over the range of $p_L$, from 0.01 to 0.3, for a fixed value of $p_H = 0.95$. The difference between the two rates is identical to the difference shown in Figure 2. Figure 3 shows the inside and outside rate separately to illustrate the magnitude of the difference relative to the two underlying rates.

The numerical results in Figure 3 indicate that the difference between the outside and inside rate along $p_L$ is proportional to the underlying outside and inside rate. In Figure 3a, the difference is the largest relative to the other two panels, but the underlying rates are also very high. The inside rate for $p_L = 0.01$ is 4000%, which is a very high interest rate because $p_L$ is so small and the weighting on $p_L$ is $1 - \theta$, which is 0.9. On the other hand, Figure 3c shows the inside and outside rate for $\theta = 0.9$. In this case, the inside rate for $p_L = 0.01$ is less than 30%. This is still high, but is not entirely implausible as an observable loan contract. At these parameter values, the difference is only about 2.5 basis points, which is small relative to the underlying rates. Therefore, the empirical implication of the theory seems to indicate minimal quantitative difference between inside and outside rates. However, along the dimension of $p_L$, the figures indicate that the difference is larger for higher levels of the inside and outside rates.

It is important to note that Figure 2 and 3 both show interest rates along the dimension of $p_L$. The exercise could also be considered along the dimension of $p_H$. A symmetric analysis of the inside and outside rates would compare differences in the rates when the outside rate is higher than the inside rate. The results for this exercise are somewhat revealed by the differences in $p_H$ in Figure 2. For a given level of $p_L$ and $\theta$, the difference appears to be larger for higher values of $p_H$. This is quite a different implication than the analysis for changes in $p_L$, because the analysis over $p_H$ suggests

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8 The analysis could also be conducted for the continuous dimension of $\theta$. The results in Figures 2b and 2c are somewhat misleading on this dimension due to the rescaling. The numerical results indicate that, if the outside rate is higher than the inside rate, the magnitude of the difference between the rates is decreasing in $\theta$. This corresponds with Proposition B, because $p(S) - p$ is decreasing in $\theta$. Closer analysis of Figures 2b and 2c indicates this point. For $p_L = 0.01$ and $p_H = 0.99$, the difference decreases in $\theta$, from 0.021 in Figure 2b to 0.011 in Figure 2c. For $p_L = 0.3$ and $p_H = 0.99$, the difference also decreases in $\theta$, from 0.003 to 0.002.
that the difference between the outside and inside rates is larger for high quality borrower pools and smaller for low quality borrower pools.

Proposition B integrates the analysis in \( p_L \) and \( p_H \) by stating the proposition in terms of \( p(S) \) and \( p \). Both the analysis in \( p_L \) and \( p_H \) indicate that the difference \( E_o(r) - E_i(r) \) is large when \( p(S) - p \) is large. Whether \( p_L \) declines relative to \( p_H \) or \( p_H \) increases relative to \( p_L \), both changes result in an increase in \( p(S) - p \). Therefore, the difference \( p(S) - p \) seems to best indicate the difference in \( E_o(r) - E_i(r) \). As discussed above, this result stems from the magnifying factor \( \frac{p(S) - p}{1 - (p(S) - p)^2} \), which itself comes from the difference between the insider and outsider proportion of medium rate contracts. The outside bank wins more medium rate contracts, because the outside bank wins more bad firms at a loss than the inside bank wins good firms at a large profit.

The difference \( p(S) - p \) can also be interpreted specifically as the difference between the probability of success of a good firm and the probability of success of the average firm (the pooling probability of success). As good firms rise in quality relative to the pool of firms, the outside bank bids more aggressively. This increases the proportion of bad firms that the outside bank wins and drives up the outside rate. Therefore, the difference between the outside rate and inside rate is large for a borrower pool in which the quality of a good firm is significantly higher than the quality of the average firm.

The empirical analysis of Black (2006) evaluates the evidence for different borrower subsamples and shows results consistent with this interpretation of the Sharpe theoretical prediction. The results indicate that the difference between outside and inside rates is largest for small firms and insignificant for large firms. This empirical result is consistent with the Sharpe prediction for differences in \( p(S) - p \) if the difference is due to lower values of \( p_L \) in small firms. This is the case if a pool of small firms has a high \( p_H \) and low \( p_L \), whereas a pool of large borrowers has both a high \( p_H \) and a high \( p_L \). If this is an accurate association, small firms have a greater difference between outside and inside rates, because \( p(S) - p \) is greater for small firms. Intuitively, a good firm in a pool of small firms is likely to have a probability of success significantly above the
average small firm, whereas a good firm in a pool of large firms may have a probability of success only marginally above the average large firm.

6. Conclusion

This paper analyzes the predictions of a benchmark model for insider vs. outsider expected interest rates. The Sharpe model (with the von Thadden correction) is a standard framework for analyzing the role of private information in the competition between bank lenders. The results of this paper show that the Sharpe model does not have a single, universal prediction for inside rates vs. outside rates. First of all, the prediction of the model depends on an assumption about which lender wins in the event of a tie bid. Secondly, the prediction of the model depends on the parameters of the model, which are the borrower characteristics. Under a split-tie assumption, the inside rate is shown analytically to be sometimes higher and sometimes lower than the outside rate. In addition, the magnitude of the difference between the inside rate and outside rate is shown numerically to be greatest for borrower pools in which the probability of success of a good firm is significantly higher than the probability of success of the average firm.

In analyzing the probabilities of winning, I find that there is a clear winner’s curse problem for the outside bank. The outside bank wins a greater proportion of F firms than is in the pool of firms. This supports the common intuition associated with the winner’s curse in bank lending. However, the implication for expected interest rates is not immediately obvious. The intuition of the winner’s curse would seem to imply that the expected interest rate at the inside bank is lower than the expected interest rate at the outside bank. The reasoning would be that outside banks win a higher proportion of bad firms, bad firms borrow at a higher rate, and, hence, the outside rate is higher. The interesting conclusion of this paper is that this reasoning does not necessarily apply. Although the outside bank wins proportionally more bad firms, the winner’s curse also causes the expected rate for each firm type to be lower at the outside bank. In other words, conditional on firm type, firms pay a lower rate at the outside bank. When this
result is combined with the winner’s curse effect, the prediction for the average interest rate at an insider vs outsider requires further analysis.

Surprisingly, it is shown that the expected interest rate at the insider may be higher than the expected interest rate at the outsider. This is shown analytically for part of the parameter space where the inside bank wins at least as many total loans as the outsider. Only in the situation where the number of high rate loan goes to zero does the result reverse. This seems to contradict the apparent intuition of the winner’s curse.

One limitation of this model is that the result depends on an assumption about tie bids. Because the bidding functions of both banks have a point mass on the highest rate, the outcome of a tie becomes important. The original assumption of Sharpe is that firms borrow from the inside bank in the event of a tie. This assumption is used in deriving the result that the expected interest rate at the inside bank is higher than at the outside bank. If this assumption is reversed, such that firms borrow from the outside bank in the event of a tie, then the predictions of the model for interest rates are also reversed. Under the assumption that the outside bank wins all ties, the expected interest rates at the inside bank are always lower.

To clarify the results, the analysis focuses on insider and outsider rates under the assumption that the banks split the firms in a tie. Analytical results show that the inside rate is higher than the outside rate when the quality of the borrower pool is low, whereas the outside rate is higher than the inside rate when the quality of the borrower pool is high. Numerical results show specifically where this threshold is crossed. In addition, the paper analyzes the size of the difference between the outside rate and inside rate when the outside rate is greater than the inside rate. Numerical results indicate that the difference between inside and outside rates is largest when the probability of success of a good firm is high relative to the probability of success of the average firm.

This paper shows that the effect of the winner’s curse on expected interest rates at the inside and outside bank have not been well established for the existing benchmark model. Despite the presence of the winner’s curse, the expected interest rates for firms borrowing from an inside bank are not necessarily lower. The predictions of the model depend on the parameters of the model, which are the borrower characteristics. This paper attempts to specify where the outside rate is higher than the inside rate and where
the difference should be expected to be greatest. This is an important step for identifying the specific predictions of the Sharpe model for insider vs. outsider interest rates in different borrower pools.
References


FIGURE 2

Figure 2a  \( \Theta = 0.1 \)  \( E_T(t) - E(t) \)

Figure 2b  \( \Theta = 0.5 \)  \( E_T(t) - E(t) \)

Figure 2c  \( \Theta = 0.9 \)  \( E_T(t) - E(t) \)
Appendix A

This appendix uses numerical calculation to compare the expected inside and outside rates under the original assumption that the inside bank wins all of the high rate contracts. The parameter space is a three-dimensional space in $(p_H, p_L, \theta)$, which is difficult to show graphically, so the results are illustrated using two cross-sections of the parameter space. Figure 1 shows the possible parameter space for $p_H$ and $p_L$. The two lines crossing the triangle identify the two cross-sections of the parameter space that are used to illustrate the results in Figures 2 and 3. Figure 2(a-d) shows the results for the cross-section of $p_L$, holding $p_H = 0.7$. Figure 3(a-d) shows the cross-section of $p_H$, holding $p_L$ constant at 0.3. Both figures illustrate the components of the probabilities of winning, beginning with $p(S)$ and $p$, and then show the expected interest rates. All results shown are for a constant $\theta = 0.5$.

Figure 2a shows $p(S)$ and $p$ over the $p_L$ cross-section. The two probabilities of success converge as $p_L \rightarrow p_H$.

Figure 2b shows the probabilities of winning for the insider and outsider. The sum of the three lines equals 1 and the probabilities are shown in such a way as to reflect the analysis of Corollary 3a. Because the probabilities of winning a low rate contract are identical, these insider and outsider probabilities are combined into “$p_{i:o} : r_L$,” which is $p p(S)$. The insider’s medium and high rate contracts are then combined into “$p_i : r_M + r_H$,” which is $p(1 - p(S)) + (1 - p)(1 - p(S))$, so that they can be compared with the outsider’s medium rate contracts of “$p_o : r_M$,” which is $(1 - p)p(S)$. It is clear that Corollary 3a applies to the parameter space to the right of the crossing point of $p_i : r_M + r_H$ and $p_o : r_M$, where $p(1 - p(S)) + (1 - p)(1 - p(S)) > (1 - p)p(S)$. It has already been shown that the inside rate is higher over this part of the parameter space.
Figure 2c is a plot of the three possible expected interest rates: \( r_L, r_M, \) and \( r_H \). Each of the expected rates are declining as \( p_L \) increases, because the expected probability of success for both types increases with \( p_L \). The highest expected rate is charged to \( F \) firms that borrow from the inside bank. This rate is significantly above the other rates, but all of the rates converge as \( p_L \rightarrow p_H \). The expected rates converge because \( r_F - r_p \rightarrow 0 \) as the two firm types approach identical default probability.

Figure 2d shows the expected interest rates at the inside and outside bank. The expected rate at the inside bank is strictly greater over the entire cross-section of \( p_L \).

Figure 3a shows \( p(S) \) and \( p \) over the \( p_H \) cross-section. The two probabilities of success diverge as \( p_H \) diverges from \( p_L \).

Figure 3b shows the probabilities of winning for the insider and outsider. Again, the sum of the three lines equals 1 and the probabilities are shown in such a way as to reflect the analysis of Corollary 3a. It is clear that Corollary 3a applies to the parameter space to the left of the crossing point of \( p_i : r_M + r_H \) and \( p_o : r_M \), where \( p(1 - p(S)) + (1 - p)(1 - p(S)) > (1 - p)p(S) \). It has already been shown that the inside rate is higher over this part of the parameter space.

Figure 3c is a plot of the three possible expected interest rates: \( r_L, r_M, \) and \( r_H \). The rates are generally decreasing in \( p_H \) except for \( r_H = r_F \), which declines and then sharply rises. The unique shape of \( r_H \) is due to the changing expectation of the firm’s basic type (\( L \) or \( H \)), given the signal of \( F \). As \( p_H \) increases from a low value, firms with a bad signal also benefit, because there is still some possibility of being an \( H \) firm. However, as \( p_H \) becomes so large that the \( F \) signal almost surely indicates an \( L \) type, then \( r_H \) begins to increase.
Figure 3d shows the expected interest rates at the inside and outside bank. The difference between the inside and outside rate is positive and increasing over the entire cross-section of $p_H$.

The important conclusion of the numerical results is in Figures 2d and 3d. In each figure, the expected rate at the inside bank is strictly greater over the entire cross-section of $p_L$ and $p_H$, respectively. This shows numerically that the inside rate is higher than the outside rate in these two cross-sections.
FIGURE FOR APPENDIX A
FIGURE FOR APPENDIX A

Figure 2a

Probability

p(S) and p

Figure 2b

Probability

p_L

Legend:
- p(S)
- p
- p_M
- p_H
- p_L

p_L

0.2 0.3 0.4 0.5 0.6 0.7

0.2 0.3 0.4 0.5 0.6 0.7
FIGURE FOR APPENDIX A

**Figure 3a**

![Graph showing the relationship between probability and $p_H$.](image)

**Figure 3b**

![Graph showing the relationship between probability and $p_H$.](image)
FIGURE FOR APPENDIX A

Figure 3c

Figure 3d