Abstract: A theoretical and computational model is presented in which heterogeneous agents choose between a publicly financed education and a private alternative. Individuals differ by income and by a preference parameter meant to represent such differences as race, religion and language. The public school is financed through a common tax rate that is chosen by the agents through majority vote. Individuals that choose to attend the public school also choose the type of education that the public school will provide. A Nash equilibrium for school choice and majority voting equilibrium for public school type is shown to exist. Computational results indicate that a majority voting equilibrium for the tax rate exists in many, but not all cases. Further computational results indicate that tax rates are (nonmonotonically) decreasing in individual heterogeneity, while public school expenditure per student tends to be higher in the presence of high levels of individual heterogeneity. These results build on the results of Alesina, Easterly, and Baqir (1999) which indicate decreasing levels of funding for public schools in the presence of ethnic fractionalization in a market with no private alternative.

JEL classification: D72, H40, I21
1. Introduction

In recent years there has been a great deal of public debate on the topic of public education. Many individuals believe that the current system is underperforming and that changes should be made to increase the quality of schooling. In some states and localities, charter schools or voucher programs have been introduced to provide parents with more choices in education. To predict the results of many of these plans one needs to understand factors that influence the choice of schooling.

Several empirical studies have been performed to explore the factors that influence student’s choices to attend private schools. Smith and Meier (1995) evaluate schools in Florida and find that religion and racial characteristics of a school are more important components of demand for schools than quality of schools. Fairlie and Resch (2002) find evidence of “white flight” from minority schoolchildren and Fairlie (2002) finds evidence of “Latino flight” from black schoolchildren. Betts and Fairlie (2003) find that for every four immigrants that enter a public secondary school, one native student switches to a private school. The findings in all of these papers indicate that factors other than school quality enter individuals’ decisions about which school to attend. In an extreme case, in which individuals are only concerned about these factors and not school quality, increased school competition will lead to demographic segregation (i.e. black schools, immigrant schools, religious schools, etc.).

Another topic that is not evaluated in these studies is how the choice of many students to attend a private school will affect the overall funding of the public school. Since public schools are often funded at a local level, one would expect that individuals who choose the private school will desire lower levels of funding for the public school.
In addition, Alesina, et al (1999) finds that local expenditures on public goods, such as education and public transportation, are decreasing in ethnic diversity. In summary, it is possible that the existence of a private alternative and large amounts of ethnic diversity could together lead to decreases in the funding available for the public school.

The purpose of this paper is to explore the impact that individual heterogeneity might have on the quality of public education. In the model developed individuals differ by income and by type. Type in this model is an artificial parameter meant to represent heterogeneity that may arise in preferences for education relating to such things as race, religion, and language. In this model individuals choose between a public school and a private alternative. The public school and the private school differ by quality and by type. School quality in this model is represented by total expenditures per student. Public school expenditures arise from a common income tax that is chosen through majority vote. Private school expenditures arise from tuition payments. In this model, school type and individual type are very closely related. Individual type represents the type of educational services that an individual demands, while school type represents the type of educational services that a school provides, such as bilingual teaching or religious education. The possibilities for both individual and school type are represented on the one dimensional interval between 0 and 1. All other things equal, individuals prefer a school type that is closer to them on the interval.

The main results of the analysis are that individuals sort between the two schools based upon their income as well as their type and that the existence of a private school at the extreme right of the type distribution leads to the public school type being left of the middle. Later a computational model is constructed to show that a majority voting
equilibrium may exist for the tax rate and that the shape of the individual type
distribution will have a significant impact on the type and funding for the public school.

The paper proceeds as follows: Section 2 describes the model. Section 3
describes the equilibrium decision-making rules for individuals in the model. Section 4
establishes a majority voting equilibrium for public school type. Section 5 introduces the
computational example and provides the results of this model for several different
parameter values. Section 6 provides the main findings of the paper and my additional
research plans.

2. Model

Consider an economy in which a large number of individuals maximize utility
through the choice of school that their children will attend. Individuals are characterized
by income and type. Household type is meant to characterize differences that arise in the
preferences for education in relation to such things as race, religion, and language. For
example, an immigrant family from Mexico may place high value on bilingual teachers
while other families in the district may not. Although individuals may differ in multiple
dimensions in this respect, in the analysis that follows differences are represented in one
type dimension. Denote the income of household \(i\) as \(y_i\) with distribution \(F(y)\), and type
as \(a_i\) with distribution \(G(a)\). Assume that household income and type are independent and
that \(F(y)\) has support \([0,\infty)\) and that \(G(a)\) has support \([0,1]\). For simplicity, the population
of the jurisdiction is normalized to one. All individuals are taxed at the same rate, \(\tau\),
which is used exclusively to fund the public school system. Note that this includes the
individuals that attend the private school and modeled as receiving no benefit from the
use of these taxes.
Schools in the economy are characterized by their quality, the type of education that is provided, and whether they are publicly or privately funded. Denote quality of school \( s \) as \( q^s \), and the education type as \( A^s \). Education type is meant to describe differences in the education services that schools may provide, such as religious education or bilingual teachers. For simplicity, assume that there is one public school, \( u \), and one private school, \( r \). Also, \( q^r \) and \( A^r \) are assumed to be exogenous. For simplicity assume that \( A^r = 1 \). The assumption that private school type and quality are exogenous is made for computational convenience. The focus of this paper is to determine the effect that aggregated decisions of individuals in the economy will have on the attributes of the public school. Allowing for the existence of endogenous private school attributes may be dealt with in future research. Comparative statics will be performed to evaluate the impact of changes in private school quality on the variables of interest.

In this economy, school quality is represented by the expenditures per student at a given school. For the private school, all revenues are spent on the students so that \( q^r \) represents both the price of attending the private school and the quality that a student at the private school receives. Since all tax revenues are used to finance the public school, public school quality can be given by the equation:

\[
(1) \quad q^u = \frac{\tau Y}{N}, \quad N > 0 \\
q^u = 0, \quad N = 0
\]

where \( N \) is the number of students attending the public school.

The utility function for household \( i \) attending school \( s \) will take the form:

\[
(2) \quad U^s_i(c,q) = \alpha \ln c^s_i + (1 - \delta_i) \ln q^s_i,
\]
where $c$ is the consumption good, $q$ is per student expenditure at school $s$, $\alpha$ is a taste
parameter capturing the relative importance between consumption and schooling in the
utility function, and $d_i = |A^s - a_i|$ is the absolute difference between an individual’s type
and the type of school that the student attends. The difference between individual type
and school type captures the negative effects of choosing a school that does not offer the
preferred type of educational services. Both individual and school type exist on the
interval $[0,1]$ by assumption, so $d_i \leq 1$. An individual that chooses the public school will
face budget constraint:

$$y_i(1 - \tau) = c_i,$$

which implies an indirect utility function of:

$$V_i^u = \alpha \ln [y_i(1 - \tau)] + (1 - |A^u - a_i|) \ln [\tau Y/N].$$

An individual that chooses to attend the private school will face the budget constraint:

$$y_i(1 - \tau) = c_i + q^r,$$

which implies (with $A^r = 1$) an indirect utility function of:

$$V_i^r = \alpha \ln [y_i(1 - \tau) - q^r] + a_i \ln [q^r].$$

3. **Equilibrium**

Before defining an equilibrium in this economy, it is important to describe the set
and timing of actions that individuals will be making. Individuals in this economy have
complete information about the attributes and utility functions of others. At each stage of
the decision-making process, individuals make decisions anticipating the current and
future actions of others. In equilibrium, all anticipations are realized. In the first stage
the tax rate, $\tau$, is chosen by individuals through majority vote. In the second stage
individuals choose whether to attend the private school or the public school. They make
this decision by choosing to the public school if the utility they receive from attending the
public school is greater than the utility they receive from attending the private school.

This is a discrete choice which results in an indirect utility function of:

\[
W_i = \max \{V_i^u, V_i^r\},
\]

which will arise from equations (4) and (6). In the last stage individuals that choose the
public school will choose the public school type, \( A^u \), by majority vote.

**Definition.** An equilibrium in the economy described above will consist of:

(i) \( \tau^* \), which is chosen by a majority vote of all citizens,

(ii) a sorting equilibrium in which no household would prefer to attend a
different school given the decisions of others and \( \tau^* \),

(iii) and public school type \( A^{u*} \), which is chosen by majority vote of the
individuals that attend the public school.

To solve for an equilibrium of this game one must use backward induction.

Through computational simulations, it has been discovered that the existence of an
equilibrium tax rate can not be guaranteed for all parameter values. However, for a given
\( \tau \) an equilibrium for stage (ii) and (iii) does exist and is unique. The first step to prove
these assertions is to show that there exists a unique \( A^{u*} \) for all values of \( N \) and \( \tau \).

**Proposition 1:** The majority voting equilibrium for public school type, \( A^{u*} \), is the equal
to the median household type attending the public school.

**Proof:** To prove Proposition 1, one need only show that (4) is single-peaked for every
household \( i \) and that the peak is at \( a_i \). Then apply the median voter theorem. **QED**

The next step is to characterize the sorting equilibrium that will arise for a given
\( \tau \). Define individual \( \bar{a} \) that is the individual who is exactly indifferent between the public
school and the private school \((V_i^u = V_i^r)\) for a given level of income, and for all values of 
\(\tau\) and \(N\). To find \(\bar{a}\), set (4) equal to (6) and solve for \(a_i\). Because of the absolute value in
(4), there will be two possible cases, one in which \(\bar{a}\) is greater than \(A^u\), \(\bar{a}_1\), and one in
which is \(\bar{a}\) less than \(A^u\), \(\bar{a}_2\):

\[
\bar{a}_1 = \frac{\alpha \ln[y(1 - t)] - \alpha \ln[y(1 - t) - q^r] + (1 + A^u) \ln[tY/N]}{\ln[q^r] + \ln[tY/N]}
\]

\[
\bar{a}_2 = \frac{\alpha \ln[y(1 - t)] - \alpha \ln[y(1 - t) - q^r] + (1 - A^u) \ln[tY/N]}{\ln[q^r] - \ln[tY/N]}
\]

**Proposition 2:** All individuals \(a_i > \bar{a}\) will choose the private school and all individuals \(a_i < \bar{a}\) will choose the public school.

**Proof:** Compare the values of equations (4) and (6) for \(a_i > \bar{a}\), and \(a_i < \bar{a}\). QED

\(A^u^*\) is determined by all individuals who attend the public school \((a_i < \bar{a})\). However, the
right-hand side of (8) depends on \(A^u\). This leads to Proposition 3:

**Proposition 3:** There exists an \(A^u^* = h(\bar{a})\) that solves (8).

**Proof:** In Appendix

Proposition 1, 2, and 3 imply that for every value of \(\tau\) and \(N\), the decision that each
household will make can be determined. Another important thing to notice is that \(\bar{a}\) is
decreasing in \(y\). This indicates that sorting occurs both across individual type and
income. Figure 1 is a graphical representation of how this sorting is characterized.

Individuals in the upper right corner of Figure 1 will attend the private school and
individuals in the lower left portion attend the public school.
Since we now know which individuals will attend the public school, we can calculate $N$:

\[(9) \quad N = \iint_U f(y) g(a) \, da \, dy,\]

where $U$ is the set of individuals that attend the public school ($a_i < \bar{a}$). Since the integral in (9) is a function of $\bar{a}$, which is a function of $N$, it is necessary to show that a unique solution exists.

**Proposition 4:** There exists an $N$ that solves (9).

**Proof:** In Appendix

Thus far, I have shown that for all values of $\tau$, stages (ii) and (iii) of the equilibrium for this economy can be characterized. At this point I must move on to a computational simulation as an equilibrium $\tau^*$ in stage (i) does not always exist. For most of the parameterizations in this model $\tau^*$ does exist and will be reported.

5. **Computational Model**

Since closed form solutions do not exist for some of the variables above (such as $N^*$) and a majority voting equilibrium for $\tau$ may not exist, many of the important implications of this model must be derived computationally. For the baseline model U.S. data is used (when applicable) to calibrate the model. $F(y)$ is assumed to be lognormal with mean = $53,000 and median = $40,000 to be similar to current U.S. statistics, which is the approach taken in Epple and Romano (1996). $G(a)$ is assumed to have a symmetric beta distribution with parameter $\rho$. This parameter $\rho$ allows for the comparison of communities with relatively homogenous tastes to those with very heterogeneous tastes. When $\rho = 1$, the distribution is uniform, as $\rho \to 0$, the distribution collapses to a mass point at each end of the distribution, and as $\rho \to \infty$, the distribution collapses to a mass
point at 0.5. In short, as one increases $\rho$, the distribution of individual types changes from being bimodal to unimodal. $q^r$ is chosen to be $3,500$ which is close to the average price of all private schools, elementary or secondary, in the US. $\alpha$ is chosen to be equal to 10 to get tax rates in the 5%-10% range in equilibrium. A sample of 25,001 individuals were selected from the distribution of individuals for each simulation of the model. 25,001 is the largest number that allows for relatively fast simulations and no ties in majority votes.

Results

Multiple simulations consistently produced similar results for the equilibrium values of $N$, $A^u$, and $\tau$. These results differ slightly due to the variation introduced from the random draws for the individual attributes. Results are reported for one of these simulations. The relationship between the endogenous variables and the parameter $\rho$ is shown in figures 2 through 5 for this simulation.

One important implication of the numerical results is the importance of the type distribution in a given school district. $N^*, A^{u*}$, $\tau^*$, and $q^{u*}$ all have very different values for low levels of $\rho$ compared to high levels of $\rho$. From Figures 2 and 3 one can see that $N$ and $A^u$ are increasing with $N \rightarrow 1$ and $A^u \rightarrow \frac{1}{2}$ as $\rho \rightarrow \infty$. In the extreme case of $\rho$ close to zero near perfect segregation of household types arises. Individuals at the extreme right of the distribution attend the private school (with the small exception of those that can’t afford the tuition), while individuals at the extreme left attend the public school.

To explain this result, compare two school districts, one with a type distribution that is less centralized ($\rho$ smaller) than the other. When one compares the school types that will be chosen in equilibrium, it is easy to see that $A^u$ will be closer to the left (lower)
for the less centralized district since there is a greater mass of individuals near the left. This leads to a median individual type that is to the left. Since $\bar{a}_1$ is increasing in $A^u$ (with a symmetric type distribution, $\bar{a} = \bar{a}_1$ as long as $q^r < (q^0)^2$, which is easily met), and $N$ is increasing in $\bar{a}$, this implies that $N$ will be higher for the more centralized district.

Gradstein and Justman (2005) claim that one of the goals of public education is social integration, that is, “assimilating people from widely varying backgrounds in a common cultural identity.” The results of this computational model seem to indicate that in the communities with the greatest need for social integration ($\rho_{low}$), public school attendance is the lowest (close to 0.5 in the extreme case) and the difference between the school types being the greatest, with the private school located at one extreme and the public school located at the other. It is also important to note that vouchers would only add to this difficulty. These results agree with some of the empirical literature (See Fairlie (2002), Betts and Fairlie (2003), and Smith and Meier (1995)) that indicates districts with large populations of Catholics or highly diverse racial compositions tend to have well attended private schools.

The main focus of this paper is to examine the effects of individual type diversity on the tax rate and public school quality. Figure 4 shows the relationship between $\tau$ and $\rho$. In this model, type diversity seems to have a decreasing affect on local tax rates. This relationship does not appear to be monotonic, however, as a large peak arises near $\rho = .35$. This result both agrees and disagrees with the results presented in Alesina, et al (1999). In this paper, the authors present a simple model in which individuals differ by type (but not income) and vote on the tax rate to fund a local public good. They report a monotonically decreasing relationship between type diversity and the tax rate. The main
difference in my paper is the existence of a private alternative. This allows dissatisfied individuals to opt out of the public good for a private alternative. The affect that these actions will have on the equilibrium tax rate can not be easily due to the large number of changes to the endogenous variables that will be caused. Furthermore, the authors use empirically study the same question and use a linear regression framework. My analysis seems to indicate that diversity and the tax rate have a nonlinear relationship.

At this point it would be easy to claim that ethnicly homogenous districts lead to higher quality schools as equilibrium tax rates are higher in homogenous districts. However, since public school quality is decreasing in N and N is increasing in \( \rho \), this is not necessarily the case. In short, total expenditures on public education will be higher in districts with a high \( \rho \), but expenditures per student may not be. The simulated data, reported in Figure 5, indicates that public school quality peaks around \( \rho = 0.4 \), which is still very diverse in terms of individual types. For values of \( \rho > 0.75 \), public school quality tends to be around $4,500, which is still higher than \( q' \). In what follows I will vary the income distribution and \( q' \) to see the effects that changes in these variables will have on the variables of interest.

**Comparative Statics**

The results of the comparative statics exercise are reported in Figures 6-13. Uniform signifies \( \rho = 1 \), for unimodal \( \rho = 1.8 \), and for bimodal \( \rho = 0.2 \). In the first group of simulations, I vary \( q' \) from $1,500 to $11,500 in increments of $2,000. Recall that \( q' \) operates not only as the quality of the private school, but also as the price of the private school. Public school attendance and type are both slightly increasing in \( q' \), with a greater increase occurring in the bimodal case than the other two cases. It is useful to
note that in the bimodal case, no equilibrium tax rate exists for $q^r = 11,500$. The
equilibrium tax rate and public school quality are unaffected by changes in $q^r$ in the
unimodal case, while there are increasing and decreasing regions for the other two cases.

In the second group of simulations, I vary income inequality by adjusting the
median income level and keeping the mean income level at $53,000$. The results indicate
that public school type and choice tend to be increasing in income inequality, with greater
increases occurring in the bimodal case. The equilibrium tax rate seems to be increasing
in the bimodal case, slightly increasing in the uniform case, and relatively constant in the
unimodal case as income inequality is increased. This leads to relatively constant values
for public school quality in the uniform and unimodal cases and increasing values for
public school quality in the bimodal case as income inequality is increased.

These simulations show that income inequality, private school quality, and type
heterogeneity all have some affect on public school quality, although type heterogeneity
seems to have the biggest impact in this specification. In all, these results seem to
disagree with the predictions in Alesina, et al (1999) that indicate a general decrease in
public good quality when high levels of ethnic diversity exist. In this model, high levels
of ethnic diversity lead to low tax rates that are offset by the subset of the population that
opts out of public schooling.

From these results some important policy implications can be drawn. It is
important to identify the nature of the school district in question before one makes any
statements about policies such as private school vouchers and public (or private) school
curriculum limits. These policies will affect not only the attendance rates for the public
schools, but also the local funding available for these schools. Take for example a school
district that has a large contingent of students who speak Spanish as their first language (small \( \rho \)). We would expect from the results of this model that either the public school or the private school would supply bilingual teaching. If private school vouchers were issued in this district, we would expect segregation to increase as the more desirable school for some individuals would become more affordable. If a law was put in place to limit public schools to teach only in English (California and Arizona) one would expect a large outflow of Hispanic students to privately funded alternatives or to charter schools that provided bilingual teaching. Some proponents of private school vouchers argue that school vouchers will lead to an increase in the overall quality of public schools. This may be the case, but is not immediately apparent since schools are differentiated by type in addition to quality. School vouchers could lead to an outflow of students from a high quality public school to a low quality private school that has a curriculum that is more favorable to the household.

6. Conclusion

In this paper, a model is presented in which individuals that differ by income and type choose whether to attend public school or a private school based upon school quality and school type. The citizens in the model decide on the type of educational services that will be provided by the public school through majority voting. In the computational model there was a majority voting equilibrium for the tax rate for most of the simulations. In this framework, when individual types were highly diverse (\( \rho \) low), public school attendance was lower, the equilibrium tax rate was lower, public school quality was higher, and school types were more distant, than in a district with less diverse individual types. Programs such as private school vouchers or public school curriculum restrictions
will not only have an impact on how many people attend each school, but also the 
makeup of the population that attends each school and the level of funding available for 
the public school. Such programs could create some difficulties for the social integration 
role of the public school. Additional computational exercises that address changes in the 
income distribution and changes in private school quality are performed and provide 
some additional predictions. My future research in this area will be empirical in nature. 
One topic that I will study will be to empirically test the predictions that are made by the 
computational model. Demographic factors such as race, religion, and gender can be 
used to capture individual heterogeneity at the district level. This project will provide 
some comparisons to the research of Alesina, Easterly, and Baqir (1999). An additional 
topic that I plan to study is to estimate the parameters of a utility function similar to that 
presented in the paper. The resulting parameter estimates and estimated utility functions 
will provide a more realistic forum to conduct policy simulations than that presented in 
this paper.
References


Appendix

Figure 1: $\hat{a}(y)$

![Figure 1: $\hat{a}(y)$](image1)

Figure 2: $N$ as a function of $\rho$ from one simulation

![Figure 2: $N$ as a function of $\rho$ from one simulation](image2)
Figure 3: Au as a function of $\rho$ from one simulation

Figure 4: $t$ as a function of $\rho$ from one simulation
Figure 5: $q^u$ as a function of $\rho$ from one simulation

Figure 6: The Affect of Private School Quality on Public School Type

Figure 7: The Affect of Private School Quality on Public School Attendance
Figure 8: The Affect of Private School Quality on the Tax Rate
Figure 9: The Affect of Private School Quality on Public School Quality

Figure 10: The Affect of Income Inequality on Public School Type
Figure 11: The Affect of Income Inequality on Public School Attendance

Figure 12: The Affect of Income Inequality on the Tax Rate
Figure 13: The Affect of Income Inequality on Public School Quality

The diagram shows the impact of income inequality on public school quality. The x-axis represents different levels of income inequality, ranging from 0.1 to 1.9, and the y-axis represents the quality of public schools. Three different types of income distributions are compared:

- **Uniform**: The line is represented by a dotted pattern.
- **Unimodal**: The line is represented by a dashed pattern.
- **Bimodal**: The line is represented by a solid pattern.

The graph illustrates how changes in income inequality affect public school quality, with different patterns suggesting varying outcomes depending on the distribution type.
Proof Appendix:

**Proposition 3**: There exists an $A^u = h(\bar{a})$ that solves (8).

Define the set $S$ as the set of individuals with $a_i < \bar{a}$. Define $a_{med}$ as the median of set $S$. Since $\bar{a}$ is continuous in $A^u$, $a_{med}$ is also continuous in $A^u$. Define $h(\bar{a}) = a_{med}$. Since $h(\bar{a})$ is continuous in $A^u$, and $A^u = a_{med}$ by Prop. 1, $h(\bar{a})$ is a continuous mapping from $A^u$ into $A^u$. A solution for $A^u = h(\bar{a})$ must exist by Brouwer’s fixed point theorem.

**Proposition 4**: There exists an $N$ that solves (9).

Notice that (9) is continuous in $\bar{a}$. $\bar{A}$ is continuous in both $A^u$ and $N$. Therefore (9) provides a continuous mapping from $N$ into $N$. Therefore a solution for $N^*$ that solves (9) exists.