Cross-market Effect of Transparency on Liquidity between the Equity and Corporate Bond Markets

Hao Yin

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* I am grateful to Craig Holden and Konstantin Tyurin for providing useful comments and suggestions. Hao Yin is from the Department of Economics, Indiana University, Bloomington, Indiana, 47405. Email address: haoyin@indiana.edu.
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Abstract

Equity and corporate bond issued by the same firm are traded simultaneously in different markets. Will the higher transparency in the corporate bond market affect the market quality of the equity market? We present a model showing that disclosure of transaction information in the corporate bond market does not only improve liquidity in its own market, but also result in better liquidity in the equity market, no matter whether the informed traders can allocate their trading activities between these two markets or not. This implies that it is necessary to account for such “cross-market” effect as well to better evaluate any transparency-related policy. In addition, we show that if the informed traders have freedom to allocate trading between markets, the choice at equilibrium will depend on the leverage ratio of the specific firm and the overall proportion of informed traders.
1. Introduction

Market transparency has been a central issue to policy debates concerning market design for a long time, and has given rise to a large number of theoretical and empirical studies\(^1\). Many researchers agree that higher transparency is associated with better liquidity, thus benefits uninformed traders. Most of the existent works, however, consider only liquidity of the market where the transparency change takes place. As we show in this paper, it is helpful to take into account the effect in other related markets in order to completely assess the role of market transparency, since there is not only a “local-market” effect but also a “cross-market” effect of higher market transparency\(^2\).

As shown in Merton (1974), both equity and corporate bond are derivatives on the value of the underlying firm, thus it is natural to expect a close relationship between the values of these two securities, and between the equity and corporate bond markets. The main finding of our paper is that the transparency change in the corporate bond market does influence the liquidity in the equity market. There are two effects which could come to play, the “information effect” and the “switching effect”. The availability of post-trade information in the corporate bond market enriches the information set of dealers not only in its own market, but also in the equity market. Correspondingly, spreads in both markets are smaller than before, which we call the “information effect”. However, if informed traders can adjust their trading strategy in response to the change, some of them may escape from one market to the other. Therefore, the more severe adverse selection problem can result in lower liquidity in the later market, which is the “switching effect”. While the “information effect’ improves liquidity under any circumstance, the net change of liquidity depends on the result of these two competing effects. And we show that the “information effect” always dominates, causing the liquidity in the both markets to improve.

The results in this paper have both policy and empirical significances. For law-makers of financial markets, it means that the impact of transparency-related policies is manifold.

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\(^1\) See Madhavan (2000) and Bias et al. (2005) for an excellent survey.

\(^2\) Liquidity is a multi-dimension concept in the microstructure literature (see e.g. Kyle (1985)). In this paper, we study only one of the measures of liquidity, spread.
More transparent market mechanism can enhance liquidity in more than the local market, and its overall performance is even more powerful as we thought if other related markets are also accounted for.

Empirically, this model also has practical application by estimating the change of liquidity in the U.S. equity market around the initiation of the TRACE (the Trade Reporting and Compliance Engine) system. Traditionally, the U.S. equity markets are quite transparent, where both pre-trade and post-trade information is disseminated to public continuously. In contrast, the corporate bond market is much more opaque, where neither pre-trade nor post-trade information is available in real time. The main reason is that unlike equity markets, which are mostly centered in organized exchanges, corporate bonds are traded over-the-counter (OTC).\(^3\) The transparency of corporate bond market did not begin to improve significantly until the initiation of the TRACE system. At 8am on July 1, 2002, NASD began to disseminate to average investors the post-trade information of about 500 eligible corporate bonds, including the date and time of transaction, price, yield, and limited quantity information\(^4\). This provides a perfect experiment to test our hypothesis\(^5\).

Although the last decade has witnessed extensive development of transparency studies in finance literature, to the best of our knowledge, this paper is the first to examine the relationship of transparency and liquidity between the equity and corporate bond markets. Based on the model of Kyle (1985), Chowdhry and Nanda (1991) propose a single-asset multi-market model, and conclude that transparency can bring in higher liquidity by “cracking down” on informed trading. Consistent with their results, Pagano and Roell (1996) also show that greater transparency reduces transaction costs incurred by uninformed traders. Introducing a risk-averse dealer, Naik et al. (1999) develop a model where better transparency can cut the inventory cost of the dealer by improving inventory risk sharing. Most of the theoretical models predict a positive relationship between transparency and liquidity; however, Madhavan (1995) provides a counter-point. By

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\(^3\) See Bessembinder et al. (2006), Biais and Green (2005).
\(^4\) See [www.nasd.com](http://www.nasd.com) for more detailed introduction to the TRACE system.
\(^5\) While NYSE is a continuous auction market, the U.S. corporate bond market was a pure dealer market before TRACE and resembles more and more to a continuous auction market after TRACE. Pagano and Roell (1996) predict a reduced transaction cost during the conversion form dealer market to continuous auction market. In this paper, we will not focus on the own-market liquidity change, but the cross-market effect instead.
introducing large liquidity traders into the model, he argues that nondisclosure may benefit certain types of market participants and thus validate market fragmentation. Moreover, in an experimental framework, Bloomfield and O’Hara suggest that while transparency is important in determining liquidity, their relationship is complex.

The empirical evidences are also mixed. While Flood et al. (1999) and Boehmer et al. (2005) document greater liquidity associated with increased transparency, Poter and Weaver (1998) and Madhavan (2005) find the opposite. Thanks to the introduction of the TRACE system, more empirical research concerning corporate bond market begin to emerge. One of the most recent studies is Bessembinder et al. (2006), who investigate the trading costs change due to the higher transparency brought by the TRACE system and find that the trading costs fell about a half for TRACE-eligible bonds. A contemporary work is by Edwards et al. (2006), who also examine determinants of cross-sectional variation in trading costs for corporate bonds using the TRACE data. Among other findings, they report lower costs for bonds with more transparent trade prices.

However, no matter what conclusions the above papers reach, they look into only the “own-market” effect of transparency, while our paper considers the cross-market effect of transparency between the equity and corporate bond markets.

Of course we are not the first to study the interplay of multi-security markets in the microstructure literature. One relevant work is by Caballe and Krishnan (1994), who generalize Kyle (1985)’s model to a multi-security heterogeneous-information setup and characterize a linear equilibrium. A similar article is Bhushan (1991), which examines the optimizing behavior of liquidity traders and emphasizes the role of uninformed traders. Our model is also close to Easley et al. (1998) who develop a model where informed traders can trade in option or equity market and examine how they choose between these two markets. They predict an important informational role of option volumes and thus option trading is not redundant, which is consistent with the findings of Back (1993) ⁶. Furthermore, compared with the substantial prior works on the connection between equity and option markets, the studies on interaction between the equity and corporate bond market in microstructure level is surprisingly sparse. One of such attempts is made

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⁶ Easley et al. (1998) concentrate on only the first period even they provide a dynamic trading model. The reason is that their primary interest is in the role of informational linkage between these two markets, not liquidity or transparency as in our model.
by Chang and Yu (2003) who propose a model where informed traders can split their trades between the corporate bond and stock markets depending on the relative liquidity of each market, and result in an optimal capital structure of the firm. However, none of these papers examines the relationship between transparency and liquidity across the markets.

This paper is organized as follows. Part 2 describes the setup of the model. Part 3 considers the exogenous informed trading proportion case, while Part 4 considers the endogenous case. And Part 5 concludes the paper.

2. The Model

The model is based on Glosten and Milgrom (1985) and in line with Easley et al. (1998). Transaction occurs in periods 1 and 2 and the interest rate between these dates is assumed to be zero. The stochastic liquidation value of a firm’s total assets, denoted by $V$, is realized in period 3 after all the trading is completed. For simplicity, we assume $V$ can take only two possible values, denoted by $V_H$ and $V_L$, corresponding respectively to a high and low state of nature with possibility of $\frac{1}{2}$ for each. Suppose that there are only two claims to the firm’s total assets, equity and corporate bond; and the face value of the corporate bond, denoted by $K$, is between $V_H$ and $V_L$. From now on, we will distinguish variables in these two markets by subscript $i (= e, c)$, where $e$ stands for the equity market and $c$ for the corporate bond market. Since corporate bond has the senior claim of a firm while equity has the residual claim, the terminal payoff of each of these securities, $V_e$ and $V_c$, can be written as

$$V_e = \max(0, V - K),$$

or,

$$V_c = V - \max(0, V - K),$$

or,

$$V_e = \begin{cases} V_H - K & \text{if } V = V_H \\ 0 & \text{if } V = V_L \end{cases},$$

$$V_c = \begin{cases} K & \text{if } V = V_H \\ V_L & \text{if } V = V_L \end{cases}.$$
Both equity and corporate bond markets are modeled as dealer markets\(^7\). All agents in the markets are assumed to be risk-neutral. At the beginning of each period, dealers set a bid and ask price indicating his will to sell one unit of security at the ask and buy one unit of security at the bid price. As in Glosten and Milgrom (1985), only one unit of security can be traded per period at each market\(^8\). Competition among the dealers in each market also leads to each dealer expecting to earn zero expected profit at the time of quoting conditional on all the information available to her.

There are two types of traders in the model, uninformed trader and informed trader. Uninformed traders trade for exogenous reasons (e.g. liquidity shocks) and they are equally likely to buy or sell one share of security in each period. It is also assumed the trading activity of uninformed traders is concentrated only in their own market, independent of the condition in the other market. Informed traders possess private information about the fundamental value of the firm’s assets, thus also the fundamental value of both securities. As in Madhavan (1995), we assume that the informed traders will buy in both markets in both periods if the firm’s assets are undervalued and sell otherwise.

Now let us consider the arrival process of traders. In each period, only one trader can be present and her type is randomly selected, unknown to the dealers in the market. We denote the probability that an informed trader is present by \(q_i (i = e, c)\), which lies between 0 and 1. We will first examine a framework where allocation of informed trading between the two markets, thus the possibility of informed trading in each market, is exogenous. Then we will allow the informed traders to allocate their trading activity across the two security markets to maximize their total profits. As will be shown in the paper, there exists dramatic difference in the effect of transparency in the corporate bond market between these two frameworks.

Figure 1 depicts the trading process for one period at one market. At the beginning of period 1 the state of nature of the firm’s underlying value is determined, either \(H\) or \(L\). Let us take the case when \(V = V_H\) as an example. Then one type of trader will

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\(^7\) U.S. corporate bond market is mostly a dealer market. Although both NYSE and NASDAQ are currently hybrid markets, the specialists and competing market venues can still be regarded as dealers for the U.S. equity market.

\(^8\) See also Madhavan (1995).
appear at period one. If it is an informed trader she will buy one unit of security at this market; if it is a uninformed trader, she will be equally likely to buy or sell one unit of security. The same process occurs for each period and each market, and the selection of trader type is also independent across both period and market\(^9\). Note that if informed traders in one market want to buy, so do informed traders in the other market. This provides an informative connection of trading activities between these two markets, and it is through this channel that transparency improvement in one market affects the quotation of dealers in the other market.

As in Glosten and Milgrom (1985) and many other sequential papers, we will consider a Bayes-Nash type equilibrium. Dealers construct bid and ask prices given their posterior beliefs about the traders’ information. Informed traders also form optimal trading strategies based on their beliefs about the dealers pricing schedules. In equilibrium, all these beliefs are consistent with the Bayes’ rule and observations.

### 3. Exogenous Informed Trading Proportion

We first consider a setting where informed traders can not allocate trading activities between the equity and corporate bond market. In practice, given private information about the value of an underlying firm, informed traders may compare between the equity and corporate bond markets and flow to the one with more desirable market condition. There are, however, some scenarios where capital flow between these two markets is forbidden or too costly, such as high entry cost or legal prohibition. Thus it is not that unrealistic to assume fixed informed trader proportion in each of the markets. Besides, the exogenous case can serve as a benchmark for comparison.

Before we study the impact of transparency improvement in the corporate bond market on the liquidity in either security market, it is useful to investigate how dealers’ information sets are affected in each of the markets, which in turn determine the quotes set by the dealers. From now on, we name the period before post-trade information in the corporate bond market becomes available as “Opaque” setting and after as “Transparent” setting. Table 1 illustrates the information set of dealers in each market for both the

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\(^9\) One implication of this assumption is that the quotes only depend on the total number of buys and sells, not the order of them. See Easley and O’Hara (1992) and Easley and O’Hara (1997).
Opaque and the Transparent settings, where $\emptyset$ denotes empty information set and $D^1_i(i = e, c)$ denotes the trading history of market $i$ in the first trading period. In the first period, dealers in either market do not observe any trading history, so their information set is empty. For Opaque setting, only post-trade information in the equity market is available to public, so dealers in both markets can only obtain this information when determining the quotes in period 2. Post-trade information in the corporate bond market, however, is inaccessible to neither the corporate bond dealers nor equity dealers. Note that unlike a single-market maker model, where the market maker possesses all the historic information of the order flow in the market, transactions conducted by one dealer are not observable to other dealers even in the same market. This resembles the U.S. corporate bond market closely, especially in the Opaque setting. When the corporate bond market becomes transparent, dealers in both markets can also update their prior beliefs based on the post-trade information of period 1 when setting prices in period 2\textsuperscript{10}. Difference in the information sets of dealers in these two transparency settings leads to the difference in liquidity, as will be seen below.

If we denote the information set of dealers in market $i(= e, c)$ for period $t(= 1, 2)$ by $I_i^t$, which is specified in Table 1, zero-profit assumption renders the dealers set the bid price $B^t_i$ and ask price $A^t_i(i = e, c)$ in period $t$ as

$$B^t_i = E[V_i \mid Sell^t_i, I^t_i],$$

$$A^t_i = E[V_i \mid Buy^t_i, I^t_i].$$

where $Sell^t_i$ and $Buy^t_i$ denote a buy order and sell order of security $i$ in period $t$, respectively\textsuperscript{11}.

Using Bayes’ rule and after some calculation, we can obtain the following lemma:

**Lemma 1.** The bid and ask prices set by the dealers in the equity and corporate bond markets in period 2 are given by

\textsuperscript{10} In practice, post-trade information does not indicate the direction of a transaction (whether it is buyer- or seller-initiated) in either of the markets. But we assume dealers can determine this by using some algorithms. See for example, Lee and Ready (1991).

\textsuperscript{11} See Glosten and Milgrom (1985), Madhavan (1995) and Easley et. al. (1998), for example.
\[
E[B_{e}^{\text{opq}2}] = \frac{(V_H - K)}{2} - \frac{q_e}{2(1 + q_e^2)} \cdot (V_H - K), \\
E[A_{e}^{\text{opq}2}] = \frac{(V_H - K)}{2} + \frac{q_e}{2(1 + q_e^2)} \cdot (V_H - K),
\]

\[
E[B_{c}^{\text{opq}2}] = \frac{(K + V_L)}{2} - \frac{(1 - q_e^2) \cdot q_e}{2(1 - q_e^2 q_e^2)} \cdot (K - V_L), \\
E[A_{c}^{\text{opq}2}] = \frac{(K + V_L)}{2} + \frac{(1 - q_e^2) \cdot q_e}{2(1 - q_e^2 q_e^2)} \cdot (K - V_L),
\]

in the Opaque setting, and

\[
E[B_{e}^{\text{trpt}2}] = \frac{(V_H - K)}{2} - \frac{q_e (1 - q_e^2)(1 + q_e^2 (1 - 2q_e^2))(V_H - K)}{2(1 + (2 - 4q_e^2)q_e^2 + q_e^4)}, \\
E[A_{e}^{\text{trpt}2}] = \frac{(V_H - K)}{2} + \frac{q_e (1 - q_e^2)(1 + q_e^2 (1 - 2q_e^2))(V_H - K)}{2(1 + (2 - 4q_e^2)q_e^2 + q_e^4)},
\]

\[
E[B_{c}^{\text{trpt}2}] = \frac{(K + V_L)}{2} - \frac{q_e (1 - q_e^2)(1 + q_e^2 (1 - 2q_e^2))(K - V_L)}{2(1 + (2 - 4q_e^2)q_e^2 + q_e^4)}, \\
E[A_{c}^{\text{trpt}2}] = \frac{(K + V_L)}{2} + \frac{q_e (1 - q_e^2)(1 + q_e^2 (1 - 2q_e^2))(K - V_L)}{2(1 + (2 - 4q_e^2)q_e^2 + q_e^4)}.
\]

in the Transparent setting, where superscript “opq” and “trpt” denotes Opaque and Transparent setting, respectively.

**Proof:** See Appendix.

As expected, the public prior beliefs about the value of a security lie right in the middle within the bid and ask prices and the spread in market \(i\) is zero on average if there is no informed trader in that market (i.e. \(q_i = 0\)).

It is then straightforward to obtain the following proposition from Lemma 1:

**Proposition 1.** If informed traders can not allocate trading activities between the equity and corporate bond markets, improved transparency in the corporate bond market will reduce transaction costs in both markets, i.e.

\[
\Psi_i^{\text{trpt}2} < \Psi_i^{\text{opq}2}, \quad i = e, c
\]
where

\[ \Psi^\text{opq}_i \equiv A^\text{opq}_i - B^\text{opq}_i , \]
\[ \Psi^\text{trpt}_i \equiv A^\text{trpt}_i - B^\text{trpt}_i , \]

is the Opaque and Transparent spread of market \( i (= e, c) \) in period 2, respectively.

**Proof:** See Appendix.

Intuitively, *ceteris paribus*, better transparency in the corporate bond market mitigates the adverse selection problem in both of the security markets. The additional post-trade information of corporate bond available to public enables the dealers to estimate the value of the security traded by them more precisely on average in the Transparent setting. Note that it is not necessary that each single spread is tighter in the Transparent setting than the Opaque setting. For example, when the underlying firm’s assets take the higher value \( (V = V_H) \), a sell order submitted by a uninformed trader in the corporate bond market will be purely a noise, causing the spreads even wider in the Transparent setting than in the Opaque setting. *On average*, however, both security markets are better off in terms of transaction cost thanks to the higher transparency in the corporate bond market.

This result accords with Pagano and Roell (1996), which theoretically show that transparency reduces own-market transaction costs. But distinguished from their paper, where only the effect of increased transparency in its own market is studied, *Proposition 1* emphasizes that improved transparency has not only own-market but also cross-market impact. Specifically, the equity and corporate bond markets possess this “spill-over” property, and the initiation of the TRACE system provided with us a natural experimental framework. Bessembinder et al. (2006) find a trading cost reduction in the U.S. corporate bond market due to the introduction of the TRACE system. Another relevant work is Edwards et al. (2006), who also examine determinants of cross-sectional variation in trading costs for corporate bonds using the TRACE data. Both of them find smaller transaction costs associated with higher transparency in the corporate bond market, which partially confirms *Proposition 1*. While the role of transparency in its own market is somehow clear, how it influences the liquidity in other related markets has not been explored, both theoretically and empirically.
4. Endogenous Informed Trading Proportion

In the previous section, it is assumed that informed traders cannot freely allocate their activities between the equity and corporate bond markets. In practice, however, those traders who possess private information about the underlying firm’s value can at least partially switch to the market with more desirable conditions, especially in the long term. Thus in this section, we will model the endogenous trading strategy taken by informed traders in these two security markets as a whole.

Suppose the total number of informed traders and uninformed traders in both the markets are constant during the whole trading periods. Informed traders can partition themselves between the equity and corporate bond market at the beginning of the date 1, but have to stick with this partition all through the trading periods. If we denote the proportion of informed traders relative to the overall trader population in both markets by \( q(0 \leq q \leq 1) \), and the percentage allocated to the equity market by \( \epsilon(0 \leq \epsilon \leq 1) \), then we have \( q_e = q \epsilon \) and \( q_c = q(1 - \epsilon) \). We assume the dealers in both markets know both \( q \) and \( \epsilon \), and thus set the quotes based on this information.

Now we investigate how transaction costs change due to the higher transparency in the corporate bond market. To study the change of spread in market \( i \), \( \Delta \Psi_i \), we define it as

\[
\Delta \Psi_i \equiv \Psi_i^{trpt} (\epsilon^{trpt}) - \Psi_i^{opq} (\epsilon^{opq}) = \Delta \Psi_i^{inf} + \Delta \Psi_i^{swt},
\]

where \( \epsilon^{opq} \) and \( \epsilon^{trpt} \) is the optimal informed trading proportion in the equity market chosen by informed traders, in the Opaque and Transparent setting respectively, and \( \Delta \Psi_i^{inf} \) and \( \Delta \Psi_i^{swt} \) is the “information effect” and “switching effect” (explained below) respectively, given by

12 This partly mirrors the fact that informed traders can not switch freely between the two markets in short term.
13 Also, we only consider the mixed-strategy equilibrium, not the pure-strategy one in this paper.
\[ \Delta \Psi_i^{\text{inf}} \equiv \Psi_i^{\text{trpt}2}(\epsilon^{\text{opq}}) - \Psi_i^{\text{opq2}}(\epsilon^{\text{opq}}), \]
\[ \Delta \Psi_i^{\text{swt}} \equiv \Psi_i^{\text{trpt}2}(\epsilon^{\text{trpt}}) - \Psi_i^{\text{trpt2}}(\epsilon^{\text{trpt}}), \]

where \( \Psi_i(\cdot) \) is a spread as a function of \( \epsilon \).

As seen above, the change of spread in either market from the Opaque to the Transparent setting consists of two parts. The “information effect” part picks up the impact on the spread due to the change of the information set of dealers. Even though there were no changes in the probability of informed trading, post-trade transparency in the corporate bond market enlarges the information set of dealers in both markets and enables them to better evaluate the security they are trading. And it has been shown in Proposition 1 that spreads in both markets are reduced by this “information effect”.

The second part of \( \Delta \Psi_i \) is the “switching effect” part, which is such named because it picks up the effect on spread of those informed traders who switch from the corporate bond market to the equity market. The sign of this effect is given by the following lemma.

**Lemma 2.** As the proportion of informed trading allocated to the equity market increases, the spread in the equity market becomes wider and the spread in the corporate bond market becomes tighter in period 2. And this is true for both the Opaque- and Transparent settings, i.e.

\[ \frac{\partial \Psi^2_e}{\partial \epsilon} > 0, \quad \frac{\partial \Psi^2_c}{\partial \epsilon} < 0. \tag{8} \]

**Proof:** See Appendix.

The intuition behind this lemma is as same as Glosten and Milgrom (1985). Basically, the adverse selection problem is worse in a market where more informed trading occurs. To avoid losses, the dealers need to set a wider spread and vice versa. Note that the spreads in the equity and corporate bond markets act oppositely with respect to \( \epsilon \); the more informed trading allocated to one market, the higher the transaction costs are in that market, and the less the profits incurred by informed traders. So unlike the “information
effect”, “switching effect” has different influence in these two markets. If the probability of informed trading is smaller in one market, it will become larger in the other market in the Transparent setting. Consequently, lesser adverse selection problem results in tighter spread in the former market but more severe adverse selection problem results in wider spread in the later market, if we only account for the “switching effect”. Thus in the market which absorbs more informed trading than before, there is competition between the “information effect” and the “switching effect”. The net effect of the improved transparency in the Transparent setting depends on which effect is larger. As we will see soon, it turns out the “information effect” always dominates.

Next we proceed to the optimization problem faced by the informed traders. For each individual informed trader, the trade in each time period is just a “one-shot” game, meaning that he does not care about the overall profit of the informed traders as whole but his own profits in these two time periods. There will always be relocation of informed traders from one market to the other until an equilibrium reaches where the expected profits for each individual informed trader across these two markets are equal. Although each individual informed trader does not make choice of \( \varepsilon \) strategically, the collective result of their choices produces an optimal \( \varepsilon \). In other words, we have the following equation in the equilibrium

\[
E[\pi_i] = E[\pi_c],
\]

where \( \pi_i (i = e, c) \) is the profit incurred in market \( i \) across two time periods for each informed trader.

Now the questions we want to address are following. First, how will the informed traders distribute between the equity and corporate bond markets before the corporate bond market becomes post-trade transparent? Second, will the improved transparency in the corporate bond market change the partition of informed traders or not? Optimizing the trading strategy leads to the following proposition:

**Proposition 2.** If informed traders can allocate trading activities between the equity and corporate bond markets, the equilibriums of the allocation of informed traders between these two markets depend on the leverage ratio of the company. For low-leverage companies, informed traders only trade in the equity market; for high-leverage
companies, informed traders only trade in the corporate bond market; for mid-leverage companies, informed traders trade in both markets. In particular, the conditions for equilibriums are given as below (see Figure 2 for illustration)

\[
\begin{align*}
\text{Pure Equilibrium (Informed trading at equity market only), } & \text{if } \frac{(V_H - K)}{(K - V_L)} \geq \frac{1}{(1 - q)^2}, \\
\text{Pure Equilibrium (Informed trading at equity market only), } & \text{if } \frac{(V_H - K)}{(K - V_L)} \leq (1 - q)^2, \\
\text{Pooled Equilibrium (Informed trading at both markets), otherwise.}
\end{align*}
\]

**Proof:** See Appendix.

As stated in Proposition 2, if given the choice between the equity and corporate bond markets, informed traders will evaluate the financial status of the company at first and act accordingly. Solvency is the key concern since the higher the leverage ratio, the higher the risk of the equity as residual claims. If the leverage ratio is too high, all the informed traders will choose to trade in the corporate bond market, which is in line with the “fly-to-quality” argument. Another interesting observation from Figure 2 is that, given the high leverage ratio of a company, as the fraction of informed traders as a whole increases, some of them are finally “crowed-out” to trade in the equity market. This is due to the worsened expected profits in one market as the competition among informed traders increases.

After we know how the informed traders are distributed between these two markets at equilibriums, it would be natural to investigate whether those equilibriums change before and after the improvement of the transparency at the corporate bond market. We have the following lemma.

**Lemma 3.** If informed traders can allocate trading activities between the equity and corporate bond markets, improved transparency in the corporate bond market only has fourth-order impact on the equilibriums of the allocation of informed traders between these two markets, i.e.
\[ \varepsilon^{\text{trpt}} \approx \varepsilon^{\text{opq}} , \]

where \( \varepsilon^{\text{opq}} \) and \( \varepsilon^{\text{trpt}} \) is the optimal informed trading proportion in the equity market chosen by informed traders, in the Opaque and Transparent setting respectively.

**Proof:** See Appendix.

Lemma 3 demonstrates that informed traders do not put as much weight to the improved transparency in the corporate bond market as they do to the other variables (such as the financial status of the company). Even though the corporate bond market is more transparent than before, the *incremental* increase in transparency only has minimal effect on the informed traders switching to other markets.

Once it is determined that the allocation of informed traders between these two markets are roughly the same before and after the corporate bond market becomes post-trade transparent, we can make use of the conclusions we draw for the exogenous informed trading proportion context. Similar to Proposition 1, we have the following statement.

**Proposition 3.** If informed traders can allocate trading activities between the equity and corporate bond markets, improved transparency in the corporate bond market will reduce transaction costs in both the corporate bond market and the equity market, i.e.

\[ \Psi^{\text{trpt}}_i < \Psi^{\text{opq}}_i , \quad i=e,c \]

where \( \Psi^{\text{trpt}}_i \) and \( \Psi^{\text{opq}}_i \) is the Opaque and Transparent spread of market \( i (= e,c) \) in period 2, respectively.

**Proof:** See Appendix.

Consistent with what we saw in the existent empirical works concerning the TRACE system, transaction costs in the corporate bond market still reduce even though the trading strategy of informed traders is endogenous. The most interesting part of Proposition 4, however, is that the higher liquidity in the corporate bond market does not
come at a cost of the lower liquidity in the equity market. A prior guess might be that more informed traders will switch to the equity market due to the higher transparency in the corporate bond market, thus causing a more severe adverse selection problem there. However Lemma 3 and Proposition 4 demonstrate that there is little such “switching” effect, and it is clear that higher transparency enhances the liquidity in both markets. Dealers in the equity market observe post-trade information of the corporate bond issued by the same firm, which enriches their information set, and this desirable result is the main driving force of the better liquidity.

In practice, it is an empirical question whether the liquidity in the equity market is enhanced or harmed by the improved transparency associated with the initiation of the TRACE system in the corporate bond market. Although we make a clear prediction of an improved liquidity in the equity market due to the introduction of the TRACE system, to the best of our knowledge, there have been no formal empirical studies on this topic.

Before we conclude, it would be favorable to examine how the profit each informed trader expects to earn is affected by the changing market transparency. If informed traders can move freely from one market to the other, at equilibrium each of them will earn the same amount of profit. Some calculation leads to the following proposition.

**Proposition 4.** If informed traders can allocate trading activities between the equity and corporate bond markets, improved transparency in the corporate bond market will reduce the profits informed traders expect to earn in both markets, i.e.

\[ E[\pi^{\text{trpt}}}] < E[\pi^{\text{opq}}], \]  

where \( E[\pi^{\text{trpt}}] \) and \( E[\pi^{\text{opq}}] \) is the profit each informed trader expects to earn in the Opaque and Transparent setting, respectively.

**Proof:** See Appendix.

As expected, higher transparency in the market reduces the expected profits to be earned by the informed traders, which is in line with the previous findings that
transparency will “crack down” on informed trading (Chowdhry and Nanda (1991)). Considering that trading is a “zero-sum” game in our model, this is also not surprising given the smaller transaction costs in the Transparent setting.

5. Conclusion

Based on Glosten and Milgrom (1985), we analyze a model where informed trading can happen simultaneously in the equity and corporate bond markets. Particularly, we examine how liquidity in these two security markets is affected by an improvement of transparency in the corporate bond market.

This issue is of large significance. Transparency lies at the heart of policy debates concerning market designing. How to evaluate the impact of transparency policy overhaul such as disclosure requirement is critical for researchers, lawmakers, practitioners and investors to correctly assess the role of transparency. The common view that transparency enhances liquidity of one market is mostly supported by studies limited to the market on which transparency takes direct effect, both theoretically and empirically. Our model shows that such liquidity enhancement in the “local market” reduces transaction cost not only in its own market but also in other related markets. In our case, the higher transparency in the corporate bond market can reduce the transaction costs in the equity market as well. So to ensure a complete assessment of the performance of transparency-related policy, the “cross-market” effect needs to be accounted for as well.

There are several important findings in this paper. First, we poses that when informed traders can not freely switch between the equity and the corporate bond markets, higher transparency in the corporate bond market will reduce the spreads set by dealers in both markets. The beneficial liquidity result in the corporate bond market has been confirmed by both Bessembinder et al. (2006) and Edwards et al. (2006).

When the trading strategy of informed traders become endogenous, something dramatic happens. The freedom of informed traders results in three different types of equilibriums, which depend on the firm-specific solvency status and the overall proportion of informed
trading. High-leverage firms are more likely to invite informed traders to trade in the corporate bond market, and low-leverage firms in the equity market.

In the between, we observe pooling equilibrium. And there are two different effects coming to play: the “information effect” and the “switching effect”. The former always widens the spread in the markets, while the later behaves differently in these two markets. We further argue that the “information effect” always dominates the “switching effect”. Consequently, the disclosure of post-trade information of corporate bond decreases the transaction costs as measured by bid-ask spread in both the corporate bond market and the equity market.

There are many potential extensions to our model. One of them is to introduce the cost of gaining private information of the firm. In our model, there is no cost for informed traders and it is always profitable to trade. However, it is imaginable that higher transparency will thin the profits further and finally drive away some high-cost informed traders. As a result, the proportion of informed traders in both markets will become endogenous too. It would be interesting to see how the model performs with this change.
Appendix:

**Proof of Lemma 1 and Proposition 1.** We begin with calculating the spreads in the Opaque setting. From equation (3) and Table 1, the bid and ask prices in period 2 are given by

\[
\begin{align*}
B_{t}^{\text{opq}2} &= E[V_{t} \mid \text{Sell}^{2}_{t}, D_{e}^{t}], \\
A_{t}^{\text{opq}2} &= E[V_{t} \mid \text{Buy}^{2}_{t}, D_{e}^{t}],
\end{align*}
\]  

(A1)

where superscript \(\text{opq}2\) stands for “period 2 in the Opaque setting”. Consider the equity market at first. There can be either a sell order or buy order in period 1, which means \(D_{e}^{1}\) can only take two possible values, \(\text{Sell}^{1}_{e}\) or \(\text{Buy}^{1}_{e}\).

If \(D_{e}^{1} = \text{Sell}^{1}_{e}\), since the dealers are Bayesian, the bid price can be written as

\[
E[B_{e}^{\text{opq}2} \mid \text{Sell}^{1}_{e}] = E[E[V_{e} \mid \text{Sell}^{2}_{e}] \mid \text{Sell}^{1}_{e}]
\]

\[
= E[V_{e} \mid \text{Sell}^{2}_{e}, \text{Sell}^{1}_{e}]
\]

\[
= (V_{H} - K) \cdot \frac{P[V_{H} \mid \text{Sell}^{2}_{e}, \text{Sell}^{1}_{e}] + 0 \cdot P[V_{L} \mid \text{Sell}^{2}_{e}, \text{Sell}^{1}_{e}]}{P[V_{H}^{2} | V_{H}] \cdot P[V_{H}^{1} | V_{H}] + P[V_{H}^{2} | V_{L}] \cdot P[V_{L}^{1} | V_{L}]}
\]

\[
= (V_{H} - K) \cdot \frac{\left(\frac{1}{2} \cdot (1 - q_{e})\right)^{2} \cdot \frac{1}{2} + \left(q_{e} + \frac{1}{2} \cdot (1 - q_{e})\right)^{2} \cdot \frac{1}{2}}{2(1 + q_{e}^{2})},
\]

(A2)

which has used the assumption of inter-temporal independence between the trading period 1 and 2.

Similarly, the ask price of one unit of equity in period 2 can be calculated as

\[
E[A_{e}^{\text{opq}2} \mid \text{Sell}^{1}_{e}] = E[E[V_{e} \mid \text{Buy}^{2}_{e}] \mid \text{Sell}^{1}_{e}]
\]

\[
= E[V_{e} \mid \text{Buy}^{2}_{e}, \text{Sell}^{1}_{e}]
\]

\[
= \frac{(V_{H} - K)}{2},
\]

(A3)

If \(D_{e}^{1} = \text{Buy}^{1}_{e}\), similar calculation yields...
Considering that

\[
E[B_e^{opq2} | \text{Buy}_e^1] = \frac{(V_H - K)}{2},
\]

\[
E[A_e^{opq2} | \text{Buy}_e^1] = (V_H - K) \cdot \frac{(1 + q_e)^2}{2(1 + q_e^2)},
\]

it is straightforward to show that

\[
E[B_e^{opq2}] = E[B_e^{opq2} | \text{Sell}_e^1] \cdot \text{Pr}[\text{Sell}_e^1] + E[B_e^{opq2} | \text{Buy}_e^1] \cdot \text{Pr}[\text{Buy}_e^1],
\]

\[
E[A_e^{opq2}] = E[A_e^{opq2} | \text{Sell}_e^1] \cdot \text{Pr}[\text{Sell}_e^1] + E[A_e^{opq2} | \text{Buy}_e^1] \cdot \text{Pr}[\text{Buy}_e^1],
\]

and

\[
\text{Pr}[\text{Sell}_e^1] = \text{Pr}[\text{Sell}_e^1 | V_H] \cdot \text{Pr}[V_H] + \text{Pr}[\text{Sell}_e^1 | V_L] \cdot \text{Pr}[V_L],
\]

\[
\text{Pr}[\text{Buy}_e^1] = \text{Pr}[\text{Buy}_e^1 | V_H] \cdot \text{Pr}[V_H] + \text{Pr}[\text{Buy}_e^1 | V_L] \cdot \text{Pr}[V_L],
\]

So the spread of equity market in period 2 in the Opaque setting is

\[
\Psi_e^{opq2} = E[A_e^{opq2}] - E[B_e^{opq2}] = (V_H - K) \cdot \frac{q_e}{(1 + q_e^2)},
\]

In the same way, we can obtain the prices and spread of the corporate bond as

\[\text{An interesting note worth pointing out is that we have}
\]

\[
E[A_e^{opq2} | S_e^1] = E[O_e^{opq2} | B_e^1] = \frac{(V_H - K)}{2},
\]

which implies that a buy order followed by a sell is equivalent to a sell order followed by a buy, neither of which has net effect on the dealers prior belief about a security’s value.
\[
E[B_{c}^{opp2}] = \frac{(1 - q_{c})(1 + q_{c}q_{c}^{2})K + (1 + q_{c})(1 - q_{c}q_{c}^{2})V_{L}}{2(1 - q_{c}^{2}q_{c}^{2})},
\]
\[
E[A_{c}^{opp2}] = \frac{(1 + q_{c})(1 - q_{c}q_{c}^{2})K + (1 + q_{c})(1 + q_{c}q_{c}^{2})V_{L}}{2(1 - q_{c}^{2}q_{c}^{2})},
\]

and

\[
\Psi_{c}^{opp2} = E[A_{c}^{opp2}] - E[B_{c}^{opp2}] = (K - V_{L}) \cdot \frac{(1 - q_{c}^{2}) \cdot q_{c}}{1 - q_{c}^{2}q_{c}^{2}}.
\]

Although the method is virtually the same in the Transparent setting, the computation is slightly more complicated. We will also take the equity market as an example. Now that the post-trade information of the corporate bond market in period one is public, (A1) becomes

\[
B_{i}^{opp2} = E[V_{i} | Sell^{2}_{i}, D^{1}_{i}, D^{1}_{c}],
\]
\[
A_{i}^{opp2} = E[V_{i} | Buy^{2}_{i}, D^{1}_{c}, D^{1}_{c}],
\]

where \(D^{1}_{i}, D^{1}_{c}\) can take four possible values, \(\{Sell^{1}_{c}, Sell^{1}_{c}\}, \{Sell^{1}_{c}, Buy^{1}_{c}\}, \{Buy^{1}_{c}, Sell^{1}_{c}\}\), and \(\{Buy^{1}_{c}, Buy^{1}_{c}\}\).

If \(D^{1}_{c}, D^{1}_{c}\) = \(\{Sell^{1}_{c}, Sell^{1}_{c}\}\), Bayes’ rule gives the bid and ask prices of these two securities as

\[
E[B_{c}^{opp2} | Sell^{1}_{c}, Sell^{1}_{c}] = E[E[V_{c} | Sell^{2}_{c}] | Sell^{1}_{c}, Sell^{1}_{c}] = (V_{H} - K) \cdot P[V_{H} | Sell^{2}_{c}, Sell^{1}_{c}, Sell^{1}_{c}] + 0 \cdot P[V_{L} | Sell^{2}_{c}, Sell^{1}_{c}, Sell^{1}_{c}]
\]
\[
= (V_{H} - K) \cdot \frac{P[Sell^{2}_{c}, Sell^{1}_{c}, Sell^{1}_{c} | V_{H}] \cdot P[V_{H}]}{P[Sell^{2}_{c}, Sell^{1}_{c}, Sell^{1}_{c} | V_{H}] \cdot P[V_{H}] + P[Sell^{2}_{c}, Sell^{1}_{c}, Sell^{1}_{c} | V_{L}] \cdot P[V_{L}]}
\]
\[
= (V_{H} - K) \cdot \frac{\left(\frac{1}{2} \cdot (1 - q_{c})\right)^{2} \cdot \left(\frac{1}{2} \cdot (1 - q_{c})\right)^{1/2}}{\left(\frac{1}{2} \cdot (1 - q_{c})\right)^{1/2} \cdot \left(\frac{1}{2} \cdot (1 - q_{c})\right)^{1/2} + \left(q_{c} + \frac{1}{2} \cdot (1 - q_{c})\right)^{2} \cdot \left(\frac{1}{2} \cdot (1 - q_{c})\right)^{1/2}}
\]
\[
= (V_{H} - K) \cdot \frac{(1 - q_{c})^{2} \cdot (1 - q_{c})}{2[1 + 2q_{c}q_{c} + q_{c}^{2}]},
\]
and

\[
E[A_{1e,1c}^2 | \text{Sell}_1, \text{Sell}_1] = (V_H - K) \frac{(1 - q_e)}{2},
\]

\[
E[B_{1e,1c}^2 | \text{Sell}_1, \text{Sell}_1] = \frac{(1 - q_e)^2 (1 - q_c)(1 + q_c)^2 (1 + q_e) K + (1 + q_e) V_L}{2(1 + 2q_e q_c + q_e^2)} ,
\]

\[
E[A_{1e,1c}^2 | \text{Sell}_1, \text{Sell}_1] = \frac{(1 - q_e) K + (1 + q_e) V_L}{2} .
\]

(A13)

To conserve space, we will not list all the prices for the cases where \( \{D_e, D_c\} \) takes other three possible values. After using the conditional possibility identities like (A5) and (A6), the expected prices charged by the dealers in period 2 can be obtained as

\[
E[B_{1e,1c}^2] = \frac{(V_H - K)}{2} - \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (V_H - K)}{2(1 + (2 - 4q_c^2) q_e^2 + q_c^4)},
\]

\[
E[A_{1e,1c}^2] = \frac{(V_H - K)}{2} + \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (V_H - K)}{2(1 + (2 - 4q_c^2) q_e^2 + q_c^4)},
\]

\[
E[B_{1e,1c}^2] = \frac{(K + V_L)}{2} - \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (K - V_L)}{2(1 + (2 - 4q_c^2) q_e^2 + q_c^4)},
\]

\[
E[A_{1e,1c}^2] = \frac{(K + V_L)}{2} + \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (K - V_L)}{2(1 + (2 - 4q_c^2) q_e^2 + q_c^4)}.
\]

(A14)

And we have the spreads in the Transparent setting in period 2 as

\[
\Psi_{1e,1c}^2 = \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (V_H - K)}{(1 + (2 - 4q_e^2) q_c^2 + q_c^4)},
\]

\[
\Psi_{1e,1c}^2 = \frac{q_e (1 - q_c^2)(1 + q_e^2 (1 - 2q_c^2)) (K - V_L)}{(1 + (2 - 4q_e^2) q_c^2 + q_c^4)}. 
\]

(A15)

Finally since the informed traders can not allocate trading between the equity and corporate bond market, \( q_e, (i = e,c) \) are the same for the Opaque and Transparent settings. If we ignore the items in the order of \( q^3 \) or higher, then from (A8), (A10) and (A15) it is easy to check that
\[ \frac{\Psi_{e \text{opq}^2}}{\Psi_{e \text{opq}^2}} = \frac{1 - q_e^2 + q_e^2}{1 + 2q_e^2}, \]

and

\[ \frac{\Psi_{e \text{opq}^2}}{\Psi_{e \text{opq}^2}} = \frac{1 + q_e^2 - q_e^2}{1 + 2q_e^2}, \]

are both less than 1 since \(0 < q_i < 1\), which concludes the proof. 

**Proof of Lemma 2.** Take the spread of corporate bond in the Opaque setting for example. Let \(q_e = q\epsilon\) and \(q_e = q(1 - \epsilon)\), and substitute into (A10), we have

\[ \Psi_{e \text{opq}^2} = (K - V_L) \cdot \frac{(1 - \epsilon^2 q^2) \cdot q \cdot (1 - \epsilon)}{1 - \epsilon^2 (1 - \epsilon^2 q^2)^4}. \]  \hspace{1cm} (A16)

Taking the first derivative with respect to \(\epsilon\), (A16) becomes

\[
\frac{\partial \Psi_{e \text{opq}^2}}{\partial \epsilon} = (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2(-2 + 3\epsilon - (1 - \epsilon)^2(-2 + 3\epsilon)q^2 + (1 - \epsilon)^2\epsilon q^4))}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)}
\]

\[
= (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2(1 - 3 + 3\epsilon - (1 - \epsilon)^2(-2 + 3\epsilon)q^2 + (1 - \epsilon)^2\epsilon q^4))}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)}
\]

\[
= (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2(1 + (1 - \epsilon)(-3 + (1 - \epsilon)(2 - 3\epsilon)q^2 + (1 - \epsilon)^2\epsilon q^4)))}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)}
\]

\[
< (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2(1 + (1 - \epsilon)(-3 + 2(1 - \epsilon)q^2 + (1 - \epsilon)^2\epsilon q^4)))}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)}
\]

\[
< (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2(1(1 - \epsilon)(-3 + 2 + 1)))}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)}
\]

\[
= (K - V_L) \cdot q \cdot \frac{(-1 + \epsilon q^2)}{(1 - \epsilon^2 (1 - \epsilon^2 q^2)^4)} < 0
\]

where we have used the fact \((0 \leq q \leq 1), (0 \leq \epsilon \leq 1)\). Similarly, the rest of (7) can be proved.

**Proof of Proposition 2.** Let us start with the Opaque setting. The equilibrium is characterized by equation (7), where the expected profit for individual informed trader at market \(i, E[\pi_i \text{opq}^2]\), is further given by

\[ E[\pi_i \text{opq}^2] = E[\pi_i \text{opq}^2 \mid \text{trade}] \cdot \text{Prob(\text{trade})}. \] \hspace{1cm} (A17)
where \( E[\pi_{\text{opq}} | \text{trade}] \) is the conditional expected profit incurred in market \( i \) given that this informed trader is picked to trade at both time periods, and \( \text{Prob}(\text{trade}) \) is the probability of being picked at one time period.

First we calculate the probability of being picked to trade at one time period for one informed trader. Without loss of generality, we assume the number of unformed traders at each market is the same, denoted by \( N \), and the number of informed traders at market \( i \) is \( M_i \). Since the probability of informed trading at market \( i \) is \( q_i \), we have \( M_i / (M_i + N) = q_i \). Solving for the total number of traders at market \( i \) yields

\[
(M_i + N) = N / (1 - q_i) \quad \text{and} \quad \text{Prob}(\text{trade}) = \frac{1}{M_i + N} = \frac{(1 - q_i)}{N}.
\] (A18)

Next we compute \( E[\pi_{\text{opq}} | \text{trade}] \). There are two possible values of \( V, V_H \) and \( V_L \).

Consider the case where \( V = V_H \) at first, then the informed traders will always want to buy a security in any of the markets. We will begin the demonstration of calculation with the equity market in period 1. Note that the probability of a successful informed transaction is \( q_e \) and the profit once the transaction is accomplished is \( (V_H - K) - E[A_{\text{eq}}^{\text{opq}} | V = V_H] \), where \( E[A_{\text{eq}}^{\text{opq}} | V = V_H] \) is the expected ask price of one unit of equity in period 1 conditional on the higher value of the firm’s assets. Since the dealers do not observe any trading history when set the ask price in period 1, we have \( E[A_{\text{eq}}^{\text{opq}} | V = V_H] = E[A_{\text{eq}}^{\text{opq}1}] \) which is given by

\[
E[A_{\text{eq}}^{\text{opq}1}] = E[V_e | B_e^1] = (V_H - K) \cdot \frac{(1 + q_e)}{2}.
\] (A19)

So we obtain the profit in the equity market in period 1, given \( V = V_H \), as

\[
E[\pi_{\text{eq}}^{\text{opq1}} | \text{trade}, V = V_H] = (V_H - K) \cdot \frac{(1 - q_e)}{2}.
\] (A20)

Similarly, we can compute the other profits given \( V = V_H \) in the Opaque setting as

\[
E[\pi_{\text{eq}}^{\text{opq2}} | \text{trade}, V = V_H] = (K - V_L) \cdot \frac{(1 - q_e)}{2},
\]

\[
E[\pi_{\text{eq}}^{\text{opq2}} | \text{trade}, V = V_H] = (V_H - K) \cdot \frac{(1 - q_e)}{2(1 + q_e^2)},
\] (A21)

\[
E[\pi_{\text{eq}}^{\text{opq2}} | \text{trade}, V = V_H] = (K - V_L) \cdot \frac{(1 - q_e)(1 - q_e^2)}{2(1 - q_e^2 q_e^2)}.
\]

Summing up (A19) and (A20) across time periods for each market gives the total expected profits of each informed trader if they observe \( V = V_H \) and successfully trade to
exploit this information. After obtaining the profits when $V = V_L$, in the same way, we have the conditional profits in the Opaque setting according to the formula

$$E[\pi_i^{opq} | \text{trade}] = E[\pi_i^{opq} | \text{trade}, V = V_H] \cdot P[V = V_H] + E[\pi_i^{opq} | \text{trade}, V = V_L] \cdot P[V = V_L],$$  \hspace{1cm} \text{(A21)}

which finally leads to, after substituting $q_e = q\varepsilon$ and $q_c = q(1 - \varepsilon)$ into the equations,

$$E[\pi_e^{opq} | \text{trade}] = (V_H - K) \cdot \left(1 - \alpha q\right) \frac{2 + \varepsilon^2 q^2}{2(1 + \varepsilon^2 q^2)},$$

$$E[\pi_c^{opq} | \text{trade}] = (K - V_L) \cdot \left(1 - q + \alpha q\right) \frac{2 - \varepsilon^2 q^2 - (1 - \varepsilon)^2 \varepsilon^2 q^4}{2(1 - (1 - \varepsilon)^2 \varepsilon^2 q^4)}.$$  \hspace{1cm} \text{(A22)}

Substituting (A17) and (A18) into (7), we get

$$E[\pi_e^{opq} | \text{trade}] \cdot (1 - \alpha q) = E[\pi_c^{opq} | \text{trade}] \cdot (1 - q + \alpha q),$$ \hspace{1cm} \text{(A23)}

which after combined with (A22) gives the equation for the equilibrium $\varepsilon$ as below (after ignoring the items in the order of $q^4$ or higher),

$$(V_H - K) \cdot (1 - \alpha q)^2 \cdot \frac{2 + \varepsilon^2 q^2}{2(1 + \varepsilon^2 q^2)} = (K - V_L) \cdot (1 - q + \alpha q)^2 \cdot \frac{2 - \varepsilon^2 q^2}{2}$$  \hspace{1cm} \text{(A24)}

If $\varepsilon = 1$, which means all informed traders choose the equity market, we have

$$\frac{(V_H - K)}{(K - V_L)} = \frac{1}{(1 - q)^2}.$$  \hspace{1cm} \text{If } \varepsilon = 0, \text{ which means all informed traders choose the corporate bond market, we have } \frac{(V_H - K)}{(K - V_L)} = (1 - q)^2. \text{ These two values set the boundaries between the pure equilibrium and the pooled equilibrium. In other words, we have (for the opaque setting)}$

$$\begin{cases} \text{Pure Equilibrium (Informed trading at equity market only), if } \frac{(V_H - K)}{(K - V_L)} \geq \frac{1}{(1 - q)^2}, \\ \text{Pure Equilibrium (Informed trading at equity market only), if } \frac{(V_H - K)}{(K - V_L)} \leq (1 - q)^2, \\ \text{Pooled Equilibrium (Informed trading at both markets), otherwise.} \end{cases}$$

The separation of these equilibriums is depicted in Figure 2 as well.
Following the same procedure, we can derive the equation of force equilibrium of the Transparent setting as below (after ignoring the items in the order of $q^4$ or higher)

\[
(V_H - K) \cdot \frac{(1 - \alpha q)^2}{2} \cdot \left(1 + \frac{(1 - \varepsilon)^2 q^2}{1 + 2 \varepsilon^2 q^2}\right)
= (K - V_L) \cdot \frac{(1 - q + \varepsilon q)^2}{2} \cdot \left(1 + \frac{(1 + \varepsilon)^2 q^2}{1 + 2(1 - \varepsilon)^2 q^2}\right).
\]

(A25)

After some calculation, it is easy to show that the separation of equilibriums in the transparent setting is the same as in the opaque setting.

**Proof of Lemma 3.** For the Opaque setting, if we further simplify (A24), we have

\[
(V_H - K) \cdot (1 - \alpha q)^2 = (K - V_L) \cdot (1 - q + \alpha q)^2 \cdot \frac{(2 - \varepsilon^2 q^2) \cdot (1 + \varepsilon^2 q^2)}{2 + \varepsilon^2 q^2},
\]

which yields, after removing the $q^4$ terms on the right-hand side,

\[
(V_H - K) \cdot (1 - \alpha q)^2 = (K - V_L) \cdot (1 - q + \alpha q)^2.
\]

(A26)

Solving (A26) for the equilibrium $\varepsilon$ gives

\[
\varepsilon_{\text{opq}} = \frac{V_H - V_L - q \cdot (K - V_L) - (2 - q) \cdot \sqrt{(V_H - K) \cdot (K - V_L)}}{q \cdot (V_H + V_L - 2K)}
\]

(A27)

For the Transparent setting, (A25) can be written as

\[
(V_H - K) \cdot \frac{(1 - \alpha q)^2}{2} \cdot \left(1 + \frac{(1 - \varepsilon)^2 q^2 + \varepsilon^2 q^2}{1 + 2 \varepsilon^2 q^2}\right) = (K - V_L) \cdot \frac{(1 - q + \varepsilon q)^2}{2} \cdot \left(1 + \frac{(1 + \varepsilon)^2 q^2}{1 + 2(1 - \varepsilon)^2 q^2}\right)
\]

or

\[
(V_H - K) \cdot \frac{(1 - \alpha q)^2}{2} \cdot \frac{2 - (1 - \varepsilon)^2 q^2 + 3\varepsilon^2 q^2}{1 + 2 \varepsilon^2 q^2} = (K - V_L) \cdot \frac{(1 - q + \varepsilon q)^2}{2} \cdot \frac{2 + 3(1 - \varepsilon)^2 q^2 - \varepsilon^2 q^2}{1 + 2(1 - \varepsilon)^2 q^2}
\]

which will lead to (A26) as well after further dropping the $q^4$ terms.

Thus we have proved that both the opaque and transparent settings result in the same equilibrium $\varepsilon$.

**Proof of Proposition 3.** Since we have proved that $\varepsilon^{\text{opq}} = \varepsilon^{\text{opq}}$ when ignoring the terms in the order of $q^4$ or higher, the proof of Proposition 3 is exactly the same as Proposition 1 where the allocation of informed trading is exogenous.
Proof of Proposition 4. Since we have proved that $\varepsilon^{\text{trpt}} = \varepsilon^{\text{opq}}$, we denote this equilibrium $\varepsilon$ by $\varepsilon^*$. At equilibrium, the expected profit earned by each informed trader is identical at the equity market and the corporate bond market, i.e.

$$E[\pi_e^{\text{opq}}(\varepsilon^*)] = E[\pi_e^{\text{trpt}}(\varepsilon^*)]$$

and

$$E[\pi_e^{\text{opq}}(\varepsilon^*)] = E[\pi_e^{\text{trpt}}(\varepsilon^*)].$$

Thus we only need to prove the expected profit is smaller in the Transparent setting at either of the market. Let us pick the equity market for example.

From (A24) and (A25), we have the expected profit at equilibrium on the equity market as

$$E[\pi_e^{\text{opq}}(\varepsilon^*)] = (V_H - K) \cdot \left(1 - \varepsilon^* q\right)^2 \cdot \frac{2 + \varepsilon^* q^2}{2(1 + \varepsilon^* q^2)}$$

(A28)

for the Opaque setting, and

$$E[\pi_e^{\text{trpt}}(\varepsilon^*)] = (V_H - K) \cdot \left(1 - \varepsilon^* q\right)^2 \cdot \frac{1 + (1 - (1 - \varepsilon^*)^2 q^2)(1 + \varepsilon^* q^2)}{1 + 2\varepsilon^* q^2}$$

(A29)

for the Transparent setting.

The difference between (A28) and (A29) is

$$E[\pi_e^{\text{opq}}(\varepsilon^*)] - E[\pi_e^{\text{trpt}}(\varepsilon^*)]$$

$$= (V_H - K) \cdot \left(1 - \varepsilon^* q\right)^2 \cdot \left(\frac{2 + \varepsilon^* q^2}{1 + \varepsilon^* q^2} - \frac{1 + (1 - (1 - \varepsilon^*)^2 q^2)(1 + \varepsilon^* q^2)}{1 + 2\varepsilon^* q^2}\right)$$

$$= (V_H - K) \cdot \left(1 - \varepsilon^* q\right)^2 \cdot \left(\frac{2 + \varepsilon^* q^2}{1 + \varepsilon^* q^2} - \frac{2 + 2\varepsilon^* q^2 - q^2 + 2\varepsilon^* q^2}{1 + 2\varepsilon^* q^2}\right)$$

$$= (V_H - K) \cdot \left(1 - \varepsilon^* q\right)^2 \cdot \frac{q^2 \cdot (1 - \varepsilon^*)^2}{(1 + \varepsilon^* q^2) \cdot (1 + 2\varepsilon^* q^2)} > 0$$

In other words, the expected profit in the Transparent setting is smaller than the one in the Opaque setting.
Reference:


Figure 1. Tree diagram of trading process

$V$ is the firm’s underlying value, $\frac{1}{2}$ is the possibility that $V$ can take $V_H$ or $V_L$, and $q_i (i = e, c)$ is the proportion of informed traders in equity or corporate bond market. An informed trader will always buy if $V = V_H$ and sell otherwise. Uninformed trader is equally likely to sell or buy. This process repeats in period 2.
Figure 2. Equilibriums of informed trading allocation
Q is the proportion of informed traders in both the equity and the corporate bond markets. $V_H$ and $V_L$ is the high and low underlying value of the firm, respectively. $K$ is the face value of the corporate bond.
### Table 1 Information Sets of Dealers When Setting Quotes

This table illustrates the information available to dealers when they are setting quotes. Ø denotes empty information set. $D_i^1 (i=e,c)$ denotes the trading history of market $i$ in the first trading period.

<table>
<thead>
<tr>
<th></th>
<th>Opaque</th>
<th></th>
<th>Transparent</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trading Period $t$</strong></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Equity Market</strong></td>
<td>Ø</td>
<td>$D_e^1$</td>
<td>Ø</td>
<td>$D_e^1, D_c^1$</td>
</tr>
<tr>
<td><strong>Corporate Bond Market</strong></td>
<td>Ø</td>
<td>$D_e^1$</td>
<td>Ø</td>
<td>$D_e^1, D_c^1$</td>
</tr>
</tbody>
</table>