Understanding the effects of government spending on Consumption and Output in the presence of Incomplete Credit Market: Reconciling theory with Empirical evidence

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Basic Question of the Paper

- What is the effect of government spending on output and consumption if there is incomplete credit market?
- Can credit market imperfections help us to reconcile the effect of government spending on consumption and output derived from a theoretical model to the results found in empirical literature?
Fatas and Mihov (2001) found that when there is an unanticipated increase in government spending, there is:

1. Persistence increase in GDP
2. Persistence increase in aggregate consumption, durables and non-durables: Crowding in
3. Immediate crowding in of Investment followed by crowding out. The effect is cyclical.
4. Persistent increase in Private employment.
5. Increase in net tax.
6. Clear positive Comovement between Durable and Non-Durable consumption.

Blanchard and Perotti (2004) found similar effect on GDP.
Theoretical Puzzles

Puzzle 1: Crowding in- Crowding out puzzle

1. Basic RBC model predicts crowding out of consumption after a G shock.

2. A large group of RBC models tried to solve the puzzle. Most failed.

3. The only notable success was achieved:
   * Either by invoking very strong and unrealistic assumptions about consumer behavior (Gali, Salido and Valles (2004)),
   ** Or by using unconventional preference structure (Linnemann (2005)).
Theoretical Puzzles

Puzzle 2: Comovement Puzzle

1. Basic RBC models predicts (I will show) a negative Comovement between durable and non-durables, unless make very strong assumption.

2. Monetary economists (Barsky (2007) and Monacelli (2008)) first pointed out this problem in MP. But the problem is generic to any policy change.
   * They showed using Collateral requirement as a borrowing constraint in a Keynesian framework can solve this problem

3. No body has yet tried to explain this in a RBC setup.

Other Puzzles


2. Some have matched the labour supply response, but failed to explain other puzzles.
I plan to develop a DSGE model with an elaborate specification for the credit market imperfections. In the theoretical environment, I will investigate the nature of the effect of government spending on output and consumption and see how the relationships depend on:

1. The deep behavioral parameters of the model.
2. The degree of credit market imperfections.
3. The nature of government financing.

I will conduct prior predictive analysis to see whether the model matches certain second order moments of the data.

In the simulation environment, I will:

1. Derive impulse response functions from the model of an unanticipated temporary change in the government spending.
3. I will show that my model can account for puzzle 1, puzzle 2 and match labor supply and cyclical investment response.
There are two types of households, named *Borrowers* and *Savers*, of measure of \((1 - F)\) and \(F\).

They are identical except for their rate of patience.

People can borrow from private credit market by issuing one period state-contingent bond.

The bonds are not enforceable. Hence the private debt market is incomplete.

Both of them face a borrowing Constraint:

\[
B_{t+1}^{a,p} \leq (1 - \phi) B_t^{a,p} + (1 - \pi) \left[ D_{t+1}^{a,p} - (1 - \delta_D) D_t^{a,p} \right]
\]

(3)

where: \(\pi\) is the fraction of a new durable good that cannot serve as collateral

\(\phi\) is the rate at which the principle is repaid. So can borrow against past borrowing.
Optimization

Let's index the consumers by \( h = a, p \). Then the general utility function of the agents look like:

\[
U(.) = \left[ \left( C_t^h \right)^{1 - \frac{1}{\sigma}} + V^h \left( D_t^h \right)^{1 - \frac{1}{\sigma}} \right]^{\frac{1 - \frac{1}{\tau}}{1 - \frac{1}{\sigma}}} - \frac{\eta}{2} \left( \frac{D_t^h - D_{t-1}^h}{D_t^h} \right)^2 + \chi^h \frac{(1 - L_t^h)^{1 - \theta} - 1}{1 - \theta}
\]

Where:

\[
C_t^*^h = C_t^h - b_h C_{t-1}^h
\]

\[
\frac{\eta}{2} \left( \frac{D_t^h - D_{t-1}^h}{D_{t-1}^h} \right)^2 = \text{Deliberation cost}
\]
The consumer chooses a sequence $C_t^h, D_t^h, B_t^h, K_t^{hs}, I_t^h, L_t^{hs}, \mu_t^h$ st:

The budget constraint:

$$C_t^h + D_t^h + X_t^{hd} + I_t^h \leq \left(1 - \tau_t^L\right) W_t L_t^{hs} + B_t^h - B_{t-1}^h R_{1t-1} + (1 - \delta_D) D_{t-1}^h$$

$$+ \bar{\delta} \tau_t^k K_{t-1}^{hs} + (1 - \tau_t^k) r_t \mu_t^h K_{t-1}^{hs} + X_{t-1}^{hd} R_{2t-1} + TR_t^h$$

The borrowing constraint:

$$B_t^h \leq (1 - \phi) B_{t-1}^h + (1 - \pi) \left[D_t^h - (1 - \delta_D) D_{t-1}^h\right]$$

and the law of motion of capital stock:

$$K_t^{hs} \leq \left\{1 - s \left(\frac{I_t^h}{I_{t-1}^h}\right)\right\} I_t^h + (1 - \delta_t^h) K_{t-1}^{hs}$$

where:

$$S(1) = S'(1) = 0, S''(1) = \gamma \geq 0, \delta_t^h = \delta \left(\mu_t^h\right)^{\omega}$$
The production function used by the firm is defined as follows:

\[ Y_t = f \left( L_t^a, L_t^p, K_{t-1}^a, K_{t-1}^p; \mu_t^a, \mu_t^p \right) \]

\[ = \left\{ \left( F(\mu_t^a K_{t-1}^a) + (1 - F)(\mu_t^p K_{t-1}^p) \right)^{\alpha} \right\} \left\{ F L_t^a + (1 - F) L_t^p \right\}^{1-\alpha} \]

The Representative firm rents capital and labor from agents to maximize profit

\[ \text{Profit} = \left\{ \left( F(\mu_t^a K_{t-1}^a) + (1 - F)(\mu_t^p K_{t-1}^p) \right)^{\alpha} \right\} \left\{ F L_t^a + (1 - F) L_t^p \right\}^{1-\alpha} \\
- r_t \left\{ \left( F(\mu_t^a K_{t-1}^a) + (1 - F)(\mu_t^p K_{t-1}^p) \right)^{\alpha} \right\} \left\{ F L_t^a + (1 - F) L_t^p \right\} \]

\[ - W_t \left\{ F L_t^a + (1 - F) L_t^p \right\} \]
\( U(.) \in C^{(2)}(R_+, R) \), strictly concave, \( U'_{ch}(.) > 0 \), \( U'_{Dh}(.) > 0 \),
\( U'_{1-Lh}(.) > 0 \), \( U''_{ch}(.) < 0 \), \( U''_{Dh}(.) < 0 \), \( U''_{1-Lh}(.) < 0 \) for all
\( C^h \succeq 0 \), \( D^h \succeq 0 \), \( L^h \succeq 0 \), and
\( \lim_{C^h \to 0} U'_{c^h}(.) = +\infty \),
\( \lim_{D^h \to 0} U'_{c^h}(.) = +\infty \), \( \lim_{L^h \to 0} U'_{c^h}(.) = +\infty \).

\( f(.) \in C^{(2)}(R_+, R) \), strictly concave, \( f'_{L_{ad}}(.) > 0 \), \( f'_{L_{pd}}(.) > 0 \),
\( f'_{K_{ad}}(.) > 0 \), \( f'_{K_{pd}}(.) > 0 \), \( f''_{L_{ad}}(.) < 0 \), \( f''_{L_{pd}}(.) < 0 \), \( f''_{K_{ad}}(.) < 0 \),
\( f''_{K_{pd}}(.) < 0 \) and
\( \lim_{L^h \to 0} f'_{L^h}(.) = +\infty \), \( \lim_{K^h \to 0} f'_{K^h}(.) = +\infty \).

So, Inada conditions are satisfied.
Government Budget Constraint and TVC

\[ G_t + X_{t-1}^s R_{2t-1} + \delta \tau_t^k \left\{ \left( F(\mu_t^a K_{t-1}^a) + (1 - F)(\mu_t^p K_{t-1}^p) \right) \right\} + TR_t = T_t + X_t^s \]

\[ T_t = T_t^l + T_t^k \tag{27} \]

\[ T_t^l = F \tau_t^L W_t L_t^a + (1 - F) \tau_t^L W_t L_t^p \tag{28} \]

\[ T_t^k = \tau_t^k \left\{ \left( F(\mu_t^a K_{t-1}^a) + (1 - F)(\mu_t^p K_{t-1}^p) \right) \right\} \tag{29} \]

\[ E_t \lim_{T \to \infty} \beta_t^{T} \lambda_{h,t+T} K_{t+T}^{h} = 0 \]

\[ E_t \lim_{T \to \infty} \beta_t^{T} \lambda_{h,t+T} B_{t+T}^{h} = 0 \]

\[ E_t \lim_{T \to \infty} \beta_t^{T} \lambda_{h,t+T} X_{t+T}^{d} = 0 \]
Fiscal Rules

\[
\ln \left( \frac{S_t^{TR^a}}{S^{TR^a}} \right) = q_{TR^a} \cdot M_M \cdot \ln \left( \frac{S_t^{B}}{S^{B}} \right), \quad q_{TR^a} \leq 0
\]

\[
\ln \left( \frac{S_t^{TR^p}}{S^{TR^p}} \right) = q_{TR^p} \cdot N_N \cdot \ln \left( \frac{S_t^{B}}{S^{B}} \right), \quad q_{TR^p} \leq 0
\]

\[
\ln G_t = \rho_G \ln G_{t-1} + \epsilon_t^G
\]

\[
\ln \left( \frac{\tau_t^L}{\tau^L} \right) = q_L \ln \left( \frac{S_t^{B}}{S^{B}} \right), \quad q_L \geq 0
\]

\[
\ln \left( \frac{\tau_t^K}{\tau^K} \right) = q_K \left( \frac{S_t^{B}}{S^{B}} \right), \quad q_K \geq 0
\]

\[M_M = \frac{TR^a}{TR} \text{ if distributionally neutral transfer adjustment and 1 otherwise} \]

\[N_N = \frac{TR^p}{TR} \text{ if distributionally neutral transfer adjustment and 1 otherwise} \]
Aggregation

\[ l_t = F l_t^a + (1 - F) l_t^p \]

\[ B_t^d = F B_t^{a \,d} + (1 - F) B_t^{p \,d} \]

\[ X_t^d = F X_t^{a \,d} + (1 - F) X_t^{p \,d}, X_t^s = F X_t^{a \,s} + (1 - F) X_t^{p \,s} \]

\[ X_t^s = F X_t^{a \,s} + (1 - F) X_t^{p \,s} \]

\[ K_t^d = F K_t^{a \,d} + (1 - F) K_t^{p \,d}, K_t^s = F K_t^{a \,s} + (1 - F) K_t^{p \,s} \]

\[ L_t^d = F L_t^{a \,d} + (1 - F) L_t^{p \,d}, L_t^s = F L_t^{a \,s} + (1 - F) L_t^{p \,s} \]

\[ C_t = F C_t^a + (1 - F) C_t^p, D_t = F D_t^a + (1 - F) D_t^p \]

\[ TR_t^a = Ftr_t^a, TR_t^p = (1 - F) tr_t^p \]
Market Clearing Conditions

Labor Market: \( L_t^d = L_t^s \)

Capital Market: \( K_t^d = K_t^s \)

Private Debt Market: \( B_t = 0 \)

Public Debt Market: \( X_t^d = X_t^s \)

Goods Market: \( C_t + I_t + G_t + D_t = Y_t + (1 - \delta_D)D_{t-1} \)

Also, some new variables:

Durable_Service = \( D_S_t = D_t - (1 - \delta_D)D_{t-1} \)

Aggregate Consumption = \( AD_C_t = C_t + D_S_t \)
Definition of Perfect Foresight competitive equilibrium

Definition

A perfect foresight competitive equilibrium (PFCE) is a pair of sequence of prices \( \{ r_t, W_t \}_{t=1}^{\infty} \), a sequence of a set of consumers’ decisions \( \{ C_t^h, D_t^h, B_t^h, K_t^h, X_t^h, l_t^h, \mu_t^h \}_{t=1}^{\infty} \), a sequence of firm’s decisions \( \{ K_t^d, L_t^d \}_{t=1}^{\infty} \), a sequence of policy variables, \( \{ X_t^s, G_t, \tau^K_t, \tau^L_t, TR_t \}_{t=1}^{\infty} \) such that, given initial levels of capital stock, private and public debt, \( K_{t-1}, B_{t-1}, X_{t-1} \), the optimization for the agents and firm’s are solved; the goods, capital, labor and the debt markets clear; the transversality conditions for capital and debts hold; the government budget constraint and at least one of the policy rules and all the aggregate conditions are satisfied. Furthermore, we will only consider the ranges of the fiscal adjustment parameters- the q’s- that are consistent with the existence of a rational expectations PFCE.
Definition of Stationary Perfect Foresight competitive equilibrium

**Definition**

We say that
\[ r, W, C^a, D^a, I^a, L^a, K^a, X^a, B^a, C^p, D^p, I^p, L^p, K^p, X^p, B^p, Y, G \]
is a stationary perfect foresight competitive equilibrium (SPFCE) if it is a perfect foresight competitive equilibrium that exhibits the following properties:

1. Both \( r \) and \( W \) are constant over time.

2. \( I^{a,p} = \delta^{a,p} K^{a,p} \) are constant over time.

3. \( C \frac{C}{Y}, G \frac{G}{Y}, K \frac{K}{Y}, X \frac{X}{Y}, B^a \frac{B^a}{Y}, B^p \frac{B^p}{Y}, D \frac{D}{Y} \) are all constant over time.

4. \( K = FK^a + (1 - F)K^p \) are constant over time where \( K \) is the unique solution to \( r = f_k(.) \).

5. \( L = FL^a + (1 - F)L^p \) is constant over time where \( L \) is the unique solution to \( W = f_L(.) \).
Assumptions

- The savers are more patient than the borrowers; $\beta^a > \beta^p$.

Axiom
In the stationary steady state, there is a positive amount of public debt held by consumers; $X > 0$.

Axiom
In the steady state, the budget constraint of both type of consumers bind; $\lambda_h > 0, h = a, p$.

Axiom
In the steady, consumption of durable and non-durables are positive; $C^a, C^p, D^a, D^p > 0$. 
**Theorem 1**

**Definition**
Let $R^K$ be the after tax-after subsidy gross return on capital in the steady state.

**Theorem**
Let assumptions 5-8 are satisfied. Then given remarks 1, 2 and the TVCs, there exists a unique SPFCE given by the following conditions:

1. $R^K = \frac{1}{\beta^a} = R_1 = R_2$.
2. $I^p = 0, K^p = 0, X^p = 0$.
3. The borrowing constraint binds for the borrower and does not bind for the saver; $\psi_a = 0, \psi_p > 0$. Also, $B^a > 0, B^p < 0$. 

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Theorem 2

Definition
Define a Log-linearized model $y_t = \Gamma_1 y_{t-1} + C + \Psi z_t + \Pi \eta_t$ where $y_t$ is a vector of log deviations of the variables of the original model from their respective steady state, $z_t$ is a vector of exogenous shocks, $\eta_t$ is a vector of expectation errors.

Theorem
*(Critical but vague for now): For a very small perturbation, the dynamics in the log-linearized model, which is approximated around the neighborhood of the SPFCE, will follow theorem 1*
Simulation Strategy

- Log-linearized the model around the steady state.
- Use Sims(2001)'s algorithm to generate impulse response functions for a unanticipated 1% increase in G.
Impulse Response: Distributionally neutral VS Non-neutral transfer adjustment

Figure 1: Response to G Shock: Transfers Adjust (Distributionally Neutral vs Non-Neutral)

- Y: Aggregate Consumption
- Non-Durable
- Durable-Flow
- L: Tax Revenue
- G: Investment
- L_a: Private Debt
- Private Debt
- Non-Durable-Saver
- Durable-Saver
- Non-Durable-Borrower
- Durable-Borrower

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Impulse Response: Labor Tax Adjust

Figure 2: Response to G Shock: Labor Tax Adjust

- Aggregate Consumption
- Non-Durable
- Durable-Flow
- Government Spending
- Aggregate Consumption
- Private Debt
- Public Debt
- Non-Durable-Saver
- Durable-Saver
- Non-Durable-Borrower
- Durable-Borrower

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Figure 3: Response to G Shock: Capital Tax Adjust

- **Y**: Aggregate Consumption
- **L**: Non-Durable
- **G**: Tax Revenue
- **L_a**: Investment
- **Private Debt**: Durable-Flow
- **L_p**: Investment
- **Public Debt**: Durable-Borrower
- **Non-Durable-Saver**: Non-Durable-Borrower
- **Durable-Saver**: Durable-Borrower
- **Non-Durable-Borrower**: Durable-Borrower

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Comparison with other Model: Basic Model without habit

Figure 4: Response to G Shock: Transfers Adjust: Comparison with Model without Habit

Graphs showing the response of various economic variables to a G shock, comparing a model with habit to one without. The variables include Aggregate Consumption, Non-Durable, Durable-Flow, L (labour), G (government spending), Tax Revenue, Investment, Private Debt, Public Debt, Non-Durable-Saver, Durable-Saver, Non-Durable-Borrower, Durable-Borrower.
Comparison with other Model: Basic Model without borrowing Constraint

Figure 5: Response to G Shock: Transfers Adjust: Comparison with model without Borrowing Constraint
Comparison with other Model: Representative agent model with habit

Figure 6: Response to G Shock: Transfers Adjust: Comparison with Rep agent model with habit

- Y
- Aggregate Consumption
- Non-Durable
- Durable-Flow
- L
- G
- Tax Revenue
- Investment
- Public Debt
Comparison with other Model: Kiyotaki and Moore(97)'s Borrowing Constraint
Comparison with other Model: Iacoviello(04) and Monacelli(08)’s Borrowing Constraint

Figure 8: Response to G Shock: Transfers Adjust: Comparison with Monacelli(2008)’s Borrowing constraint

Y

Aggregate Consumption

Non-Durable

Durable-Flow

L

G

Tax Revenue

Investment

L_a

Private Debt

L_p

Public Debt

Non-Durable-Saver

Durable-Saver

Non-Durable-Borrower

Durable-Borrower