Financial Development and the Patterns of International Capital Flows

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Abstract

We develop a tractable two-country overlapping generations model and show analytically that the cross-country differences in financial development help explain three recent empirical facts characterizing international capital flows: financial capital flows from relatively poor to relatively rich countries while FDI flows in the opposite direction; net capital flows are from poor to rich countries; despite of its negative net positions on international investment, the United States receives a positive net investment income. We also show that how the patterns of capital flows may reverse along the growth path of a developing country.

According to Matsuyama (Econometrica, 2004), in the presence of credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output. We show that this symmetry-breaking property depends crucially on the assumption of the fixed investment size of entrepreneurial projects.

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1 Introduction

Standard international macroeconomics predicts that capital should flow from capital-rich countries, where the marginal return on investment is low, to capital-poor countries, where the marginal return is high. Furthermore, there would be no difference between gross and net capital flows, as capital movements are unidirectional.

The patterns of international capital flows observed in the past 20 years, however, stand in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2006, 2007). First, Prasad, Rajan, and Subramanian (2006, 2007) show that since 1998, the average per-capita income of countries running current account surpluses has actually been below that of the deficit countries, i.e., net capital flows have been “uphill” from poor to rich countries. Second, Ju and Wei (2007) observe that many developing economies, including China, Malaysia, and South Africa, are net importers of foreign direct investment (hereafter, FDI) and net exporters of financial capital at the same time, while developed countries such as France, the United Kingdom, and the United States follow the opposite pattern. Third, despite of negative net positions on international investment since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

Two strands of literature have recently offered explanations for these empirical puzzles. One is based on the risk-sharing that investors can achieve by diversifying investment globally (Devereux and Sutherland, 2009; Tille and van Wincoop, 2008a,b). In these models, the cross-correlation patterns of aggregate shocks hitting individual economies determine the patterns of international portfolio investment. Mendoza, Quadrini, and Ríos-Rull (2009) analyze the joint determination of financial capital flows and FDI in a heterogeneous-agent model with idiosyncratic endowment and investment risks. The precautionary savings motive is the main driving force of two-way capital flows in their model. Apart from Mendoza, Quadrini, and Ríos-Rull (2009), there is no substantive difference between FDI and portfolio investment in this literature. The credit-supply side is the center of the analysis here.

The other strand of literature focuses on the behavior on the credit-demand side. Matsuyama (2004) shows that in the presence of credit market imperfection, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output, a result he calls “symmetry breaking”. In the steady state, financial capital flows from the poor to the rich country. Aoki, Benigno, and Kiyotaki (2007) analyze how the adjustment process to capital account liberalization depends on the degree of domestic financial development. Caballero, Farhi, and Gourinchas (2008) show that cross-regional differences in the ability of asset supply can shape global capital flows, interest rates and portfolios, in line with the em-
pirical facts. Ju and Wei (2007) show in a static model that when both FDI and financial capital flows are allowed, all financial capital leaves the country where credit market imperfections are more severe, while FDI flows into this country. Thus, capital mobility allows investors to fully bypass the underdeveloped financial system.

Our paper extends the second strand of literature and provides a tractable, two-country, overlapping generations model to explain the three empirical facts mentioned above. Our model builds on the notion that individuals in an economy differ in their ability of using physical capital for production (Kiyotaki and Moore, 1997). From an efficiency perspective, it would be desirable to transfer all capital to the most productive individuals to maximize aggregate output. Due to financial frictions, however, the most productive individuals are subject to borrowing constraints. The constraint on the aggregate credit demand has a general equilibrium effect, keeping the rate of return on loans (hereafter, the loan rate) lower and, thus, the rate of return on equity capital (hereafter, the equity rate) higher than the marginal return on investment. In the absence of financial frictions, these two interest rates should be both equal to the marginal return on investment. Thus, financial frictions generate an equity premium in this deterministic model. The overlapping-generations framework together with certain assumptions allows us to keep the aggregate credit supply perfectly inelastic to the loan rate\(^1\) so that we can fully isolate the effect of financial frictions on the aggregate credit demand and interest rates.

Following Matsuyama (2004), we take the tightness of the borrowing constraints as a measure of a country’s level of financial development. In a financially more developed country, credit contracts can be enforced and borrowers can be monitored more effectively, allowing the most productive individuals to borrow more from financial institutions. The two countries in our model differ fundamentally only in the level of financial development. Under international financial autarky, the equilibrium level of interest rates depends on two factors. First, a lower aggregate capital-labor ratio implies a higher marginal return on investment and a higher equity and loan rate. We call this the neoclassical effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. Second, for a given capital-labor ratio, a lower level of financial development implies a lower aggregate demand for loans, which leads to a lower loan rate and a higher equity rate. We call this the credit-demand effect of financial development. Under certain assumptions, financial frictions in equilibrium only distort the two interest rates but not production efficiency in our model.\(^2\) This way, financial development affects

\(^{1}\)The assumption in Caballero, Farhi, and Gourinchas (2008) that agents have a constant probability of death also delivers the feature of perfectly inelastic credit supply.

\(^{2}\)As shown in von Hagen and Zhang (2009), our qualitative results also hold in an extension where financial frictions in equilibrium distort production efficiency as well as interest rates. However, such an extension undermines the model tractability.
the two interest rates only through the credit-demand channel but not the neoclassical channel. Under international financial autarky, the neoclassical effect is muted in the steady state so that the loan rate is higher while the equity rate is lower in the more financially developed country.

Under full capital mobility, the cross-country interest rate differentials drive international flows of financial capital and FDI. Intuitively, the credit market in the more financially developed country has a larger capacity so that capital in the net term flows from the less to the more financially developed country. Due to net capital outflow, the less financially developed country endogenously becomes poor, while the other one becomes rich. Given the cross-country patterns of the two interest rates under international financial autarky mentioned above, financial capital flows “uphill” from the poor to the rich country\(^3\) while FDI flows “downhill”. Despite of its negative net international investment position, the rich country receives a positive net investment income from international investment, because it earns a higher return on its FDI than it pays out on its foreign debts. In fact, the more financially developed country “exports” its service of financial intermediation in the form of two-way capital flows and it receives a positive net reward accordingly. As our first contribution, we show that the cross-country differences in financial development help explain the three empirical facts.

Ju and Wei (2007) assume cross-country differences in terms of capital and labor endowment, financial development, corporate governance, and property right protection for generating two-way capital flows, while the cross-country differences in financial development are sufficient to generate two-way capital flows in our model. The static model of Ju and Wei (2007) is useful for analyzing the immediate impacts of capital account liberalization, but not for studying the transitional and long-run effects, while our overlapping-generations model facilitates the discussion on the short-run and the long-run effects.

We also analyze a more general and realistic scenario where one country is more financially developed and in the steady state (e.g., the developed country) while the other country is less financially developed and below its steady state (e.g., the developing country) before capital account liberalization. Here, the credit-demand effect and the neoclassical effect jointly determine the cross-country interest rate differentials. If the initial capital-labor ratio in the developing country is very low, the neoclassical effect dominates the credit-demand effect so that the loan rate in the developing country is higher than in the developed country under international financial autarky. Immediately after capital account liberalization, both financial capital and FDI flow into the developing country. Thus, there are one-way gross capital flows and “downhill” net capital flows. If the initial

\(^3\)“Uphill” financial capital flows take place between two countries with the same level of financial development in Matsuyama (2004), while the two countries differ in the level of financial development in our model.
capital-labor ratio in the developing country is moderately low, the credit-demand effect dominates the neoclassical effect. Immediately after capital account liberalization, financial capital flows “uphill” but its magnitude is dominated by “downhill” FDI. Thus, there are two-way gross capital flows and “downhill” net capital flows. In both cases, capital account liberalization facilitates net capital inflows and speeds up capital accumulation, which increases the growth rate of the developing country.

If the initial capital-labor ratio in the developing country is slightly below the steady state, “uphill” financial capital flows dominates “downhill” FDI immediately after capital account liberalization. Thus, there are two-way gross capital flows and “uphill” net capital flows. Net capital outflows hamper capital accumulation and eventually keeps the developing country converging to the steady state lower than under international financial autarky. Thus, as our second contribution, we show that for a developing country, the patterns of capital flows may change or even reverse along its growth process and capital mobility has opposite effects on capital accumulation as well as welfare at the different stages of its growth process.

In our model, financial capital flows as well as FDI affect the owners of credit capital and equity capital differently. The welfare predictions of capital account liberalization depend on the levels of financial development in the two countries. In our OLG framework, liberalizing capital flows affects the intergenerational distribution of income due to transitional effects. This way, our model explains why capital account liberalization often encounters both support and opposition in a given country.

Our model setting differs from Matsuyama (2004) in only one aspect. We assume that the mass of individuals in a country who can invest is fixed, while the investment size of each project is endogenously determined. Thus, changes in aggregate investment takes place on the intensive margin instead of on the extensive margin as in Matsuyama (2004). We prove that under capital mobility, there exists a unique and stable steady-state equilibrium in the presence of credit market imperfections in our model. Thus, countries with identical fundamentals have the same and unique steady state under capital mobility. As our third contribution, we show that Matsuyama’s symmetry-breaking property depends critically on the assumption of a fixed investment size of entrepreneurial projects and, thus, changes in aggregate investment taking place only on the extensive margin.

The rest of the paper is structured as follows. Section 2 sets up the model and discusses the long-run patterns of interest rates with respect to financial development under international financial autarky. Section 3 analyzes two scenarios of capital mobility. Section 4 concludes with the main findings. Appendix collects the proofs of propositions and other relevant discussions.
2 The Model under International Financial Autarky

We use an overlapping generations model closely related to Matsuyama (2004). The world economy consists of two countries, Home (H) and Foreign (F). There are two types of goods, a final good, which is internationally tradable and serves as the numeraire, and a capital good, which is not traded internationally. The price of the capital good in country \( i \in \{ H, F \} \) and period \( t \) is denoted by \( v_i^t \). The final good can be either consumed or transformed into capital goods. At the beginning of each period, final goods \( Y_i^t \) are produced with capital goods \( K_i^t \) and labor \( L_i^t \) in a Cobb-Douglas fashion. Capital goods fully depreciate after production. Capital goods and labor are priced at their respective marginal products in terms of final goods. To summarize,

\[
Y_i^t = \left( \frac{K_i^t}{\alpha} \right) \alpha \left( \frac{L_i^t}{1 - \alpha} \right)^{1-\alpha}, \quad \text{where} \quad \alpha \in (0, 1) \quad (1)
\]

\[
v_i^t K_i^t = \alpha Y_i^t \quad \text{and} \quad w_i^t L_i^t = (1 - \alpha) Y_i^t. \quad (2)
\]

There is no uncertainty in the economy. In this section, we assume that capital flows are not allowed between the two countries.

In both countries, the population consists of two generations, the old and the young, which live for two periods each. There is no population growth and the population size of each generation in each country is normalized to one. Agents consume only when old.\(^4\) Young agents are endowed with a unit of labor which they supply inelastically to the production of final goods \( L_i^t = 1 \) at the wage rate \( w_i^t \) in period \( t \). Each generation consists of two types of agents of mass \( \eta \) and \( 1 - \eta \), respectively, which we call entrepreneurs and workers. Only young entrepreneurs are endowed with the productive projects and it takes one period to produce capital goods using final goods.

Consider any particular worker born in period \( t \). With no other investment opportunity available to him,\(^5\) the worker has to lend his entire labor income inelastically to the credit market at a gross interest rate of \( r_i^t \) in period \( t \) to finance his consumption in period \( t + 1 \),

\[
c_{t+1}^{i,w} = w_i^t r_i^t. \quad (3)
\]

Consider any particular entrepreneur born in period \( t \). The entrepreneur invests \( i_i^t \) units of final goods into his project in period \( t \) and produces \( R_i^t \) units of capital goods in period \( t + 1 \). Given the gross loan rate of \( r_i^t \), he finances the investment with own fund \( w_i^t \)

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\(^4\)Allowing agents to have concave utility with respect to consumption in both periods does not change our results but makes the analysis more complicated.

\(^5\)Excluding workers from other savings alternatives facilitates the closed-form solutions, but it may seem implausible. von Hagen and Zhang (2009) show that allowing workers to have other investment opportunity does not change our results qualitatively but undermines the model tractability.
and debt $z^i_t$. Due to limited commitment problems, however, he can borrow only against a fraction of the project revenues,

$$r^i_t z^i_t = r^i_t (i^i_t - w^i_t) \leq \theta^i Rv^i_{t+1}. \quad (4)$$

Following Matsuyama (2004, 2007, 2008), we regard $\theta^i \in (0, 1]$ as a measure of the level of financial development in country $i$. That is, $\theta^i$ is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market. Thus, $\theta^i$ captures a wide range of institutional factors. \(^6\) We assume that countries $H$ and $F$ differ only in the level of financial development, i.e., $0 < \theta^H < \theta^F \leq 1$.

Let $\lambda^i_t = \frac{z^i_t}{w^i_t}$ denote the investment-to-equity ratio of the entrepreneurial project and $I^i_t$ denotes the aggregate project investment in country $i$ and period $t$. Under international financial autarky, the credit market equilibrium condition,

$$\eta(i^i_t - w^i_t) = (1 - \eta)w^i_t, \quad \Rightarrow \quad I^i_t = \eta i^i_t = w^i_t, \quad (5)$$

implies that the aggregate labor income in period $t$ is invested by the young entrepreneurs. Thus, the investment-to-equity ratio is constant at $\lambda^i_t = \frac{1}{\eta}$ and the degree of financial development $\theta^i$ does not affect aggregate investment. Intuitively, in the country with a lower level of financial development, the aggregate credit demand is lower. Given the perfectly inelastic aggregate credit supply, the credit market clears at a lower loan rate.

After repaying the debt in period $t + 1$, the entrepreneur gets $Rv^i_{t+1} - r^i_t z^i_t$ as the return on equity capital (the own funds that he invested in period $t$), $w^i_t$. The equity rate is defined as the rate of return on equity capital,

$$\Gamma^i_t \equiv \frac{Rv^i_{t+1} - r^i_t z^i_t}{w^i_t} = Rv^i_{t+1} + (Rv^i_{t+1} - r^i_t) \left(1 - \frac{1}{\eta}\right) \eta \geq r^i_t. \quad (6)$$

Intuitively, for each unit of own funds that the entrepreneur invests in the project, he gets $Rv^i_{t+1}$ as the marginal return to his own funds. Additionally, he can borrow $(\lambda^i_t - 1) = \frac{1 - \eta}{\eta}$ units of debt which provides him an extra rate of return $(Rv^i_{t+1} - r^i_t)$. The term $(Rv^i_{t+1} - r^i_t) \left(1 - \frac{1}{\eta}\right)$ captures the leverage effect. In equilibrium, the equity rate should be no less than the loan rate; otherwise, he would rather lend than borrow. The inequality in (6) is equivalent to $r^i_t \leq Rv^i_{t+1}$ and can be considered as his participation constraint.

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\(^6\)The pledgeability, $\theta^i$, can be justified in various forms of agency costs, e.g., inalienable human capital by Hart and Moore (1994), costly state verification by Townsend (1979), or unobservable project (effort) choices by Holmstrom and Tirole (1997). See Tirole (2006) for a comprehensive overview. In order to compare our results with Matsuyama (2004), we minimize the deviation of our model setting from his by choosing this simplest form of borrowing constraints. The pledgeability of individual projects may depend on idiosyncratic features. Since we focus on the aggregate implications of financial development, we assume that entrepreneurs investing in country $i$ are subject to the same $\theta^i$ for simplicity.
If \( r_i^t < R v_i^t+1 \), the entrepreneur borrows to the limit, i.e., he finances the investment \( i_t^i \) using \( z_t^i = \frac{\theta^i R v_i^t}{r_i^t} \) units of debt and \( w_t^i \) units of equity capital in period \( t \). After repaying the debt in period \( t+1 \), he gets \((1-\theta^i)R v_i^t v_{t+1}\) as the project return. Given the investment-to-equity ratio at \( \lambda_t^i = \frac{1}{\eta} \), the equity rate has a reduced-form solution,

\[
\Gamma_t^i = (1 - \theta^i)R v_i^t v_{t+1} = \frac{(1 - \theta^i)R v_i^t+1}{\eta}. \tag{7}
\]

Combining equations (6) and (7), we get a reduced-form solution for the loan rate,

\[
r_t^i = \frac{\theta^i R v_i^t+1}{1 - \eta}. \tag{8}
\]

If \( r_t^i = R v_i^t+1 \), the entrepreneur does not borrow to the limit. According to equation (6), the equity rate is equal to the loan rate, \( \Gamma_t^i = r_t^i = R v_i^t+1 \). Lemma 1 summarizes the interest rate patterns with respect to the level of financial development.

**Lemma 1.** Let \( \bar{\theta} \equiv 1 - \eta \). For \( \theta^i \in (\bar{\theta}, 1] \), the equity rate is equal to the loan rate at \( \Gamma_t^i = r_t^i = R v_i^t+1 \) and, thus, the borrowing constraints are not binding; for \( \theta^i \in (0, \bar{\theta}) \), the equity rate is strictly larger than the loan rate, \( \Gamma_t^i = \frac{(1-\theta^i)R v_i^t+1}{\eta} > R v_i^t+1 > \frac{\theta^i R v_i^t+1}{1-\eta} = r_t^i \), and, thus, the borrowing constraints are binding.

Given the labor income \( w_t^i \), the entrepreneur chooses the project investment \( i_t^i \) in period \( t \) to maximize his consumption when old in period \( t+1 \),

\[
c_t^{i,e} = v_{t+1}^i R i_t^i - r_t^i z_t^i = w_t^i \Gamma_t^i, \tag{9}
\]

subject to the borrowing constraint (4) and the participation constraint (6). Note that only one of the two constraints can be strictly binding in equilibrium.

Since the aggregate labor income is invested in the entrepreneurial projects in period \( t \), the aggregate output of capital goods available for production in period \( t+1 \) is

\[
K_{t+1} = R I_t^i = R w_t^i. \tag{10}
\]

The market-clearing condition for final goods in period \( t \) is

\[
C_t^i + I_t^i = Y_t^i, \tag{11}
\]

where \( C_t^i = \eta c_t^{i,e} + (1-\eta)c_t^{i,w} \) is the aggregate consumption of the old generation in period \( t \). We measure the social welfare of the generation born in period \( t \) and country \( i \) using its aggregate consumption when old, \( C_t^{i+1} \).

**Definition 1.** Given the level of financial development \( \theta^i \), market equilibrium in country \( i \in \{H, F\} \) under international financial autarky is a set of allocations of workers, \( \{c_t^{i,w}\} \), entrepreneurs, \( \{i_t^i, z_t^i, c_t^{i,e}\} \), and aggregate variables, \( \{Y_t^i, K_t^i, C_t^i, w_t^i, v_t^i, r_t^i, \Gamma_t^i\} \), satisfying equations (1)-(5) and (9)-(11) as well as Lemma 1.
Given the size of the working population normalized at one, the capital-labor ratio coincides with the aggregate capital stock, $K^i_t$. For simplicity, the capital-labor ratio is also denoted by $K^i_t$.

According to equations (1), (2), and (10), the model dynamics can be characterized by a first-order difference equation on the wage dynamics,

$$w^i_{t+1} = (1 - \alpha)Y^i_{t+1} = \left(\frac{K^i_{t+1}}{\rho}\right)^\alpha = \left(\frac{Rw^i_t}{\rho}\right)^\alpha, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1 - \alpha}. \quad (12)$$

Given $\alpha \in (0, 1)$, the phase diagram of wages is concave and starts from the origin. Its slope, $\frac{dw^i_{t+1}}{dw^i_t} = \alpha \left(\frac{R}{\rho}\right)^\alpha$, converges to $+\infty$ for $w^i_t \to 0$ and to 0 for $w^i_t \to +\infty$. Thus, there exists a unique and stable non-zero steady state with the wage at,

$$w_{IFA} = \left(\frac{R}{\rho}\right)^\rho, \quad (13)$$

where the subscript $IFA$ denotes the steady-state value of a particular variable under international financial autarky. According to equations (12) and (13), the wage dynamics are independent of the level of financial development $\theta^i$ and, thus, the wage converges to the same steady state in the two countries. So do aggregate output and capital.\(^7\)

According to Lemma 1, for $\theta^i \in [1 - \eta, 1]$, the two interest rates are equal to the marginal return on investment, $r^i_t = \Gamma^i_t = Rv^i_{t+1} = R^{\alpha \rho^{1-\alpha}}(K^i_t)^{\alpha(\alpha-1)}$, depending negatively on the capital-labor ratio, $K^i_t$. Thus, the two interest rates are higher in the country with a lower capital-labor ratio. We call this the neoclassical effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. It is independent of the level of financial development $\theta^i$ and, thus, the wage converges to the same steady state in the two countries. So do aggregate output and capital.\(^7\)

According to Lemma 1, for $\theta^i \in [1 - \eta, 1]$, the two interest rates are equal to the marginal return on investment, $r^i_t = \Gamma^i_t = Rv^i_{t+1} = R^{\alpha \rho^{1-\alpha}}(K^i_t)^{\alpha(\alpha-1)}$, depending negatively on the capital-labor ratio, $K^i_t$. Thus, the two interest rates are higher in the country with a lower capital-labor ratio. We call this the neoclassical effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. It is independent of the level of financial development. For $\theta^i \in (0, 1 - \eta)$, besides the neoclassical effect, the loan rate, $r^i_t = Rv^i_{t+1} \frac{\theta^i}{(1 - \eta)}$, is additionally affected by the level of financial development. Given the capital-labor ratio, the loan rate is higher in the country with a higher $\theta$, reflecting the general equilibrium effect of the larger aggregate credit demand. We call this the credit-demand effect of financial development.\(^7\)

According to equation (6), besides the neoclassical effect, the equity rate is additionally affected by the leverage effect. Given the capital-labor ratio, in the country with a lower $\theta$, the lower loan rate keeps the spread higher, $(Rv^i_{t+1} - r^i_t)$. Thus, the leverage effect is larger in the country with a lower $\theta$, as the debt-to-equity ratio is constant at $\frac{(1 - \eta)}{\eta}$. This is confirmed by the reduced-form solution of the equity rate, $Rv^i_{t+1} \frac{(1 - \theta^i)}{\eta}$. Thus, financial development affects the equity rate via the cost-of-funds effect.

\(^7\)The phase diagram of the capital-labor ratio is normally used to prove the existence, uniqueness, and stability of the steady-state equilibrium (Matsuyama, 2004). Our model dynamics can also be represented by the phase diagram of the capital-labor ratio $K^i_{t+1} = Rw^i_t = R \left(\frac{K^i_t}{\rho}\right)^\alpha$, which has the concavity property similar as that of wages. For notational convenience, we use the phase diagram of wages.
The financial frictions do not distort production efficiency but distort the two interest rates in our model.\(^8\) The distortions on the interest rates have a distributional effect on the welfare of borrowers (entrepreneurs) and lenders (workers).

Entrepreneurs invest the aggregate labor income into their projects in period \(t\), \(I_t^i = w_t^i = (1 - \alpha)Y_t^i\), and produce capital goods with the value of \(v_{t+1}^i K_{t+1}^i = \alpha Y_{t+1}^i\) in period \(t + 1\). In the steady state, \(Y_{t+1}^i = Y_t^i = Y^i\), and the marginal return on investment is \(v^i R = \frac{v^i K^i}{w^i} = \frac{\alpha Y^i}{(1 - \alpha)Y^i} = \rho\). Plugging it into Lemma 1, we get the steady-state patterns of interest rates, which is summarized in Proposition 1.

**Proposition 1.** In the unique steady state, the two interest rates are equal to the marginal return on investment, \(r^i = \Gamma^i = \rho\), for \(\theta^i \in (\bar{\theta}, 1]\). The loan rate rises and the equity rate declines in the level of financial development, \(r^i = \frac{\theta^i \rho}{1 - \eta} < \Gamma^i = \frac{(1 - \theta^i) \rho}{\eta}\), for \(\theta^i \in (0, \bar{\theta})\).

\[\begin{array}{c}
\text{Figure 1: Steady-State Patterns under International Financial Autarky}
\end{array}\]

Figure 1 shows the steady-state patterns of output, wages, and interest rates with respect to the level of financial development, where \(\theta^U = \bar{\theta} = 1 - \eta\). Since \(\theta^i\) does not affect production efficiency, the neoclassical effect is muted in the steady state in the sense that the marginal return on investment is constant at \(\rho\). Thus, the cross-country loan rate (equity rate) differentials only depend on the credit-demand (leverage) effect.

According to Proposition 1, for \(\theta^H \in [0, \bar{\theta})\) and \(\theta^F \in (\theta^H, 1]\), the loan rate is strictly lower while the equity rate higher in country \(H\) than in county \(F\); for \(\bar{\theta} \leq \theta^H < \theta^F \leq 1\),

\[\text{The equity premium, } \Gamma^i - r^i > 0, \text{ in the case of } \theta^i \in (0, \bar{\theta}) \text{ results from two factors, i.e., the difference in the ability of using capital productively and the binding borrowing constraints. For } \theta \in (0, \bar{\theta}), \text{ due to the constraint on aggregate credit demand, the credit market clears at a rate below the marginal return on investment. In this case, the positive equity premium is the reward to the exclusive project endowment of entrepreneurs. For } \theta \in (\bar{\theta}, 1], \text{ however, the unconstrained aggregate credit demand pushes up the loan rate to the marginal return on investment and, thus, the equity premium vanishes.}\]
the borrowing constraints are not binding in the two countries so that the credit-demand effect and the leverage effect are muted. Then, the two interest rates coincide with the marginal return on investment, which is same in the two countries.

3 The Model under International Capital Mobility

We consider three scenarios of capital mobility, free mobility of financial capital under which individuals are allowed to lend abroad but not to make equity investments abroad, free mobility of FDI under which entrepreneurs are allowed to make equity investments abroad but individuals are not allowed to lend abroad, and full capital mobility under which individuals are allowed to lend abroad and entrepreneurs are allowed to make equity investments abroad.

In subsections 3.1, 3.2, and 3.3, we assume that the two countries are in the steady state under international financial autarky before capital mobility is allowed from period \( t = 0 \) on in the three scenarios, respectively. Thus, we analyze capital flows between two countries with the same initial steady-state per capita output. In subsections 3.4, we assume that country F is initially in the steady state while country H is below its steady state before capital mobility is allowed from period \( t = 0 \) on. This assumption allows us to analyze a more general and realistic case of capital flows between initially rich and poor countries, or between developed and developing countries.

Let \( \Upsilon_i^t \) and \( \Omega_i^t \) denote financial capital and FDI outflows from country \( i \) in period \( t \), respectively, with negative values indicating capital inflows. Financial capital flows affect the domestic credit supply, \((1 - \eta)w_i^t - \Upsilon_i^t\), while FDI flows affect the amount of equity capital available for domestic investment, \( \eta w_i^t - \Omega_i^t \). We assume that entrepreneurs making FDI borrow only from the host country and are subject to the borrowing constraints in the host country, too. As a result, FDI inflows increase the aggregate credit demand in the host country and reduce that in the parent country. With these changes, the analysis

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9We assume that workers do not have relevant technology required for FDI. FDI may involves entrepreneurs moving their projects to produce abroad, which is analytically equivalent to the approach chosen here for simplicity.

10We assume that the two countries differ only in \( \theta \) and only entrepreneurs can produce capital goods. In equilibrium, the two countries have the same per capita income in the steady state. Introducing cross-country differences in other aspects or allowing workers to produce capital goods using less productive projects (von Hagen and Zhang, 2009) can generate cross-country differences in per capita output in the steady state but undermines the model tractability. Furthermore, our assumptions help identify the role of the cross-country differences in the level of financial development in affecting capital flows.

11Generally, in the case of the debt default, the creditors have to liquidate the project at the local market and the liquidation value depends on the efficiency of the law enforcement and the asset market in the host country. Alternatively, we can also assume entrepreneurs making FDI can borrow from their parent country. In this case, our qualitative results still hold but the model tractability is undermined.
in section 2 carries through under various scenarios of capital mobility, due to the linearity of the preferences, the productive projects, and the borrowing constraints.

Without loss of generality, we focus on the case of \( \theta^H \in (0, \bar{\theta}) \) and \( \theta^F \in (\bar{\theta}, 1] \). Proposition 1 shows that for \( \theta^H \in (\bar{\theta}, 1) \) and \( \theta^F \in (\bar{\theta}, 1] \), the two interest rates are equal to \( \rho \) in both countries and, thus, there are no capital flows in the steady state.

Let \( r^i_{IFA} \equiv \frac{\theta^i \rho}{1 - \eta} \) and \( \Gamma^i_{IFA} \equiv \frac{(1 - \theta^i) \rho}{1 - \theta^i} \) denote the steady-state values of the two interest rates in country \( i \) under international financial autarky, for \( \theta^i \in (0, \bar{\theta}) \).

### 3.1 Free Mobility of Financial Capital

Free mobility of financial capital equalizes the loan rates across the border, \( r^H_t = r^F_t = r^*_t \). Given the world loan rate at \( r^*_t \), we first prove the existence, uniqueness, and stability of the steady-state equilibrium. Then, we analyze how financial capital flows affect the two interest rates in the two countries. Finally, we discuss how financial capital flows affect production and welfare on the individual, country, and world levels.

In the following analysis, we first consider the case where the borrowing constraints are binding in country \( i \). In subsection 3.1.2, we will analyze under what conditions the borrowing constraints are binding. The Cobb-Douglas production function implies,

\[
v^i_{t+1} = (w^i_{t+1})^{\frac{1}{\alpha}} \quad \text{and} \quad I^i_t = \frac{K^i_{t+1}}{R} = \frac{\rho}{R} (w^i_{t+1})^{\frac{1}{\alpha}}.
\]  

(14)

The investment-to-equity ratio is by definition \( \lambda^i_t = \frac{i^i_t}{n^i_t} = \frac{1 - \theta^i R^i_t}{r^i_t} \). Given the domestic equity capital \( \eta w^i_t \), the aggregate investment in the domestic projects is,

\[
I^i_t = \lambda^i_t \eta w^i_t = \frac{\eta w^i_t}{1 - \frac{\theta^i R^i_t}{r^i_t}}.
\]

(15)

Using equations (14) and (15) to substitute away \( I^i_t \) and \( v^i_{t+1} \), we get

\[
\eta w^i_t = \frac{\rho}{R} (w^i_{t+1})^{\frac{1}{\alpha}} - \frac{\theta^i \rho}{r^*_t w^i_{t+1}}.
\]

(16)

### 3.1.1 Existence, Uniqueness, and Stability of the Steady-State Equilibrium

**Proposition 2.** Under free mobility of financial capital, there is a unique and stable non-zero steady state equilibrium with wages at \( w^*_{FCF} = \eta + (1 - \eta) \frac{r^*_{IFA}}{r^*_{FCF}} \), where the subscript \( FCF \) denotes the steady-state value of a particular variable under free flows of financial capital.

**Proof.** See appendix B. \( \square \)
The solid line in the left panel of figure 2 shows the phase diagram of wages under international financial autarky, while the dash-dotted line shows the phase diagram under free mobility of financial capital, given a fixed world loan rate at \( r^* = r_{IFA} \). In both cases, wages converge monotonically and globally to a unique steady state (point A).

Figure 2: The Phase Diagrams of Wage

Our model setting differs from Matsuyama (2004) in only one aspect. He assumes that the investment size of every entrepreneurial project is fixed at \( i_t = 1 \), while the mass of individuals in a country who become entrepreneurs is endogenously determined. In contrast, we assume that the mass of entrepreneurs in a country is fixed at \( \eta \), while the investment size of any entrepreneurial project \( i_t \) is endogenously determined. The difference in assumptions makes a big difference in the property of the steady-state equilibrium. Matsuyama (2004) shows that, at a given world loan rate, free mobility of financial capital may lead to an equilibrium with multiple steady states. In contrast, there is a unique steady-state equilibrium in our model.

Under free mobility of financial capital and a given world loan rate, \( r^*_t \), the borrowing constraints, if binding, take the same form in both models,

\[
 r^*_t (1 - \frac{w_t}{i_t}) = \theta^* R v_t^{i+1} = \theta^* R (w_{t+1}^i) - \frac{1}{\rho}.
\] (17)

**Lemma 2.** Given the world loan rate \( r^*_t \), for \( w_t^i \in [0, 1 - \theta^i] \), the phase diagram of wages in Matsuyama (2004) described by \( r^*_t (1 - w_t^i) = \theta^* R (w_{t+1}^i) - \frac{1}{\rho} \) is strictly convex, and \( w_{t+1}^i \) monotonically increases in \( w_t^i \) with an intercept on the vertical axis at \( w_{t+1}^i = \left[ \frac{\theta^* R}{r^*_t} \right]^\rho \); for \( w_t^i > 1 - \theta^i \), the phase diagram of wages is flat with \( w_{t+1}^i = \left( \frac{R}{r^*_t} \right)^\rho \).

**Proof.** See appendix B.

The solid line in the right panel of figure 2 shows the phase diagram of wages under international financial autarky in Matsuyama (2004), which is exactly same as in our
model. The dash-dotted line shows the phase diagram under free mobility of financial capital in his model, given a fixed world loan rate \( r^*_t = r^i_{iFA} \). The phase diagram is convex for wages below a threshold value. Thus, the steady state at point A, which is stable under international financial autarky, becomes unstable under free mobility of financial capital, because the slope of the phase diagram at point A is larger than one. There are two stable steady states at points B and G. This implies that countries with the identical fundamentals (including the level of financial development) and, thus, the same steady state under international financial autarky may end up with different levels of per capita output and welfare under free mobility of financial capital. Thus, Matsuyama (2004) claims that in the presence of credit market imperfection, financial capital flows may result in the symmetry breaking.\(^\text{12}\)

Intuitively, according to equation (17), given a world loan rate and a fixed size of project investment as in Matsuyama (2004), a marginal increase in the current wage reduces the credit demand of each borrower, \((1 - w^i_t)\), and the debt-investment ratio, \( \frac{z^i_t}{q^i_t} = (1 - \frac{w^i_t}{q^i_t}) = (1 - w^i_t) \). More domestic individuals can borrow at the prevailing world loan rate and produce capital goods as entrepreneurs. The higher the initial wage level \( w^i_0 \), the lower the debt-investment ratio, the larger the expansion of the extensive margin of aggregate investment, and consequently, the larger the increase in aggregate output and the wage in period \( t + 1 \). This explains the convexity of the phase diagram of wages in his model and the possibility of the equilibrium with multiple steady states.

In contrast, given a constant world loan rate and a fixed mass of entrepreneurs in our model, a marginal increase in the current wage enables entrepreneurs to borrow and invest more. According to equation (17), the increase in \( i^i_t \) partially offsets the effects of the marginal increase in \( w^i_t \) on the debt-investment ratio, \( \frac{z^i_t}{q^i_t} = (1 - \frac{w^i_t}{q^i_t}) \), and then on \( w^i_{t+1} \). The higher the initial wage level \( w^i_0 \), the smaller the expansion of the intensive margin of aggregate investment, and consequently, the smaller the increase in the production of capital goods and the wage in period \( t + 1 \). This explains the concavity of the phase diagram of wages in our model and then the uniqueness of the steady-state equilibrium.

### 3.1.2 Interest Rates and Capital Flows

Since the world economy is initially in the steady state under international financial autarky, the loan rate is lower in country H than in country F. From period \( t = 0 \) on, individuals are allowed to lend abroad. The initial cross-country loan rate differentials drive financial capital unambiguously flowing from country H to country F and the loan rate in country H (F) adjusts from below (above) to the world level.

\(^{12}\)The symmetry-breaking property depends on the specific value of the world loan rate and the steady-state equilibrium may be unique under other values of world loan rate. See Matsuyama (2004) for details.
Proposition 3. Under free mobility of financial capital, there exists a unique world loan rate that clears the world credit market. In the steady state, the world loan rate is $r^*_F \in (r^*_{IFA}, r^*_{FCAF})$, where $r^* = \frac{r^*_{IFA} + r^*_{FCAF}}{2}$.

Proof. See appendix B.

Proposition 4. Under free mobility of financial capital, if the borrowing constraints are binding in country $i$, the equity rate is $\Gamma^i_t = \frac{(1-\theta^i)\rho w^i_{t+1}}{w^i_t}$. In the steady state, the equity rate is same as under international financial autarky, $\Gamma^i_{FCAF} = \frac{(1-\theta^i)\rho}{\eta} = \Gamma^i_{IFA}$.

Proof. See appendix B.

Since the borrowing constraints are binding, entrepreneurs borrow $\frac{\theta^i Rv^i_{t+1}}{r^*_t}$ unit of loans for each unit of their project investment in period $t$ and have to invest $\frac{(1-\theta^i)Rv^i_{t+1}}{\Gamma^i_t}$ units of equity capital. Thus, the project-financing equation can be written as

$$\frac{1}{Rv^i_{t+1}} = \frac{\theta^i}{r^*_t} + \frac{1 - \theta^i}{\Gamma^i_t}.$$  \hfill (18)

Financial capital flows affect the equity rate in two ways. Consider country H. On the one hand, financial capital outflows raise the loan rate and the higher borrowing costs tend to reduce the equity rate in country H. On the other hand, financial capital outflow have a general equilibrium effect, i.e., all entrepreneurs reduce their project investment and the decline in the aggregate output of capital goods leads to the rise in the price of capital good in period $t+1$. The equity rate in country H tends to rise due to the increase in the project’s marginal return $Rv^i_{t+1}$.

In period $t = 0$, the first effect dominates the second and the equity rate falls below the previous level under international financial autarky. It is confirmed by the reduced-form solution, $\Gamma^i_0 = \frac{(1-\theta^i)\rho w^i_0}{w^i_0} < \frac{(1-\theta^i)\rho}{\eta} = \Gamma^i_{IFA}$, given the predetermined labor income in period $t = 0$ and the decline in the labor income in period $t = 1$, $w^i_1 < w^i_0 = w_{IFA}$. As the economy converges to the new steady state, the price of capital good rises further and the equity rate converges back to its initial level. In the new steady state, the equity rate is same as under international financial autarky, because the loan-rate effect is fully offset by the price-of-capital effect.

We now discuss under what conditions the borrowing constraints are binding in the steady state.

Proposition 5. Given $\theta^H \in (0, \bar{\theta})$, there exists $\bar{\theta}^F_{FCAF} \in (\bar{\theta}, 1 - \frac{\theta^H \eta}{1-\eta})$ as the function of $\theta^H$ such that for $\theta^F \in (\bar{\theta}^H, \bar{\theta}^F_{FCAF})$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\bar{\theta}^F_{FCAF}, 1]$, the borrowing constraints are binding in country H but not in country F and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}^F_{FCAF}$.
Figure 3 illustrates these results. The horizontal and vertical axes denote the levels of financial development in country H and in country F, $\theta^i \in (0, 1]$, respectively.

![Figure 3: Free Mobility of Financial Capital: Threshold Values](image)

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under international financial autarky. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, the loan rates are equal to the marginal return on investment, which is same in the two countries, according to Proposition 1. In these two cases, there are no financial capital flows even if allowed. The curve splitting regions B and D represents the threshold value $\bar{\theta}^F_{FCF}$ as the function of $\theta^H$ described by equation (28). For the parameters on the curve, the equity rate in country F is equal to the world loan rate, $\Gamma^F_{FCF} = \frac{(1-\bar{\theta}^F_{FCF})}{\eta} = r^*_{FCF}$. Similarly, the curve splitting region $B'$ and $D'$ represents the threshold value $\bar{\theta}^H_{FCF}$ as the function of $\theta^F$. For the parameters on the curve, the equity rate in country H is equal to the world loan rate, $\Gamma^H_{FCF} = \frac{(1-\bar{\theta}^H_{FCF})}{\eta} = r^*_{FCF}$.

**Proposition 6.** For the parameters in region D of figure 3, financial capital flows have a closed-form solution in the steady state, $\Upsilon^i_{FCF} = (r^*_{FCF} - r^i_{IFA}) \frac{(1-\eta)w_{FCF}}{r^*_{FCF}}$ and are from country H to country F, $\Upsilon^H_{FCF} > 0 > \Upsilon^F_{FCF}$; for the parameters in region B, financial capital flows are same as in the case of $\theta^H$ and $\bar{\theta}^F_{FCF}$.

**Proof.** See the proof of Proposition 3 and equation (26) in appendix B.

Table 1 summarizes the steady-state patterns of financial capital flows and the equity premium in the five regions of figure 3. Note that $\Upsilon^F = -\Upsilon^H$. $\Upsilon^H (\theta^i)$ implies that given the parameters in region $B$ and $B'$, financial capital flows depend only on $\theta^i$ not on $\theta^m$, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if
the equity premium is positive. In the following analysis, we only focus on the parameters in region $D$ where the borrowing constraints are binding in both countries.

### 3.1.3 Production and Welfare

Due to financial capital flows, the aggregate investment in country $H$ ($F$) is lower (higher) than under international financial autarky from period $t = 0$ on and so is aggregate output. In the steady state, according to Propositions 2 and 3, the wage is

$$w_{IFA} = \left[\eta + (1 - \eta) \frac{r_{IFA}}{r_{FCF}}\right]^{\rho}$$

and the loan rate is $r^* \in (\tilde{\omega}, r_{IFA})$, implying that $w_{FCF} < w_{IFA} < w_{FCF}$. Thus, aggregate output, which is proportional to the wage, is lower in the steady state in country $H$ than in country $F$, $Y_{H,FCF} < Y_{IFA} < Y_{F,FCF}$, where $Y_{IFA} = \frac{w_{IFA}}{1 - \alpha}$.

**Proposition 7.** Suppose that the world economy is in the steady state under international financial autarky before financial capital mobility is allowed from period $t = 0$ on. For $\theta^H \in (0, \bar{\theta})$ and $\theta^F \in (\theta^H, 1)$, world output is strictly lower from period $t = 1$ on than its initial value, $Y_t^{H,FCF} + Y_t^{F,FCF} < 2Y_{IFA}$.

**Proof.** See appendix B.

As mentioned in section 2, production is efficient and identical in the two countries in the steady state under international financial autarky. From period $t = 0$ on, financial capital flows reallocate aggregate investment among the two countries, which moves the world economy away from the efficient allocation. In our model, the world output losses essentially result from the concavity of the aggregate production with respect to the capital-labor ratio on the country level, $\alpha \in (0, 1)$, and the effect of the Jensen’s Inequality. This also explains the world output losses in Matsuyama (2004). More generally, this is a typical result of the theory of second best. In the presence of domestic credit market imperfections, capital account liberalization causes financial capital flowing to the country where the loan rate is higher rather than to the country where the marginal product of capital is higher. In our model, the more (less) financially developed country endogenously becomes rich (poor), due to capital inflows (outflows). In equilibrium, financial capital flows from the poor country where the marginal product of capital is higher to the rich country where the marginal product of capital is lower. Thus, world output strictly falls.
The welfare of entrepreneurs born in period $t$ and country $i$ is measured by their consumption when old, which, according to Proposition 4, is proportional to the wage in period $t+1$, $c_{t+1}^i = w_t^i \Gamma_t^i = w_t^i \left( \frac{1-\theta^i}{\eta} \right)$. This reflects the joint effect of financial capital flows on the labor income and the equity rate in period $t$. From period $t = 0$ on, due to financial capital flows, aggregate investment in country H (F) is strictly lower (higher) than its initial value and so is the wage in period $t+1$, $w_{t+1}^H < w_{IFA}^H < w_{t+1}^F$. From period $t = 0$ on, entrepreneurs born in country H (F) is strictly worse (better) off than before period 0. Thus, entrepreneurs in the less (more) financially developed country have a strong incentive to oppose (support) policies favoring financial capital mobility.

Given their labor income $w_0^i$ predetermined by aggregate investment in period $t = -1$, workers born in country H (F) and period $t = 0$ are strictly better (worse) off, $c_{0,w}^i = w_0^i \tau^*_{0}$, due to the rise (decline) in the loan rate, $r_{IFA}^H < r^*_0 < r_{IFA}^F$. From period $t = 1$ on, financial capital flows affect the welfare of workers born in country H (F) and period $t$, $c_t^H = w_t^H \tau^*_t$, ($c_t^F = w_t^F \tau^*_t$) through the negative (positive) effect on the labor income, $w_t^H < w_{IFA}^H < w_t^F$, and the positive (negative) effect on the loan rate, $r_{IFA}^H < r_t^* < r_{IFA}^F$. The net welfare effect on workers is ambiguous and depends on the levels of financial development in the two countries. Free mobility of financial capital also has an ambiguous effect on the social welfare on the country level. Since free mobility of financial capital generates world output losses, its welfare implications on the world level is strictly negative in our model. Obviously, under free mobility of financial capital, it is impossible for any public transfer policy to achieve a world-level Pareto improvement, in comparison with the steady-state allocation under international financial autarky.

**Proposition 8.** In comparison with the steady state under international financial autarky, entrepreneurs in country H (F) are strictly worse (better) off, while the implications to the welfare of workers and the social welfare on the country level depend on the specific parameter values.

**Proof.** See appendix B.

Table 2 summarizes the welfare implications of free mobility of financial capital to workers under various parameter constellations, where $c_{FCF}^i$ denotes the steady-state consumption of workers born in country $i$ under free mobility of financial capital.

### Table 2: Welfare Implications of Free Mobility of Financial Capital to Workers

<table>
<thead>
<tr>
<th></th>
<th>$\frac{(\sigma-1)(1-\eta)}{\eta}$</th>
<th>$(-\infty, \frac{\theta^H+\theta^F}{2\theta^F})$</th>
<th>$\left( \frac{\theta^H+\theta^F}{2\theta^F} , 1 \right)$</th>
<th>$[1, \frac{\theta^F}{\theta^H}]$</th>
<th>$(\frac{\theta^F}{\theta^H}, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H,h$</td>
<td>$c_{FCF}^H - c_{IFA}^H$</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$F,h$</td>
<td>$c_{FCF}^F - c_{IFA}^F$</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

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Thus, in some cases, entrepreneurs and workers in the same generation are affected by free mobility of financial capital in opposite ways.

**Proposition 9.** Workers of different generations born in the same country may be affected by financial capital flows in opposite ways during the transitional process from international financial autarky to free mobility of financial capital.

**Proof.** As mentioned above, workers born in country H and period \( t = 0 \) are strictly better off from the higher loan rate. For \( \frac{\rho - 1}{\eta} (1 - \eta) \geq \frac{\rho F}{\rho F} \) in Table 2, financial capital outflows reduce the domestic investment in country H and the resulting decline in the labor income dominates the rise in the loan rate in the long run. Thus, workers born in country H strictly lose from free mobility of financial capital in the long run. Such opposite welfare patterns may also exist for workers in early and later generations born in country F, under certain parameter values. Thus, the introduction of free mobility of financial capital may have opposite welfare effects across generations.

### 3.2 Free Mobility of FDI

The analysis for the scenario under free mobility of FDI highly resembles that under free mobility of financial capital. For simplicity, we briefly summarize the main results here and put the detailed analysis in appendix A.

Under free mobility of FDI, there exists a unique and stable steady state with the wage at \( w_{ FD1 } = w_{ IFA } \left[ 1 - \eta + \eta \frac{\Gamma_{ IFA }^*}{\Gamma_{ FD1 }^*} \right] \rho \), where the subscript FDI denotes the steady-state value of a particular variable under free flows of FDI. The steady-state world equity rate is \( \Gamma_{ FD1 }^* \in (\Gamma_{ IFA }^*, \Gamma_{ IFA }^H) \), where \( \Gamma_{ IFA }^* = \Gamma_{ IFA }^H + \frac{1}{2} \Gamma_{ IFA }^F \). FDI flows from country F to country H with the steady-state value, \( \Omega_{ FD1 }^i = (\Gamma_{ FD1 }^* - \Gamma_{ IFA }^H) \frac{w_{ FD1 }^i}{\Gamma_{ FD1 }^*} \). If the borrowing constraints are binding in country \( i \), the domestic loan rate has a closed-form solution, \( r_{ FD1 }^i = \frac{\rho F}{\rho F} \frac{w_{ FD1 }^i}{\eta w_{ FD1 }^i} \), and its steady-state value has the same form as under international financial autarky, \( r_{ FD1 }^i = \frac{\rho F}{\rho F} \frac{w_{ FD1 }^i}{\eta w_{ FD1 }^i} \). We also show under what conditions the borrowing constraints are binding.

Due to FDI flows, aggregate investment is higher (lower) in country H (F) than under international financial autarky and so is aggregate output. Country H endogenously become richer than country F. FDI flows “uphill” from country F and country H and world output is strictly lower than under international financial autarky, as the marginal return on investment is lower in country H than in country F.

The welfare of workers born in period \( t \) and country \( i \) is measured by their consumption when old, which is proportional to the wage in period \( t + 1 \), \( c_{ i+1 }^{ w } = w_{ FD1 }^i r_{ t }^i = w_{ i+1 }^i \frac{\rho F}{\rho F} \frac{1}{1 - \eta} \), and the second equality is obtainable using the solution of the loan rate mentioned above. This reflects the joint effect of FDI flows on the labor income and the loan rate in period \( t \). From period \( t = 0 \) on, due to FDI flows, aggregate investment in country H (F) is strictly
higher (lower) than its initial value and so is the wage in period \( t+1, w^H_{t+1} > w^IFA > w^F_{t+1} \).
From period \( t = 0 \) on, workers born in country H (F) is strictly better (worse) off than before period 0. Thus, workers in the less (more) financially developed country have a strong incentive to support (oppose) policies favoring international mobility of FDI.

Given their labor income \( w^0_i \) predetermined by aggregate investment in period \( t = -1 \), entrepreneurs born in country H (F) and period \( t = 0 \) are strictly worse (better) off, \( c^H_0 = w^H_0 \Gamma^*_0 \), due to the decline (rise) in the equity rate, \( r^H_{IFA} < r^*_0 < r^F_{IFA} \). From period \( t = 1 \) on, FDI flows affect the welfare of entrepreneurs born in country H (F) and period \( t, c^H_t = w^H_t \Gamma^*_t \), (\( c^F_t = w^F_t \Gamma^*_t \)) through the positive (negative) effect on the labor income, \( w^H_t > w^IFA > w^F_t \) and the negative (positive) effect on the equity rate, \( \Gamma^H_{IFA} > \Gamma^*_t > \Gamma^F_{IFA} \). The net welfare effect on entrepreneurs is ambiguous and depends on the levels of financial development in the two countries. Free mobility of FDI also has an ambiguous effect on the social welfare on the country level. Since free mobility of FDI generates world output losses, its welfare implications on the world level is strictly negative in our model. Obviously, under free mobility of FDI, it is impossible for any public transfer policy to achieve a world-level Pareto improvement, in comparison with the steady-state allocation under international financial autarky.

3.3 Full Capital Mobility

Full capital mobility equalizes the loan rates and the equity rates across the border, \( r^H_t = r^F_t = r^*_t \) and \( \Gamma^H_t = \Gamma^F_t = \Gamma^*_t \). Given the world loan rate and the world equity rate at \( r^*_t \) and \( \Gamma^*_t \), we first prove the existence, uniqueness, and stability of the steady-state equilibrium. Then, we analyze how financial capital and FDI flows affect the loan rate and the equity rate in the two countries. Finally, we discuss how financial capital and FDI flows affect production and welfare on the individual, country, and world levels.

In the following analysis, we first consider the case where the borrowing constraints are binding in country \( i \). In subsection 3.3.2, we will analyze under what conditions the borrowing constraints are binding. Using equation (14) to substitute away \( v^i_{t+1} \) from the project-financing equation (18), we get

\[
(w^i_{t+1})^\frac{\rho}{\rho} = R \left[ \frac{(1 - \theta^i)\rho}{\Gamma^*_t} + \frac{\theta^i\rho}{r^*_t} \right], \quad \text{where} \quad \frac{\partial w^i_{t+1}}{\partial \Gamma^*_t} < 0, \quad \frac{\partial w^i_{t+1}}{\partial r^*_t} < 0. \quad (19)
\]

3.3.1 Existence, Uniqueness, and Stability of the Steady-State Equilibrium

**Proposition 10.** Under full capital mobility, there exists a non-zero steady state equilibrium which is unique and stable, given the world loan rate and the world equity rate at \( r^*_\text{FCM} \) and \( \Gamma^*_\text{FCM} \), respectively. The steady-state wage is \( w^i_{\text{FCM}} = w^IFA \left( \frac{(1 - \theta)^\rho}{\Gamma^*_\text{FCM} + \frac{\theta^i\rho}{r^*_\text{FCM}}} \right)^\rho \),
where the subscript $FCM$ denotes the steady-state value of a particular variable under full capital mobility.

Proof. According to equation (19), $w_{t+1}$ is determined only by $\Gamma_t^*$ and $r_t^*$. The phase diagram of wages is flat and crosses the 45 degree line only once and from the left. □

3.3.2 Interest Rates and Capital Flows

Since the world economy is initially in the steady state under international financial autarky, the loan rate is lower while the equity rate is higher in country H than in country F. From period $t = 0$ on, international flows of financial capital and FDI are allowed. The initial cross-country interest rate differentials drive financial capital flowing from country H to country F while FDI flowing in the opposite direction. As a result, the loan rate in country H (F) adjusts from below (above) to the world level while the equity rate in country H (F) adjusts from above (below) to the world level.$^{13}$

Proposition 11. Under full capital mobility, there exists the unique world equity rate and world loan rate that clear the world equity market and credit market, respectively. In the steady state, the world equity rate and the world loan rate are $\Gamma_{FCM}^* \in (\Gamma_{IFA}^F, \Gamma^*)$ and $r_{FCM}^* \in (r^*, r_{IFA}^F)$.

Proof. See appendix B. □

We now discuss under what conditions the borrowing constraints are binding in the steady state.

Proposition 12. Given $\theta^H \in (\max\{1 - 2\eta, 0\}, 1 - \eta)$, there exists a threshold value $\tilde{\theta}_{FCM}^F = 2(1 - \eta) - \theta^H$ such that for $\theta^F \in (\theta^H, \tilde{\theta}_{FCM}^F)$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\tilde{\theta}_{FCM}^F, 1)$, the world loan rate and equity rate are same as the marginal return to investment, $\Gamma^* = r^* = \rho$, in the steady state, the borrowing constraints are not binding in both countries, and the economic allocation is same as that in the case of $\theta^F = \tilde{\theta}_{FCM}^F$.

Proof. See appendix B. □

Figure 4 illustrates the results. The horizontal and vertical axes denote the levels of financial development in country H and in country F, $\theta^i \in (0, 1]$, respectively.

$^{13}$In order to show the equilibrium result of two-way capital flows in a more intuitive way, we can also assume that the world economy is initially in the steady state under free mobility of financial capital. The loan rate is equalized in the two countries and, according to Proposition 4, the equity rate in country H is higher than in country F. From period $t = 0$ on, entrepreneurs are additionally allowed to make equity investments abroad. The initial cross-country equity rate differentials drive FDI flowing unambiguously from country F to country H. As shown later, FDI do not reverse the pattern of financial capital flows.
Figure 4: Full Capital Mobility: Threshold Values

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under international financial autarky and so are the equity rates. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, according to Proposition 1, the loan rate and the equity rate under international financial autarky are equal to the marginal return on investment, which is same in the two countries, $r^*_{IFA} = \Gamma^*_{IFA} = \rho$. In these two cases, there are no financial capital flows or FDI even if allowed. The line splitting region B and D represents the threshold value of $\bar{\theta}_{FCM}$ as the function of $\theta^H$, while the line splitting region $B'$ and $D'$ represents the threshold value of $\bar{\theta}_{FCM}$ as the function of $\theta^F$. For the parameters on the two lines, the world loan rate is equal to the world equity rate, $r^* = \Gamma^* = \rho$.

**Proposition 13.** For the parameters in region D of figure 4, financial capital flows and FDI have closed-form solutions in the steady state, $\Upsilon^i_{FCM} = (r^i_{FCM} - r^i_{IFA}) \frac{(1-\eta)w^i_{FCM}}{r^i_{FCM}}$ and $\Omega^i_{FCM} = (\Gamma^i_{FCM} - \Gamma^i_{IFA}) \frac{\eta w^i_{FCM}}{\Gamma^i_{FCM}}$. Financial capital flows from country H to country F, $\Upsilon^H_{FCM} > 0 > \Upsilon^F_{FCM}$, while FDI flows in the opposite direction, $\Omega^H_{FCM} < 0 < \Omega^F_{FCM}$. Net capital flows are from country H to country F, $\Upsilon^H_{FCM} + \Omega^H_{FCM} > 0 > \Upsilon^F_{FCM} + \Omega^F_{FCM}$.

For the parameters in region B, financial capital flows and FDI are same as in the case of $\theta^H$ and $\bar{\theta}_{FCM}$. Financial capital flows and FDI cancel out and, thus, net capital flows are zero, $\Upsilon^i_{FCM} + \Omega^i_{FCM} = 0$.

**Proof.** See the proofs of Propositions 11 and 14 in appendix B.

Table 3 summarizes the steady-state patterns of capital flows and the equity premium in the five regions of figure 4. Note that $\Upsilon^F = -\Upsilon^H$ and $\Omega^F = -\Omega^H$. $\Upsilon^H(\theta^i)$ and $\Omega^H(\theta^i)$ implies that given the parameters in region B and $B'$, financial capital flows and FDI
depend only on \(\theta^i\) not on \(\theta^m\), where \(i, m \in \{H, F\}\) and \(i \neq m\). The borrowing constraints are strictly binding only if the equity premium is positive.

<table>
<thead>
<tr>
<th>Region</th>
<th>(A)</th>
<th>(B)</th>
<th>(B')</th>
<th>(D)</th>
<th>(D')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Upsilon^H)</td>
<td>0</td>
<td>(\Upsilon^H(\theta^H) &gt; 0)</td>
<td>(\Upsilon^H(\theta^F) &lt; 0)</td>
<td>((0, \Upsilon^H(\theta^H)))</td>
<td>((\Upsilon^H(\theta^F), 0))</td>
</tr>
<tr>
<td>(\Omega^H)</td>
<td>0</td>
<td>(\Omega^H(\theta^H) &lt; 0)</td>
<td>(\Omega^H(\theta^F) &gt; 0)</td>
<td>((\Omega^H(\theta^H), 0))</td>
<td>((0, \Omega^H(\theta^F)))</td>
</tr>
<tr>
<td>(\Omega^H + \Upsilon^H)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(\Gamma^* - r^*)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Intuitively, the credit market in country F has a larger capacity than that in country H so that capital in the net term flows from country H to country F.

3.3.3 Production and Welfare

**Proposition 14.** In the steady state, for the parameter combination in region \(D\) of figure 4, due to non-zero net capital flows, aggregate output is lower in the steady state in country H than in country F, \(Y_{FCM}^H < Y_{IFA} < Y_{FCM}^F\), and world output is strictly lower than under international financial autarky, \(Y_{FCM}^H + Y_{FCM}^F < 2Y_{IFA}\); for the parameter combination in region \(B\), due to zero net capital flows, aggregate investment and output are identical as under international financial autarky on the country level and on the world level.

**Proof.** See appendix B.

It is the net capital flows that matters for world output losses.

For the parameters in region \(D\) of figure 4, the borrowing constraints are binding and the equity premium is positive in the steady state, \(\Gamma^*_i > r^*_i\). Despite of its status as the net debtor, country F earns a higher return on its equity investment abroad (FDI) than its pays out on foreign debt. Although the closed-form solutions for the two interest rates in the steady state are not available, we can prove \(r_{FCM}^* Y_{FCM}^i + \Gamma_{FCM}^* \Omega_{FCM}^i = 0\). Thus, The net interest income from the international investment of country F,

\[
(r_{FCM}^* - 1)Y_{FCM}^F + (\Gamma_{FCM}^* - 1)\Omega_{FCM}^F = -(Y_{FCM}^F + \Omega_{FCM}^F) = Y_{FCM}^H + \Omega_{FCM}^H,
\]

is fully financed by net capital outflow from country H.

**Proposition 15.** For the parameters in region \(D\) of figure 4, country F has a negative net position on international investment, \(Y_{FCM}^F + \Omega_{FCM}^F < 0\), but receives a positive net investment income, \((r_{FCM}^* - 1)Y_{FCM}^F + (\Gamma_{FCM}^* - 1)\Omega_{FCM}^F > 0\).
For the parameters in region B, country i has zero net position on international investment, \( \Upsilon^i_{FCM} + \Omega^i_{FCM} = 0 \). Since \( r^*_{FCM} = \Gamma^*_{FCM} \), the net investment income is zero, \( (r^*_{FCM} - 1) \Upsilon^i_{FCM} + (\Gamma^*_{FCM} - 1) \Omega^i_{FCM} = 0 \).

Proof. See appendix B.

Country F has a competitive advantage in financial intermediation. By exporting the financial service via two-way capital flows, it receives the positive net interest income.

**Proposition 16.** For the parameters in region D of figure 4, due to the decline (rise) in the labor income and the equity rate in country H (F), entrepreneurs in country H (F) are strictly worse (better) off in comparison with the steady state under international financial autarky. In addition, country H (F) as a whole is strictly worse (better) off.

For the parameters in region B, due to the unaffected labor income and the decline (rise) in the equity rate and the rise (decline) in the loan rate in country H (F), entrepreneurs are strictly worse (better) off while workers are strictly better (worse) off in country H (F). In addition, the welfare on the country level is unaffected.

Proof. See appendix B.

Country H as a whole is never better off from full capital mobility in the steady state. Thus, full capital mobility is never an option for country H to make Pareto improvement for its citizens upon the steady-state allocation under international financial autarky, no matter with what kinds of public transfer policy. In contrast, country F as a whole is never worse off in the steady state. Thus, full capital mobility might a good option for country F to make Pareto improvement for its citizens, if implemented with some appropriately designed public transfer policies.

For parameters in region D of figure 4, world output is strictly lower in the steady state than under international financial autarky. In this case, full capital mobility can never achieve Pareto improvement for all individuals in the two countries upon the steady-state allocation under international financial autarky, no matter with any public transfer policy both on the country level and on the world level.

### 3.4 Capital Mobility between Initially Poor and Rich Countries

In this subsection, we assume that country F is financially developed, \( \theta^F = \bar{\theta} \), and in the steady state while country H is financially underdeveloped, \( 0 < \theta^H < \bar{\theta} \), and below the steady state until capital mobility is allowed in period \( t = 0 \), \( 0 < w^H_0 < w^F_0 = w_{IFA} \). We analyze the patterns of capital flows in the three scenarios as well as how capital flows affect aggregate investment and output in country H in period \( t = 0 \).
3.4.1 Free Mobility of Financial Capital

We first define a counterfactual case where the world economy is still under international financial autarky in period $t = 0$. It helps us to identify the (underlying) cross-country interest rate differentials that drive capital flows in period $t = 0$ in the actual case.

According to Lemma 1 and equations (12) and (14), given $0 < \theta^H < \bar{\theta}$, there exist a threshold value of the capital-labor ratio in period 0, $\tilde{K}^H_0 = \left(\frac{\theta^H}{\bar{\theta}}\right)^{\frac{1}{1-\alpha}} K_{IFA}$, such that for $K^H_0 = \tilde{K}^H_0$, the loan rate in country H is equal to that in country F in the counterfactual case, $r^H_0 = r^H_0 = \rho$. For $K^H_0 < \tilde{K}^H_0$, the neoclassical effect dominates the credit-demand effect so that $r^H_0 > r^H_0$ in the counterfactual case and, thus, in the actual case of financial capital mobility from period $t = 0$ on, the (underlying) loan rate differentials drive financial capital flowing “downhill” from country F to country H in period $t = 0$. Otherwise, for $K^H_0 > \tilde{K}^H_0$, the credit-demand effect dominates and, thus, financial capital flows “uphill” in period $t = 0$.

The dashed curve in figure 5 shows the threshold value, $\tilde{K}^H_0$, as a function of $\theta^H$. The horizontal axis denotes the level of financial development in country H, and the vertical axis denotes the capital-labor ratio in country H and period 0. The solid curve in figure 5 represents the steady-state value of the capital-labor ratio in country H, $K^H_{FCF}$, as a function of $\theta^H$, while the upper bound of figure 5 represents $K_{IFA}$ independent of $\theta^H$. Given $\theta^H$, for $K^H_0 \in (0, K_{IFA})$, the capital-labor ratio in country H eventually converges, from period 0 on, to $K^H_{FCF}$ via capital accumulation, which is lower than $K_{IFA}$.

![Figure 5: Financial Capital Flows between Initially Poor and Rich Countries](image)

Intuitively, for a developing country, at its early stage of economic growth, its capital-labor ratio is so low that the (underlying) loan rate in the counterfactual case is higher than the world loan rate. Free mobility of financial capital facilitates financial capital inflows which speed up its capital accumulation. However, as long as the capital-labor
ratio exceeds a threshold value so that the (underlying) loan rate in the counterfactual case falls below the world loan rate, free mobility of financial capital leads to financial capital outflows which hamper aggregate domestic investment. As a result, the country converges to a steady state lower than under international financial autarky. Thus, for a developing country, the patterns of financial capital flows may reverse along its growth path. Furthermore, free mobility of financial capital has opposite effects on its aggregate production at the different stages of economic growth.

Since the marginal return on investment is higher in the poor country (country H) than in the rich country (country F), in the case of “downhill” (“uphill”) financial capital flows, world output in period \( t = 0 \) is strictly higher (lower) than in the counterfactual case under international financial autarky.

### 3.4.2 Free Mobility of FDI

According to Lemma 1, both the neoclassical effect and the spread make the equity rate strictly higher in country H than in country F in the counterfactual case in period 0. In the actual case of free mobility of FDI from period 0 on, FDI flows unambiguously “downhill”, which speeds up capital accumulation in country H. Eventually, country H converges to a steady state higher than that under international financial autarky.

The solid curve denoted by \( K_{FDI}^H \) in figure 6 represents the steady-state value of the capital-labor ratio in country H as a function of \( \theta^H \), while the horizontal solid line represents \( K_{IFA} \) independent of \( \theta^H \). The horizontal axis denotes the level of financial development in country H, and the vertical axis denotes the capital-labor ratio in country H and period 0.

![Figure 6: Free Mobility of FDI and World Output](image)

Since the marginal return on investment is higher in the poor country (country H)
than in the rich country (country F), “downhill” FDI flows tend to increase world output. However, according to Proposition 7, capital flows tend to reduce world output, due to the Jensen’s Inequality effect. The net effect of FDI flows on world output depends on the relative magnitude of the two effects. The dashed line in figure 6 represents the threshold value \( \hat{K}_0^H \) as a function of \( \theta^H \) such that given \( \theta^H \), for \( 0 < K_0^H < \hat{K}_0^H \), world output in period \( t = 0 \) is higher than that in the counterfactual case, because the efficiency gains due to “downhill” capital flows dominates the efficiency loss due to the Jensen’s-Inequality effect; otherwise, given \( \theta^H \), for \( \hat{K}_0^H < K_0^H < K_{IFA} \), world output is lower.

### 3.4.3 Full Capital Mobility

For the illustrative purpose, we defined a second counterfactual case where free mobility of FDI is allowed from period \( t = 0 \) on, besides the first one defined in subsection 3.4.1.

In the second counterfactual case, due to FDI flows, the loan rate in period \( t = 0 \) is higher (lower) in country H (F) than that in the first counterfactual case. There is a threshold value, \( \tilde{K}_0^H \), as a function of \( \theta^H \), such that for \( K_0^H = \tilde{K}_0^H \), the loan rates in period 0 coincide in the two countries in the second counterfactual case and, thus, in the actual case of full capital mobility, there are no financial capital flows in period 0. For \( 0 < K_0^H < \tilde{K}_0^H \), the neoclassical effect dominates and the loan rate in period 0 is higher in country H than in country F in the second counterfactual case and, thus, in the actual case of full capital mobility from period 0 on, both financial capital and FDI flow “downhill” in period 0. For \( \tilde{K}_0^H < K_0^H < K_{IFA} \), the credit-demand effect dominates and, thus, financial capital flows “uphill” while FDI flows “downhill” in period 0.

Besides \( \hat{K}_0^H \), there is another threshold value, \( \tilde{K}_0^H > \hat{K}_0^H \), as a function of \( \theta^H \). For \( K_0^H \in (\tilde{K}_0^H, \hat{K}_0^H) \), “downhill” FDI flows dominates “uphill” financial capital flows and net capital flows in period 0 are “downhill”; For \( K_0^H \in (\hat{K}_0^H, K_{IFA}) \), “uphill” financial capital flows dominate “downhill” FDI flows and net capital flows are “uphill” in period 0.

The dash-dotted line and the dashed line in figure 7 show the threshold values of \( \hat{K}_0^H \) and \( \tilde{K}_0^H \) as functions of \( \theta^H \in (0, \bar{\theta}) \), respectively. The horizontal axis denotes the level of financial development in country H, and the vertical axis denotes the capital-labor ratio in country H and period 0. In region D-O, which is below the dash-dotted line, net capital flows are “Downhill” with financial capital and FDI flowing in One direction; in region D-T, which is between the dashed and the dash-dotted lines, net capital flows are “Downhill” with financial capital and FDI flowing in Two ways; in region U-T, which is above the dashed line, net capital flows are “Uphill” with financial capital and FDI flowing in Two ways. The solid curve represents the steady-state value of the capital-labor ratio in country H, \( K_{FCM}^H \), as a function of \( \theta^H \), while the upper bound of figure 7 represents \( K_{IFA} \) independent of \( \theta^H \). Given \( \theta^H \), for \( K_0^H \in (0, K_{IFA}) \), the capital-labor
ratio in country H eventually converges to $K_{FCM}^H$, which is lower than $K_{IFA}$.

Figure 7 helps us understand how the patterns of capital flows may reverse along the growth path of a developing country. Suppose that this country is initially very poor with the capital-labor ratio in region $D-O$. Under full capital mobility, both financial capital and FDI flow into this country in period 0, which speeds up its capital accumulation. As the capital-labor ratio sequentially exceeds the two threshold values and moves from region $D-O$ to region $D-T$ and then to $U-T$, the direction of financial capital flows first reverses from “downhill” to “uphill”, and then, the magnitude of “uphill” financial capital flows exceeds that of “downhill” FDI and the direction of net capital flows reverses from “downhill” to “uphill”. Eventually, the country converges to a new steady state with the capital-labor ratio $K_{FCM}^H < K_{IFA}$. Thus, full capital mobility helps a developing country to speed up capital accumulation in the early stage of its growth process but at the cost of a lower steady-state output.

![Figure 7: Full Capital Mobility between Initially Poor and Rich Countries](image)

As long as capital in the net term flows “downhill”, i.e., in regions $D-O$ and $D-T$ of figure 7, world output is higher than under international financial autarky because the marginal product of capital is higher in the poor country (country H) than in the rich country (country F); otherwise, world output is lower.

4 Conclusion

We develop a two-country, overlapping-generations model in which the cross-country differences in financial development can explain three recent empirical facts. Intuitively, financial development can be considered as an endowment for a country, which does not change in the short run. In the country with more developed financial sector, the ag-
aggregate credit demand is, ceteris paribus, higher and the loan rate tends to be higher while the equity rate is lower under international financial autarky. Under full capital mobility, the cross-country interest rate differentials drive financial capital flows and FDI. Ceteris paribus, the country with competitive advantage in the financial sector “exports” its financial service by borrowing financial capital from abroad at a low interest rate and making foreign direct investment for a high rate of return. Although this country becomes a net debtor in equilibrium, it receives a positive net investment income, due to the interest-rate spread on its assets and liabilities. We also discuss how the patterns of capital flows change or even reverse along the growth path of a developing country and how capital mobility affects the growth rate along its equilibrium growth process.

For simplicity, we take the level of financial development as given and analyze how the cross-country differences in financial development affect capital flows. A related question would be how various forms of capital flows affect financial development along the process of economic development. We leave this issue for future research.

References


Free mobility of FDI equalizes the equity rates across the border, $\Gamma^H_t = \Gamma^F_t = \Gamma^*_t$. Given the world equity rate at $\Gamma^*_t$, we first prove the existence, uniqueness, and stability of the steady-state equilibrium. Then, we analyze how FDI flows affect the two interest rates in the two countries. Finally, we discuss how FDI flows affect production and welfare on the individual, country, and world levels.
In the following analysis, we first consider the case where the borrowing constraints are binding in country \( i \). In subsection A.2, we will analyze under what conditions the borrowing constraints are binding. According to the credit market equilibrium, domestic equity capital and investment in country \( i \) are

\[
\eta w_i - \Omega_i = \frac{(1-\eta) w_i^i}{\lambda_i - 1} \quad \text{and} \quad I_i = \lambda_i (\eta w_i - \Omega_i) = \frac{\lambda_i (1-\eta) w_i}{\lambda_i - 1} \quad \Rightarrow \frac{\theta^i Rv_{i+1}^i}{r_i^i} = \frac{(1-\eta) w_i^i}{I_i^i}.
\]

The project-financing equation can be transformed

\[
1 = \frac{\theta^i Rv_{i+1}^i + (1-\theta^i) Rv_{i+1}^i}{\Gamma_i^*} \Rightarrow 1 = (1-\eta) w_i^i + \frac{1-\theta^i) Rv_{i+1}^i}{\Gamma_i^*}.
\]

Using equation (14) to substitute away \( v_{i+1}^i \) and \( I_i^i \) in equation (20), we get

\[
(1-\eta) w_i^i = \frac{\rho}{R} (w_{i+1}^i)^{\frac{\alpha}{\alpha - 1}} - \frac{(1-\theta^i) \rho w_{i+1}^i}{\Gamma_i^*}.
\]

**A.1 Existence, Uniqueness, and Stability of the Steady-State Equilibrium**

**Proposition 17.** Under free mobility of FDI, there exists a unique and stable non-zero steady-state equilibrium with the wage at \( w_{FDI}^i = w_{IFA} [1-\eta + \eta \frac{\Gamma_{IFA}}{\Gamma_{FDI}}]^\rho \), where the subscript FDI denotes the steady-state value of a particular variable under free flows of FDI.

**Proof.** See appendix B.

The solid line in figure 8 shows the phase diagram of wages under international financial autarky, while the dash-dotted line shows the phase diagram under free mobility of FDI, given a fixed world equity rate at \( \Gamma_i^* = \Gamma_{IFA}^i \). In both cases, the wage converges monotonically and globally to the unique steady state (point A).

**A.2 Interest Rates and Capital Flows**

Since the world economy is initially in the steady state under international financial autarky, the equity rate is higher in country H than in country F. From period \( t = 0 \) on, entrepreneurs are allowed to make equity investment abroad. The initial cross-country equity rate differentials drive FDI unambiguously flowing from country F to country H and the equity rate in country H (F) adjusts from above (below) to the world level.

**Proposition 18.** Under free mobility of FDI, there exists a unique world equity rate that clears the world equity market. In the steady state, the world equity rate is \( \Gamma_{FDI}^* \in (\Gamma^*, \Gamma_{IFA}^H) \), where \( \Gamma^* = \frac{\Gamma_{IFA}^H + \Gamma_{IFA}^F}{2} \).

**Proof.** See appendix B.
Proposition 19. Under free mobility of FDI, if the borrowing constraints are binding in country $i$, the loan rate is $r_i^t = \frac{\theta_i}{\rho_i} (1 - \eta) w_{i+1}^t w_i^t$. In the steady state, the loan rate is same as under international financial autarky, $r_{FDI}^i = \frac{\theta_i}{1 - \eta}$.

The proof resembles that of Proposition 4. In the steady-state, the equity-rate effect and the price-of-capital effect cancel out.

We now discuss under what conditions the borrowing constraints are binding in the steady state.

Proposition 20. If $\eta \in \left[ \frac{2\rho}{1+2(\rho+1)}, 1 \right)$, given $\theta^H \in (0, \bar{\theta})$, there exists $\bar{\theta}_{FDI}^F \in (\bar{\theta}, 1)$ as the function of $\theta^H$ such that for $\theta^F \in (\theta^H, \bar{\theta}_{FDI}^F)$, the borrowing constraints are binding in country $F$ in the steady state; for $\theta^F \in (\bar{\theta}_{FDI}^F, 1]$, the borrowing constraints are not binding in country $F$ and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}_{FDI}^F$.

If $\eta \in (0, \frac{2\rho}{1+2(\rho+1)})$, there exists $\bar{\theta}^H$ such that given $\theta^H \in [\theta^H, \bar{\theta})$, there exists $\bar{\theta}_{FDI}^F \in (\bar{\theta}, 1)$ as the function of $\theta^H$ such that for $\theta^F \in (\theta^H, \bar{\theta}_{FDI}^F)$, the borrowing constraints are binding in country $F$ in the steady state; for $\theta^F \in (\bar{\theta}_{FDI}^F, 1]$, the borrowing constraints are not binding in country $F$ and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}_{FDI}^F$. Given $\theta^H \in (0, \bar{\theta}^H)$, the borrowing constraints are always binding in country $F$ for $\theta^F \in (\theta^H, 1]$.

Proof. See appendix B.

Figure 9 illustrates these results in the cases of $\eta < \frac{2\rho}{1+2(\rho+1)}$ and $\eta > \frac{2\rho}{1+2(\rho+1)}$ respectively. The horizontal and vertical axes denote the levels of financial development in country H and in country F, $\theta^i \in (0, 1]$, respectively.

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the equity rate is same in the two countries under international financial autarky. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e.,
the parameters in region $A$, according to Proposition 1, the equity rates are equal to the marginal return on investment, which is same in the two countries. In these two cases, there are no FDI flows even if allowed. The curve splitting regions $B$ and $D$ represents the threshold value of $\tilde{\theta}^F_{FDI}$ as the function of $\theta^H$ described by equation (45). For the parameters on the curve, the loan rate in country $F$ is equal to the world equity rate $r^F_{FDI} = \frac{\tilde{\theta}^F_{FDI}}{1-\eta} = \Gamma^*_{FDI}$. Similarly, the curve splitting regions $B'$ and $D'$ represents the threshold value of $\tilde{\theta}^H_{FDI}$ as the function of $\theta^F$. For the parameters on the curve, the loan rate in country $H$ is equal to the world equity rate, $r^H_{FDI} = \frac{\tilde{\theta}^H_{FDI}}{1-\eta} = \Gamma^*_{FDI}$.

**Proposition 21.** For the parameters in region $D$ of figure 9, FDI flows have a closed-form solution in the steady state, $\Omega^i_{FDI} = (\Gamma^*_{FDI} - \Gamma^i_{IFA}) \frac{\eta_{w,F}}{\Gamma^*_{FDI}}$ and are from country $F$ to country $H$, $\Omega^H_{FCF} < 0 < \Omega^F_{FCF}$; for the parameters in region $B$, FDI flows are same as in the case of $\theta^H$ and $\tilde{\theta}^F_{FDI}$.

**Proof.** See the proof of Proposition 18 and equation (43).

**Table 4: FDI Flows and Equity Premium in the Steady State**

<table>
<thead>
<tr>
<th>Region</th>
<th>$A$</th>
<th>$B$</th>
<th>$B'$</th>
<th>$D$</th>
<th>$D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega^H$</td>
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<td>$\Omega^H(\theta^H) &lt; 0$</td>
<td>$\Omega^H(\theta^F) &gt; 0$</td>
<td>$(\Omega^H(\theta^H), 0)$</td>
<td>$(0, \Omega^H(\theta^F))$</td>
</tr>
<tr>
<td>$\Gamma^H - r^*$</td>
<td>$0$</td>
<td>$+$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\Gamma^F - r^*$</td>
<td>$0$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Table 4 summarizes the steady-state values of FDI flows and the equity premium in the five regions. Note that $\Omega^F = -\Omega^H$. $\Omega^H(\theta^i)$ implies that given the parameters in
region $B$ and $B'$, FDI flows depend only on $\theta^i$ not on $\theta^m$, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive. In the following analysis, we only focus on the parameters in region $D$ where the borrowing constraints are binding in both countries.

### A.3 Production and Welfare

Due to FDI flows, the aggregate investment in country H (F) is higher (lower) than under international financial autarky from period $t = 0$ on and so is aggregate output. In the steady state, according to Propositions 17 and 18, the wage is $w^H_{\text{FDI}} = w_{\text{IFA}} \left[ 1 - \eta + \frac{\eta \Gamma^H_{\text{IFA}}}{\Gamma^H_{\text{FDI}}} \right]^{\rho}$ and the equity rate is $\Gamma^* \in (\Gamma^H_{\text{IFA}}, \Gamma^F_{\text{IFA}})$, implying that $w^H_{\text{FDI}} > w^F_{\text{FDI}}$. Thus, aggregate output, which is proportional to the wage, is higher in the steady state in country H than in country F, $Y^H_{\text{FDI}} > Y_{\text{IFA}} > Y^F_{\text{FDI}}$.

**Proposition 22.** Suppose that the world economy is in the steady state under international financial autarky before FDI is allowed from period $t = 0$ on. For $\theta^H \in (0, \bar{\theta})$ and $\theta^F \in (\theta^H, 1)$, world output is strictly lower from period $t = 1$ on than its initial value, $Y^H_t + Y^F_t < 2Y_{\text{IFA}}$.

The proof follows that of Proposition 7.

The welfare implications are discussed briefly in subsection 3.2 and summarized in Proposition 23.

**Proposition 23.** In comparison with the steady state under international financial autarky, workers in country H (F) are strictly better (worse) off under free mobility of FDI, while the implications to the welfare of entrepreneurs and the social welfare on the country level depend on the specific parameter values.

**Proof.** See appendix B. \openbox

Table 5: Welfare Implications of Free Mobility of FDI to Entrepreneurs

<table>
<thead>
<tr>
<th></th>
<th>$(\rho - 1) \eta \left[ \frac{1 - \theta^H + 1 - \theta^F}{2(1 - \rho^H)} \right]$</th>
<th>$(-\infty, \frac{1 - \theta^H + 1 - \theta^F}{2(1 - \rho^H)}]$</th>
<th>$\left[ \frac{1 - \theta^H + 1 - \theta^F}{2(1 - \rho^H)} , 1 \right]$</th>
<th>$[1, 1 - \rho^H]$</th>
<th>$[1 - \rho^H, 1]$</th>
<th>$(1 - \rho^H, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^H_{\text{FDI}} - c^H_{\text{IFA}}$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$c^F_{\text{FDI}} - c^F_{\text{IFA}}$</td>
<td>$+$</td>
<td>$-$</td>
<td>$?$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 5 summarizes the welfare implications of free mobility of FDI to entrepreneurs under various parameter constellations, where $c^i_{\text{FDI}}$ denotes the steady-state consumption of entrepreneurs born in country $i$ under free mobility of FDI.
Proposition 24. Entrepreneurs of different generations born in the same country may be affected by FDI flows in opposite ways during the transitional process from international financial autarky to free mobility of FDI.

Proof. As mentioned in subsection 3.2, entrepreneurs born in country H and period \( t = 0 \) are strictly worse off from the lower equity rate. For \( \left( \frac{p - 1}{1 - \eta} \right) \geq 1 \) in Table 5, FDI inflows increases the domestic investment in country H and the resulting rise in the labor income dominates the decline in the equity rate in the long run. Thus, entrepreneurs born in country H strictly benefit from free mobility of FDI in the long run. Such opposite welfare patterns may also exist for entrepreneurs in early and later generations born in country F, under certain parameter values. Thus, the introduction of free mobility of FDI may have opposite welfare effects across generations.

B Proofs of Propositions

Proof of Proposition 2

Proof. Take the world loan rate \( r^*_t \) as given. According to equation (16), \( w_{i+1}^t \) can be considered as a function of \( w_i^t \). For \( w_i^t \in \left[ 0, \frac{(1 - \theta)\rho}{R_t} \right] \), take the first derivative of equation (16) with respect to \( w_i^t \),

\[
\eta = \left[ \frac{\rho}{R\alpha} \left( \frac{w_{i+1}^t}{w_i^t} \right)^{\frac{1}{2}} - \frac{\theta^i}{r_0^t} \right] \frac{dw_{i+1}^t}{dw_i^t}. \tag{22}
\]

According to equation (16), for \( w_i^t = 0 \), there is a non-zero solution of \( w_{i+1}^t = \left[ \frac{\theta^i R}{r_0^t} \right]^\rho \). The slope of the phase diagram at the point \( (0, \left[ \frac{\theta^i R}{r_0^t} \right]^\rho) \) is \( \frac{\eta}{\theta^i} > 0 \). In other words, \( w_{i+1}^t \geq \left( \frac{\theta^i R}{r_0^t} \right)^\rho \). Thus, according to equation (22), the phase diagram of wages has the positive slope, \( \frac{dw_{i+1}^t}{dw_i^t} > 0 \). Take the second derivative of equation (16) with respect to \( w_i^t \),

\[
0 = \left[ \frac{\rho}{R\alpha} \left( \frac{w_{i+1}^t}{w_i^t} \right)^{\frac{1}{2}} - \frac{\theta^i}{r_0^t} \right] \frac{dw_{i+1}^t}{dw_i^t} + \left( \frac{dw_{i+1}^t}{dw_i^t} \right)^2 \frac{1}{R\alpha} \left( \frac{w_{i+1}^t}{w_i^t} \right)^{\frac{1-2\alpha}{\alpha}}, \quad \Rightarrow \quad \frac{dw_{i+1}^t}{dw_i^t} < 0.
\]

Thus, the phase diagram of wages is concave for \( w_i^t \in \left[ 0, \frac{(1 - \theta)\rho}{R_t} \right] \), and \( w_{i+1}^t \) monotonically increases in \( w_i^t \) with an intercept on the vertical axis at \( w_{i+1}^t = \left[ \frac{\theta^i R}{r_0^t} \right]^\rho \).

For \( w_i^t > \left( \frac{1 - \theta)\rho}{R_t} \right)^{\frac{1}{2}} \), the marginal return on investment is equal to the world loan rate, \( Rv_{i+1}^t = R^*_t \), and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages \( w_{i+1}^t = (v_{i+1}^t)^{-\frac{1}{2}} = \left( \frac{R}{r_0^t} \right)^\rho \) is flat and independent of \( w_i^t \).

Overall, the phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. The stability and the uniqueness of the non-zero steady-state equilibrium is proved. \( \square \)
Proof of Lemma 2

Proof. Take the world loan rate \( r^*_t \) as given. For \( w^i_t \in (0, 1 - \theta^i) \) and \( i^*_t = 1 \), take the first and second derivatives of equation (17) with respect to \( w^i_t \),

\[
\frac{dw^i_{t+1}}{dw^i_t} = \frac{\rho r^*_t}{\theta^i R} (w^i_{t+1})^{\frac{1}{2}} > 0, \quad \text{and}, \quad \frac{d^2 w^i_{t+1}}{d^2 w^i_t} = \frac{\rho r^*_t}{\theta^i R} \alpha (w^i_{t+1})^{\frac{3}{2}} \frac{dw^i_{t+1}}{dw^i_t} > 0. \tag{23}
\]

The convexity of the phase diagram of wages is proved for \( w^i_t \in (0, 1 - \theta^i) \). Similarly as in Proposition 2, the phase diagram of wages has the intercept on the vertical axis at \( w^i_{t+1} = \left[ \frac{\theta^i R}{r^*_t} \right]^\rho \). For \( w^i_t > 1 - \theta^i \), the marginal return on investment is equal to the world loan rate, \( R w^i_{t+1} = r^*_t \), and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages \( w^i_{t+1} = (v^i_t)^{-\frac{1}{2}} = \left( \frac{R}{r^*_t} \right)^\rho \) is flat and independent of \( w^i_t \). \( \square \)

Proof of Proposition 3

Proof. The world loan rate is determined by the equilibrium of financial capital flows, \( Y^H_t + Y^F_t = 0 \). We first prove the existence of a unique world loan rate clearing the world credit market every period and then derive the world loan rate in the steady state.

Suppose that the borrowing constraints are binding in country \( i \). Given the predetermined \( w^i_t \), equation (16) shows that \( w^i_{t+1} \) is a function of \( r^*_i \). Take the first derivative of equation (16) with respect to \( r^*_i \),

\[
0 = \left[ \frac{\rho}{R \alpha} (w^i_{t+1})^{\frac{1}{2}} - \frac{\theta^i \rho}{r^*_t} \right] \frac{dw^i_{t+1}}{dr^*_i} + \frac{\theta^i \rho}{(r^*_t)^2}.
\]

As shown in the proof of Proposition 2, \( w^i_{t+1} \geq \left( \frac{\theta^i R}{r^*_t} \right)^\rho \) so that the term in the square bracket is positive. An increase in the world loan rate enhances financial capital outflows and reduces domestic investment. Thus, wage in the next period declines, \( \frac{dw^i_{t+1}}{dr^*_i} < 0 \).

Capital outflows represent the gap between domestic savings and investment,

\[
Y^i_t = w^i_t - I^i_t = w^i_t - \frac{\rho}{R} (w^i_{t+1})^{\frac{1}{2}} = (1 - \eta) w^i_t - \frac{\rho}{r^*_t} \theta^i w^i_{t+1}. \tag{24}
\]

The world credit market equilibrium implies

\[
Y^H_t + Y^F_t = 0, \quad \Rightarrow \quad (1 - \eta)(w^H_t + w^F_t) = \frac{\rho}{r^*_t} (\theta^H w^H_{t+1} + \theta^F w^F_{t+1}). \tag{25}
\]

Take the wage \( w^i_t \) as given. Suppose that \( \theta^H \in (0, \bar{\theta}) \) and \( \theta^F \in (\theta^H, \bar{\theta}) \). As shown in figure 1, the loan rate in country \( H \) is lower than in country \( F \) before period \( t \), \( r^H_{tFA} < r^F_{tFA} \). The world loan rate in period \( t \) must be \( r^*_t \in (r^H_{tFA}, r^F_{tFA}) \). The proof is by contradiction. If \( r^*_t > r^F_{tFA} > r^H_{tFA} \), \( w^i_{t+1} \) would be lower than its value under international financial autarky in both countries as \( \frac{dw^i_{t+1}}{dr^*_t} < 0 \), and hence equation (25) would not hold. The same argument applies to the case of \( r^*_t < r^H_{tFA} < r^F_{tFA} \). Since \( w^i_{t+1} \) is a monotonically
decreasing function of \( r^*_t \), there exists a unique solution of \( r^*_t \in (r^H_{IFA}, r^F_{IFA}) \) that clears the world credit market and financial capital flows from country \( H \) to country \( F \), given the predetermined \( w^H_t \) and \( w^F_t \).

In the next step, we assume that there exists a unique world loan rate in the steady state and then prove its uniqueness.

Given \( r^* \), Proposition 2 shows that there is a unique and stable steady state where \( w^i = \left( \frac{\rho}{\rho} \right)^{\theta} \left( \eta + \frac{\rho_r}{\rho_r} \right)^{\rho} \). According to equation (24), the steady-state financial capital flows are

\[
\Upsilon^i = w^i \left[ (1 - \eta) - \frac{\theta^i \rho}{r^*} \right].
\]

According to equation (25), the world loan rate in the steady state is determined by

\[
\Upsilon^H + \Upsilon^F = 0, \quad \text{or} \quad \frac{\theta^F \rho}{r^* - \rho r^*} = \frac{w^H}{w^F}, \quad \text{or} \quad \frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} = \left( \frac{\eta r^* + \theta^H \rho}{\eta r^* + \theta^F \rho} \right)^{\rho}.
\]  

(27)

For \( \theta^H \in (0, \bar{\theta}) \) and \( \theta^F > \theta^H \), the right-hand side of equation (27) is less than one,

\[
\frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} < 1, \quad \Rightarrow \quad r^* \in (\bar{\Upsilon}^*, r^F_{IFA}).
\]

Let \( \mathcal{N}(r^*) \equiv \frac{r^F_{IFA} - r^H_{IFA}}{r^* - r^H_{IFA}} - 1 \) and \( \mathcal{R}(r^*) \equiv \left[ 1 - \frac{(\theta^F - \theta^H) \rho}{\eta r^* + \theta^F \rho} \right]^{\rho} \) denote the left-hand and the right-hand sides of equation (27) as the functions of \( r^* \). For \( r^* \in (\bar{\Upsilon}^*, r^F_{IFA}) \),

\[
\mathcal{N}'(r^*) < 0 < \mathcal{R}'(r^*), \\
\mathcal{N}(r^* = \bar{\Upsilon}^*) = 1 > \mathcal{R}(r^* = \bar{\Upsilon}^*), \\
\mathcal{N}(r^* = r^F_{IFA}) = 0 < \mathcal{R}(r^* = r^F_{IFA}).
\]

Thus, \( \mathcal{N}(r^*) \) decreases while \( \mathcal{R}(r^*) \) increases monotonically in \( r^* \); the two functions cross once and only once for \( r^* \in (\bar{\Upsilon}^*, r^F_{IFA}) \). Therefore, there exists a steady-state equilibrium under free mobility of financial capital, which is also unique.

\( \square \)

**Proof of Proposition 4**

*Proof*. Under free mobility of financial capital, the equity rate is \( \Gamma^i_t = \frac{(1-\theta)Rv^i_{t+1}}{1-r^t} \) if the borrowing constraints are binding in country \( i \). Using equations (16), we can rewrite the equity rate as \( \Gamma^i_t = \frac{(1-\theta)Rv^i_{t+1}}{\eta} = \frac{(1-\theta)Rv^i_{t+1}}{w^i_{t+1}} \). Take country \( H \) as an example. Financial capital outflow reduces domestic investment in period \( t \) and the wage declines in period \( t + 1 \). Thus, the equity rate drops in period \( t \). As time goes on, the wage converges downwards and hence, the equity rate converges upwards to their respective steady-state level. In the new steady state, \( w^i_{t+1} = w^i_t \) and the equity rate \( \Gamma^i_{FCF} = \frac{(1-\theta)\rho}{\eta} \) has the same form as under international financial autarky.

\( \square \)
Proof of Proposition 5

Proof. According to Proposition 4, if the borrowing constraints are binding in the two countries under free mobility of financial capital, the steady-state equity rate $\Gamma^i = \frac{(1-\theta^i)^\rho}{\eta}$ has the same form as under international financial autarky. Given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^{FCF}$, the equity rate in country F is equal to the world loan rate and the borrowing constraints are weakly binding in the steady state, $\Gamma^F = \frac{\rho(1-\bar{\theta}^{FCF})}{\eta} = r^*$. Thus, $\bar{\theta}^{FCF}_F$ is the solution to the following equation,

$$
\frac{\bar{\theta}^{FCF}_F - \frac{1-a}{\eta}(1-\bar{\theta}^{FCF}_F)}{\frac{1-a}{\eta}(1-\bar{\theta}^{FCF}_F) - \theta^H} = (1 - \bar{\theta}^{FCF}_F + \theta^H)\rho.
$$

(28)

Let $\mathcal{N}(\bar{\theta}^{FCF}_F) \equiv \frac{\bar{\theta}^{FCF}_F - \theta^H}{\eta(1-\bar{\theta}^{FCF}_F) - \theta^H} - 1$ and $\mathcal{R}(\bar{\theta}^{FCF}_F) \equiv (1 - \bar{\theta}^{FCF}_F + \theta^H)^\rho$ denote the left-hand and the right-hand sides of equation (28) as the functions of $\bar{\theta}^{FCF}_F$. For $\bar{\theta}^{FCF}_F \in (\bar{\theta}, 1 - \frac{\theta^H\eta}{1-\eta})$,

$$
\mathcal{N}'(\bar{\theta}^{FCF}_F) > 0 > \mathcal{R}'(\bar{\theta}^{FCF}_F),
$$

$$
\mathcal{N}(\bar{\theta}^{FCF}_F = \bar{\theta}) = 0 < (\eta + \theta^H)^\rho = \mathcal{R}(\bar{\theta}^{FCF}_F = \bar{\theta}),
$$

$$
\mathcal{N}(\bar{\theta}^{FCF}_F = 1 - \frac{\theta^H\eta}{1-\eta}) \to +\infty > \left( \frac{\theta^H}{1-\eta} \right)^\rho = \mathcal{R}(\bar{\theta}^{FCF}_F = 1 - \frac{\theta^H\eta}{1-\eta}).
$$

Thus, $\mathcal{N}(\bar{\theta}^{FCF}_F)$ monotonically increases while $\mathcal{R}(\bar{\theta}^{FCF}_F)$ monotonically decreases in $\bar{\theta}^{FCF}_F$; the two functions cross once and only once for $\bar{\theta}^{FCF}_F \in (\bar{\theta}, 1 - \frac{\theta^H\eta}{1-\eta})$. Therefore, the threshold value of $\bar{\theta}^{FCF}_F \in (\bar{\theta}, 1 - \frac{\theta^H\eta}{1-\eta})$ exists and is unique.

For $\theta^F \in (\bar{\theta}^{FCF}_F, 1]$, $\Gamma^F = r^*$ in the steady state and the borrowing constraints are not binding in country F. The economic allocation is same as in the case of $\theta^F = \bar{\theta}^{FCF}_F$. 

Proof of Proposition 7

Proof. Let $a_t \equiv \frac{w^H_t + w^F_t}{2w^H_{IFA}}$ and $b_t \equiv \frac{w^F_t - w^H_t}{2w^H_{IFA}} + \frac{\Upsilon^H_t}{w^H_{IFA}}$, where $t = 0, 1, 2, 3, \ldots$. According to Proposition 6 and the aggregate resource constraint in country H, $0 < \Upsilon^H_t < w^H_{IFA}$, we get $b_t \in (0, a_t)$. In period t, the aggregate project investment in country H and in country F are $I^H_t = w^H_t - \Upsilon^H_t = (a_t - b_t)w^H_{IFA}$ and $I^F_t = w^F_t + \Upsilon^H_t = (a_t + b_t)w^H_{IFA}$, respectively. Given the share of capital goods in the aggregate production, $\alpha \in (0, 1)$, and $b_t \in (0, a_t)$, the world-average wage in period $t+1$ can be reformulated into a condensed form with the following property,

$$
\frac{w^H_{t+1} + w^H_t}{2} = \left( \frac{R}{\rho} \right)^\alpha \left[ \frac{(I^H_t)^\alpha + (I^F_t)^\alpha}{2} \right] \Leftrightarrow a_{t+1} = \frac{(a_t - b_t)^\alpha + (a_t + b_t)^\alpha}{2} < (a_t)^\alpha, \quad (29)
$$

due to the Jensen’s Inequality. Since the world economy is in the steady state under international financial autarky before period 0, the wage in period 0 is same in the two countries, $w^H_0 = w^F_0 = w^H_{IFA}$, and, thus, $a_0 = 1$. From period 0 on, financial capital flows
are allowed. According to the inequality in equation (29), we get \( a_1 < 1 \). For \( t = 1, 2, 3, \ldots, \) given \( b_t \in (0, a_t) \), we have \( a_{t+1} < (a_t)^\alpha \) and, thus, the time series of \( a_t \) is strictly below 1, or equivalently, \( w^H_i + w^F_i < w_{RF} \). Thus, world output is lower than its steady-state value under international financial autarky, \( Y_t^H + Y_t^F = \frac{w^H_i + w^F_i}{1-a} < \frac{2w_{RF}}{1-a} = Y_{RF}^H + Y_{RF}^F \). \( \square \)

**Proof of Proposition 8**

Proof. If the borrowing constraints are binding, the steady-state workers’ consumption is

\[
c_i^i = w^t r^* = \left( \frac{R}{\rho} \right)^\rho \left( r^* \eta + \theta^i \right)^\eta (r^*)^{1-\rho},
\]

\[
d \log c_i^i = \frac{d \ln c_i^i}{dr^*} = \frac{r^* \eta + \theta^i \rho - \theta^i \rho^2}{(r^* \eta + \theta^i \rho)^2}.
\]

As an analytical solution of the world loan rate is not obtainable, we provide sufficient conditions for the welfare changes as follows.

Evaluate \( \frac{d \ln c^H_i}{dr^*} \) at \( r^* = r_{IF}^H \) and \( r^* = r_{IF}^F \). For \( \frac{(\rho-1)(1-\eta)}{\eta} \leq 1, \frac{d \ln c^H_i}{dr^*} \mid_{r^*=r_{IF}^H} > \frac{d \ln c^H_i}{dr^*} \mid_{r^*=r_{IF}^F} \geq 0 \) implies that workers born in country H is better off than under international financial autarky since the positive loan rate effect dominates the negative wage effect; for \( \frac{(\rho-1)(1-\eta)}{\eta} \geq \frac{\theta^F}{\theta^H} \), \( \frac{d \ln c^H_i}{dr^*} \mid_{r^*=r_{IF}^H} < \frac{d \ln c^H_i}{dr^*} \mid_{r^*=r_{IF}^F} \leq 0 \) implies that workers born in country H is worse off since the positive loan rate effect is dominated by the negative wage effect; for \( \frac{(\rho-1)(1-\eta)}{\eta} \in (1, \frac{\theta^F}{\theta^H}) \), the numerical solutions are required for the welfare evaluation.

Evaluate \( \frac{d \ln c^F_i}{dr^*} \) at \( r^* = r^* \) and \( r^* = r_{IF}^F \). For \( \frac{(\rho-1)(1-\eta)}{\eta} \leq \frac{\theta^H + \theta^F}{2\theta^H} \), \( \frac{d \ln c^F_i}{dr^*} \mid_{r^*=r_{IF}^F} > \frac{d \ln c^F_i}{dr^*} \mid_{r^*=r^*} \geq 0 \) implies that workers born in country F is worse off since the negative loan rate effect dominates the positive wage effect; for \( \frac{(\rho-1)(1-\eta)}{\eta} \geq 1 \), \( \frac{d \ln c^F_i}{dr^*} \mid_{r^*=r_{IF}^F} < \frac{d \ln c^F_i}{dr^*} \mid_{r^*=r^*} \leq 0 \) implies that workers born in country F is better off since the negative loan rate effect is dominated by the positive wage effect; for \( \frac{(\rho-1)(1-\eta)}{\eta} \in (\frac{\theta^H + \theta^F}{2\theta^H}, 1) \), the numerical solutions are required for the welfare evaluation.

Social welfare is defined as the weighted sum of consumption of individuals born in country \( i \). In the steady state,

\[
C_i^i \equiv \eta c_i^i + (1-\eta) c_i^j = w^t [\eta \Gamma^i + (1-\eta) r^*] = \left( \frac{R}{\rho} \right)^\rho \left( r^* \eta + \theta^i \right)^\eta (r^*)^{1-\rho},
\]

\[
d \log C_i^i = \frac{d \ln C_i^i}{dr^*} = \frac{1-\eta}{r^* \eta + \theta^i \rho - (1-\theta^i) \rho + (1-\eta) r^*}.
\]

Evaluate \( \frac{d \ln C^H_i}{dr^*} \) at \( r^* = r_{IF}^H \) and \( r^* = r_{IF}^F \). For \( \rho \in (0, \frac{\theta^H}{1-\eta}] \), \( \frac{d \ln C^H_i}{dr^*} \mid_{r^*=r_{IF}^H} > \frac{d \ln C^H_i}{dr^*} \mid_{r^*=r_{IF}^F} \geq 0 \) implies that the workers’ welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for \( \rho \in (\frac{\theta^F}{1-\eta} \frac{\theta^H + \theta^F}{\theta^H}, \infty) \), \( \frac{d \ln C^H_i}{dr^*} \mid_{r^*=r_{IF}^H} < \frac{d \ln C^H_i}{dr^*} \mid_{r^*=r_{IF}^F} \leq 0 \) implies that both workers
and entrepreneurs are worse off or the workers’ welfare gains are dominated by the welfare losses of entrepreneurs and hence, country H as a whole loses from free mobility of financial capital; for \( \rho \in (\frac{\theta_F}{1-\eta}, \frac{\theta_F}{\alpha + \eta(\theta_F - \theta_H)}) \), the numerical solutions are required for the welfare evaluation.

Evaluate \( \frac{d \ln C_F}{dr^*} \) at \( r^* = \rho \) and \( r^* = r_{FA}^* \). For \( \rho \in (0, \frac{\theta_F}{2(\theta_H - \eta)} \left[ \frac{\theta_F - \eta \theta_H}{1 - \eta} \right]) \), \( \frac{d \ln C_F}{dr^*} \bigg|_{r^*=\rho} \geq 0 \) implies that both workers and entrepreneurs are worse off or the workers’ welfare gains are dominated by the welfare losses of entrepreneurs and hence, country F as a whole loses from free mobility of financial capital; for \( \rho \in [\frac{\theta_F}{1-\eta}, \infty) \), \( \frac{d \ln C_F}{dr^*} \bigg|_{r^*=\rho} \leq 0 \) implies that the workers’ welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for \( \rho \in (\frac{\theta_F}{\alpha + \eta(\theta_F - \theta_H)}, \frac{\theta_F}{2(\theta_H - \eta)} \left[ \frac{\theta_F - \eta \theta_H}{1 - \eta} \right]) \), the numerical solutions are required for the welfare evaluation.

**Proof of Proposition 11**

Proof. The world equity rate \( \Gamma_t^* \) is determined by the identity of FDI flows, \( \Omega_{Ht}^* + \Omega_{It}^* = 0 \) and the world loan rate \( r_t^* \) by \( \Upsilon_t^H + \Upsilon_t^F = 0 \). We first prove the existence of a unique world equity rate and a unique world loan rate clearing the world equity market and the world credit market every period, respectively, and then derive the two interest rates in the steady state.

According to the domestic credit market equilibrium and the Cobb-Douglas production function, FDI and financial capital flows are solved as

\[
\lambda^i_t \left( \eta w^i_t - \Omega^i_t \right) = \frac{\lambda^i_t}{\lambda^i_t} \left[ (1-\eta)w^i_t - \Upsilon^i_t \right] = I^i_t = \frac{\rho}{R}(w^i_{t+1})^{\frac{1}{2}}, \quad (30)
\]

\[
\Omega^i_t = \eta w^i_t - \frac{(1-\theta^i)\rho}{\Gamma_t^*} w^i_{t+1}, \quad (31)
\]

\[
\Upsilon^i_t = (1-\eta)w^i_t - \frac{\theta^i \rho}{r^*_t} w^i_{t+1}, \quad (32)
\]

\[
\Omega^i_t + \Upsilon^i_t = w^i_t - I^i_t = w^i_t - \frac{\rho}{R}(w^i_{t+1})^{\frac{1}{2}}. \quad (33)
\]

Take \( w^i_t \) as given. For the world interest rate at \( \Gamma_t^* \) and \( r_t^* \), the wage \( w^i_{t+1} \) is uniquely determined under full capital mobility and so are \( \Upsilon^i_t \) and \( \Omega^i_t \). Take first derivative of equations (31) and (32) with respect to the two interest rates, respectively,

\[
\frac{d \Omega^i_t}{d \Gamma_t^*} = (1 - \theta^i)\rho w^i_{t+1} - (1 - \theta^i)\rho \frac{dw^i_{t+1}}{d \Gamma_t^*} > 0,
\]

\[
\frac{d \Omega^i_t}{d r_t^*} = -(1 - \theta^i)\rho \frac{dw^i_{t+1}}{d r_t^*} > 0,
\]

\[
\frac{d \Upsilon^i_t}{d \Gamma_t^*} = \frac{\theta^i \rho}{(r_t^*)^2} w^i_{t+1} - \frac{\theta^i \rho}{r_t^*} \frac{dw^i_{t+1}}{d \Gamma_t^*} > 0,
\]

\[
\frac{d \Upsilon^i_t}{d r_t^*} = \frac{\theta^i \rho}{r_t^*} \frac{dw^i_{t+1}}{d r_t^*} > 0.
\]

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Let \( w \) be the left-hand and the right-hand sides of equation (37). Following the same logic of the proof in Proposition 3, there exists a unique world equity rate and world loan rate that clear the world credit and equity markets, \( \Gamma^* \in (\Gamma^F_{FA}, \Gamma^H_{FA}) \) and \( r^* \in (r^H_{FA}, r^F_{FA}) \). Given \( w^i \), the world interest rates \( \Gamma^i_t \) and \( r^i_t \) are uniquely determined by the two equilibrium conditions, i.e., \( \Omega^H_t + \Omega^F_t = 0 \) and \( \Upsilon^H_t + \Upsilon^F_t = 0 \).

\[
\eta(w^H_t + w^F_t) = \frac{\rho}{\Gamma^i_t} \left[ (1 - \theta^H_t) R^\rho \left( 1 - \theta^H_t \Gamma^i_t + \frac{\theta^H_t}{r^H_t} \right)^\rho + (1 - \theta^F_t) R^\rho \left( 1 - \theta^F_t \Gamma^i_t + \frac{\theta^F_t}{r^F_t} \right)^\rho \right],
\]

(34)

\[
(1 - \eta)(w^H_t + w^F_t) = \frac{\rho}{r^F_t} \left[ \theta^H R^\rho \left( 1 - \theta^H_t \Gamma^i_t + \frac{\theta^H_t}{r^H_t} \right)^\rho + \theta^F R^\rho \left( 1 - \theta^F_t \Gamma^i_t + \frac{\theta^F_t}{r^F_t} \right)^\rho \right].
\]

In the next step, we assume that there exists a unique world loan rate and a unique world equity rate and then prove that they are unique.

In the steady state, \( w^i_{t+1} = w^i_t \). According to equations (31) and (32), the equilibrium conditions of FDI and financial capital flows, \( \Omega^H_t + \Omega^F_t = \Upsilon^H_t + \Upsilon^F_t = 0 \), are rewritten as,

\[
\frac{\eta - (1-\theta^F_t)^\rho}{(1-\theta^H_t)^\rho - \eta} = \frac{w^H_t}{w^F_t} = \frac{\theta^F_t}{\Gamma^i_t} - (1 - \eta) \frac{1}{r^F_t - \theta^H_t \rho} - 1,
\]

\[
\frac{(\theta^H_t - \theta^H_t)^\rho}{(\rho - \eta \Gamma^i_t) - \theta^H_t \rho} - 1 = \frac{(\theta^F_t - \theta^H_t)^\rho}{(1 - \eta) r^* - \theta^H_t \rho} - 1,
\]

\[(\rho - \eta \Gamma^i_t) - \theta^H_t \rho = (1 - \eta) r^* - \theta^H_t \rho, \quad \Rightarrow \quad \Gamma^* = \frac{\rho}{\eta} - \frac{1 - \eta}{\eta} r^*.
\]

(35)

(36)

In the case of the binding borrowing constraints, \( \frac{\partial \ln w^i}{\partial \theta^i} = \frac{\rho (\Gamma^* - r^*)}{\Gamma^i_t + \theta^i_t (1 - r^*)} > 0 \) implies \( w^H < w^F \). According to equation (34),

\[
\frac{\eta - (1-\theta^F_t)^\rho}{(1-\theta^H_t)^\rho - \eta} = \frac{\theta^F_t}{\Gamma^i_t} - (1 - \eta) \frac{1}{r^F_t - \theta^H_t \rho} = \frac{w^H_t}{w^F_t} < 1, \quad \Rightarrow \quad \Gamma^* < \Gamma^i_t \text{ and } \quad r^* > r^F_t.
\]

Thus, the steady-state values of the world equity rate and the world loan rate are \( \Gamma^* \in (\Gamma^F_{FA}, \Gamma^H_{FA}) \) and \( r^* \in (r^F_{FA}, r^F_{FA}) \), respectively.

Substitute equation (36) and \( w^i_t = R^\rho \left[ 1 - \frac{\theta^i_t}{\Gamma^i_t} + \theta^i_t \rho \right] \) into equation (34), \( r^* \) solves the following equation,

\[
\frac{(\theta^H_t - \theta^H_t)^\rho}{(1 - \eta) r^* - \theta^H_t \rho} - 1 = \left[ 1 - \frac{(\theta^F - \theta^H_t)^\rho}{\frac{\eta}{\Gamma^i_t + \theta^i_t}} + \theta^F_t \right] \rho.
\]

(37)

Let \( \mathcal{N}(r^*) \equiv \frac{(\theta^F - \theta^H_t)^\rho}{(1 - \eta) r^* - \theta^H_t \rho} - 1 \) and \( \mathcal{R}(r^*) \equiv \left[ 1 - \frac{(\theta^F - \theta^H_t)^\rho}{\frac{\eta}{\Gamma^i_t + \theta^i_t}} + \theta^F_t \right] \rho \) denote the functions of \( r^* \) defined by the left-hand and the right-hand sides of equation (37).

Given \( \theta^F > \theta^H_t \) and \( r^* \in (r^F_{FA}, r^F_{FA}) \), we get

\[
\mathcal{N}'(r^*) < 0 < \mathcal{R}'(r^*),
\]

\[
\mathcal{N}(r^* = r^F_{FA}) = 1 > \mathcal{R}(r^* = r^F_{FA}),
\]

\[
\mathcal{N}(r^* = r^F_{FA}) = 0 < \mathcal{R}(r^* = r^F_{FA}).
\]
Thus, there must exist a unique world loan rate in the steady state $r^* \in (\bar{r}^*, r^*_{IFA})$ that solves equation (37).

**Proof of Proposition 12**

*Proof.* Suppose that for $\theta^H \in (\max\{1 - 2\eta, 0\}, 1 - \eta)$ and $\theta^F = \bar{\theta}^F_{FCM}$, the borrowing constraints are binding and the loan rate is equal to the equity rate in both countries. According to equation (36), $\Gamma^* = r^* = \rho$. The wage is same in the two countries, $w^i = \left(\frac{R}{\eta}\right)^{\bar{\rho}}$. According to equation (34),

$$\frac{\theta^F \rho - (1 - \eta)}{(1 - \eta) - \frac{\theta^F \rho}{\bar{\rho}}} = \frac{w^H}{w^F} = 1, \iff \bar{\theta}^F_{FCM} = 2(1 - \eta) - \theta^H. \tag{38}$$

For $\theta^F \in (\bar{\theta}^F_{FCM}, 1)$, the borrowing constraints are not binding and the loan rate is equal to the equity rate at $\Gamma^* = r^* = \rho$.

**Proof of Proposition 14**

*Proof.* According to equations (31) and (32), the steady-state values of FDI and financial capital flows are $\Omega^i = (\Gamma^* - \Gamma^*_{IFA}) \frac{w^i}{\bar{\rho}}$ and $\Upsilon^i = (r^* - r^*_{IFA}) \frac{(1 - \eta)w^i}{\bar{\rho}}$, respectively. Since $r^* \in (\bar{r}^*, r^*_{IFA})$, financial capital flows from country H to country F, $\Upsilon^H > 0 > \Upsilon^F$; since $\Gamma^* \in (\Gamma^*_{IFA}, \Gamma^*)$, FDI flows in the opposite direction, $\Omega^H < 0 < \Omega^F$. The direction of capital flows is same as under free mobility of FDI and financial capital, respectively.

According to equations (33), net capital flows are $\Omega^i + \Upsilon^i = w^i \left[1 - \frac{\rho}{\bar{\rho}}(w^i)^{1 + \bar{\rho}}\right]$ in the steady state. The identity of net capital flows $\Omega^H + \Upsilon^H + \Omega^F + \Upsilon^F = 0$ implies

$$\sum_{i \in \{H,F\}} w^i \left[1 - \frac{\rho}{\bar{\rho}}(w^i)^{1 + \bar{\rho}}\right] = 0 \iff \left[1 - \frac{\rho}{\bar{\rho}}(w^H)^{1 + \bar{\rho}}\right] \left[1 - \frac{\rho}{\bar{\rho}}(w^F)^{1 + \bar{\rho}}\right] \leq 0. \tag{39}$$

If $\Gamma^* > r^*$, the borrowing constraints are binding and the steady-state wage is lower in country H than in country F, $w^H \leq w^F$. Thus, $1 - \frac{\rho}{\bar{\rho}}(w^H)^{1 + \bar{\rho}} > 1 - \frac{\rho}{\bar{\rho}}(w^F)^{1 + \bar{\rho}}$. According to equation (39), $1 - \frac{\rho}{\bar{\rho}}(w^H)^{1 + \bar{\rho}} > 0 > 1 - \frac{\rho}{\bar{\rho}}(w^F)^{1 + \bar{\rho}}$ and the net capital flows are from country H to country F in the steady state, $\Omega^H + \Upsilon^H > 0 > \Omega^F + \Upsilon^F$.

If $\Gamma^* = r^*$, the borrowing constraints are not binding and the steady-state wage is same in the two countries with zero net capital flows, $w^i = \left(\frac{R}{\rho}\right)^{\bar{\rho}}$ and $\Omega^i + \Upsilon^i = 0$. Economic allocation is almost same as under international financial autarky except that the interest rates in country H are different, $\Gamma^* = \rho < \Gamma^*_{IFA}$ and $r^* = \rho > r^*_{IFA}$.

**Proof of Proposition 15**

*Proof.* According to equations (31) and (32), in the steady state, $r^* \Upsilon^i + \Gamma^* \Omega^i = w^i[(1 - \eta)r^* + \eta \Gamma^*] - \rho w^i$. According to equation (36), $(1 - \eta)r^* + \eta \Gamma^* = \rho$, and, thus, $r^* \Upsilon^i + \Gamma^* \Omega^i = 0$. This way, country F is a net debtor, $\Upsilon^F + \Omega^F < 0$, and still receives a positive net
international investment income, \( NII^F \equiv (r^* - 1)Y^F + (\Gamma^* - 1)\Omega^F = 0 - (\Upsilon^F + \Omega^F) > 0 \). Intuitively, the net interest income received by entrepreneurs from investing abroad, \(|(\Gamma^* - 1)\Omega^F|\) dominates the net interest income paid to foreign workers, \(|(r^* - 1)\Upsilon^F|\). It mainly results from the positive equity premium.

**Proof of Proposition 16**

*Proof.* According to Proposition 14, \( w_{FCM}^H < w_{IFA}^H \leq w_{FCM}^F \). Given the world equity rate \( \Gamma_{FCM}^* \in (\Gamma_{IFA}^F, \Gamma^*) \), entrepreneurs in country \( H \) (\( F \)) are strictly worse (better) off than under international financial autarky due to the decline (increase) in both the wage and the equity rate, \( c^{i,e} = w^i\Gamma^i \).

Social welfare in country \( i \) is \( C^i = \eta c^{i,e} + (1 - \eta)c^{i,w} = w^i[\eta \Gamma^* + (1 - \eta)r^*] \) in the steady state. According to equation (36), \( \eta \Gamma^* + (1 - \eta)r^* = \rho \), and, thus, social welfare is proportional to the wage in country \( i \), \( C^i = w^i\rho \). In comparison with international financial autarky, net capital outflow from country \( H \) reduces aggregate investment in country \( H \). Thus, the wage is lower and so is the social welfare of country \( H \). Similar argument applies to country \( F \).

**Proof of Proposition 17**

*Proof.* Take the world equity rate \( \Gamma^* \) as given. According to equation (21), \( \beta_{t+1}^i \) is considered as a function of \( \beta_t^i \). For \( \beta_t^i \in [0, \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1}{1 - \alpha}}] \), the marginal return on investment is equal to the world equity rate, \( R\beta_{t+1}^i = \Gamma_t^* \), and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages is flat at \( \beta_{t+1}^i = \left( \frac{R}{\Gamma_t^i} \right)^{\rho} \), independent of \( \beta_t^i \).

For \( \beta_t^i \geq \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1}{1 - \alpha}} \), take the first derivative of equation (21) with respect to \( \beta_t^i \),

\[
1 - \eta = \left[ \frac{\rho}{R\alpha} \frac{\beta_t^i}{\Gamma_t^*} - \frac{(1 - \theta_i)^2}{\Gamma_t^*} \right] \frac{d\beta_{t+1}^i}{d\beta_t^i}.
\]  \( \text{(40)} \)

For \( \beta_t^i = \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1}{1 - \alpha}} \), there is a non-zero solution \( \beta_{t+1}^i = \left( \frac{R}{\Gamma_t^i} \right)^{\rho} \). The slope of the phase diagram at the point \( \left( \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1}{1 - \alpha}}, \left( \frac{R}{\Gamma_t^i} \right)^{\rho} \right) \) is \( \frac{\Gamma_t^*}{\rho_{\frac{1}{1 - \alpha}}(1 - \rho_{\frac{1}{1 - \alpha}})^{\frac{1 - \alpha}{1 - \rho_{\frac{1}{1 - \alpha}}}}} > 0 \). In other words, \( \beta_{t+1}^i \geq \left( \frac{\theta_i}{R(1 - \eta)} \right)^\rho \). Thus, according to equation (40), the phase diagram has the positive slope, \( \frac{d\beta_{t+1}^i}{d\beta_t^i} > 0 \). Take the second derivative of equation (21) with respect to \( \beta_t^i \),

\[
0 = \left[ \frac{\rho}{R\alpha} \frac{\beta_t^i}{\Gamma_t^*} - \frac{(1 - \theta_i)^2}{\Gamma_t^*} \right] \frac{d^2\beta_{t+1}^i}{d\beta_t^i} + \left( \frac{d\beta_{t+1}^i}{d\beta_t^i} \right)^2 \frac{1}{R\alpha} \left( \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1 - 2\alpha}{1 - \alpha}} \right), \Rightarrow \frac{d\beta_{t+1}^i}{d\beta_t^i} < 0.
\]

The phase diagram of wages is concave for \( \beta_t^i > \frac{\theta_i}{R(1 - \eta)} \left( \frac{R}{\Gamma_t^i} \right)^{\frac{1}{1 - \alpha}} \) and \( \beta_{t+1}^i \) monotonically increases in \( \beta_t^i \).
Overall, the phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. The stability and the uniqueness of the non-zero steady-state equilibrium is proved.

Proof of Proposition 18

Proof. The world equity rate is determined by the equilibrium of FDI flows, \( \Omega^H_t + \Omega^F_t = 0 \). We first prove the existence of a unique world equity rate clearing the world equity market every period and then derive the world equity rate in the steady state.

Suppose that the borrowing constraints are binding in country \( i \). Given the predetermined \( w^i_t \), equation (21) shows that \( w^i_{t+1} \) is a function of \( \Gamma^*_t \). Take the first derivative of equation (21) with respect to \( \Gamma^*_t \),

\[
0 = \left[ \frac{\rho}{R \alpha} (w^i_{t+1})^{\frac{1}{\rho}} - \frac{(1 - \theta^i)\rho}{\Gamma^*_t} \right] \frac{dw^i_{t+1}}{d\Gamma^*_t} + \frac{(1 - \theta^i)\rho}{(\Gamma^*_t)^2}.
\]

As shown in the proof of Proposition 17, \( w^i_{t+1} \geq \left( \frac{R}{\Gamma^*_t} \right)^{\rho} \). The term in square brackets is positive. Thus, an increase in the world equity rate enhances FDI outflows, which reduce current domestic investment and hence the wage in the next period declines, \( \frac{dw^i_{t+1}}{d\Gamma^*_t} < 0 \).

Capital outflows represent the gap between domestic savings and investment,

\[
\Omega^i_t = w^i_t - I^i_t = w^i_t - \frac{\rho}{R} (w^i_{t+1})^{\frac{1}{\rho}} = \eta \Gamma^*_t (1 - \theta^i)w^i_{t+1}.
\]

The world equity market equilibrium implies

\[
\Omega^H_t + \Omega^F_t = 0, \Rightarrow \eta (w^H_t + w^F_t) = \frac{\rho}{\Gamma^*_t} \left[ (1 - \theta^H)w^H_{t+1} + (1 - \theta^F)w^F_{t+1} \right].
\]

Take the wage \( w^i_t \) as given. Suppose \( \theta^H \in (0, \bar{\theta}) \) and \( \theta^F \in (\theta^H, \bar{\theta}) \). As shown in figure 1, the equity rate in country H is higher than in country F before period \( t \), \( \Gamma^*_t > \Gamma^*_{FA} \). The world equity rate in period \( t \) must be \( \Gamma^*_t \in (\Gamma^*_FA, \Gamma^*_FA) \) and FDI flows from country F to country H, given the predetermined \( \Gamma^*_{FA} \) and \( \Gamma^*_{FA} \). The proof is by contradiction similar as in the proof of Proposition 3.

In the next step, we assume that there exists a unique world equity rate in the steady state and then prove its uniqueness.

Given \( \Gamma^* \), Proposition 17 shows that there is a unique and stable steady state where \( w^i = \left( \frac{R}{\rho} \right)^{\rho} \left[ (1 - \eta) + \frac{(1 - \theta^i)\rho}{\Gamma^*} \right]^{\frac{1}{\rho}} \). According to equation (41), FDI in the steady state is

\[
\Omega^i = w^i \left[ \eta - \frac{(1 - \theta^i)\rho}{\Gamma^*} \right].
\]

According to equation (42), the world equity rate in the steady state is determined by

\[
\frac{\Omega^H}{w^H} = \frac{\Omega^F}{w^F} = \frac{\Gamma^* - \Gamma^*_{FA}}{\Gamma^*_{FA} - \Gamma^*} = \left[ \frac{\Gamma^* + \frac{1 - \theta^H}{1 - \eta} \rho}{\Gamma^* + \frac{1 - \theta^F}{1 - \eta} \rho} \right]^\rho.
\]
For $\theta^H \in (0, \bar{\theta})$ and $\theta^F > \theta^H$, the right-hand side of equation (44) is larger than one,

$$\frac{\Gamma^* - \Gamma^F_{IFA}}{\Gamma^H_{IFA} - \Gamma^*} > 1,$$

or $\Gamma^* \in (\Gamma^*, \Gamma^H_{IFA})$.

Let $\mathcal{N}(\Gamma^*) \equiv \frac{\Gamma^H_{IFA} - \Gamma^F_{IFA}}{\Gamma^H_{IFA} - \Gamma^*} - 1$ and $\mathcal{R}(\Gamma^*) \equiv \left[1 + \frac{(\theta^F - \theta^H)\rho}{(1 - \eta)\rho + (1 - \theta^F)\rho}\right]^{\rho}$ denote the left-hand and the right-hand sides of equation (44) as the functions of $\Gamma^*$. For $\Gamma^* \in (\Gamma^*, \Gamma^H_{IFA})$,

$$\mathcal{N}(\Gamma^*) > 0 > \mathcal{R}(\Gamma^*),$$

$$\mathcal{N}(\Gamma^* = \Gamma^*) = 0 < \mathcal{R}(\Gamma^* = \Gamma^*),$$

$$\mathcal{N}(\Gamma^* = \Gamma^H_{IFA}) \to -\infty > \mathcal{R}(\Gamma^* = \Gamma^H_{IFA}).$$

Thus, $\mathcal{N}(\Gamma^*)$ decreases while $\mathcal{R}(\Gamma^*)$ increases monotonically in $\Gamma^*$; the two functions cross once and only once for $\Gamma^* \in (\Gamma^*, \Gamma^H_{IFA})$. Therefore, there exists a steady-state equilibrium under free mobility of FDI, which is also unique. \qed

**Proof of Proposition 20**

*Proof.* If the borrowing constraints are binding in the two countries under free mobility of FDI, the steady-state loan rate $r^i = \frac{\theta^i\rho}{(1 - \eta)}$ has the same form as under international financial autarky. Suppose that given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^F_{FDI}$, the borrowing constraints are binding and the loan rate in country F is equal to the world equity rate, $r^F = \frac{\theta^F\rho}{1 - \eta} = \Gamma^*$. Substitute it into equation (44),

$$\frac{\eta - (1 - \bar{\theta}^F_{FDI})}{(1 - \eta)(1 - \theta^H) - \theta^F\eta} = (1 + \bar{\theta}^F_{FDI} - \theta^H)^{\rho}. \quad (45)$$

It can be shown for $\eta \in \left[\frac{\rho}{1+2\rho+1}, 1\right]$, given $\theta^H \in (0, \bar{\theta})$, there exist a $\bar{\theta}^F_{FDI} \in (\bar{\theta}, 1)$ that solve equation (45). For $\eta \in (0, \frac{\rho}{1+2\rho+1})$, there exists a $\theta^H$ that solves equation (46),

$$\frac{\eta}{(1 - \eta)(1 - \theta^H) - \eta} = (2 - \theta^H)^\rho. \quad (46)$$

For $\theta^H \in [\bar{\theta}^H, \bar{\theta})$, there exists $\bar{\theta}^F_{FDI}$ that solves equation (45); for $\theta^H \in (0, \bar{\theta}^H)$, the borrowing constraints are always binding in country F for $\theta^F \in (\theta^H, 1)$. \qed

**Proof of Proposition 23**

*Proof.* If the borrowing constraints are binding, the steady-state consumption of entrepreneurs is

$$c^{i,e} = w^i\Gamma^* = \left(\frac{R}{\rho}\right)^\rho \left[\Gamma^*(1 - \eta) + (1 - \theta^i)\rho\right]^{\rho} (\Gamma^*)^{1-\rho},$$

$$\frac{d\ln c^{i,e}}{d\Gamma^*} = \frac{\Gamma^*(1 - \eta) + (1 - \theta^i)\rho(1 - \rho)}{\left[\Gamma^*(1 - \eta) + (1 - \theta^i)\rho\right]^{\rho}}.$$
As the analytical solution of the world loan rate is not obtainable, we provide the sufficient conditions of welfare changes as follows.

Evaluate \( \frac{dlnc^{I,e}}{dt} \) at \( \Gamma^* = \Gamma^H_{IFA} \) and \( \Gamma^* = \Gamma^* \). For \( \frac{(\rho-1)\eta}{(1-\eta)} \leq \frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)} \), \( \frac{dlnc^{I,e}}{dt} \) \( |\Gamma^* = \Gamma^H_{IFA} > \frac{dlnc^{I,e}}{dt} \) \( |\Gamma^* = \Gamma^* \geq 0 \) implies that entrepreneurs born in country \( H \) is worse off since the negative labor income effect; for \( \frac{(\rho-1)\eta}{(1-\eta)} \geq \frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)} \), the numerical solutions are required for the welfare evaluation.

Evaluate \( \frac{dlnc^{F,e}}{dt} \) at \( \Gamma^* = \Gamma^H_{IFA} \) and \( \Gamma^* = \Gamma^F_{IFA} \). For \( \frac{(\rho-1)\eta}{(1-\eta)} \leq 1 \), \( \frac{dlnc^{F,e}}{dt} \) \( |\Gamma^* = \Gamma^H_{IFA} > \frac{dlnc^{F,e}}{dt} \) \( |\Gamma^* = \Gamma^F_{IFA} \geq 0 \) implies that entrepreneurs born in country \( F \) is worse off since the negative equity rate effect dominates the positive wage effect; for \( \frac{(\rho-1)\eta}{(1-\eta)} > 1 \), \( \frac{dlnc^{F,e}}{dt} \) \( |\Gamma^* = \Gamma^H_{IFA} < \frac{dlnc^{F,e}}{dt} \) \( |\Gamma^* = \Gamma^F_{IFA} \leq 0 \) implies that both workers and entrepreneurs are worse off or the welfare gains of entrepreneurs is dominated by the welfare losses of workers and hence, country \( H \) as a whole loses from free mobility of FDI; for \( \frac{(\rho-1)\eta}{(1-\eta)} \) \( (\frac{1-\theta^H}{1-\theta^F}) \), the numerical solutions are required for the welfare evaluation.

The steady-state social welfare of country \( i \) is

\[
C^i \equiv \eta c^{i,e} + (1 - \eta)c^{i,w} = \rho \Gamma^*(1 - \eta) + (1 - \theta^i)\rho \Gamma^*(\eta \Gamma^* + \theta^i \rho),
\]

\[
\frac{dlnc^i}{dt} = \left( \frac{R^j}{\rho} \right)^\rho \Gamma^*(1 - \eta) + (1 - \theta^i)\rho \Gamma^*(\eta \Gamma^* + \theta^i \rho).
\]

Evaluate \( \frac{dlnc^H}{dt} \) at \( \Gamma^* = \Gamma^H_{IFA} \) and \( \Gamma^* = \Gamma^* \). For \( \rho \in (0, \frac{(2-\theta^H-\theta^F)(2-\theta^H-\theta^F+\eta(\theta^F-\theta^H))}{2(1-\theta^H)(2-\theta^H-\theta^F)}) \), \( \frac{dlnc^H}{dt} \) \( |\Gamma^* = \Gamma^H_{IFA} > \frac{dlnc^H}{dt} \) \( |\Gamma^* = \Gamma^* \geq 0 \) implies that the welfare loss of entrepreneurs dominates the welfare gains of workers and hence, country \( H \) as a whole losses from free mobility of FDI; for \( \rho \in \frac{(2-\theta^H-\theta^F)(2-\theta^H-\theta^F+\eta(\theta^F-\theta^H))}{2(1-\theta^H)(2-\theta^H-\theta^F)} \), (\( 1-\theta^H \)), the numerical solutions are required for the welfare evaluation.

Evaluate \( \frac{dlnc^F}{dt} \) at \( \Gamma^* = \Gamma^H_{IFA} \) and \( \Gamma^* = \Gamma^F_{IFA} \). For \( \rho \in (0, \frac{1-\theta^H}{\eta}) \), \( \frac{dlnc^F}{dt} \) \( |\Gamma^* = \Gamma^H_{IFA} > \frac{dlnc^F}{dt} \) \( |\Gamma^* = \Gamma^F_{IFA} \geq 0 \) implies that the welfare gains of entrepreneurs dominates the welfare losses of workers and hence, country \( F \) as a whole benefits from free mobility of FDI; for \( \rho \in \frac{(1-\theta^H)(1-\theta^H-\eta(\theta^F-\theta^H))}{\eta(1-\theta^H)(1+\theta^F-\theta^H)} \), the numerical solutions are required for the welfare evaluation.

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