North-South Trade with Multinational Firms 
and Increasing Product Variety

Peter Gustafsson 
National Institute of Economic Research

Paul S. Segerstrom  
Stockholm School of Economics

Current version: June 29, 2010

Abstract: We present a model of North-South trade with multinational firms and increasing 
product variety. Firms engage in innovative R&D to develop new product varieties in the North 
and foreign affiliates of multinational firms engage in adaptive R&D to learn how to produce product 
varieties in the South. We find that a shift to stronger protection of intellectual property rights 
in the South induces foreign affiliates of multinational firms to increase their R&D expenditures, 
results in a faster rate of technology transfer within multinational firms and increases long-run 
consumer welfare in both regions.

JEL classification: F12, F23, F43, 031, O34.

Keywords: Multinational firms, North-South trade, Intellectual Property Rights, Foreign Direct 
Investment, Product cycles, Economic Growth.

Acknowledgments: We thank Elias Dinopoulos, Rikard Forslid, Damian Migueles Chazarreta, 
Yoichi Sugita, seminar participants at the Stockholm School of Economics, Stockholm University, 
University of Vienna, Cambridge University, Uppsala University and three anonymous referees for 
helpful comments. Financial support from the Wallander Foundation is gratefully acknowledged.

Author: Peter Gustafsson, National Institute of Economic Research, Box 3116, 103 62 Stockholm, 
Sweden (E-mail: peter.gustafsson@konj.se, Tel: +46-8-4535958).

Corresponding Author: Paul S. Segerstrom, Stockholm School of Economics, Department of 
Economics, Box 6501, 11383 Stockholm, Sweden (E-mail: paul.segerstrom@hhs.se, Tel: +46-8- 
7369203, Fax: +46-8-313207).

This is a preprint of an Article accepted for publication in International Economic Review © 2003 
by the Economics Department of the University of Pennsylvania and the Osaka University Institute 
You shall not steal. You shall not covet anything that belongs to your neighbor.
Keep the words of this covenant to do them, that you may prosper in all that you do.

_Deuteronomy 5:19,21; 29:9_

1 Introduction

In recent years, many developing countries have pursued patent reform aimed at strengthening intellectual property rights (IPR) protection.\(^1\) As a result, an intense debate has arisen about the welfare implications of stronger IPR protection in developing countries. IPR advocates argue that patent reform promotes innovation in the global economy and benefits developing countries by fostering more rapid economic growth. They also claim that a strengthening of IPR protection stimulates the transfer of technology from developed to developing countries, a further channel through which developing countries benefit. IPR opponents counter that this reform leads to neither faster economic growth nor faster international technology transfer, but mainly results in the transfer of rents to multinational corporate patent holders headquartered in the world’s most advanced countries.\(^2\)

In an important paper, Branstetter, Fisman and Foley (2006) present new evidence that is directly relevant to this debate. They examine how technology transfer within US-based multinational firms has changed in response to a series of patent reforms undertaken by sixteen mainly developing countries over the 1982-1999 time period. They find that royalty payments for the use of intangible assets made by affiliates to parent firms, which reflect the value of technology transfer, increased in the wake of stronger patent regimes. R&D spending by affiliates—usually viewed as a complement to technology imports from parent firms—also increased after patent reform. The increases in affiliate royalties and R&D were concentrated among affiliates of firms that make extensive use of the US patent system prior to reforms and were therefore likely to value reforms the most. For these patent-intensive firms, there was a 34 percent increase in affiliate royalty payments and a 23 percent increase in affiliate R&D spending. Branstetter et al. (2006) conclude that improvements in IPR protection resulted in significant increases in technology transfer from US-based multinationals to their affiliates in reforming countries.

---

\(^1\)Dinopoulos and Kotarridi (2008, Table 1) document that during the time period 1960-2000, IPR protection increased on average by 70 percent in a sample of developing countries, using a popular index of patent rights protection constructed by Ginarte and Park (1997).

\(^2\)For example, according to McCalman (2001), the implied income transfers caused by the TRIPs-driven stronger IPR protection benefited the US, Germany, France, Italy, Sweden and Switzerland, and harmed all other countries.
This new evidence represents a challenge for the literature on multinational firms and trade between the North (developed countries) and the South (developing countries). In North-South trade models developed by Glass and Saggi (2002), Sener (2006) and Glass and Wu (2007), stronger IPR protection in the South leads to an unambiguously lower rate of technology transfer, the exact opposite of what Branstetter et al. (2006) find empirically.\(^3\) The observed increase in the rate of technology transfer that results from stronger IPR protection is consistent with the the implications of North-South trade models developed by Helpman (1993), Lai (1998), and Branstetter and Saggi (2009). However these papers all assume that international technology transfer within multinational firms is costless and thus cannot account for the observed increase in R&D spending by foreign affiliates of US multinationals. In these papers there is no R&D spending by affiliates, while several empirical studies have documented that R&D conducted by affiliates in developing countries is focused on the absorption of parent-firm technology and on its modification for local markets (Kuemmerle, 1999).\(^4\)

A second challenge for the literature is to explain large wage differences between the North and the South. For example, when Sener (2006, Table 2b) solves his North-South trade model numerically for plausible parameter values (the benchmark case), he finds that the northern wage rate is only 7 percent higher than the southern wage rate. Real world wage differences between developed and developing countries are obviously much larger.

In this paper, we present a model of North-South trade that is consistent with the above-mentioned evidence about multinational firms and can generate large North-South wage differences for plausible parameter values. Unlike in the North-South trade models developed by Lai (1998) and Branstetter and Saggi (2009), this model does not have the counterfactual scale effect property discussed in Jones (1995, 2005). We use this model to study both the equilibrium effects and the welfare effects of stronger IPR protection.\(^5\)

In the model, firms engage in innovative R&D to develop new product varieties in the North and

\(^3\)In another paper of interest, Parello (2008) finds that stronger IPR protection in the South has an ambiguous effect on the rate of technology transfer within multinational firms.

\(^4\)In a recent paper, Sener and Zhao (2009) present a North-South trade model where northern production can be skipped. Firms can do R&D in the North to develop higher quality products and then immediately produce these products in the South. Sener and Zhao find that stronger IPR protection leads to faster technology transfer to the South but there is no R&D spending by foreign affiliates (even though FDI is costly). They do not allow for products to be first produced in the North and then later produced in the South.

\(^5\)We find that solving for the effects of stronger IPR protection is easier when scale effects are not present. Lai (1998) has to work quite hard to solve his model analytically assuming costless technology transfer (see his Appendix D) and when his model is extended to allow for costly technology transfer, it becomes analytically intractable. In contrast, our model reduces to a simple system of 2 equations in 2 unknowns and the effects of stronger IPR protection are easily determined graphically.
foreign affiliates of these northern multinationals engage in adaptive R&D to learn how to produce these product varieties in the South. All shifts in production go from North to South, resulting in one-way product cycles in international trade. New product varieties are produced in the North and exported to the South, while old product varieties are produced in the South and exported to the North. The profit flows earned by firms jump up when they are successful in transferring their production to the South and each production transfer is associated with a royalty payment from the foreign affiliate to its parent for the use of the parent firm’s technology. When firms are successful in transferring their production to the South, they also become exposed to a positive rate of imitation by southern firms. Stronger IPR protection in the South is modeled as a reduction in the rate at which southern firms imitate the product varieties produced by foreign affiliates.

We find that stronger IPR protection in the South (e.g., patent reform in developing countries) leads to a permanent increase in the rate of technology transfer to the South within multinational firms and a permanent increase in adaptive R&D spending in the South by foreign affiliates. These two effects are connected because the increase in adaptive R&D spending is what drives the increase in the rate of technology transfer within multinational firms. Thus, the model is consistent with the two main empirical findings in Branstetter et al. (2006), that patent reform is associated with increased royalty payments to parent firms from their foreign affiliates and increased R&D spending by these foreign affiliates. Furthermore, we find that stronger IPR protection in the South leads to a temporary increase in the northern innovation rate and a permanent decrease in the northern relative wage. Thus, this paper provides theoretical support for the argument that patent reform in developing countries promotes innovation in the global economy and identifies an additional wage channel through which developing countries can potentially benefit from stronger IPR protection.

To gain some insights about the long-run welfare implications of the model, we turn to computer simulations. We calibrate the model based on evidence about the US and Mexico. We find that increasing IPR protection in the South monotonically increases long-run consumer welfare in the South. Furthermore, the long-run welfare effects are surprisingly large. When IPR protection goes from very weak (one out of five varieties are copied each year) to very strong (varieties are never copied), real consumption per capita in the South increases by 119 percent. Northern consumers also benefit from stronger IPR protection, with a corresponding percentage increase in real consumption per capita of 45 percent.

This paper is closely related to two companion papers about North-South trade and technology transfer. Dinopoulos and Segerstrom (2010) show that the same comparative steady-state equi-
librium properties that we derive assuming that innovations are increases in product variety also hold when innovations are increases in product quality. However, the Dinopoulos-Segerstrom model generates two-way product cycles and consequently, can only account for small North-South wage differences for plausible parameter values. Gustafsson and Segerstrom (2010) study what happens when all technology transfer takes the form of southern firms copying products produced in the North. In this model without multinational firms and FDI, we find that stronger IPR protection in the South reduces the rate of technology transfer, reduces the northern innovation rate and makes southern consumers unambiguously worse off in the long run. We conclude that the effects of stronger IPR protection are much more beneficial for southern consumers when technology transfer takes place within multinational firms.

The rest of the paper is organized as follows: In Section 2, we explain why models with two-way products cycles can only account for small North-South wages differences, to motivate the analysis that follows. In Section 3, we present the model and in Section 4, we show that it always has a unique steady-state equilibrium with one-way product cycles. We solve the model analytically for its steady-state equilibrium properties in Section 5 and to learn more about the long-run welfare implications, we solve the model numerically in Section 6. In Section 7, we discuss why previous papers have different results about the effects of stronger IPR protection. In Section 8, we explore how the properties of the model change when imitation is costly and the imitation rate is endogenously determined. Finally, we offer some concluding remarks in Section 9.

2 Motivation

In this section, we explain why North-South trade models with two-way products cycles can only account for small wage differences between the North and the South. We use Sener (2006) to illustrate the issues since he solves his model numerically. Without fully specifying his model, we focus on key conditions that must be satisfied for the existence of a steady-state equilibrium with two-way product cycles.

In Sener (2006), firms in the North engage in innovative R&D to develop higher quality versions of existing products, foreign affiliates of these firms engage in adaptive R&D to transfer production to the low-wage South, and southern firms engage in imitative R&D to copy the products of foreign affiliates. There is a fixed number of industries and in each industry, labor is the only factor of production. Labor markets are perfectly competitive in both regions. The wage rate of northern
labor is $w_N$ and the wage rate of southern labor is $w_S$. There is constant returns to scale and for any northern firm that knows how to produce a product, one unit of northern labor produces one unit of output. For a foreign affiliate, $\alpha > 1$ units of southern labor produce one unit of output and for a southern firm, one unit of southern labor produces one unit of output. Thus, the constant marginal cost of production for a firm in the North is $w_N$, the constant marginal cost of production for a foreign affiliate in the South is $\alpha w_S$, and the constant marginal cost of production for a southern firm is $w_S$. There is costless trade between the two regions.

In each industry, production jumps from the North to the South when a foreign affiliate is successful in adaptive R&D and production jumps from the South to the North when some northern firm develops a higher quality version of a southern firm’s product. From each region where production takes place, products are exported to the other region. Production only jumps from the North to the South following successful adaptive R&D if the foreign affiliate has a lower marginal cost of production than the northern parent firm. Thus, one condition that must hold in equilibrium is $w_N > \alpha w_S$. There is no incentive for a foreign affiliate to incur the positive costs of adaptive R&D if its resulting marginal cost is not lower than the marginal cost of the parent firm in the North. Furthermore, production only jumps from the South to the North when a northern firm innovates if, when taking into account the size of the quality improvement $\lambda > 1$, the northern firm has a lower effective marginal cost than its southern rival. There is no incentive for northern firms to incur the positive costs of innovating if they cannot then compete against southern firms with lower wage costs. Thus, a second condition that must hold in equilibrium is $w_S > w_N/\lambda$. Putting these two conditions together, we obtain

$$\lambda > \frac{w_N}{w_S} > \alpha.$$  

We can already see why many North-South trade models have difficulty explaining large wage differences between the North and the South: the size of the quality improvement limits the North-South wage ratio that these models are capable of explaining. For example, if innovations are assumed to be 25 percent improvements in product quality ($\lambda = 1.25$), then these models can explain at most a northern wage rate that is 25 percent higher than the southern wage rate. Based on empirical evidence about markups of price over marginal cost, Sener (2006) chooses $\lambda = 1.25$ when he solves his model numerically and for plausible parameter values (the benchmark case in Table 2b), he obtains a northern wage rate that is only 7 percent higher than the southern wage.
rate.\(^6\)

In this paper, we present a model of one-way product cycles, where all production shifts go from North to South. In our model, we set \(\alpha = 1\) and the resulting equilibrium condition \(\frac{w_N}{w_S} > 1\) has to hold for production to shift to the South following adaptive R&D success. But production never shifts back to the North, so the equilibrium condition \(\lambda > \frac{w_N}{w_S}\) no longer applies. Thus, this model with one-way product cycles can potentially account for much larger wage differences between the North and the South.

3 The Model

3.1 Overview

We consider a global economy consisting of two regions: the North and the South. In both regions, labor is the only factor used to manufacture product varieties and to do R&D. Labor is perfectly mobile across activities within a region but cannot move across regions. Since labor markets are perfectly competitive, there is a single wage rate paid to all northern workers and a single wage rate paid to all southern workers. Labor is employed in three distinct activities: manufacturing of final consumption goods, innovative R&D and adaptive R&D. All innovative R&D is done in the North and all adaptive R&D is done in the South. There is costless trade between the two regions.

In this global economy, firms can hire northern workers to engage in innovative R&D with the goal of learning how to produce new product varieties. A successful firm earns global monopoly profits from producing a new product variety and selling it to consumers in both regions. We call such a firm a northern firm because all production is located in the North. A northern firm can hire southern workers to engage in adaptive R&D with the aim of transferring its manufacturing operations to the low-wage South. When successful in adaptive R&D, a firm earns even higher global monopoly profits because of the lower wage costs in the South. We call such a firm a foreign affiliate because production takes place in the South but a fraction of its profits is repatriated back to its northern stockholders. Adaptive R&D can be interpreted as an index of FDI (foreign direct investment) because it represents the cost that northern firms incur to transfer their technology to foreign affiliates, and even when financed by southern savings, northern firms control the amount of adaptive R&D in order to maximize their global profits. When firms transfer their production

---

\(^6\)Sener and Zhao (2009, Table 2b) also solve their model numerically, this time assuming that \(\lambda = 1.5\) and find that the northern wage is only 27 percent higher than the southern wage.
to the South, they become exposed to a positive rate of imitation by southern firms, firms that are owned and operate in the South. We model stronger IPR protection in the South as a reduction in this imitation rate.\textsuperscript{7}

As illustrated in Figure 1, each product variety experiences a one-way FDI-driven product cycle over time: the variety is initially developed and produced by a northern firm, its production later shifts to a foreign affiliate as a result of adaptive R&D, and eventually shifts to southern firms that are successful in imitating the production technology.\textsuperscript{8}

### 3.2 Households

In both the North and the South, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate $g_L > 0$, the population growth rate. Let $L_{Nt} = L_{N0}e^{gLt}$ denote the supply of labor in the North at time $t$, let $L_{St} = L_{S0}e^{gLs}$ denote the corresponding supply of labor in the South, and let $L_t = L_{Nt} + L_{St}$ denote the world supply of labor. In addition to wage income, households also receive asset income from their ownership of firms. We assume that R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings, which is consistent with the Feldstein and Horioka (1980) finding that domestic savings finances domestic investments.

Households in both the North and the South share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^\infty e^{-(\rho - g_L)t} \ln[u_t] \, dt$$

where $\rho > g_L$ is the subjective discount rate and $u_t$ is the static utility of an individual at time $t$.

\textsuperscript{7}We do not allow a multinational firm to take advantage of lower costs by having certain tasks or operations carried out outside the firm. Grossman and Helpman (2003) present a model where firms need specialized components and can either produce them in a subsidiary that is located in a foreign country or can import them from a foreign supplier. They study the trade-off between FDI and outsourcing from abroad when contracts with foreign suppliers are incomplete.

\textsuperscript{8}Lu (2007) presents a model of innovation and FDI where industries differ in terms of innovation size. Lu finds that product cycles only emerge for medium-tech industries. In high-tech industries, firms only do innovative R&D so production stays in the North and in low-tech industries, innovative R&D stops after production moves to the South. Unlike in this paper, Lu does not allow firms that produce in the North and earn positive profits to engage in FDI, so all FDI is done by non-producing northern firms (followers). There is no imitation in Lu’s model and IPR protection issues are not studied.
Innovation

Production by Northern Firm
  $n_N$ varieties

FDI
  $\phi$

Production by Foreign Affiliate
  $n_F$ varieties

Imitation
  $\iota$

Production by Southern Firms
  $n_S$ varieties

Figure 1: The FDI-driven product cycle.
The static constant elasticity of substitution (CES) utility function is given by

$$u_t = \left[ \int_0^{n_t} x_t(\omega)^\alpha d\omega \right]^{\frac{1}{\alpha}} \quad 0 < \alpha < 1$$  \hspace{1cm} (2)

where \( x_t(\omega) \) is the per capita quantity demanded of the product variety \( \omega \) at time \( t \), \( n_{N_t} \) is the number of varieties produced by northern firms, \( n_{F_t} \) is the number of varieties produced in foreign affiliates, \( n_{S_t} \) is the number of varieties produced by southern firms, and \( n_t = n_{N_t} + n_{F_t} + n_{S_t} \) is the number of varieties available on the world market. We assume that varieties are gross substitutes. Then with \( \alpha \) measuring the degree of product differentiation, the elasticity of substitution between product varieties is \( \sigma \equiv \frac{1}{1-\alpha} > 1 \).

Solving the static consumer optimization problem yields the familiar demand function

$$x_t(\omega) = \frac{P_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}}$$  \hspace{1cm} (3)

where \( c_t \) is the individual consumer expenditure at time \( t \), \( p_t(\omega) \) is the price of variety \( \omega \) and \( P_t = \left[ \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \) is an index of consumer prices. Maximizing (1) subject to (2) where (3) has been used to substitute for \( x_t(\omega) \) yields the intertemporal optimization condition

$$\frac{\dot{c}_t}{c_t} = r_t - \rho$$  \hspace{1cm} (4)

implying that individual consumer expenditure grows over time only if the market interest rate \( r_t \) exceeds the subjective discount rate \( \rho \).

Let \( w_N \) and \( w_S \) denote the equilibrium wage rates in the North and South, respectively. Likewise, let \( c_N \) and \( c_S \) denote the representative consumer’s expenditure in the North and South, respectively. We treat the southern wage as the numeraire price \( (w_S = 1) \), that is, we measure all prices relative to the price of southern labor. Furthermore, we solve the model for a steady-state equilibrium where \( w_N, w_S, c_N \) and \( c_S \) are all constant over time. Then (4) implies that the steady-state market interest rate is also constant over time and given by \( r_t = \rho \).\(^9\)

\(^9\)Our earlier assumption that “R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings” implies that there is no international capital mobility. Thus the two regions typically have different interest rates along the transition path leading to a new steady-state equilibrium. But in a steady-state equilibrium, the two regions must have the same interest rate because consumers in both regions have the same subjective discount rate \( \rho \).
3.3 Product Markets

The firms producing different product varieties compete in prices and maximize profits. The production of output is characterized by constant returns to scale. For each firm that knows how to produce a product variety, one unit of labor produces one unit of output, regardless of the location of production. Thus, each northern firm has a constant marginal cost of production equal to \( w_N \) and each foreign affiliate (as well as each southern firm) has a constant marginal cost of production equal to \( w_S \). Production only shifts from the North to the South over time if foreign affiliates have lower marginal costs of production than northern firms: \( w_N > w_S \). We solve the model for a steady-state equilibrium where this inequality holds, that is, northern workers earn a higher wage than southern workers.

A northern firm earns the flow of global profits \( \pi_{Nt} = (p_N - w_N)(x_{Nt}L_{Nt} + x_{Nt}^*L_{S,t}) \) where \( p_N \) is the price charged, \( x_{Nt} = p_N^{-\sigma}c_N/P_t^{1-\sigma} \) is the quantity demanded by the typical northern consumer, and \( x_{Nt}^* = p_N^{-\sigma}c_S/P_t^{1-\sigma} \) is the quantity demanded by the typical southern consumer.

It is straightforward to verify that the profit-maximizing price is \( p_N = \frac{w_N}{\alpha} \), that is, each northern firm charges the standard monopoly markup of price over marginal cost. Taking this into account, we can write northern profits more simply as

\[
\pi_{Nt} = \frac{w_N\bar{x}_{Nt}L_t}{\sigma - 1}
\]

where

\[
\bar{x}_{Nt} = \frac{p_N^{-\sigma}\bar{c}}{P_t^{1-\sigma}}
\]

is the average quantity demanded of northern varieties by world consumers and \( \bar{c} \equiv (c_NL_{Nt} + c_SL_{S,t})/L_t \) is the average consumer expenditure in the world.

Similarly, a foreign affiliate earns the flow of global profits \( \pi_{Ft} = (p_F - w_S)(x_{Ft}^*L_{Nt} + x_{Ft}L_{S,t}) \) where \( p_F \) is the price charged, \( x_{Ft}^* = p_F^{-\sigma}c_N/P_t^{1-\sigma} \) is the quantity demanded by the typical northern consumer, and \( x_{Ft} = p_F^{-\sigma}c_S/P_t^{1-\sigma} \) is the quantity demanded by the typical southern consumer. It is straightforward to verify that the profit-maximizing price of each foreign affiliate is \( p_F = \frac{w_S}{\alpha} \). Taking this into account, we can write foreign affiliate profits more simply as

\[
\pi_{Ft} = \frac{w_S\bar{x}_{Ft}L_t}{\sigma - 1}
\]
where

\[
\bar{x}_{Ft} = \frac{p_{Ft}^{-\sigma} \bar{c}}{P_{t}^{1-\sigma}}
\]  

(8)

is the average quantity demanded of foreign affiliate varieties by world consumers.

After a variety that a foreign affiliate produces is imitated by southern firms, price competition guarantees that the market price falls to marginal cost. Thus the price charged for each variety produced by southern firms is \( p_S = w_S \) and southern firms earn zero economic profits. For each southern variety, the quantity demanded by the typical northern consumer is \( x^*_{St} = p_S^{-\sigma} c_N / P_{t}^{1-\sigma} \), the quantity demanded by the typical southern consumer is \( x_{St} = p_S^{-\sigma} c_N / P_{t}^{1-\sigma} \), and the average quantity demanded by world consumers is

\[
\bar{x}_{St} = \frac{p_S^{-\sigma} \bar{c}}{P_{t}^{1-\sigma}}.
\]  

(9)

The above analysis implies that \( p_N > p_F > p_S \), that is, as a product shifts from being produced by a northern firm to its foreign affiliate and then to a southern firm, the equilibrium price of the product declines. This price pattern is consistent with Vernon’s (1966) description of the product life cycle, in which multinational firms play a central role.

### 3.4 Innovation, FDI and Imitation

There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology. To innovate and develop a new product variety, a representative northern firm \( i \) must devote \( a_N / K_t^\theta \) units of labor to innovative R&D, where \( a_N \) is an innovative R&D productivity parameter, \( K_t \) is the disembodied stock of knowledge at time \( t \) and \( \theta \) is an intertemporal knowledge spillover parameter. The disembodied stock of knowledge grows over time and is available to all firms in the world economy. We assume that it is proportional to the total number of varieties that have been developed in the past and choose units so that \( K_t = n_t \). The intertemporal knowledge spillover parameter \( \theta \) can be positive or negative. For \( \theta > 0 \), R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knowledge increases. In contrast, innovating becomes more difficult as time passes when \( \theta < 0 \). Grossman and Helpman (1991) assume that intertemporal knowledge spillovers are quite strong and set \( \theta = 1 \). We will instead follow Jones (1995) by assuming that intertemporal knowledge spillovers are weaker and satisfy \( \theta < 1 \). This assumption is the key to ruling out strong scale effects.
Given the innovative R&D technology, the flow of new products developed by northern firm $i$ is

$$
\dot{n}_{it} = \frac{l_{Rit}}{a_N/n_t^\theta} = \frac{n_t^\theta l_{Rit}}{a_N}
$$

where $\dot{n}_{it}$ is the time derivative of $n_{it}$ and $l_{Rit}$ is the labor used for innovative R&D by firm $i$ ("R" for R&D). Summing over individual northern firms, the aggregate flow of new products developed in the North is

$$
\dot{n}_t = \frac{n_t^\theta L_{Rt}}{a_N}
$$

(10)

where $L_{Rt} = \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities.

Similarly, to learn how to produce a northern variety in the South, the foreign affiliate of a northern firm must devote $a_F/n_t^\theta$ units of labor to adaptive R&D, where $a_F$ is an adaptive R&D productivity parameter.\(^{10}\) We interpret $a_F$ as a parameter measuring the friendliness of southern FDI-related policies. A recipient economy where “FDI-friendly” policies have been adopted would then correspond to a relatively low value of $a_F$, so that successful adaptation requires relatively few resources. Taking into account that adaptation is followed by imitation, the number of varieties that have been successfully adapted for southern production by firm $i$ increases over time according to

$$
\dot{n}_{Fit} + \dot{n}_{St} = \frac{l_{Fit}}{a_F/n_t^\theta} = \frac{n_t^\theta l_{Fit}}{a_F}
$$

where $\dot{n}_{Fit} + \dot{n}_{St}$ is the time derivative of the number of varieties that firm $i$ is responsible for moving to the South and $l_{Fit}$ is the labor used for adaptive R&D by firm $i$ ("F" for FDI). Summing over individual foreign affiliates, the aggregate flow of varieties to the South is given by

$$
\dot{n}_{Ft} + \dot{n}_{St} = \frac{n_t^\theta L_{Ft}}{a_F}
$$

(11)

where $L_{Ft} = \sum_i l_{Fit}$ is the total amount of southern labor employed in adaptive R&D activities.\(^{11}\)

Multinational firms that move their production to the South expose themselves to a positive rate of imitation by southern firms. We let $\iota \equiv \dot{n}_{St}/n_{Ft}$ denote this imitation rate. As in Helpman (1993) and Lai (1998), we assume that the imitation rate is exogenously given and model stronger

\(^{10}\)In the Appendix, we explore how the model's properties change when adaptive R&D also uses northern labor.

\(^{11}\)To see that the term $\dot{n}_{St}$ should be included in equation (11), it is helpful to think about a bathtub with an open drain that is being filled with water from a faucet. The flow of water coming out of the faucet into the bathtub equals the rate of change in the volume of water in the bathtub plus the flow of water going down the open drain. Likewise, the flow number of varieties that firms transfer to the South through FDI ($\dot{n}_{Fit} + \dot{n}_{St}$) equals the rate of change in the number of varieties produced by foreign affiliates ($\dot{n}_{Fit}$) plus the flow number of foreign affiliate varieties that are imitated by southern firms ($\dot{n}_{St}$).
IPR protection in the South as a reduction in the parameter \( \iota \).\(^{12}\)

### 3.5 R&D Incentives

Let \( v_{Nt} \) denote the expected discounted profits associated with innovating in the North at time \( t \). From (10), the R&D labor used to develop one new variety is \( a_N/n_t^\theta \) and the cost of developing this variety is \( w_Na_N/n_t^\theta \). Since there is free entry into innovative R&D activities in the North, the cost of innovating must be exactly balanced by the benefit of innovating in equilibrium:

\[
v_{Nt} = \frac{w_N a_N}{n_t^\theta}.
\]  

(12)

Let \( v_{Ft} \) denote the expected discounted profits that a foreign affiliate earns from producing a variety in the South at time \( t \). From (11), the R&D labor that a foreign affiliate uses to transfer one variety to the South is \( a_F/n_t^\theta \) and the cost of this transfer is \( w_S a_F/n_t^\theta \). The benefit of the transfer is not the expected discounted profits that a firm could earn from moving its production to the South \( v_{Ft} \) but the gain in expected discounted profits \( v_{Ft} - v_{Nt} \), since the firm is already earning profits from producing in the North. Since the cost of technology transfer must be exactly balanced by the benefit in steady-state equilibrium, we obtain

\[
v_{Ft} - v_{Nt} = \frac{w_S a_F}{n_t^\theta}.
\]  

(13)

When technology transfer occurs, each foreign affiliate pays its parent firm the royalty payment \( v_{Nt} \) for the use of its technology in the South, since the adaptive R&D accounts for the increment in the firm’s value \( v_{Ft} - v_{Nt} \) which is less than the foreign affiliate’s market value \( v_{Ft} \).

We assume that there is a stock market that channels consumer savings to firms that engage in R&D in each region and helps households to diversify the risk of holding stocks issued by these firms. Since there is no aggregate risk in each region, it is possible for households to earn a safe return by holding the market portfolio in each region. Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate \( \rho \).

\(^{12}\)It does not follow from equation (11) that \( \dot{n}_{St} \) increases when firms devote more resources to adaptive R&D. Using the definition of \( \iota \), (11) can be rewritten as \( \dot{n}_{Ft} + \iota n_{Ft} = n_t^\theta L_{Ft}/a_F \). At time \( t \), \( n_{Ft} \) and \( n_t \) are state variables that cannot be influenced by current decision-making and \( \iota \) is a given parameter value, so an increase in \( L_{Ft} \) causes \( \dot{n}_{Ft} \) to increase but leads to no change in \( \dot{n}_{St} = \iota n_{Ft} \).
For a northern firm \( i \), the relevant no-arbitrage condition is

\[
(\pi_{Nt} - w_{SlFt})dt + \dot{v}_{Nt}dt + (\dot{n}_{Fit} + \dot{n}_{Sit})dt(v_{Ft} - v_{Nt}) = \rho v_{Nt}dt. 
\]

The northern firm earns the profit flow \( \pi_{Nt} \) during the time interval \( dt \) but also incurs the adaptive R&D expenditure flow \( w_{SlFt} \) during this time interval. In addition, the firm experiences the gradual capital gain \( \dot{v}_{Nt}dt \) during the time interval \( dt \) and its market value jumps up by \( v_{Ft} - v_{Nt} \) for each product that it succeeds in moving to the South. The firm succeeds in moving \( (\dot{n}_{Fit} + \dot{n}_{Sit})dt \) products to the South during the time interval \( dt \). To rule out arbitrage opportunities for investors, the rate of return for the northern firm must be the same as the return on an equal sized investment in a riskless bond \( \rho v_{Nt}dt \). Now (13) implies that \( (\dot{n}_{Fit} + \dot{n}_{Sit})(v_{Ft} - v_{Nt}) = w_{SlFt} \) and (12) implies that the gradual capital gain satisfies \( \dot{v}_{Nt}/v_{Nt} = -\theta g \), where \( g \equiv \dot{n}_t/n_t \) is the steady-state growth rate of varieties. Thus, the no-arbitrage condition simplifies to \( \pi_{Nt}/v_{Nt} - g = \rho \) or \( v_{Nt} = \pi_{Nt}/(\rho + \theta g) \).

Combining this equation with (12), the northern no-arbitrage condition can be written as

\[
\frac{\pi_{Nt}}{\rho + \theta g} = \frac{w_{NaN}}{n_t^\rho}. 
\]  

(14)

In equation (14), the left-hand-side is the expected discounted profits from innovating and the right-hand-side is the cost of innovating. The northern firm’s expected discounted profits or market value is equal to its current profit flow \( \pi_{Nt} \) appropriately discounted by the market interest rate \( \rho \) and the capital loss term \( \theta g \) (capital gain if \( \theta \) is negative).

For a foreign affiliate, the relevant no-arbitrage condition is

\[
\pi_{Ft}dt + \dot{v}_{Ft}dt - (\eta dt)v_{Ft} = \rho v_{Ft}dt. 
\]

The foreign affiliate earns the profit flow \( \pi_{Ft} \) and experiences the gradual capital gain \( \dot{v}_{Ft}dt \) during the time interval \( dt \). However, it is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if imitated, which occurs with probability \( \eta dt \). Equations (12) and (13) imply that the gradual capital gain satisfies \( \dot{v}_{Ft}/v_{Ft} = -\theta g \). Thus, the no-arbitrage condition simplifies to \( \pi_{Ft}/v_{Ft} - \theta g - \eta = \rho \) or \( v_{Ft} = \pi_{Ft}/(\rho + \theta g + \eta) \). Using (12) and (13), the foreign affiliate no-arbitrage condition can be written as

\[
\frac{\pi_{Ft}}{\rho + \theta g + \eta} - \frac{w_{NaF}}{n_t^\rho} = \frac{w_{SaF}}{n_t^\rho}. 
\]  

(15)
where the left-hand-side is the increase in expected discounted profits from moving production to the South and the right-hand-side is the adaptive R&D cost. The expected discounted profits or market value of the foreign affiliate is equal to its current profit flow $\pi_{Ft}$ appropriately discounted by the market interest rate $\rho$, the capital loss term $\theta g$ and the imitation rate $\iota$.

### 3.6 Labor Markets

Labor markets are perfectly competitive and wages adjust instantaneously to equate labor demand and labor supply. In the North, labor is employed in production or in innovative R&D. Each innovation requires $a_N/n_t^\theta$ units of labor, so the total employment of labor in innovative R&D is $a_N\hat{n}_t/n_t^\theta$. Northern firms use $\bar{x}_NtL_t$ units of labor for each variety produced and there are $n_{Nt}$ varieties produced by northern firms, so the total employment of labor in northern production is $\bar{x}_NtL_t n_{Nt}$. As $L_{Nt}$ denotes the supply of labor in the North, full employment requires that

$$L_{Nt} = \frac{a_N\hat{n}_t}{n_t^\theta} + X_{Nt} L_t$$

where $X_{Nt} \equiv \bar{x}_Ntn_{Nt}$ is the per capita world demand for all northern varieties.

In the South, labor is employed in adaptive R&D, in production by foreign affiliates and in production by southern firms. Each variety transferred to the South requires $a_F/n_t^\theta$ units of labor, so the total employment of labor in adaptive R&D is $a_F(\hat{n}_{St} + \hat{n}_{St})/n_t^\theta$. A foreign affiliate uses $\bar{x}_{Ft}L_t$ units of labor for each variety produced and there are $n_{Ft}$ varieties produced by foreign affiliates, so the total employment of labor in affiliate production is $\bar{x}_{Ft}L_t n_{Ft}$. Likewise, a southern firm uses $\bar{x}_{St}L_t$ units of labor for each variety produced and there are $n_{St}$ varieties produced by southern firms, so the total employment of labor in southern production is $\bar{x}_{St}L_t n_{St}$. As $L_{St}$ denotes the supply of labor in the South, full employment requires that

$$L_{St} = \frac{a_F(\hat{n}_{Ft} + \hat{n}_{St})}{n_t^\theta} + X_{Ft} L_t + X_{St} L_t$$

where $X_{Ft} \equiv \bar{x}_{Ft}n_{Ft}$ and $X_{St} \equiv \bar{x}_{St}n_{St}$ are the per capita world demands for all foreign affiliate and southern varieties, respectively.

This completes the description of the model.
4 Solving the model

In this section, we solve the model for a unique steady-state (or balanced growth) equilibrium where all endogenous variables grow over time at constant (not necessarily identical) rates. As we will show, solving the model for a steady-state equilibrium reduces to solving a simple system of two equations in two unknowns, where the two unknowns are the FDI rate \( \phi \) and the level of relative R&D difficulty \( \delta_N \). The FDI rate is defined by \( \phi \equiv (\dot{n}_{Ft} + \dot{n}_{St})/n_{Nt} \), the rate at which northern varieties shift to the South as a result of the adaptive R&D done by foreign affiliates.\(^{13}\) This is the same definition as in Lai (1998) and Branstetter and Saggi (2009).\(^{14}\) The concept of relative R&D difficulty will be defined shortly.

In any steady-state equilibrium, the share of labor employed in R&D activities must be constant over time (in both the North and the South). Given that the supply of labor in each of the two regions grows at the population growth rate \( g_L \), northern R&D employment \( L_{Rt} \) and foreign affiliate R&D employment \( L_{Ft} \) must grow at this rate as well. Since the steady-state growth rate of the number of varieties is \( g \equiv \dot{n}_t/n_t \), dividing both sides of (10) by \( n_t \) yields \( g = n_t^{\theta-1} L_{Rt}/a_N \). It follows that \( g \) can only be constant over time if

\[
g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1 - \theta}.
\]

Thus, the steady-state rate of innovation \( g \) is pinned down by parameter values and is proportional to the population growth rate \( g_L \). As in Jones (1995), the parameter restriction \( \theta < 1 \) is needed to guarantee that the steady-state rate of innovation is positive and finite (given that there is positive population growth).

Equation (18) has two important implications, given that the innovation rate \( g \) is proportional to the economic growth rate (as we will show later in the paper). First, equation (18) implies that public policy changes like stronger intellectual property rights protection (a decrease in \( \iota \)) or a shift

\(^{13}\)Substituting for \( \iota \equiv \dot{n}_{St}/n_{Pt} \) into the FDI rate definition, we obtain \( \phi = (\dot{n}_{Ft} + \dot{n}_{Pt})/n_{Nt} \). It does not follow from this equation that an increase in \( \iota \) causes an increase in \( \phi \) because \( \dot{n}_{Pt} \) can fall enough in response to an increase in \( \iota \) so that \( \phi \) decreases. Indeed, this is what we will show (Theorem 2 establishes that a permanent increase in \( \iota \) generates a permanent decrease in \( \phi \)).

\(^{14}\)If the FDI rate is defined differently as \( \phi \equiv \dot{n}_{Pt}/n_{Nt} \), then following the same procedure as we use to obtain equation (19), it is possible to derive that \( n_t/n_{Nt} = 1 + [\phi(\iota + g)/g^2] \) holds in steady-state equilibrium. Thus holding fixed the innovation rate \( g \equiv \dot{n}_t/n_t \) and the FDI rate \( \phi \), an increase in the imitation rate \( \iota \) decreases the proportion of products produced in the North \( n_{Nt}/n_t \). This property does not make sense. An increase in the imitation rate within the South should only affect the composition of products produced within the South (between foreign affiliates and southern firms) and should have no effect on the proportion of products produced in the North. We conclude that it is a conceptual mistake to define the FDI rate as \( \phi \equiv \dot{n}_{Pt}/n_{Nt} \).
to more FDI-friendly policies (a decrease in $a_F$) have no effect on the steady-state rate of innovation $g$ and hence the steady-state rate of economic growth. In this model, growth is “semi-endogenous”. We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well. This data leads us to be skeptical about models where public policy changes have large long-run growth effects.\textsuperscript{15} Second, equation (18) implies that the level of per capita income in the long run is an increasing function of the size of the economy (because positive population growth is associated with positive economic growth). Jones (2005) has a lengthy discussion of this “weak scale effect” property and cites Alcala and Ciccone (2004) as providing the best empirical support. Controlling for both trade and institutional quality, Alcala and Ciccone find that a 10 percent increase in the size of the workforce in the long run is associated with 2.5 percent higher GDP per worker.\textsuperscript{16}

Next we derive some steady-state equilibrium implications of the variety condition $n_{Nt} + n_{Ft} + n_{St} = n_t$. First, the number of varieties produced by each type of firm $n_{Nt}$, $n_{Ft}$, and $n_{St}$ must grow at the same rate $g$. Second, the variety shares $\gamma_N \equiv \frac{n_{Nt}}{n_t}$, $\gamma_F \equiv \frac{n_{Ft}}{n_t}$, and $\gamma_S \equiv \frac{n_{St}}{n_t}$ are necessarily constant over time and satisfy $\gamma_N + \gamma_F + \gamma_S = 1$. Third, the FDI rate $\phi$ is also necessarily constant over time in any steady-state equilibrium since $\phi = \frac{\dot{n}_{Ft}}{n_{Ft}} \frac{n_{Ft}}{n_{Nt}} + \frac{\dot{n}_{St}}{n_{St}} \frac{n_{St}}{n_{Nt}} = gc_1 + gc_2$ where $c_1$ and $c_2$ are both constant over time. Given these properties, we can solve for $\gamma_N$. By differentiating the variety condition, we obtain that $\frac{\dot{n}_t}{n_t} = \frac{\dot{n}_{Nt}}{n_{Nt}} \frac{n_{Nt}}{n_t} + \frac{\dot{n}_{Ft}}{n_{Ft}} \frac{n_{Ft}}{n_t} + \frac{\dot{n}_{St}}{n_{St}} \frac{n_{St}}{n_t}$ or $g = g\gamma_N + \phi\gamma_N$, and solving for $\gamma_N$ yields

$$\gamma_N = \frac{g}{g + \phi}. \quad (19)$$

\textsuperscript{15}In spite of the arguments in Jones (2005), the issue of whether economic growth is “semi-endogenous” (public policies have no long-run growth effects) or “fully-endogenous” (public policies have long-run growth effects) remains controversial. For example, Ha and Howitt (2007) present evidence that fully-endogenous growth models have better empirical support than semi-endogenous growth models. We think that their analysis has an important limitation. Ha and Howitt implicitly assume that convergence to steady-state is fast, so that with 50 years of data, they can just focus on the steady-state implications of growth models. This assumption is called into question in Steger (2003), who calibrates a semi-endogenous growth model using US data and finds that convergence to steady-state is slow: it takes almost 40 years to go half the distance to the steady-state. With such slow convergence, we think that future tests of semi-endogenous growth theory should take into account the transition path implications of the models.\textsuperscript{16}One piece of evidence that is often interpreted as going against the weak scale effects prediction is the negative coefficient on population growth in typical cross-country growth regressions, such as Mankiw, Romer and Weil (1992). Other things being equal, countries with higher population growth rates tend to have lower per capita incomes. This evidence is discussed at length in Jones (2005) and he concludes that it is not inconsistent with models in which weak scale effects play a role.
An increase in the FDI rate $\phi$ naturally results in a lower share of varieties produced in the North and a higher share of varieties produced in the South. We can also solve for $\gamma_F$ and $\gamma_S$. Since $t = \frac{n_S}{n_F} = \frac{n_S}{n_F} \frac{n_t}{n_F}$, we obtain $\gamma_F = \frac{\gamma_S}{t}$ and substituting into $\gamma_F + \gamma_S = \frac{\phi}{1+\sigma}$ yields

$$\gamma_F = \left(\frac{\phi}{g + \theta}\right) \left(\frac{g}{g + t}\right) \quad \text{and} \quad \gamma_S = \left(\frac{\phi}{g + \theta}\right) \left(\frac{t}{g + t}\right) \quad (20)$$

An increase in the imitation rate $\lambda$ naturally results in a lower share of varieties produced by foreign affiliates $\gamma_F$ and a higher share of varieties produced by southern firms $\gamma_S$.

Given the variety shares, we can solve for the profits earned by firms using (5), (6), (7), (8), and the definition of $P_t$. Straightforward calculations yield that $\pi_{Nt} = \left(\frac{wn_F}{\sigma-1}\right) \frac{L_{NL}}{n_t}$ and $\pi_{Ft} = \left(w_P/\phi\right) \frac{L_{NL}}{n_t}$, where $\Phi_N = \frac{\gamma_N(p_N)^{1-\sigma}\epsilon\log/L_{NL}}{\gamma_N(p_N)^{1-\sigma}\epsilon+p_F(p_F)^{1-\sigma}+\gamma_S(p_S)^{1-\sigma}}$ and $\Phi_F = \frac{(p_F)^{1-\sigma}\epsilon\log/L_{NL}}{\gamma_N(p_N)^{1-\sigma}\epsilon+p_F(p_F)^{1-\sigma}+\gamma_S(p_S)^{1-\sigma}}$ are constant in steady-state. Consequently, $\pi_{Nt}$ and $\pi_{Ft}$ are both proportional to $L_{NL}/n_t$ and only change over time based on how $L_{NL}/n_t$ changes over time. We interpret $L_{NL}/n_t$ as a measure of the size of the market that firms sell to. Population growth increases the size of the market for firms but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.

We are now in a position to define relative R&D difficulty. Let the term $n_t^{-\theta}$ in (12) and (13) be a measure of (absolute) R&D difficulty. It increases over time if $\theta < 0$ and decreases over time if $0 < \theta < 1$. By taking the ratio of R&D difficulty and the market size term $L_{NL}/n_t$, we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market):

$$\delta_N \equiv \frac{n_t^{-\theta}}{L_{NL}/n_t} = n_t^{-\theta} \frac{L_{NL}}{n_t}. \quad (21)$$

Log-differentiating (21) using (18), it follows immediately that $\delta_N$ is constant over time in any steady-state equilibrium. In any steady-state equilibrium with decreasing R&D difficulty ($0 < \theta < 1$), it follows from (21) that each firm faces a shrinking market size since population growth is exceeded by variety growth ($L_{NL}/n_t$ decreases over time). If R&D difficulty increases over time ($\theta < 0$), then each firm faces an expanding market size since variety growth is exceeded by population growth ($L_{NL}/n_t$ increases over time).17

---

17 We define relative R&D difficulty using the Northern labor force (instead of the Southern labor force or the world labor force) in order to facilitate the comparative steady-state analysis in the next section. There, we study the long-run effects of an initial increase in the size of the South and because this increase does not cause $\delta_N$ to jump, $\delta_N$ can play the role of a state variable in the analysis. To study the steady-state effects of an initial increase in the size of the North, it is convenient to instead measure relative R&D difficulty as $\delta_S \equiv n_t^{1-\theta}/L_{SL}$. 

---

18
From the definition of the price index, \( P_t^{1-\sigma} = [\gamma_N(p_N)^{1-\sigma} + \gamma_F(p_F)^{1-\sigma} + \gamma_S(p_S)^{1-\sigma}] n_t \) must grow over time at the rate \( g \) in any steady-state equilibrium. It then follows from (6), (8) and (9) that \( \bar{x}_{Nt}, \bar{x}_{Ft} \) and \( \bar{x}_{St} \) all grow at the rate \(-g\) over time. Consequently, \( X_N \equiv \bar{x}_{Nt} n_{Nt} \), \( X_F \equiv \bar{x}_{Ft} n_{Ft} \) and \( X_S \equiv \bar{x}_{St} n_{St} \) must all be constant over time.

We can now derive a steady-state northern innovation condition. Substituting into (14) for \( \pi_{Nt} \) using (5), dividing both sides by \( w_N \) and then by the market size term \( L_{Nt}/n_t \) yields the steady-state northern innovation condition

\[
\frac{X_N L_0}{(\sigma-1)\gamma_N L_{N0}} = a_N \delta_N. \tag{22}
\]

The left-hand-side of (22) is the market size-adjusted benefit from innovating and the right-hand-side is the market size-adjusted cost of innovating. In steady-state calculations, we need to adjust for market size because market size changes over time if \( \theta \neq 0 \). The market size-adjusted benefit from innovating is higher when the average world consumer buys more of each northern variety \((X_N/\gamma_N)↑\), there are more consumers in the world to sell to \((L_0)↑\), future profits are less heavily discounted \((\rho ↓)\) and northern firms experiences larger capital gains over time \((\theta g ↓)\). The market size-adjusted cost of innovating is higher when northern researchers are less productive \((a_N ↑)\) and innovating is relatively more difficult \((\delta_N ↓)\).

Similarly, we can derive a steady-state southern adaptation condition. Substituting into (15) for \( \pi_{Ft} \) using (7), dividing both sides by \( w_S \) and then by the market size term \( L_{Nt}/n_t \) yields the steady-state southern adaptation condition

\[
\frac{X_F L_0}{(\sigma-1)\gamma_F L_{N0}} - \omega a_N \delta_N = a_F \delta_N. \tag{23}
\]

where \( \omega \equiv w_N/w_S \) is the Northern relative wage or North-South wage ratio. The left-hand-side of (23) is the market size-adjusted benefit from southern adaptation and the right-hand-side is the market size-adjusted cost of southern adaptation. The market size-adjusted benefit is higher when the average world consumer buys more of each foreign affiliate variety \((X_F/\gamma_F)↑\), there are more consumers in the world to sell to \((L_0)↑\), future profits are less heavily discounted \((\rho ↓)\), foreign affiliates experiences larger capital gains over time \((\theta g ↓)\), and foreign affiliates are exposed to a lower imitation rate \((\iota ↓)\). The market size-adjusted cost is higher when foreign affiliate researchers are less productive \((a_F ↑)\) and adaptation is relatively more difficult \((\delta_N ↓)\).
The northern innovation condition (22) can be combined with the northern labor market condition (16) to yield a single steady-state condition describing the North. Using (21) and evaluating at time $t = 0$, the northern labor condition (16) can be rewritten as $L_{N0} = a_N\delta_N g L_{N0} + X_N L_0$. Substituting for $X_N L_0$ using (22), substituting for $\gamma_N$ using (19), and dividing both sides by $L_{N0}$ yields the *northern steady-state equation*

$$1 = a_N \delta_N g \left[ 1 + \left( \frac{\rho + \theta g}{g + \phi} \right) (\sigma - 1) \right]. \quad (24)$$

Equation (24) is a northern full employment condition that takes into account the implications of profit-maximizing R&D behavior by northern firms. The first term on the right-hand-side is the share of northern labor devoted to R&D while the second term represents the share of northern labor devoted to manufacturing production. Since the only two unknowns are $\delta_N$ and $\phi$, the right-hand-side is increasing in $\delta_N$ and the right-hand-side is decreasing in $\phi$, the northern steady-state equation is globally upward-sloping in $(\delta_N, \phi)$ space. It also has a strictly positive $\delta_N$ intercept.

The intuition behind the positive slope of the northern steady-state equation is as follows: An increase in relative R&D difficulty $\delta_N$ increases the demand for innovative R&D labor (more researchers are needed to maintain the steady-state innovation rate $g$) and the demand for northern labor employed in manufacturing production (stronger consumer demand for varieties is required to justify the greater R&D effort). In contrast, an increase in the FDI rate $\phi$ decreases northern manufacturing employment by shifting production to the South. Consequently, to satisfy both northern profit-maximization and full-employment conditions, any increase in relative R&D difficulty $\delta_N$ (which raises both northern R&D and production employment) must be matched by an increase in the FDI rate $\phi$ (which reduces northern production employment).

The southern adaptation condition (23) can be combined with the southern labor market condition (17) to yield a single steady-state equation describing the South. Using (21) and evaluating at time $t = 0$, the southern labor condition (17) can be written as $L_{S0} = L_{N0} a_F \delta_N \phi \gamma_N + X_F L_0 + X_S L_0$. It is easy to verify that $X_S/X_F = (p_F/p_S)^\sigma \gamma_S/\gamma_F = (1/\alpha)^{\sigma} \theta \rho / g$, so the southern labor condition can be rewritten as $L_{S0} = L_{N0} a_F \delta_N \phi \gamma_N + X_F L_0 [1 + (1/\alpha)^{\sigma} \theta \rho / g]$. Now substituting for $X_F L_0$ using (23) and substituting for $\gamma_N$ using (19) yields the *southern steady-state equation*

$$1 = \frac{L_{N0} \delta_N \phi}{L_{S0} (g + \phi)} \left[ a_F g + (a_F + \omega a_N) (\rho + \theta g + \phi) (\sigma - 1) \Phi(\iota) \right], \quad (25)$$

where $\Phi(\iota) \equiv \frac{\rho}{g + \phi} \left( \frac{1}{\alpha} \right)^{\sigma} \frac{\iota}{g + \phi}$. Equation (25) is a southern full employment condition that takes into
account the implications of profit-maximizing R&D behavior by foreign affiliates. The first term on the right-hand-side is the share of southern labor devoted to R&D while the second term represents the share of southern labor devoted to production. Since the right-hand-side is increasing in both \( \delta_N \) and \( \phi \), the southern steady-state equation is globally downward-sloping in \((\delta_N, \phi)\) space for any given value of \( \omega \). It has no intercepts since neither \( \delta_N \) nor \( \phi \) can equal zero.

The intuition behind the negative slope of the southern steady-state equation is as follows: An increase in relative R&D difficulty \( \delta_N \) increases the demand for adaptive R&D labor (more southern researchers are needed to maintain any given FDI rate \( \phi \)) and the demand for southern production labor (stronger consumer demand is needed to justify the greater R&D effort). In contrast, a decrease in the FDI rate \( \phi \) reduces the R&D employment of foreign affiliates in the South and also reduces the demand for southern production workers since less production shifts to the South. Consequently, to satisfy the southern full-employment condition, any increase in relative R&D difficulty \( \delta_N \) (which raises both southern production and R&D employment) must be matched by a decrease in the FDI rate \( \phi \) (which reduces both southern production and R&D employment).

We can solve for the steady-state value of the North-South wage ratio \( \omega = w_N/w_S \) using the two R&D conditions. It is straightforward to verify that \( X_N/X_F = (p_F/p_N)\gamma_N/\gamma_F = \omega^{-\sigma}(g+\iota)/\phi \). Substituting for \( X_N = X_F\omega^{-\sigma}(g+\iota)/\phi \) in (22), then solving for \( X_F \) and substituting into (23) yields the steady-state wage equation

\[
\omega^\sigma \left[ \frac{\rho + \theta g}{\rho + \theta g + \iota} \right] - \omega = \frac{a_F}{a_N} \tag{26}
\]

Since the left-hand-side of (26) is a continuous function of \( \omega \) only, takes on a negative value when \( \omega = 1 \), has a positive first derivative when \( \omega \) is sufficiently large, has a positive second derivative for all \( \omega > 1 \) and the right-hand-side is a positive constant, there must exist a value of \( \omega > 1 \) that satisfies (26) and this value is uniquely determined.

Given the value of \( \omega \) determined by (26), solving for a steady-state equilibrium reduces to solving a system of two equations in two unknowns: the northern and southern steady-state equations. These equations are illustrated in Figure 2 and are labeled “North” and “South,” respectively. Because the northern steady-state equation is globally upward-sloping with a strictly positive \( \delta_N \) intercept, the southern steady-state equation is globally downward-sloping with no intercepts, and the southern steady-state equation asymptotically approaches the \( \delta_N \) axis, these two curves must
have a unique intersection (as illustrated). Thus, the steady-state equilibrium values of $\delta_N$ and $\phi$ are uniquely determined.

![Figure 2: Existence of Steady-State Equilibrium.](image)

To verify that the intersection in Figure 2 really does represent a steady-state equilibrium, we need to solve for the remaining endogenous variables of the model. We proceed by solving for asset holdings, consumer expenditures, consumer utility and the economic growth rate.

Let $A_{Nt}$ and $A_{St}$ denote the aggregate value of northern and southern financial assets, respectively. Let $A_t = n_{Nt}v_{Nt} + n_{Ft}v_{Ft}$ denote the aggregate value of all financial assets. Since consumer savings within the South finance the R&D investments in the South, $A_{St} = n_{Ft}(v_{Ft} - v_{Nt}) = n_{Ft}w_SA_F/n_t^\theta = \gamma_F w_SA_F \delta_N L_{Nt}$ and it follows that $A_{Nt} = A_t - A_{St} = (n_{Nt} + n_{Ft})v_{Nt} = (n_{Nt} + n_{Ft})w_N a_N \delta_N L_{Nt}$.

Let $a_{it}$ denote the financial asset holdings of the typical consumer in region $i$ ($i = N, S$). The intertemporal budget constraint of a typical consumer in region $i$ is $\dot{a}_{it} = w_i + \rho a_{it} - c_i - g_L a_{it}$. In any steady-state equilibrium where the wage rates $w_i$ are constant over time, we must have that $\dot{a}_{it} = 0$ and it follows that $c_i = w_i + (\rho - g_L)a_{it}$. For the typical northern consumer, $a_{Nt} = A_{Nt}/L_{Nt}$ and for the typical southern consumer, $a_{St} = A_{St}/L_{St}$. It follows that typical northern and southern consumer expenditure levels are given by

$$c_N = w_N \left[1 + (\rho - g_L)(\gamma_N + \gamma_F)a_N \delta_N \right]$$

(27)
\[ c_S = w_S \left[ 1 + (\rho - g_L) \gamma_F a_F \delta_N L_{N0} / L_{S0} \right] \tag{28} \]

The first term on the right-hand-side of these equations is labor income and the second term represents the return to asset ownership.

Having pinned down individual consumer expenditure levels, we are in a position to solve for steady-state consumer utility levels. For the typical northern consumer, (2) implies that

\[ u_{Nt} = \left[ n_{Nt} (x_{Nt})^\alpha + n_{Ft} (x_{Ft})^\alpha + n_{St} (x_{St})^\alpha \right]^{1/\sigma}. \]

Substituting into this equation for \( x_{Nt} \), \( x_{Ft}^* \), and \( x_{St}^* \), we obtain a simple expression for northern consumer utility

\[ u_{Nt} = \frac{c_N}{P_t}. \tag{29} \]

Likewise, for the typical southern consumer, (2) implies that

\[ u_{St} = \left[ n_{Nt} (x_{Nt})^\alpha + n_{Ft} (x_{Ft})^\alpha + n_{St} (x_{St})^\alpha \right]^{1/\sigma}. \]

Substituting into this equation for \( x_{Nt}^* \), \( x_{Ft} \), and \( x_{St} \), we obtain a simple expression for southern consumer utility

\[ u_{St} = \frac{c_S}{P_t}. \tag{30} \]

In any steady-state equilibrium, individual consumer expenditure is constant over time but consumer utility nevertheless grows because the price index falls over time. Along the steady-state equilibrium path, the price index is given by

\[ P_t = \left[ \gamma_N (p_N)^{1-\sigma} + \gamma_F (p_F)^{1-\sigma} + \gamma_S (p_S)^{1-\sigma} \right]^{1/(1-\sigma)} n_t^{1/(1-\sigma)}. \tag{31} \]

The term in brackets is constant over time but the price index \( P_t \) falls because the number of varieties \( n_t \) increases and \( \sigma > 1 \).

Finally, we can solve for the economic growth rate in each region. Taking logs and differentiating (29) and (30) using (27), (28) and (31) yields

\[ g_u = \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = \frac{g}{\sigma - 1}. \tag{32} \]

Since consumer utility is proportional in consumer expenditure holding prices fixed, consumer
utility growth equals real wage growth and we use it as our measure of economic growth. Thus, the steady-state economic growth rate given by (32) is proportional to the innovation rate \( g \), as we earlier claimed.

It is time to summarize what we have accomplished. The steady-state market interest rate satisfies \( r_t = \rho \) and the steady-state innovation rate \( g \equiv \dot{n}_t/n_t \) is pinned down by (18). Given \( g \), the steady-state wage equation (26) pins down the North-South wage ratio \( \omega \) and \( w_N \) equals \( \omega \) because we have made the numeraire choice \( w_S = 1 \). Given \( g \) and \( \omega \), the northern steady-state equation (24) and the southern steady-state equation (25) together determine steady-state values of the FDI rate \( \phi \) and relative R&D difficulty \( \delta_N \). Given these values, (19) and (20) determine \( \gamma_N \), \( \gamma_F \) and \( \gamma_S \). Since \( n_t^{1-\theta} = \delta_N n_{Nt}, n_{Nt} = \gamma_N n_t, n_{Ft} = \gamma_F n_t \) and \( n_{St} = \gamma_S n_t \), the entire steady-state time paths of \( n_t, n_{Nt}, n_{Ft} \) and \( n_{St} \) are pinned down. All the remaining endogenous variables are then uniquely determined and we have established

**Theorem 1** For all parameter values, the model has a unique steady-state equilibrium where the northern wage rate \( w_N \) is higher than the southern wage rate \( w_{S} \).

We can now answer the question: Why do northern workers earn a higher wage rate than southern workers given that workers in both regions are equally productive (one unit of labor produces one unit of output)? The answer is that workers in both regions are not really equally productive since southern products sell for lower prices than northern products and this is reflected in lower wages for southern workers (\( p_N = w_N/\alpha > p_F = w_{S}/\alpha > p_S = w_S \)). Southern products sell for lower prices because the South has relatively few products to sell and increased production of a limited number of products drives down the prices of those products. And the South has relatively few products to sell because it is dependent on multinational firms transferring their production technologies.

### 5 Steady-State Properties of the Model

In this section, we study analytically the steady-state properties of the model. The main focus of this paper is on the steady-state equilibrium effects of stronger IPR protection in the South, captured by a permanent decrease in \( \iota \). The Trade-Related Intellectual Property Rights (TRIPs) agreement which was signed as part of the Uruguay round of multilateral trade negotiations in 1994 calls for the establishment of minimum standards of IPR protection by all World Trade Organization
(WTO) members by 2006. The burden of policy adjustment, however, has fallen on the shoulders of developing countries because developed countries already have higher levels of IPR protection (Maskus, 2000).

Since the effects are almost the same, we consider at the same time the steady-state equilibrium effects of the South adopting more FDI-friendly policies, captured by a permanent decrease in $a_F$. Hill (2005) reports that in the period 1991-2001 about 95 percent of the 1395 changes in FDI laws and regulations created a more favorable environment for multinational firms. In addition, many countries have encouraged more FDI by engaging in a number of bilateral investment treaties designed to protect and promote investment between countries. As of 2002, there were 2,099 such bilateral investment treaties in the world involving more than 160 countries.

![Figure 3: Comparative Steady-State Effects.](image)

A decrease in $t$ causes the left-hand-side of the steady-state wage equation (26) to shift up, while having no effect on the right-hand-side, so the steady-state value of $\omega$ unambiguously falls. The northern steady-state equation (24) is not affected by a decrease in $t$ but the southern steady-state equation (25) is affected. By definition, $\Phi$ is a weighted average of 1 and $(1/\alpha)^\sigma > 1$, so $\Phi$ is an increasing function of $t$. Thus when $t$ decreases and $\omega$ consequently decreases, the bracketed term in (25) falls, and for each value of $\phi$, $\delta_N$ must increase. As is illustrated in Figure 3, the entire southern steady-state equation shifts to the right in $(\delta_N, \phi)$ space, resulting in higher steady-state values of both $\delta_N$ and $\phi$. The measure of relative R&D difficulty $\delta_N \equiv n_t^{1-\theta}/L_{Nt}$ can only permanently
increase if the number of products $n_t$ temporarily grows at a faster than usual rate. This means that a permanent increase in $\delta_N$ is associated with a temporary increase in the northern innovation rate $\dot{n}_N/n_t$ above its steady-state value given by (18).

A decrease in $a_F$ also causes the steady-state value of $\omega$ to unambiguously fall, this time because the right-hand-side of the steady-state wage equation (26) shifts down, while there is no effect on the left-hand-side. The northern steady-state equation (24) is not affected by a decrease in $a_F$ but the southern steady-state equation (25) is affected. When $a_F$ decreases and consequently $\omega$ decreases, the bracketed term in (25) falls, and for each value of $\phi$, $\delta_N$ must increase. As is illustrated in Figure 3, the entire southern steady-state equation shifts to the right in $(\delta_N, \phi)$ space, resulting in higher steady-state values of both $\delta_N$ and $\phi$.

Although both policy changes result in a permanent increase in $\phi$, the question remains, what happens to R&D employment by foreign affiliates in the South? From (11), $L_{Ft} = a_F(\dot{n}_{Ft} + \dot{n}_{St})/n_t^\theta$, which can be written more usefully as $L_{Ft} = a_F \phi \delta_N \frac{\phi}{g+\phi} L_N t$. Thus a decrease in $\iota$ permanently increases R&D employment by foreign affiliates $L_{Ft}$ because $\phi$ and $\delta_N$ increase and there is no permanent change in $g$. When $a_F$ decreases, the effect on $L_{Ft}$ is ambiguous because $a_F$ also appears in the $L_{Ft}$ equation. We have established

**Theorem 2** The adoption of stronger IPR protection in the South $(\iota \downarrow)$ generates a permanent increase in the rate of technology transfer to the South within multinational firms $(\phi \uparrow)$ and a permanent increase in R&D employment by foreign affiliates $(L_{Ft} \uparrow)$, a permanent decrease in the North-South wage ratio $(\omega \equiv w_N/w_S \downarrow)$ and a temporary increase in the northern innovation rate $(\delta_N \uparrow)$. The adoption of more FDI-friendly policies by the South $(a_F \downarrow)$ generates almost the same steady-state equilibrium effects, the only difference being that the effect on R&D employment by foreign affiliates $L_{Ft}$ is ambiguous.

When faced with stronger IPR protection in the South, multinational firms find it profitable to increase the adaptive R&D spending of their foreign affiliates and transfer their manufacturing production to the low-wage South more quickly $(\phi \uparrow)$. The more rapid technology transfer from

---

18Solving the model in Lai (1998) reduces to solving a system of 3 steady-state equations in 3 unknowns, where one of the equations is (26) and one of the unknowns is the innovation rate $g$. When technology transfer is costless ($a_F = 0$), Lai can use (26) to obtain a closed-form solution for $\omega$ as a function of $g$ and $\iota$, which he can then substitute into the other 2 equations and analytically solve the model for the effects of decreasing $\iota$. But when technology transfer is costly ($a_F > 0$), no closed form solution for $\omega$ can be obtained and Lai’s model becomes analytically intractable. What helps us to solve our model is that there are no scale effects, so $g$ is pinned down by parameter values. With $g$ known, (26) implies that a decrease in $\iota$ unambiguously lowers $\omega$. We do not need a closed form solution for $\omega$ to determine the equilibrium effects of stronger IPR protection.
North to South in turn increases the demand for production labor in the South and decreases the
demand for production labor in the North. These two effects cause a permanent decline in the
Northern relative wage \( w_N/w_S \) and make it more attractive for firms to engage in innovative
R&D in the North. Firms respond by innovating more frequently; relative R&D difficulty increases
and this increase causes the innovation rate to slow down. The permanent increase in relative
R&D difficulty \( \delta_N \uparrow \) is associated with a temporary increase in the innovation rate \( g \) above its
steady-state value \( g = g_L/(1 - \theta) \).

The effects of increased IPR protection summarized in Theorem 2 contrast with results derived
in the earlier literature. In particular, Glass and Saggi (2002), Sener (2006) and Glass and Wu
(2007) study the same issue but find that stronger IPR protection reduces the rates of innovation
and technology transfer. This paper presents a model that is consistent with the empirical evidence
in Branstetter et al. (2006) that multinational firms increase their R&D spending in developing
countries that offer stronger IPR protection and increase their technology transfer to these reforming
countries.

In the model of North-South trade with increasing variety by Grossman and Helpman (1991),
the central steady-state property is that an increase in the size of either region (the North or the
South) increases the economic growth rate in both regions. This property does not hold in our
model where both regions are increasing in size over time and the common economic growth rate
is constant over time. However, our model does have a related property. For the third and final
comparative steady-state exercise, we consider what happens when there is a one-time jump up in
the size of the South (e.g., countries like China joining the world trading system). In the model,
this is captured by an increase in \( L_{S0} \).

An increase in \( L_{S0} \) has no effect on the steady-state wage equation (26) so \( \omega \) is unchanged
and has no effect on the northern steady-state equation (24). However, the southern steady-state
equation (25) shifts to the right in \( (\delta_N, \phi) \) space since \( \delta_N \) must increase for any given \( \phi \). Thus,
as illustrated in Figure 3, we obtain higher steady-state values of both \( \delta_N \) and \( \phi \). The permanent
increase in \( \delta_N \) is associated with a temporary increase in the innovation rate above its steady-
state value given by (18). Also, the increase in \( L_{S0} \) is associated with a permanent increase in
\( L_{F1} = a_F\phi\delta_N\frac{g}{g+\phi}L_{N1} \) since both \( \delta_N \) and \( \phi \) increase. We have established

\textbf{Theorem 3} An increase in the initial size of the South \( (L_{S0} \uparrow) \) generates a temporary increase in
the northern innovation rate \( (\delta_N \uparrow) \), as well as a permanent increase in the rate of technology trans-
er to the South within multinational firms \( (\phi \uparrow) \) and a permanent increase in R&D employment
by foreign affiliates \((L_{Ft}\uparrow)\). There is no change in the steady-state wage ratio \(\omega \equiv w_N/w_S\).

An increase in the initial size of the South does increase the rate of technological change affecting both regions \((n_t/n_0)\), but because this effect is temporary in nature, it does not lead to a steady-state increase in the economic growth rate \(g_n\) as in Grossman and Helpman (1991).

The absence of any steady-state effect of increasing \(L_{S0}\) on the North-South wage ratio is surprising. This result follows directly from the wage equation (26) but to understand it intuitively, it is helpful to refer back to the no-arbitrage conditions (14) and (15). It follows from these conditions that an increase in \(L_{S0}\) has no long-run effect on the R&D difficulty-adjusted profits of firms \((\pi_{Nt}/n_t^{-\theta} \text{ and } \pi_{Ft}/n_t^{-\theta})\), the expected duration of these profits or the R&D difficulty-adjusted costs of obtaining these profits. Adjusting for R&D difficulty, the R&D incentives of firms are not affected by an increase in \(L_{S0}\) and it is these R&D incentives that determine the long-run wage ratio \(\omega\).\(^{19}\)

We turn now to studying the welfare effects of policy changes. Does stronger IPR protection in developing countries make people better off? Or does patent reform in developing countries mainly benefit the owners of multinational firms in the North? It is beyond the scope of this paper to assess whether or not discounted consumer utility increases at the time of policy change. To do so, we would need to take into account how consumer utility evolves along the entire transition path leading to a new steady-state equilibrium. Instead, we pursue the more modest objective of trying to determine the long-run welfare effects of policy changes. We ask the question: do these changes make consumers better off in the long run? To answer this question, it suffices to compare steady-state utility paths before and after a policy change. Furthermore, since policy changes do not affect the steady-state rate of utility growth and different steady-state utility paths share a common utility growth rate, it suffices to compare steady-state consumer utility levels at time \(t = 0\) to determine whether consumers benefit in the long run from a policy change. For example, to determine the long-run welfare effects of stronger IPR protection in the South, we solve for how the steady-state equilibrium values of \(u_{N0}\) and \(u_{S0}\) change when \(\iota\) decreases.

From (29) and (30), \(u_{N0} = c_N/P_0\) and \(u_{S0} = c_S/P_0\). The steady-state consumer expenditure levels \(c_N\) and \(c_S\) are given by (27) and (28), respectively. To determine the initial value of the price index, we first evaluate (21) at \(t = 0\) to obtain \(n_0 = (\delta_N L_{N0})^{1/(1-\theta)}\) and then substitute this

\(^{19}\)One question for future research is the following: under what alternative assumptions does the equilibrium wage ratio \(w_N/w_S\) become responsive to changes in relative labor supply \(L_{N0}/L_{S0}\)?
expression into (31) to obtain

\[ P_0 = \left[ \gamma_N(p_N)^{1-\sigma} + \gamma_F(p_F)^{1-\sigma} + \gamma_S(p_S)^{1-\sigma} \right]^{1/(1-\sigma)} (\delta_N L_N 0)^{1/(1-\theta)(1-\sigma)}. \] (33)

Consider now the effect of a decrease in \( \iota \) on \( u_{S0} = c_S/P_0 \). From Theorem 2, a decrease in \( \iota \) increases both \( \phi \) and \( \delta_N \). It then follows from (20) that a decrease in \( \iota \) increases \( \gamma_F \) and taking into account that \( w_S = 1 \), (20) implies that southern consumer expenditure \( c_S \) increases. However, the effect of a decrease in \( \iota \) on the price index \( P_0 \) is less clear. Because a decrease in \( \iota \) leads to an increase in \( \phi \) by Theorem 2, the combined effect on \( \gamma_S = \frac{\phi}{g+\phi g+\iota} \) can go either way and consequently the price index \( P_0 \) can either increase or decrease. When \( \iota \) decreases, southern consumers benefit from the increase in their asset-generated income (\( c_S \uparrow \)), from the temporary increase in the rate of technological change (\( \delta_N \uparrow \)), from the lower prices of imported varieties (\( w_N \downarrow \)) and from the shift in production from higher-priced northern firms to lower-priced foreign affiliates (\( \gamma_N \downarrow \)). However, southern consumers can be hurt if there is a shift in production from lower-priced southern firms to higher-priced foreign affiliates (\( \gamma_S \downarrow \)) and this possibility cannot be ruled out. We conclude that the overall effect on long-run southern consumer welfare of stronger southern IPR protection is theoretically ambiguous.

6 Numerical Results

To learn more about the long-run welfare implications of the model, we turn to computer simulations. In this section, we report results obtained from solving the model numerically.

In our computer simulations, we used the following benchmark parameter values: \( \rho = 0.07 \), \( \alpha = 0.714 \), \( g_L = 0.014 \), \( L_{N0} = 1 \), \( L_{S0} = 2 \), \( \theta = 0.72 \), \( \iota = 0.05 \), \( a_N = 1 \) and \( a_F = 4.21 \). The subjective discount rate \( \rho \) was set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985). The preference parameter \( \alpha = 0.714 \) determines the markup of price over marginal cost \( 1/\alpha \) and was chosen to generate a northern markup of 40 percent, which is within the range of estimates reported in Basu (1996) and Norrbin (1993). The average southern markup is lower because imitation results in competitive pricing. The population growth rate \( g_L = 0.014 \) equals the annual rate of world population growth between 1991 and 2000 according to the World Development Indicators (World Bank, 2003). Since only the ratio \( L_{S0}/L_{N0} \) plays any role in determining the model’s steady-state equilibrium outcome (other than the initial level of development \( n_0 \)), we set

29
$L_{N0} = 1$ and then chose $L_{S0} = 2$ so that $L_{S0}/L_{N0}$ equals the ratio of the working age population in middle income countries to that in high income countries—as defined by the World Bank (2003). The R&D difficulty parameter $\theta = 0.72$ was chosen to generate a steady-state economic growth rate of 2 percent using $g_u = \frac{q_u}{(\sigma-1)(1-\theta)}$, which is consistent with the average US GDP per capita growth rate from 1950 to 1994 reported in Jones (2005). The imitation rate $\iota = 0.05$ represents an intermediate rate of copying in the South: one out of twenty varieties produced by foreign affiliates are copied each year. Since only the ratio $a_F/a_N$ plays any role in determining the model’s steady-state equilibrium outcome (other than the initial level of development $n_0$ and $\delta_N$), we set $a_N = 1$. Finally, $a_F = 4.21$ was chosen to guarantee that the North-South income ratio $c_N/c_S$ is 2.17, which is the same as the US-Mexico GDP per worker ratio according to Jones (2002).

The results from the computer simulations are reported in Tables 1, 2 and 3. In each table, the first column shows different endogenous variables that we solved for and the middle column of numbers shows the results for the benchmark parameter values. Table 1 shows how the values of different endogenous variables change when $\iota$ decreases from 0.2 to 0 (the South adopts stronger IPR protection). Table 2 shows how the values of endogenous variables change when $a_F$ decreases from 16 to 1 (the South adopts more FDI-friendly policies) and Table 3 show how the values of endogenous variables change when $L_{S0}$ increases from 1 to 3 (the South increases in size). There are several conclusions that we draw from studying these tables.

<table>
<thead>
<tr>
<th>$\iota$</th>
<th>0.2</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_N/w_S$</td>
<td>2.31</td>
<td>2.04</td>
<td>1.87</td>
<td>1.70</td>
<td>1.65</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>3.29</td>
<td>3.38</td>
<td>3.50</td>
<td>3.83</td>
<td>4.13</td>
</tr>
<tr>
<td>$\phi$</td>
<td>.002</td>
<td>.004</td>
<td>.006</td>
<td>.013</td>
<td>.019</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>0.95</td>
<td>0.93</td>
<td>0.89</td>
<td>0.80</td>
<td>0.72</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0.17</td>
<td>0.27</td>
</tr>
<tr>
<td>$c_N/c_S$</td>
<td>2.72</td>
<td>2.39</td>
<td>2.17</td>
<td>1.91</td>
<td>1.80</td>
</tr>
<tr>
<td>$u_{N0}$</td>
<td>5.69</td>
<td>5.99</td>
<td>6.35</td>
<td>7.34</td>
<td>8.25</td>
</tr>
<tr>
<td>$u_{S0}$</td>
<td>2.09</td>
<td>2.51</td>
<td>2.93</td>
<td>3.84</td>
<td>4.58</td>
</tr>
</tbody>
</table>

First, for plausible parameter values, the model can account for large wage differences between the North and the South. When Sener (2006, Table 2b) solved his model numerically for plausible parameter values, he found that the northern wage rate was only 7 percent higher than the southern wage rate. In contrast, we find that the northern wage rate is 87 percent higher than the southern wage rate ($w_N/w_S = 1.87$) in the benchmark parameter case (column 4 in each table). Furthermore,
Table 2: More FDI-Friendly Policies ($a_F \downarrow$)

<table>
<thead>
<tr>
<th>$a_F$</th>
<th>16</th>
<th>8</th>
<th>4.21</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_N/w_S$</td>
<td>2.58</td>
<td>2.16</td>
<td>1.87</td>
<td>1.61</td>
<td>1.44</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>3.28</td>
<td>3.37</td>
<td>3.50</td>
<td>3.72</td>
<td>3.99</td>
</tr>
<tr>
<td>$\phi$</td>
<td>.002</td>
<td>.004</td>
<td>.006</td>
<td>.010</td>
<td>.016</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>0.96</td>
<td>0.93</td>
<td>0.89</td>
<td>0.83</td>
<td>0.76</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>$c_N/c_S$</td>
<td>2.96</td>
<td>2.50</td>
<td>2.17</td>
<td>1.89</td>
<td>1.70</td>
</tr>
<tr>
<td>$u_{N0}$</td>
<td>5.60</td>
<td>5.93</td>
<td>6.35</td>
<td>7.00</td>
<td>7.74</td>
</tr>
<tr>
<td>$u_{S0}$</td>
<td>1.89</td>
<td>2.37</td>
<td>2.93</td>
<td>3.72</td>
<td>4.55</td>
</tr>
</tbody>
</table>

the model can account for even higher North-South wage ratios if we assume weaker IPR protection (when $\iota$ increases to 0.2, $w_N/w_S$ increases to 2.31) or less FDI-friendly policies (when $a_F$ increases to 16, $w_N/w_S$ increases to 2.58).

Second, we find that increasing IPR protection in the South has surprisingly large and positive long-run welfare effects. When IPR protection goes from very weak (one out of five varieties are copied each year) to very strong (varieties are never copied), real consumption per capita in the South increases by 119 percent (when $\iota$ decreases from 0.2 to 0, $u_{S0}$ increases from 2.09 to 4.58). Northern consumers also benefit from stronger IPR protection, with a corresponding percentage increase in real consumption per capita of 45 percent ($u_{N0}$ rises from 5.69 to 8.25).

Third, we find that the adoption of more FDI-friendly policies in the South also has large and positive long-run welfare effects. When the South goes from very FDI-unfriendly ($a_F = 16$) to very FDI-friendly ($a_F = 1$), real consumption per capita in the South increases by 140 percent ($u_{S0}$ rises from 1.89 to 4.55) and real consumption per capita in the North increases by 38 percent ($u_{N0}$ rises from 5.60 to 7.74). Northern consumers do not benefit as much as southern consumers because the adoption of more FDI-friendly policies leads to a significant shift in production from the North to the South ($\gamma_N$ falls from 0.96 to 0.76) and the increased demand for southern production workers leads to a significant drop in the Northern relative wage ($w_N/w_S$ falls from 2.58 to 1.44).

Finally, we find that increases in the initial size of the South generate more balanced welfare benefits. When the initial size of the South $L_{S0}$ is increased from 1.0 (the same size as the North) to 3.0 (three times the size of the North), real consumption per capita in the South increases by 34 percent ($u_{S0}$ rises from 2.50 to 3.34) and real consumption per capita in the North increases by 35 percent ($u_{N0}$ rises from 5.40 to 7.28). The reason for the more balanced welfare benefits is that there is no change in the Northern relative wage ($w_N/w_S$ remains unchanged at 1.87). Having
more southern consumers/workers increases the demand for the products of northern innovators ($\delta_N$ rises from 3.34 to 3.66) but also results in increased transfer of production within multinational firms ($\phi$ rises from .003 to .009).

### 7 Related Literature

In this section, we discuss why earlier papers obtain different results about the effects of stronger IPR protection.

Compared with this paper, Glass and Wu (2007) find opposite effects of stronger IPR protection on innovation and FDI. Their model is similar to ours in many respects but they find that lowering the exogenous imitation rate causes a decline in the rates of FDI and innovation. The key difference is that while we assume that there is free entry into innovative R&D races and all northern firms have access to the same R&D technology, Glass and Wu assume that industry leaders (the firms producing the highest quality products in each industry) are sufficiently more productive at innovating so that all innovative R&D is done by industry leaders. Glass and Wu’s assumption has strong implications because industry leaders only engage in innovative R&D when they have the most to gain by innovating, namely, after their products have been imitated and they are not earning economic profits. In their model, no innovative R&D takes place in industries where northern firms or multinational firms produce: innovative R&D is targeted at industries where products are produced by southern firms under perfect competition.\footnote{Our discussion of Glass and Wu (2007) focuses on their main model presented in sections 2 and 3. Glass and Wu also present a model in section 5 where innovative R&D takes place in all industries and find that stronger IPR protection can increase FDI if the imitation rate is sufficiently high.}

In the steady-state equilibrium that Glass and Wu (2007) solve for, imitation must occur before further innovation, FDI must occur before imitation, and innovation must occur before FDI. Be-
cause the expected inflow of product lines into each of these three states must be balanced by the

corresponding outflow, the model has the implication that the aggregate rates of innovation, FDI

and imitation must be identical, so any policy that reduces the rate of imitation (such as stronger

IPR protection) must also reduce the aggregate rates of innovation and FDI. This implication is not

supported by the evidence reported in Branstetter et al. (2006). By contrast, in our model, because

firms are always doing innovative R&D, the aggregate rates of innovation, FDI, and imitation are

not identical and are allowed to move in different directions in response to policy changes.


and costly FDI, but they make the same simplifying assumptions about innovative R&D. As they

state on page 392, “For simplicity, we do not allow northern innovation to target other northern

firms by making the necessary assumptions for such innovation to fail to earn the market rate of

return.” When Glass and Saggi solve their model, they find that there are two types of equilibrium

outcomes with FDI. In the case where there is imitation of multinationals by southern firms but

not imitation of northern firms, they obtain the same results as in Glass and Wu (2007) and for the

same reasons. Since the steady-state rates of imitation, FDI, and innovation must be identical, and

stronger IPR protection must lower the rate of imitation, it reduces the aggregate rates of innovation

and FDI. In the case where there is imitation of both multinationals and northern firms, Glass and

Saggi’s model is more complicated but it continues to be true that stronger IPR protection leads

to lower rates of imitation, innovation and FDI due to the R&D targeting property.

Sener (2006) presents a more general model where there is costly innovation, costly FDI, costly

imitation of products produced by northern firms, and costly imitation of products produced by

foreign affiliates in the South. Unlike the strong R&D targeting assumption of Glass and Saggi

(2002) and Glass and Wu (2007), Sener allows firms engaging in innovative R&D to target all

northern industries. Nevertheless, he finds that stronger IPR protection leads to less FDI, the

exact opposite of what we find in this paper.

To better understand what drives the different result in Sener (2006), we extend our model

to allow for costly imitation in Section 8. This extended model is closely comparable to Sener’s

model and yields imitation rates that are endogenously determined based on profit-maximization

considerations. For a wide range of parameter values, we find that stronger IPR protection leads

to more FDI in our endogenous imitation model. The only exception occurs when the degree of

decreasing returns to R&D activities (measured by parameter $\beta$) is very low. For values of $\beta$

that are considerably below the lowest point estimate from the empirical literature on patents and R&D,
we find that stronger IPR protection leads to less FDI. Sener (2006) assumes that $\beta = 0$ throughout his analysis and for that low $\beta$ value, we also obtain that stronger IPR protection leads to less FDI. Thus, Sener’s result appears to be driven by the commonly made but unrealistic assumption that R&D activities are not subject to decreasing returns ($\beta = 0$).

In our exogenous imitation model, FDI is the only mode of international technology transfer. In the Section 8 model, we also allow for imitation of northern products as a second mode of international technology transfer. When the R&D costs of imitating northern products (measured by parameter $d$) are sufficiently low relative to the costs of transferring technology through FDI, the endogenous imitation model has fundamentally different properties. A marginal increase in IPR protection then leads to less innovation and a higher North-South wage gap. These results are exactly what we establish analytically in the companion paper Gustafsson and Segerstrom (2010), where we study a North-South trade model with costly innovation, costly imitation but no FDI. When there is little FDI and most technology transfer occurs through imitation of northern products, then stronger IPR protection slows the rate of technology transfer because it leads to a lower imitation rate of northern products. This means that more production remains in the North and the “excess” production increases the demand for northern labor and the North-South wage ratio. Northern firms respond to the higher costs of R&D by decreasing their R&D investment, and this leads to a decline in the rate of innovation. We conclude that stronger IPR protection has very different marginal effects when southern firms can imitate northern products and the costs of doing so are sufficiently low.

8 Endogenous Imitation

In this section, we explore how the model’s properties change when imitation is costly and profit-maximization considerations determine how quickly products are imitated. We allow southern firms to copy both products produced in the South and products produced in the North.

8.1 New Assumptions

Suppose now that due to their lack of familiarity with the southern economic environment, foreign affiliates have higher production costs than southern firms. Whereas one unit of labor produces one unit of output for a foreign affiliate operating in the South, $\zeta \in (0, 1)$ units of labor produce one unit of output for a southern firm. Then each northern firm has marginal cost $w_N$, each foreign
affiliate has marginal cost $w_S$, each southern firm has marginal cost $\zeta w_S$, and these marginal cost levels satisfy $w_N > w_S > \zeta w_S$.

After a variety that a foreign affiliate produces is imitated by a southern firm, the southern firm earns the flow of global profits $\pi_{It} = (p_I - \zeta w_S)(x_{It}^* N_t + x_{It} L_{St})$ by charging the limit price $p_I = w_S$, where $x_{It}^* = p_I^{-\sigma} c_N / P_t^{1-\sigma}$ is the quantity demanded by the typical northern consumer, and $x_{It} = p_I^{-\sigma} c_S / P_t^{1-\sigma}$ is the quantity demanded by the typical southern consumer ("I" for imitation). We assume that $\zeta > \alpha$, which guarantees that the limit price $w_S$ is less than the monopoly price $\zeta w_S / \alpha$ and it is profit-maximizing for the southern firm to practice limit-pricing. Then

$$\pi_{It} = w_S (1 - \zeta) \bar{x}_{It} L_t$$

where $\bar{x}_{It} = p_I^{-\sigma} \bar{c} / P_t^{1-\sigma}$.

After a variety that a northern firm produces is imitated by a southern firm, the southern firm earns the flow of global profits $\pi_{Ct} = (p_C - \zeta w_S)(x_{Ct}^* N_t + x_{Ct} L_{St})$ by charging the monopoly price $p_C = \zeta w_S / \alpha$, where $x_{Ct}^* = p_C^{-\sigma} c_N / P_t^{1-\sigma}$ is the quantity demanded by the typical northern consumer, and $x_{Ct} = p_C^{-\sigma} c_S / P_t^{1-\sigma}$ is the quantity demanded by the typical southern consumer ("C" for copying). We restrict attention to steady-state equilibria where $w_N / w_S > \zeta / \alpha$, to guarantee that the limit price $w_N$ is greater than the monopoly price $\zeta w_S / \alpha$ and it is profit-maximizing for the southern firm to practice monopoly-pricing. Then

$$\pi_{Ct} = \frac{w_S \zeta \bar{x}_{Ct} L_t}{\sigma - 1}$$

where $\bar{x}_{Ct} = p_C^{-\sigma} \bar{c} / P_t^{1-\sigma}$.

In what follows, we let $n_{Nt}$ denote the number of varieties produced by northern firms, $n_{Ft}$ denote the number of varieties produced by foreign affiliates, $n_{It}$ denote the number of varieties produced by southern firms that have imitated foreign affiliates, and $n_{Ct}$ denote the number of varieties produced by southern firms that have imitated northern firms. Then the total number of varieties $n_t$ satisfies $n_t = n_{Nt} + n_{Ft} + n_{It} + n_{Ct}$, the innovation rate is $g \equiv \dot{n}_t / n_t$, the FDI rate is $\phi \equiv (\dot{n}_{Ft} + \dot{n}_{It}) / n_{Nt}$, the imitation rate of southern-produced varieties is $\iota_S \equiv \dot{n}_{It} / n_{Ft}$ and the imitation rate of northern-produced varieties is $\iota_N \equiv \dot{n}_{Ct} / n_{Nt}$.

To innovate and develop a new product variety, a northern firm must devote $a_N g^\beta / n_t^\theta$ units of labor to innovative R&D, where $\beta > 0$ is a new externality parameter that captures the duplicative nature of innovative R&D. When all firms do more innovative R&D ($g \equiv \dot{n}_t / n_t$ is higher), $\beta > 0$
means that the individual firm must do more innovative R&D in order to develop a new product variety. Given this technology, the flow of new products developed by northern firm \( i \) is

\[
\dot{n}_{it} = \frac{l_{Rit}}{[a_Ng^\beta/n_t^\theta]} = n_t^\theta l_{Rit}/[a_Ng^\beta]
\]

where \( l_{Rit} \) is the labor used for innovative R&D by firm \( i \). Summing over individual northern firms, the total flow of new products developed in the North is

\[
\dot{n}_t = \frac{n_t^\theta L_{Rt}}{a_Ng^\beta}
\]

where \( L_{Rt} = \sum_i l_{Rit} \) is the total amount of northern labor employed in innovative activities. Taking into account the definition of \( g \), this expression can be rewritten as

\[
\dot{n}_t = (n_t^\theta + 1) L_{Rt}/a_N \right)^{1/(1+\beta)}
\]

Thus the parameter \( \beta \) determines the degree of decreasing returns to innovative R&D at the industry level. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the industry level (point estimates of \( 1/(1 + \beta) \) lie between 0.1 and 0.6 according to Kortum (1993), which corresponds to \( \beta \) values between .66 and 9).

To learn how to produce a northern variety in the South, the foreign affiliate of a northern firm must devote \( a_F\phi^\beta/n_t^\theta \) units of labor to adaptive R&D, where the externality parameter \( \beta > 0 \) now captures the duplicative nature of adaptive R&D. When all firms do more adaptive R&D (\( \phi \) is higher), \( \beta > 0 \) means that the individual firm must do more adaptive R&D in order to produce a northern variety in the South. Given this technology, the number of varieties that have been successfully adapted for southern production by firm \( i \) increases over time according to

\[
\dot{n}_{Fii} + \dot{n}_{Iit} = l_{Fii}/[a_F\phi^\beta/n_t^\theta] = n_t^\theta l_{Fii}/[a_F\phi^\beta]
\]

where \( \dot{n}_{Fii} + \dot{n}_{Iit} \) is the time derivative of the number of varieties that firm \( i \) is responsible for moving to the South and \( l_{Fii} \) is the labor used for adaptive R&D by firm \( i \). Summing over individual foreign affiliates, the aggregate flow of varieties to the South is given by

\[
\dot{n}_{Ft} + \dot{n}_{It} = \frac{n_t^\theta L_{Ft}}{a_F\phi^\beta}
\]

where \( L_{Ft} = \sum_i l_{Fii} \) is the total amount of southern labor employed in adaptive R&D activities.

To learn how to produce a foreign affiliate variety, a southern firm must devote \( a_I\iota_S^\beta/n_t^\theta \) units of labor to imitative R&D, where \( a_I > 0 \) is an imitative R&D productivity parameter and the externality parameter \( \beta > 0 \) captures the duplicative nature of imitative R&D. We interpret \( a_I \) as measuring the strength of IPR protection in the South and study what happens when \( a_I \) increases. When all firms do more imitative R&D (\( \iota_S \) is higher), \( \beta > 0 \) means that the individual southern firm must do more imitative R&D to learn how to produce a foreign affiliate variety. Given this technology, the flow of foreign affiliate varieties that southern firm \( i \) imitates is

\[
\dot{n}_{Iit} = l_{Iit}/[a_I\iota_S^\beta/n_t^\theta] = n_t^\theta l_{Iit}/[a_I\iota_S^\beta]
\]
where $l_{It}$ is the labor used for imitative R&D by firm $i$. Summing over individual southern firms, the aggregate flow of foreign affiliate varieties that southern firms imitate is

$$\dot{n}_{It} = \frac{n_t^\vartheta L_{It}}{a_{It}^\vartheta S},$$

where $L_{It} = \sum_i l_{It}$ is the total amount of southern labor employed in imitating foreign affiliate varieties.

Similarly, to learn how to produce a northern variety in the South, a southern firm must devote $da_{It}^\vartheta N/n_t^\vartheta$ units of labor to imitative R&D, where $d > 1$ is a “distance” parameter that captures the extra cost of imitating northern-produced products. Given this technology, the flow of northern varieties that southern firm $i$ imitates is

$$\dot{n}_{Cit} = \dot{l}_{Cit} / [da_{It}^\vartheta N/n_t^\vartheta] = n_t^\vartheta \dot{l}_{Cit} / [da_{It}^\vartheta N]$$

where $l_{Cit}$ is the labor used for imitative R&D by firm $i$. Summing over individual southern firms, the aggregate flow of northern varieties that southern firms imitate is

$$\dot{n}_{Ct} = \frac{n_t^\vartheta L_{Ct}}{da_{It}^\vartheta N}$$

where $L_{Ct} = \sum_i l_{Ct}$ is the total amount of southern labor employed in imitating northern varieties.

### 8.2 Solving the Model with Endogenous Imitation

Let $v_{It}$ denote the expected discounted profits associated with imitating a foreign affiliate variety and let $v_{Ct}$ denote the expected discounted profits associated with imitating a northern variety. Following the same procedure as in section 3, we obtain four no-arbitrage conditions:

$$v_{Nt} = \frac{\pi_{Nt}}{\rho + \theta g + \iota_N} = \frac{w_N a_N g^\beta}{n_t^\vartheta}$$

$$v_{Ft} - v_{Nt} = \frac{\pi_{Ft}}{\rho + \theta g + \iota_S} - \frac{w_N a_N g^\beta}{n_t^\vartheta} = \frac{w_S a_F \varphi^\beta}{n_t^\vartheta}$$

$$v_{It} = \frac{\pi_{It}}{\rho + \theta g} = \frac{w_S a_{It}^\vartheta S}{n_t^\vartheta}$$

$$v_{Ct} = \frac{\pi_{Ct}}{\rho + \theta g} = \frac{w_S d a_{It}^\vartheta N}{n_t^\vartheta}.$$
For the North, we need to derive a new full employment of labor condition since innovative R&D is now subject to decreasing returns. Equation (16) becomes

\[ L_{Nt} = \frac{a_N g^\beta \dot{n}_t}{n_t^\rho} + X_{Nt} L_t. \]

Evaluating this equation at time \( t = 0 \) using (21), we obtain the steady-state full employment of labor condition for the North:

\[ L_{N0} = a_N \delta_N g^{1+\beta} L_{N0} + X_N L_0 \tag{34} \]

For the South, we need to derive a new full employment of labor condition since labor is now employed in imitative R&D activities. Equation (17) becomes

\[ L_{St} = \frac{a_F (\dot{n}_{Ft} + \dot{n}_{It}) \phi^\beta + a_I \dot{n}_{It} \phi^\beta + d a_I \dot{n}_{Cl} \phi^\beta}{n_t^\rho} + L_t (X_{Ft} + \zeta X_{It} + \zeta X_{Ct}) \]

where \( X_{It} \equiv \bar{x}_{It} n_{It} \) and \( X_{Ct} \equiv \bar{x}_{Ct} n_{Ct} \). Evaluating this equation at time \( t = 0 \) using (21) and the FDI rate \( \phi \equiv (\dot{n}_{Ft} + \dot{n}_{It})/n_{Nt} \), we obtain the steady-state full employment of labor condition for the South

\[ L_{S0} = \delta_N L_{N0} [a_F \gamma_N \phi^{1+\beta} + a_I \gamma_F \phi^{1+\beta} + d a_I \gamma_N \phi^{1+\beta}] + L_0 (X_F + \zeta X_I + \zeta X_C). \tag{35} \]

Following the same procedure as in section 4, we obtain the steady-state rate of innovation

\[ g \equiv \frac{\dot{n}_t}{n_t} = \frac{gL}{1 - \theta}, \]

steady-state versions of the four no-arbitrage conditions

\[ \frac{X_{Nt} L_0}{(\sigma - 1) \gamma_N L_{N0}} = a_N \delta_N g^\beta \tag{36} \]

\[ \frac{X_{Ft} L_0}{(\sigma - 1) \gamma_F L_{N0}} = \omega a_N \delta_N g^\beta = a_F \delta_N \phi^\beta \tag{37} \]

\[ \frac{(1 - \zeta) X_{It} L_0}{\gamma I L_{N0}} = a_I \delta_N \phi^I_S \tag{38} \]
\[ \frac{\zeta X_{C,L_0}}{(\sigma - 1)\gamma C L_{N_0}} = d a_N\delta N \frac{\beta }{\rho + \theta g}, \quad (39) \]

and steady-state variety shares

\[ \gamma_N \equiv \frac{n_{Nt}}{n_t} = \frac{\gamma N}{g + \phi + \iota_N} \quad \gamma_C \equiv \frac{n_{Ct}}{n_t} = \frac{\gamma N}{g + \phi + \iota_N} \quad \gamma_I \equiv \frac{n_{It}}{n_t} = \left( \frac{\gamma N}{g + \phi + \iota_N} \right) \left( \frac{\iota_S}{g + \iota_S} \right). \]

Furthermore, since \( X_I/X_F = \tilde{x}_I n_{It}/(\tilde{x}_F n_{Ft}) = (p_I/p_F)^{-\sigma}(\gamma_I/\gamma_F) = (1/\alpha)^{\sigma}I_S/g \), \( X_C/X_F = \tilde{x}_C n_{Ct}/(\tilde{x}_F n_{Ft}) = (p_C/p_F)^{-\sigma}(\gamma_C/\gamma_F) = (1/\zeta)^{\sigma}I_N(g + \iota_S)/(\phi g) \) and \( X_N/X_F = \tilde{x}_N n_{Nt}/(\tilde{x}_F n_{Ft}) = (p_N/p_F)^{-\sigma}(\gamma_N/\gamma_F) = \omega^{-\sigma}(g + \iota_S)/\phi \), the steady-state values of \( X_I, X_C \) and \( X_N \) are given by

\[ X_I = X_F \frac{\alpha^{-\sigma}I_S}{g} \quad X_C = X_F \frac{\zeta^{-\sigma}I_N(g + \iota_S)}{\phi g} \quad X_N = X_F \frac{\omega^{-\sigma}(g + \iota_S)}{\phi}. \]

Thus, solving for a steady-state equilibrium reduces to solving a system of 6 equations \([(34), (35), (36), (37), (38), (39)]\) in 6 unknowns \( [\omega, \delta N, \phi, \iota_S, \iota_N, X_F] \).

Following the same procedure as in sections 4 and 5, we obtain consumer expenditure levels \( c_N = \omega [1 + (\rho - g_L) (\gamma_N + \gamma_F) a_N \delta N g^2 \] \) and \( c_S = 1 + (\rho - g_L) \delta N \gamma_F a_F \phi^2 + \gamma a_I \phi^2 + \gamma a_l \delta N] L_{N0} / L_{S0}, \) the price index \( P_I = [\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_C p_C^{1-\sigma} + \gamma I p_I^{1-\sigma}]^{1/(1-\sigma)} n_t^{1/(1-\sigma)}, \) consumer utility levels \( u_{Nt} = c_N / P_I \) and \( u_{St} = c_S / P_I, \) the initial number of varieties \( n_0 = (\delta N L_{N0})^{1/(1-\sigma)}, \) and the initial R&D employment by foreign affiliates \( L_{F0} = \delta N L_{N0} a_F \gamma_N \delta N g^{1+\beta}. \)

### 8.3 Numerical Results

It is straightforward to solve the system of 6 equations in 6 unknowns numerically. In our computer simulations, we use the following benchmark parameter values \( \rho = 0.07, \alpha = 0.714, g_L = 0.014, L_{N0} = 1, L_{S0} = 2, \theta = 0.72, \zeta = 0.9, \beta = 1, d = 10,000, a_N = 20, a_F = 925, \) and \( a_I = 167. \) The first 6 parameter choices are the same as in section 6. The cost parameter \( \zeta = 0.9 \) was chosen so southern firms have 10 percent lower production costs than foreign affiliates. The parameter \( \beta \) determines the degree of decreasing returns due to duplicative R&D. Given that point estimates of \( 1/(1 + \beta) \) lie between 0.1 and 0.6 in the empirical literature on patents and R&D, we use the intermediate value \( 1/(1 + \beta) = 0.5, \) which corresponds to \( \beta = 1. \) To facilitate comparison with the exogenous imitation model where \( \iota_N = 0, \) we chose the high value \( d = 10,000 \) to guarantee that \( \iota_N \) is very close to zero. Given the new innovative R&D technology, we use the same type
of normalization as before by setting $a_N g^2 = 1$. Since $g = g_L/(1 - \theta) = .05$, this equation yields $a_N = 20$. Finally, we chose $a_F = 925$, and $a_I = 167$ to guarantee that $c_N/c_S = 2.17$ (the North-South income ratio is the same as the US-Mexico GDP per worker ratio) and $\nu_S = 0.05$ (an intermediate rate of copying in the South).

Table 4: Stronger IPR protection with endogenous imitation

<table>
<thead>
<tr>
<th>$a_I$</th>
<th>$d = 10,000$</th>
<th>$d = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_N/w_S$</td>
<td>2.22 1.95 1.98</td>
<td>1.99 1.92 1.98</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>3.33 3.47 3.68</td>
<td>3.39 3.53 3.68</td>
</tr>
<tr>
<td>$\phi$</td>
<td>.003 .005 .009</td>
<td>.002 .006 .009</td>
</tr>
<tr>
<td>$\nu_S$</td>
<td>.263 .050 .001</td>
<td>.185 .026 .001</td>
</tr>
<tr>
<td>$\nu_N$</td>
<td>.000 .000 .000</td>
<td>.003 .000 .000</td>
</tr>
<tr>
<td>$L_{F0}$</td>
<td>.020 .087 .257</td>
<td>.016 .118 .254</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>.95 .90 .84</td>
<td>.90 .88 .84</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>.01 .05 .16</td>
<td>.01 .07 .16</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>.04 .05 .00</td>
<td>.03 .04 .00</td>
</tr>
<tr>
<td>$\gamma_C$</td>
<td>.00 .00 .00</td>
<td>.05 .01 .00</td>
</tr>
<tr>
<td>$c_N/c_S$</td>
<td>2.48 2.17 2.08</td>
<td>2.11 2.10 2.08</td>
</tr>
<tr>
<td>$u_{N0}$</td>
<td>5.83 6.30 6.90</td>
<td>6.02 6.46 6.89</td>
</tr>
<tr>
<td>$u_{S0}$</td>
<td>2.35 2.90 3.31</td>
<td>2.85 3.07 3.31</td>
</tr>
</tbody>
</table>

Results from the computer simulations are reported in Table 4. Column 3 shows the results for the benchmark parameter values and columns 2 to 4 show how the values of different endogenous variables change when $a_I$ is increased (the South adopts stronger IPR protection). What is striking is that the model with endogenous imitation has almost the same qualitative properties as the model with exogenous imitation. When the South adopts stronger IPR protection ($a_I$ increases from 50 to 10,000), the rate of technology transfer to the South within multinational firms goes up ($\phi$ increases from .003 to .009), R&D employment by foreign affiliates rises ($L_{F0}$ increases from .020 to .257), and northern innovation is stimulated ($\delta_N$ increases from 3.33 to 3.68). Furthermore, there are large positive long-run welfare effects of stronger IPR protection: real consumption per capita rises by 41 percent in the South ($u_{S0}$ increases from 2.35 to 3.31) and by 18 percent in the North ($u_{N0}$ increases from 5.83 to 6.90). The only change is that the North-South wage ratio $w_N/w_S$ does not monotonically fall when $a_I$ is increased, as in the model with exogenous imitation. Instead we find a non-monotonic relationship ($w_N/w_S$ falls from 2.22 to 1.95 but then rises to 1.98).

In the benchmark parameter case (column 3), FDI is a much more important mode of interna-

---

21In our computer simulation results, the conditions $\zeta > \alpha$ and $w_N/w_S > \zeta/\alpha$ are easily satisfied.
tional technology transfer than imitation ($\phi = .005$ is much higher than $\iota_N = .000$). In column 5, we make different parameter choices ($d = 100$ and $a_I = 50$) and obtain that imitation is the more important mode of international technology transfer ($\iota_N = .003$ is higher than $\phi = .002$). Surprisingly, even starting from this case, we obtain the same qualitative effects from strengthening IPR protection. As shown in columns 5 to 7, when $a_I$ increases from 50 to 8000, the rate of technology transfer to the South within multinational firms goes up ($\phi$ increases from .002 to .009), R&D employment by foreign affiliates rises ($L_{F0}$ increases from .016 to .254), and northern innovation is stimulated ($\delta_N$ increases from 3.39 to 3.68).

We have run many computer simulations with different parameter choices and find that the qualitative results reported in Table 4 hold for a wide range of parameter values. We have only found two exceptions.

First, we obtain different effects of stronger IPR protection when $d$ is sufficiently low, that is, when imitation is a sufficiently important mode of international technology transfer. For example, when $d = 10$ and $a_I \leq 140$ (all other parameters taking on the benchmark values), we find that stronger IPR protection leads to less innovation, less imitation and a higher northern relative wage ($a_I \uparrow \Rightarrow \iota_N \downarrow, \delta_N \downarrow, \omega \uparrow$). These are the same marginal effects as derived in Theorem 3 of Gustafsson and Segerstrom (2010), where imitation is the only mode of international technology transfer. Even when $d$ is low, we find that stronger IPR protection leads to more FDI ($a_I \uparrow \Rightarrow \phi \downarrow$).

Second, we obtain different effects of stronger IPR protection when $\beta$ is very low, that is, the degree of decreasing returns to R&D activities is very low. Table 5 shows what can happen in this case (for the results reported in Table 5, $d = 2$, $\beta$ and $a_I$ take on specified values, and the other parameters take on the benchmark values). As shown in columns 2 to 4, we find that stronger IPR protection leads to less FDI and more southern imitation when $\beta = 0$ ($a_I \uparrow \Rightarrow \phi \downarrow, \iota_S \downarrow$). As shown in columns 5 to 7, stronger IPR protection has the same effects when $\beta = .05$. However, as shown in columns 8 to 10, stronger IPR protection leads to more FDI and less southern imitation when $\beta = .1$ ($a_I \uparrow \Rightarrow \phi \uparrow, \iota_S \downarrow$). Given that point estimates of $\beta$ lie between .66 and 9 in the empirical literature on patents and R&D, the parameter values $\beta = 0$, $\beta = .05$ and $\beta = .1$ are all well outside of this range on the low side.

We conclude that the main property of the model with exogenous imitation (stronger IPR protection leads to more FDI) holds for a wide range of parameter values in the model with endogenous imitation and only fails to hold when the degree of decreasing returns to R&D activities is unrealistically low.
9 Concluding Comments

This paper presents a model of North-South trade with multinational firms and increasing product variety. Firms engage in innovative R&D to develop new product varieties in the North and foreign affiliates of these northern multinationals engage in adaptive R&D to learn how to produce these product varieties in the South. The model generates one-way product cycles and large North-South wage differences for plausible parameter values.

The main focus of the paper is on studying the steady-state equilibrium effects of stronger IPR protection in the South (e.g., patent reform in developing countries). We find that stronger IPR protection in the South induces foreign affiliates of multinational firms to increase their R&D expenditures and results in a faster rate of technology transfer with these multinational firms, consistent with the empirical evidence in Branstetter, et al. (2006). Stronger IPR protection in the South also stimulates innovative R&D spending by northern firms and decreases the relative wage of northern workers, additional channels through which southern consumers benefit. When we solve the model numerically for plausible parameter values, we find that stronger IPR protection in the South significantly increases real consumption per capita in both regions, with the biggest percentage gains going to southern consumers.

We also study the effects of more FDI-friendly policies that reduce the costs of multinational firms transferring their production and the effects of increasing the initial size of the South (e.g., developing countries like China joining the world trading system). The adoption of more FDI-friendly

<table>
<thead>
<tr>
<th>$a_I$</th>
<th>$\beta = 0$</th>
<th>$\beta = .05$</th>
<th>$\beta = .1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_C/w_S$</td>
<td>1.90 2.02 2.81</td>
<td>1.90 2.04 2.39</td>
<td>1.87 1.85 1.84</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>.188 .174 .162</td>
<td>.212 .199 .191</td>
<td>.243 .253 .259</td>
</tr>
<tr>
<td>$\phi$</td>
<td>.011 .006 .001</td>
<td>.009 .004 .001</td>
<td>.007 .011 .013</td>
</tr>
<tr>
<td>$\iota_S$</td>
<td>.000 .024 .274</td>
<td>.008 .043 .161</td>
<td>.013 .004 .000</td>
</tr>
<tr>
<td>$\iota_N$</td>
<td>.000 .000 .000</td>
<td>.000 .001 .002</td>
<td>.001 .000 .000</td>
</tr>
<tr>
<td>$L_{F0}$</td>
<td>.257 .134 .027</td>
<td>.188 .091 .019</td>
<td>.141 .210 .254</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>0.82 0.90 0.98</td>
<td>0.85 0.91 0.94</td>
<td>0.85 0.82 0.80</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>0.18 0.07 0.00</td>
<td>0.13 0.04 0.00</td>
<td>0.10 0.16 0.20</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>0.00 0.03 0.02</td>
<td>0.02 0.04 0.01</td>
<td>0.03 0.01 0.00</td>
</tr>
<tr>
<td>$\gamma_C$</td>
<td>0.00 0.00 0.00</td>
<td>0.00 0.01 0.04</td>
<td>0.02 0.01 0.00</td>
</tr>
<tr>
<td>$c_N/c_S$</td>
<td>2.01 2.28 3.25</td>
<td>2.06 2.24 2.51</td>
<td>2.01 1.98 1.95</td>
</tr>
<tr>
<td>$u_{N0}$</td>
<td>.099 .087 .076</td>
<td>.117 .105 .097</td>
<td>.141 .151 .157</td>
</tr>
<tr>
<td>$u_{S0}$</td>
<td>.049 .038 .023</td>
<td>.057 .047 .039</td>
<td>.070 .076 .080</td>
</tr>
</tbody>
</table>
policies has similar steady-state effects to stronger IPR protection. There are some significant differences, however, when it comes to the effects of increasing the initial size of the South. We find that increasing the size of the South leads to a faster rate of technology transfer within multinational firms and stimulates innovative R&D spending by northern firms but has no long-run effect on the relative wage of northern workers and leads to much more balanced increases in real consumption per capita in the two regions.

References


Appendix: Adaptive R&D Uses Northern Labor

In this appendix, we explore how the model’s properties change when adaptive R&D uses both northern and southern labor. In the main text, we assumed that adaptive R&D uses only southern labor. But it is easy to envision that multinational firms use both northern and southern labor when they explore technology transfer to the South.

New Assumptions

Suppose now that in order to learn how to produce a northern variety in the South, the northern firm must devote $a_F/n_F^\theta$ units of labor to adaptive R&D in the South and $a_{FN}/n_{FN}^\theta$ units of labor to adaptive R&D in the North, where $a_F > 0$ and $a_{FN} > 0$ are adaptive R&D productivity parameters. In the main text, we implicitly assumed that $a_{FN} = 0$. 


[34] World Bank (2003), *World Development Indicators*, Washington, D.C.
Solving The Model When Adaptive R&D Uses Northern Labor

Following the same procedure as in section 3, we obtain two equilibrium conditions that are the same as before:

\[ v_{Nt} = \frac{\pi_{Nt}}{\rho + \theta g} = \frac{w_{NaN}}{n_t^\theta} \]

\[ L_{St} = \frac{a_F (\dot{n}_{Fl} + \dot{n}_{Sl})}{n_t^\theta} + X_{Fl}L_t + X_{Sl}L_t \]

and two equilibrium conditions that involve new terms:

\[ v_{Lt} - v_{Nt} = \frac{\pi_{Lt}}{\rho + \theta g + \iota} - \frac{w_{NaN}}{n_t^\theta} = \frac{w_{Saf}}{n_t^\theta} + \frac{w_{NaFN}}{n_t^\theta} \]

\[ L_{Lt} = \frac{a_N \dot{n}_{lt}}{n_t^\theta} + \frac{a_{FN} (\dot{n}_{Fl} + \dot{n}_{Sl})}{n_t^\theta} + X_{Nl}L_t. \]

Note that technology transfer now involve both southern and northern wage costs, and that northern workers are employed in adaptive R&D (in addition to innovative R&D and production employment).

Following the same procedure as in section 4, we obtain the steady-state northern innovation condition

\[ \frac{X_{NL0} \rho}{(\sigma - 1) \gamma N L_{N0}} = a_N \delta_N, \]

the steady-state southern adaptation condition

\[ \frac{X_{FL0} \rho}{(\sigma - 1) \gamma F L_{N0}} - \omega a_N \delta_N = a_F \delta_N + \omega a_{FN} \delta_N, \]

the northern steady-state equation

\[ 1 = a_N \delta_N g \left[ 1 + \left( \frac{\rho + \theta g}{g + \phi} \right) (\sigma - 1) \right] + a_{FN} \delta_N g \frac{\phi}{g + \phi}, \] (40)

the southern steady-state equation

\[ 1 = \frac{L_{N0} \delta_N \phi}{L_{S0} (g + \phi)} \{ a_F g + [a_F + \omega (a_{FN} + a_N)] (\rho + \theta g + \iota) (\sigma - 1) \Phi(\iota) \}, \] (41)
and the steady-state wage equation

$$\omega^\sigma \left[ \frac{\rho + \theta \gamma}{\rho + \theta \gamma + \tau} \right] - \omega \left[ 1 + \frac{a_{FN}}{a_N} \right] = \frac{a_F}{a_N}. \quad (42)$$

Solving for a steady-state equilibrium reduces to solving a system of 3 equations [(40), (41), (42)] in 3 unknowns [\(\omega, \delta_N, \phi\)].

Consider first the properties of the northern steady-state equation (40). The RHS of (40) is unambiguously increasing in \(\delta_N\). Things are more complicated when it comes to an increase in \(\phi\):

$$\frac{\partial \text{RHS}}{\partial \phi} = \frac{\delta_N g}{(g + \phi)^2} \{ a_{FN} g - a_N (\rho + \theta g) (\sigma - 1) \}$$

is less than zero if \(a_{FN}\) is small \([a_{FN} < a_N (\rho + \theta g) (\sigma - 1)/g]\) and is greater than zero if \(a_{FN}\) is large \([a_{FN} > a_N (\rho + \theta g) (\sigma - 1)/g]\). Thus, the northern steady-state equation (40) is globally upward-sloping in \((\delta_N, \phi)\) space when \(a_{FN}\) is small (as is the case in section 5) but is globally downward-sloping in \((\delta_N, \phi)\) space when \(a_{FN}\) is large. In both cases, there is a strictly positive \(\delta_N\) intercept and changes in \(\iota, a_F S\) or \(L_{S0}\) have no effect on the graph of the northern steady-state equation.

The southern steady-state equation (41) has exactly the same properties as in section 5. It is globally downward-sloping in \((\delta_N, \phi)\) space for any given value of \(\omega\) and has no intercepts since neither \(\delta_N\) or \(\phi\) can equal zero. As in Figure 3, the southern steady-state equation shifts to the right when either \(\iota\) decreases, \(a_F\) decreases or \(L_{S0}\) increases.

The steady-state wage equation (42) uniquely determines \(\omega > 1\) and also has exactly the same properties as in section 5. A decrease in \(\iota\) causes \(\omega\) to fall, a decrease in \(a_F\) causes \(\omega\) to fall, and an increase in \(L_{S0}\) has no effect on \(\omega\).

We conclude that all of the results derived in section 5 (assuming that \(a_{FN}\) equals zero) continue to hold when \(a_{FN}\) is positive but small. In particular, the adoption of stronger IPR protection in the South (\(\iota \downarrow\)) generates a permanent increase in the rate of technology transfer to the South within multinational firms (\(\phi \uparrow\)), a permanent increase in R&D employment by foreign affiliates (\(L_{Ft} = a_F \delta_N g L_{Nt} \phi / (g + \phi) \uparrow\)), a permanent decrease in the North-South wage ratio \((\omega \equiv w_N/w_S \downarrow)\) and a temporary increase in the northern innovation rate \((\delta_N \uparrow)\).

However, not all of the results derived in section 5 continue to hold when \(a_{FN}\) is large. What changes when \(a_{FN}\) becomes large is that the northern steady-state equation becomes downward-sloping as illustrated in Figure 4 (which should be contrasted with the upward-sloping northern steady-state equation illustrated in Figure 3). Because of this difference, the results derived in section 5 concerning the northern innovation rate are reversed when \(a_{FN}\) is large. In particular,
the adoption of stronger IPR protection in the South ($t \downarrow$) generates a temporary decrease in the northern innovation rate ($\delta_N \downarrow$).