Time varying effects of public investment
and a Stone-Geary production technology

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Abstract

This paper starts with presenting a new empirical evidence based on time-varying parameter VAR that, in Japan, the effect of public investment on output has fallen over time while its crowding out effect on private investment has become more pronounced. Does this mean that we could not expect too much of a stimulative impact from today’s public investment, even in the aftermath of massive capital destruction (such as a strong earthquake)? I try to answer this question by building a growth model in which the effect of public investment varies endogenously, with the level of capital accumulation. The model is based on a specification of production technology of a Stone-Geary type, which generates a time varying degree of complementarity between private and public capital.
1 Introduction

Recent empirical evidence indicates that the output enhancing effect of public investment has fallen over time in Japan. Such a tendency is confirmed in this paper by a new approach based on the time-varying parameter VAR (TVP-VAR). This brings up two natural questions. The first one has to do with the cause of the decline: is there an endogenous mechanism that governs the policy effectiveness? The second question is if public investment will remain ineffective even in the aftermath of massive destruction of both private and public capital due, for example, to a natural disaster. Finding the correct answer to the latter question is important in formulating an appropriate policy reaction to such destruction.

This paper is a first step toward answering those questions. In this paper, I build a growth model in which the effect of public investment on output varies with the level of both private and public capital. This is done through endogenizing the elasticity of substitution between private and public capital. For that purpose, I introduce a novel specification of production technology. Under the proposed functional form, private and public capital are more complementary when the levels of capital are low. As both types of capital are accumulated, they become more substitutable. I incorporate this feature into an otherwise standard neoclassical growth (real business cycles) model. I show that the model is capable of producing a much larger impact of public investment on output when the initial levels of private and public capital are low, either because the country is at an early stage of capital accumulation or because it has experienced large destruction of both private and public capital, compared with when the economy starts from a neighborhood of the steady state.

The rest of the paper is organized as follows. Section 2 reviews the empirical literature on the output effect of public investment in Japan. Section 3 confirms previous findings with a new econometric approach and updated data. Section 4 develops a neoclassical growth model with public capital, in which the elasticity of substitution between private and public capital is exogenously given. The purpose of this section is to highlight the importance of this elasticity in determining the output effect of public
investment. Section 5 introduces the new specification of technology. Section 6 incorporates this technology into the growth model. Section 7 concludes and contemplates directions for future research.

2. Background

This paper is motivated by two strands of empirical literature that are closely related with each other but nonetheless have developed more or less independently over time. The first one tries to evaluate contribution of public capital to output through estimating either a production function that incorporates public capital, often utilizing regional data (Mera (1973) and Asako and Wakasugi (1984) are pioneering work in this tradition, that precedes the much debated work by Aschauer (1989)) or an income convergence equation (Shioji (2001a), among others). It has been found that the contribution has been declining over time (refer, for example, to Shioji (2000, 2001b, 2008)). The second literature employs time series techniques and estimates effects of a fiscal expansion using aggregate data. Ihori, Nakazato and Kawade (2003) estimate a VAR model for Japan and conclude that the effectiveness of fiscal policy has declined in the 1990s. Fujii (2008) estimates a structural VAR model and performs a sequence of tests for a structural break, and determines that there was a break in the fourth quarter of 1999. After that, the effect of fiscal policy is diminished. Eguchi and Hiraga (2010) employ a smooth transition model and reach a similar conclusion. Morita (2011), following the intuition of Ramey (2011), estimates an “expectation augmented” VAR: he finds evidence in favor of a structural change.

A traditional interpretation is that the former approach captures the long run, productivity enhancing effect (or the “supply side” effect) of public capital, while the latter deals mostly with its business cycle effect (or the “demand side” effect). However, recent studies in the New Keynesian literature suggest that such dichotomy may not be appropriate. For example, Gali et. al. (2007) search for conditions under which a New Keynesian macroeconomic model generates an effect of a fiscal expansion (which is assumed to have no direct impact on productivity) that is
comparable in size to the one found in data analyses. They find that they need to assume a very high population share of “rule of thumb” consumers and also to introduce a strong assumption on the degree of labor market rigidity. On the other hand, models which admit productive roles of public spending can yield much larger effects of a fiscal expansion without resorting to those strong assumptions. Hence, the finding that public investment has lost its productivity enhancing effect and the result that its short run stimulus effect has been diminished are likely to be related. Based on those observations, in this paper, I introduce public capital into a relatively simple neoclassical growth model without various “frictions”, as I believe they are less essential (though could still be useful) in understanding the main issue of the paper.

3. New evidence based on a time varying parameter VAR

In this section, I will try to see if findings from the previous studies are confirmed with a new empirical technique and an updated data set. The methodology employed here is the time varying parameter VAR (TVP-VAR from here on) utilized by Primiceri (2005) and Nakajima (2011), among others. In contrast to the more traditional time varying coefficient VAR, which allows only the reduced form VAR coefficients to vary over time, this method also allows the error variance covariance matrix to be time varying. I estimate a TVP-VAR model on the Japanese data which consists of the following three variables: public investment (denoted $I_C$), private investment ($I_P$) and GDP ($Y$) (all from the National Accounts statistics). The sample period is 1955Q3-2011Q1. I take logs and then take first differences of all three series. They are multiplied by 100 so that they can be interpreted as percentage changes. The number of lags is set at 4.

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2 Shioji, Vu and Takeuchi (2011) develop a small open economy New Keynesian model and arrive at a similar conclusion.
3 I utilize a collection of matlab codes provided by Jouchi Nakajima (a Duke graduate student in statistics).
4 In a related ongoing work, Hiroko Takeuchi (Yokohama National University) estimates a TVP-VAR model for Japan which consists of government consumption, public investment and GDP.
Figure 1: Time variations in intercept terms (horizontal axis = time): $I_G$, $I_P$ and $Y$

Figure 2: Impulse responses of public investment ($I_G$) to its own shock, at selected points in time (dashed lines are the 16% and the 84% quantiles)
Figure 3: Impulse responses of private investment ($I_p$) to public investment shock, at selected points in time (dashed lines are the 16% and the 84% quantiles)

Figure 4: Impulse responses of GDP ($Y$) to public investment shock, at selected points in time (dashed lines are the 16% and the 84% quantiles)
In the VAR model, it is assumed that the contemporaneous relationship between the three variables is characterized by a recursive structure. I put $I_G$ at the top of the causal ordering, followed by $I_P$ and then $Y$.

Figure 1 shows time variations in the estimated constant term for each of the three equations, along with the 68% error bands. Note that, for $Y$, it captures the slowdown in Japanese growth over time. Figures 2 to 4 report the estimated impulse responses, computed for the first quarter of selected years (with about ten years interval between each other), together with the 68% error bands. They correspond to responses to the public investment ($I_G$) shock. Figure 2 shows responses of public investment to its own shock. We see very little time variation in those responses. That is, there was little change in the dynamics of public investment over time. Thus, if we find time variations in responses of $I_P$ and $Y$ (as we do), that is not because the time series nature of $I_G$ changed over time.

Figure 3 contains responses of private investment ($I_P$) to the $I_G$ shock. Between 1957 and 1975, it is insignificant at the impact, while it turns significantly positive in the medium run. However, in 1985, the impact response is at the borderline between significance and insignificance. In 1995 and 2005, those effects are significantly negative. The results confirm that the crowding-out effect on private investment became more pronounced over time.

Figure 4 demonstrates responses of GDP ($Y$). The impact is significantly positive and is around 0.5 between 1957 and 1975. However, in 1985 and 1995, the effect becomes much weaker and is only borderly significant. In 2005, the effect makes a slight come-back and it is significant again. Thus, the overall picture is consistent with the belief that the effect of public investment on output has become weaker over time.

4. Analysis based on the standard (CES) production function

4-1 CES production function with public capital

This section considers a standard neoclassical growth model augmented with public capital. Consider a closed economy. Time is discrete. Assume that there is a continuum
of identical firms along the interval \([0, 1]\). For the sake of simplicity of exposition, I will omit the time subscript \(t\) in this sub-section.

Production involves three types of factors of production, namely private capital (denoted by \(K_P\)), public capital (\(K_G\)) and labor (\(L\)). Consider firm \(i\) which employs \(K_{Pi}\) units of physical capital and \(L_i\) units of labor. Aggregate amount of public capital in this economy is simply denoted as \(K_G\). Then the firm’s production function is assumed to take the following form:

\[
y_i = A \cdot \left[ aK_{Pi}^{\rho} + (1 - a)K_G^{\rho} \right]^{\frac{\alpha}{\rho} - \alpha} \cdot L_i^{1 - \alpha}
\]

Note that public capital enters into this function not in “per firm” units but as an aggregate quantity. This reflects the view that public capital is not a private input but a public good, and that it influences the firm’s production through an external effect. In equation (1), \(A\) is a positive constant and denotes the Total Factor Productivity, and \(0 < \alpha < 1\). Also, \(\rho ( < 1)\) is a key parameter in this section, and it determines the elasticity of substitution between private and public capital, which is \(1/(1 - \rho)\). A question is how to set the value for the parameter \(b\). Setting \(b=1\) would imply that the production exhibits constant returns to scale with respect to the three factors of production, namely \(K_{Pi}\), \(K_G\) and \(L_i\). This in turn implies it has the property of diminishing returns to private inputs, namely \(K_{Pi}\) and \(L_i\). On the other hand, suppose

\[a = 0.5.\]  

Suppose also the government fixes the value of public investment at a constant number, denoted \(I^*_G\), in such a way that the marginal products of private and public capital are equalized at the steady state. This would imply:

\[K_{P_1}^* = K_G^*\]  

where “\(^*\)” denotes the steady state. In this case, it can be shown that, if I set

\[b=2,\]  

the production function would have the property of constant returns to private inputs, in the neighborhood of the steady state. From now on, I shall assume (2)-(4). That is, the production function is specified as:
\[ Y_t = A \left[ 0.5K_{Pt}^\rho + 0.5K_G^\rho \right]^{\alpha/\rho} \cdot L_t^{1-\alpha}. \]  

(5)

4-2 The rest of the model

We abstract from population growth and technological progress. The representative household’s lifetime utility takes the following form:

\[ U_0 = \sum_{t=0}^{\infty} \beta^{-t} u(C_t, L_t) \]  

(6)

where

\[ u(C_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \mu L_t^{1+\lambda}, \quad \text{and} \quad \sigma > 0, \quad \lambda > 0, \quad \mu > 0. \]  

(7)

Here, \( C_t \) is consumption and \( L_t \) is labor. As the parameter \( \lambda \) becomes smaller, labor supply becomes more elastic. The household supplies labor to firms and rents private capital. It also owns shares of the firms, which are not traded. The budget constraint for each period is as follows:

\[ C_t + K_{Pt+1} + T_t = w_t L_t + \Pi_t + (1 + r_t - \delta)K_{Pt}. \]  

(8)

In the above, \( K_{Pt} \) is the total amount of private capital at the beginning of period \( t \), \( T_t \) represents lump sum taxes to finance public investment, \( w_t \) is the real wage, \( \Pi_t \) denotes distribution of profits from firms, \( r_t \) is the real rental rate of private capital and \( \delta \) is the depreciation rate for capital (\( 0 < \delta < 1 \)). Private capital cannot be negative, and the household is also subject to the Non-Ponzi Game constraint. Firms take the real wage and the real rental rate of private capital as given, and maximize profits each period.

The government levies the lump sum tax on the household and conducts public investment:

\[ I_{Gt} = T_t, \quad \text{and} \quad K_{Gt+1} = I_{Gt} + (1 - \delta)K_{Gt}. \]  

(9)

In the above, \( I_{Gt} \) denotes public investment. It is assumed that the same depreciation rate is applicable to both private and public capital, which simplifies the analysis to some extent.
4-3 Equilibrium

Under the above specification, using the symmetry of the firms, the equilibrium conditions are derived as follows:

\[
Y_t = A \left[ 0.5K_{p_t}^\rho + 0.5K_{G_t}^\rho \right]^{2a/\rho} \cdot L_t^{1-\alpha}, \quad (10-1)
\]

\[
Y_t = C_t + I_{p_t} + I_{G_t}, \quad (10-2)
\]

\[
K_{p_{t+1}} = I_{p_t} + (1-\delta)K_{p_t}, \quad (10-3)
\]

\[
K_{G_{t+1}} = I_{G_t} + (1-\delta)K_{G_t}, \quad (10-4)
\]

\[
\left( \frac{C_{t+1}}{C_t} \right)^{\sigma} = \beta [1 + MPK_{p_{t+1}} - \delta], \quad (10-5)
\]

\[
MPL_t = \mu C_t^{\sigma} L_t^{\lambda}, \quad (10-6)
\]

\[
MPK_{p_t} = 2\alpha A \left[ 0.5K_{p_t}^\rho + 0.5K_{G_t}^\rho \right]^{(2a/\rho)-1} \left( 0.5K_{p_t}^{\rho-1} \right) \cdot L_t^{1-\alpha}, \quad (10-7)
\]

and \( MPL_t = (1-\alpha)A \cdot \left[ 0.5K_{p_t}^\rho + 0.5K_{G_t}^\rho \right]^{2a/\rho} \cdot L_t^{-\alpha}. \) \( (10-8) \)

4-4 Steady state

As already stated, the government fixes public investment at a constant value, \( I^*_G, \) in such a way that the marginal products of private and public capital will be equalized in the steady state. This implies:

\[
K^* = K_p^* = K_G^*, \quad (11-1)
\]

and also \( I^*_p = I^*_G = \delta K^*. \) \( (11-2) \)

The rest of the steady state conditions are as follows:

\[
Y^* = A \cdot K^*^{2\alpha} \cdot L^{*^{1-\alpha}}, \quad (11-3)
\]

\[
Y^* = C^* + 2\delta K^*, \quad (11-4)
\]

\[
MPK^* - \delta = 1/\beta - 1, \quad (11-5)
\]

\[
MPL^* = \mu C^*^{\sigma} L^*^{\lambda}, \quad (11-6)
\]

\[
MPK^* = \alpha A \cdot K^*^{2\alpha-1} \cdot L^{*^{1-\alpha}}, \quad (11-7)
\]

and \( MPL^* = (1-\alpha)A \cdot K^*^{2\alpha} \cdot L^{*^{-\alpha}}. \) \( (11-8) \)
4-5 Shock to public investment

I simulate the dynamic path of this economy following a shock to public investment. I will interpret one “period” in the model as a quarter. Suppose that, in period 1, the government unexpectedly increases public investment by ten percent:

\[ \ln(I_{G1}) - \ln(I_G^*) = 0.1. \]  

(12-1)

After this period, public investment gradually goes back to its steady state value following an AR1 process:

\[ \ln(I_G^t) - \ln(I_G^{*t}) = \phi \left[ \ln(I_{Gt-1}) - \ln(I_G^{*t}) \right] \quad (t \geq 2). \]  

(12-2)

Here, I set the AR1 parameter to be \( \phi = 0.68 \). I derive this value by estimating the AR1 model on Japanese public investment, detrended by the Hodrick Prescott Filter. Figure 5 depicts the impulse responses of public investment and public capital to this shock. The former shows a relatively weak persistence.

Figure 5: Impulse responses of public investment (left) and public capital (right)

4-6 Numerical analysis

The analysis in this sub-section aims to shed light on the role of the elasticity of substitution between two types of capital goods in determining the effects of public investment. I fix some of the parameter values as follows:

\[ \alpha = \lambda = 1/3, \beta = 0.99, \delta = 0.023, \sigma = 1 \]

Those values are fairly standard ones, except for \( \lambda \), whose value is chosen for
convenience of numerical analyses\(^5\). Values for \(A\) and \(\mu\) are chosen endogenously to satisfy the following steady state conditions:

\[ K^* = 150, L^* = 1. \]

This enables me to always start the economy from the same steady state irrespective of the parameter values. As for the substitutability between two types of capital goods, which is the crucial parameter under consideration here, I consider three alternative cases: \(\rho = 0.5, 0.01, \) and \(-1\). They correspond to elasticities of 2, about 1, and 0.5, respectively.

I solve for the dynamic equilibrium path using the shooting algorithm\(^6\). Figure 6 demonstrates how endogenous variables respond to a public investment shock, starting from the steady state. Each panel corresponds to responses of output, private capital, private investment, consumption, and labor, respectively. Different lines within each panel correspond to three different underlying values for the substitution elasticity. We can see that qualitative results do not depend on the elasticity. In all cases, output increases. Both private capital and private investment respond strongly negatively in the short run, and then the responses turn positive. Consumption also decreases in the short run and then increases. Labor increases in the short run.

The results can be interpreted as follows. In the short run, the productivity effect of public investment is still small, as public capital accumulates only slowly over time. On the other hand, the household immediately realizes that its disposable income is reduced by the tax increase. The household also understands that the future increase in public capital will increase its future disposable income, but this effect is dominated by the tax effect (because the degree of persistence in the increase in public investment is set relatively small). Thus the household’s life time disposable income is revised downward. Thus consumption decreases in the short run, and the demand for leisure also goes down (i.e., labor supply increases). Labor supply increases also through intertemporal substitution prompted by a higher real interest rate. The short run

\(^5\) When both \(\alpha\) and \(\lambda\) are equal to 1/3, the equilibrium condition for each period becomes a quadratic equation, and can be solved analytically, making it possible to avoid a crucial difficulty in the numerical analysis.

\(^6\) I utilized a matlab code supplied by Masaru Inaba (Kansai University) on the web.
increase in output is primarily due to this increase in labor supply. In the short run, there is a crowding-out of private investment due to the higher real interest rate.

In the medium to long runs, the productivity effect of public capital becomes more important. The response of private capital turns positive because the larger stock of public capital enhances the productivity of private capital as well. Due to the increases in both types of capital, output remains persistently high. Private consumption turns positive due to the income effect. The marginal product of labor also increases due to the increases in the two types of capital, but, on the other hand, the household reduces labor supply due to the income effect. As a consequence of those two opposing effects, the response of labor turns out to be near zero or slightly negative.

The substitutability between the two types of capital has substantial quantitative influences on those impulse responses. Comparing the three impulse responses in each panel, that correspond to different values of the elasticity of substitution, we learn the following. As private and public capital become more complementary, the short run crowding-out of private investment becomes smaller and shorter, as an increase in public capital tends to enhance productivity of private capital more strongly. This increase in private capital also pushes up the marginal product of labor, resulting in a larger positive response of labor. As private capital decreases to a lesser extent and the size of the positive response of labor becomes larger, the positive response of output becomes much more pronounced. The response of consumption is more complicated. When the two types of capital become more complementary, as the short run crowding-out effect of private investment becomes weaker, the crowding-out effect of private consumption becomes stronger. In the medium to long runs, however, as disposable income of the household increases more strongly under a low substitutability, the increase in consumption becomes more pronounced.
Figure 6: Responses to an increase in public investment: the CES case.
Vertical axis: rate of deviation from the steady state.
Horizontal axis: number of periods since the shock.

5. Stone Geary production technology and the substitution elasticity

The previous section has demonstrated the importance of the substitution elasticity between private and public capital in determining the effect of public investment. This section looks for a way to endogenize it. One class of production function that delivers such a result is known as the “Variable Elasticity of Substitution” or VES class. Bairam (1994) presents four types of functions that belong to this category. All of them
share a common feature that the elasticity of substitution between two types of inputs depends on the ratio between the quantities of the two. For example, if we were to consider a production function that has a VES feature between private and public capital, then the elasticity of substitution between the two would be a monotonically increasing (or decreasing) function in the ratio between the volumes of the two\(^7\). As a consequence, when the two types of capital grow at the same rate, keeping the ratio between the two constant, the elasticity will also remain unchanged.

Thus, the above production function does not represent the notion that, as both private and public capital accumulate, they become more substitutable. This paper proposes another class of production technology which yields such an outcome. Consider the following Stone Geary type technology which generalizes (5):

\[
y_i = A \cdot \left[ 0.5 \left( K_{p_i} - \bar{K}_p \right)^\rho + 0.5 \left( K_G - \bar{K}_G \right)^\rho \right]^{2\alpha/\rho} \cdot L_i^{1-\alpha}, \bar{K}_p \geq 0, \bar{K}_G \geq 0. \quad (13)
\]

When \( \bar{K}_p \) and \( \bar{K}_G \), the minimum requirement for private and public capital, are both equal to zero, this function is reduced to the CES form in equation (5). In the following analysis, I will assume \( \bar{K}_p = \bar{K}_G \equiv \bar{K} \), which will simplify the above expression into:

\[
y_i = A \cdot \left[ 0.5 \left( K_{p_i} - \bar{K} \right)^\rho + 0.5 \left( K_G - \bar{K} \right)^\rho \right]^{2\alpha/\rho} \cdot L_i^{1-\alpha}, \bar{K} \geq 0. \quad (14)
\]

Figure 7 displays the isoquants under this specification. In the figure, I set \( \rho = -0.5 \), and \( \bar{K} = 1 \). The isoquants have exactly the same shapes as those for the CES function, except that they look as if the origin was “shifted northeast”. Note that, when either private or public capital takes a value close to 1, which is the threshold value here, the

\(^7\) For example, consider the following functional form that is based on the one specified in Revankar (1971) and Karagiannis, Palivos and Papageorgiou (2005):

\[
y_i = A \cdot \left[ K_{p_i}^a \left( K_G + baK_{p_i} \right)^{(1-a)} \right]^\alpha \cdot L_i^{1-\alpha}
\]

where \( 0 < a \leq 1, \ b > -1 \) and the economy is assumed to be in a region that satisfies \( K_G / K_{p_i} \geq -b \). This function generalizes the Cobb-Douglas form, and converges to that form as the parameter \( b \) goes to zero. Under this specification, the elasticity of substitution between private and public capital is equal to \( 1 + bK_{p_i} / K_G \), which is linear in the ratio between the two. If \( b \) is positive, private and public capital becomes more substitutable over time when the former grows faster than the latter.
isoquant looks much like that of the Leontief production function. As we increase the values of both simultaneously, the effect of the threshold diminishes, and the isoquants start to look more and more like those of the regular CES function. This indicates that the elasticity of substitution is declining in the amount of both capital. In fact, it can be shown that, along the 45 degree line, namely when \( K_p = K_g = K \), the elasticity of substitution between private and public capital can be written as:

\[
\frac{K}{K-K'} \cdot \frac{1}{1-\rho}.
\]

As \( K \) tends to infinity, this converges to the elasticity under the CES specification.

As \( K \) tends to infinity, this converges to the elasticity under the CES specification.

\[
\begin{align*}
\frac{K}{K-K'} \cdot \frac{1}{1-\rho}.
\end{align*}
\]

Figure 7: Isoquants under the Stone Geary production technology

6. Effects of public investment under the Stone Geary technology

6-1 Equilibrium and steady state conditions

In this section, I redo the analysis in section 3 with the new form of production function in equation (14). Unlike the CES production function in (5), which had the property of constant returns to private inputs, at least locally around the steady state, the specification in (14) implies local increasing
for the expressions for the marginal products:

\[
MPK_{Pt} = 2\alpha A \cdot \left[ 0.5 \left( K_{Pt} - K \right)^{\alpha} + 0.5 \left( K_{Gt} - K \right)^{\alpha} \right]^{(2\alpha/\rho)-1} \left( 0.5 \left( K_{Pt} - K \right)^{\alpha-1} \right) \cdot L^{-\alpha}.
\] (10-7')

and

\[
MPL_t = (1 - \alpha) A \cdot \left[ 0.5 \left( K_{Pt} - K \right)^{\alpha} + 0.5 \left( K_{Gt} - K \right)^{\alpha} \right]^{2\alpha/\rho} \cdot L^{-\alpha}.
\] (10-8')

The steady state conditions are also the same as before except for the following three:

\[
Y^* = A \cdot \left( K^* - K \right)^{2\alpha} \cdot L^{*\alpha}.
\] (11-3')

\[
MPK^* = \alpha A \cdot \left( K^* - K \right)^{2\alpha-1} \cdot L^{*\alpha},
\] (11-7')

and

\[
MPL^* = (1 - \alpha) A \cdot \left( K^* - K \right)^{2\alpha} \cdot L^{-\alpha}.
\] (11-8')

It is important to note that (11-5) continues to hold despite the change in the technology specification. That is, the steady state marginal product of private capital is independent of \( K \): this is because it is primarily determined by the rate of discount of the representative household. Thus, although effects of public investment turn out to depend on the value of \( K \), as we will see in the next sub-section, the differences in the policy effects are not due to differences in the steady state marginal products of capital.

6-2 Simulation analysis

Now I simulate the model under the new technology specification and compare the results with those that are obtained under the standard CES specification. I will pay a particular attention to how the effects differ between the case in which the policy action is introduced in the neighborhood of the steady state as opposed to the case in which the initial condition was far below the steady state (due either to a low state of returns to those factors of production. In this paper, I will simply assume that there are a fixed number of firms and that all of them operate at the same scale. It is an important future task to come up with a market setup (such as monopolistic competition) which will support such an outcome as a stable equilibrium.
development or to a recent destruction of capital caused by a disaster). For that purpose, a “local” solution and simulation methodologies are inappropriate, and I will continue to utilize the shooting algorithm.

I consider the following two technology specifications:

(CES case) The production function is of the CES form, namely $\bar{K} = 0$.

(SG case) The function is of the Stone Geary (SG) form: I assume $\bar{K} = 50$.

The rest of the parameters are the same as in section 3: the substitutability parameter is set at $\rho = 0.5$. Thus, the elasticity of substitution between two types of capital is exactly 2 in the CES case and it is asymptotically equal to 2, as both types of capital become larger, in the SG case. I consider the following two sets of initial conditions:

(Start from the steady state) $K_{p0} = K_{G0} = K^*$

(Start from half of the steady state) $K_{p0} = K_{G0} = 0.5K^*$

Impulse responses for the latter case are computed in the following manner. First, I derive the equilibrium path of the economy when there is no shock to public investment: that is, the government fixes its value at the steady state level, $I_G^*$. Second, I derive the equilibrium path under the assumption that there was a shock to public investment as we saw in section 3. Then, for each variable, I take differences of its paths between the two scenarios, and normalize them by its steady state value.

Figure 8 depicts the resulting impulse responses. In the first panel, the response of output is not much different between the CES and the SG cases, as long as the economy starts from the steady state. In both cases, the effects become larger when the starting point is far below the steady state, because of diminishing returns to public capital. The difference in the effects due to different starting points is small in the CES case. However, in the SG case, the effect becomes about three times larger when we start from a half way to the steady state as opposed to from the steady state. In the second and third panels, in the SG case, the short run crowding-out effect on private investment is smaller and the medium to long run crowding-in effect is larger when the economy starts from far below the steady state. This is because, in this technology specification, the two types of capital goods become more complementary at a lower level of capital accumulation. As a consequence, the same amount of increase in public
investment induces a greater reaction of private investment, and the increased level of private capital, in turn, reinforces the productivity of public capital. In the fourth panel, there is not much difference in the response of consumption, irrespective of the initial conditions, as long as we are in the CES case. However, in the SG case, the difference is more noticeable: when we start from far below the steady state, the initial crowding-out effect is limited, the response of consumption turns positive quickly, and the medium to long run crowding-in effect is much stronger. The response of labor shown in the fifth panel is the most striking: in the SG case, it is five times larger when the initial values of capital are half of the steady state values than when the economy is at the steady state initially.

Figure 8: Effects of public investment under the CES/SG technologies ($\rho=0.5$)
Vertical axis: (deviation from the path under no policy change)/(steady state)
Horizontal axis: number of periods since the shock
7. Conclusions and future tasks

In this paper, I have first established that the effect of public investment on output has been declining over time, while the crowding-out effect on private investment has become more pronounced. To explain those findings, the paper has developed a growth model in which the elasticity of substitution between private and public capital changes endogenously as both types of capital accumulate. This model can potentially provide an explanation on why output enhancing effect of public investment has fallen in recent years in Japan. The same model predicts that, in the aftermath of massive capital destruction, the effects of public investment can be much larger than under normal circumstances.

To gain policy implications that are more relevant for capital destruction caused by a natural disaster such as an earthquake, an important future task would be to extend the model to an open economy setup. With possible exceptions of small island nations, it is more often the case that a natural disaster, even a large one, hits a particular region of a country disproportionately severely, rather than causing large destruction evenly throughout the country. To understand the role of public investment in such a situation, I would have to build a two or (multi) region growth model in which one of the regions is hit by a disaster shock.
References


