Tiebout Sorting and Inefficiency*

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1. Introduction.

The analogy between competition among firms in providing private goods and “Tiebout competition” among jurisdictions in providing local public goods is central to the economic study of local public finance. The basic idea is that household mobility will induce jurisdictions to provide efficient mixes of local public goods and taxes, or they will fail to attract residents. A large literature examines when the analogy is sufficiently compelling so that inter-jurisdictional competition is efficient and the nature of departures from efficiency when these conditions are not fulfilled.¹ For efficiency, essentially the tax system and housing-market price must control any externalities in residential choice with also efficient governmental choice of the level(s) of the local public good(s). While standard models frequently fail to meet the conditions for efficiency, economic intuition suggests that some Tiebout competition is better overall than none: The alternative of centralized provision will do nothing to match heterogeneous preferences to provision of local public goods. This paper challenges this intuition by showing that Tiebout competition can lead to an aggregate welfare loss in a standard model for realistic parameter values.

The case we consider is not contrived. A metropolitan area is made up of multiple jurisdictions with given boundaries. Households differ by income with CES utility function over numeraire consumption, housing consumption, and the level of the local congested public good (e.g., per student educational expenditure). The local public good is financed by a property tax that is chosen by majority vote of residents of the

¹ See Epple and Nechyba (2004) and Scotchmer (2002) for recent surveys of this literature.
jurisdiction. Households choose where to reside, and then vote in their jurisdiction and consume. Our findings regard cases when an income-stratified equilibrium exists, i.e., when a Tiebout-type equilibrium arises.\(^2\)

We show that welfare in aggregate, measured by aggregate compensating variation, is lower than in the analogous centralized equilibrium with the same political process for reasonably specified parameters. We know a priori that the Tiebout equilibrium will not be efficient. First, majority choice of the tax level satisfies the median resident’s preference and will not generally satisfy the Samuelsonian condition for efficient provision of the local public good. Second, the property tax causes a distortion in the housing market, while a head tax would be non-distorting and efficient. Third, the latter distortions imply externalities in individual residential choice. With local head taxes chosen efficiently and equilibrium household choices of jurisdictions, the modified Tiebout allocation would generate substantial welfare gains. With the imperfect system, these potential welfare gains are not just lost, but reversed. We go on to show that the most costly inefficiency is the externality in residential choices. Too many relatively poorer households move into richer jurisdictions. Efficient sorting would be more exclusive than arises in equilibrium. It is rather surprising that getting part way there is frequently less efficient than no sorting.

Section 2 presents the theoretical model. The calibration of a counterpart computational model is discussed in Section 3, followed by presentation of the main findings. Robustness is then analyzed. Section 4 concludes. An appendix contains some of the detail.

\(^2\) As described in detail below, such an equilibrium will arise for realistic parameter values where a standard single-crossing condition is satisfied.
2. The Model.

a. Elements of the Model. Our intent is to examine a very standard model of a metropolitan area with property taxation. Households differ by endowed income $y$ and have the same quasi-concave utility function over numeraire consumption $x$, housing consumption $h$, and the level of the local public good $g$ measured in dollars. We assume a CES utility function:

$$U = [\alpha_x x^\rho + \alpha_h h^\rho + \alpha_g g^\rho]^{1/\rho}, \rho < 1;$$

which allows us to examine how results vary with the elasticity of substitution, $1/(1-\rho)$, and the share parameters, $\alpha$. The distribution of endowed income is continuous with c.d.f. $F(y)$ and p.d.f. $f(y)$.

We compare a Tiebout-type equilibrium having the metropolitan area divided into jurisdictions to the counterpart single-jurisdiction centralized equilibrium. Focusing first on the former case, the metropolitan area is divided into $J$ jurisdictions, each with constant elasticity housing supply function $H_j(p^s_j)$, where $p^s_j$ denotes the net-of-tax or supplier price of housing, and $j = 1,2,\ldots,J$ henceforth unless indicated otherwise. Such a housing supply function arises if units of housing are produced competitively by combining a jurisdiction’s inelastically supplied $l_j$ and, $L_j$, with an elastically supplied factor, $q$, according to constant-returns production function: $h = L^\gamma q^{1-\gamma}$, $\gamma \in (0,1)$.

Specifically,

$$H_j^s = L_j \left(p^s_j\right)^{1-\gamma} \left(\frac{1-\gamma}{w}\right)^{1-\gamma},$$

3 Provide evidence on prevalence of property taxation.
where \( w \) is the given price of input \( q \). The quantity of housing available at given housing price then varies across jurisdictions proportionately to their land endowments.

We assume absentee land owners, but will account for their rents in our welfare calculations. We assume absentee land owners simply because it is most standard.

Equilibrium is determined in three stages. First, households purchase a home in a jurisdiction. Second, they vote in their jurisdiction for a property tax that is used to finance the local public good. Last, the local public good is determined from local governmental budget balance, and households consume (although their housing consumption is determined in the first stage). Households have rational expectations, thus anticipate all continuation equilibrium values.

This specification conforms to the case sometimes called “myopic voting,” because households take as given residences, housing consumption, and the supplier price of housing when voting, which are all established in the first stage.\(^4\) We examine this case because it is historically the most standard case in the literature. We show in the robustness analysis of Section 3 that the welfare loss we find from Tiebout sorting increases with other standard specifications of the timing of choices and thus voter beliefs that are perhaps more appealing.

b. Properties of Equilibrium. To provide a formal description of equilibrium, begin with the third stage. Let \( f_j(y) \) denote the density of household types living in jurisdiction \( j \), \( t_j \) the property tax rate, and \( h_j(y) \) housing consumption of household with income \( y \), all of which are given in the third stage. The gross housing price \( (p^*_j) \), local public good level

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\(^4\) The label “myopic voting” is potentially confusing since voters are fully rational given residence and housing consumption have been committed in the first stage. The “myopia” interpretation arises if households could move after voting, but voters fail to recognize this. Equilibrium is the same with either interpretation because no one moves in equilibrium in either case.
(g), and household numeraire consumption are determined in the third stage, satisfying respectively:

\[ p_i^t = (1 + t_j)p_i^j; \]  

\[ g_j \int_0^\infty f_j(y)dy = t_j p_i^j H_j^i(p_i^j); \]  

and

\[ x = y - (1 + t_j)p_i^j h_j(y); \]

where \( p_i^j \) is also given, established in the first stage. The congestion assumption about the public good implicit in (4) is also fairly standard, as for public schooling, and avoids issues of economies in providing local public goods. Obviously, the third stage values exist and are unique for any input vector (see the previous footnote).

Now consider the second, voting stage. Substitute (5) into the utility function and write indirect utility of household with income \( y \) as a function of \( (p_i^j, g_j) \):

\[ V(p_i^j, g_j; y) = [\alpha_s (y - p_i^j h_j(y))^p + \alpha_g (h_j(y))^p + \alpha_{g_j} g_j^p]^{1/p}. \]

When voting on the property tax rate, households maximize \( V(\cdot) \) while correctly anticipating that \( (p_i^j, g_j) \) will satisfy (3)-(4), taking as given \( (f_j(y), h_j(y), p_i^j) \). To show the properties of voting equilibrium (including existence), it is useful to determine the first stage choice of housing. Optimal housing consumption is given by ordinary demand, for the correctly anticipated value of the gross housing price (satisfying (3)):

\[ A \text{ negative numeraire will never arise as explained later.} \]
\begin{align*}
h_j(y) &= h_d(p_j^i, y) \equiv yz(p_j^i); \quad z(p_j^i) = \left[ p_j^i + \left( \frac{\alpha_s}{\alpha_h} \right)^{\frac{1}{1-\rho}} (p_j^i)^{\frac{1}{1-\rho}} \right]^{-1}; \quad (7) \\
\text{where the supplier price of housing satisfies market clearance:}
\end{align*}

\[ \int_0^\infty h_d((1 + t_j)p_j^i, y)f_j(y)dy = H_j^i(p_j^i). \quad (8) \]

Voting preferences satisfy a single-crossing condition.\(^6\) Dropping the \(j\) designation on variables, a voter’s indifference curve in the \((g, p_t)\) plane is defined by

\[ V(p_t, g; y) = \text{constant}. \] It has slope given by:

\[ \frac{dp_t}{dg} \bigg|_{V=\text{const.}} = \frac{\alpha_s g^{\rho-1}}{\alpha_s (y - p_t h(y))^{\rho-1}} = \frac{\alpha_s g^{\rho-1} y^{-\rho}}{\alpha_s (1 - p_t z)^{p-1} z}; \quad (9) \]

where the second equality uses \((7)\). Then:

\[ \text{sign} \left\{ \frac{dp_t}{dg} \bigg|_{V=\text{const.}} \right\} / dy = - \text{sign} \rho. \quad (10) \]

Voting preferences satisfy single crossing: If \(\rho < (>) 0\), then the slope of indifference curves in the \((g, p_t)\) plane increases (decreases) with income, which we refer to as SRI (SDI).\(^7\) If \(\rho = 0\), then the slope of indifference curves is invariant to income and thus preferences are indistinguishable. Single crossing implies:

\begin{quote}
**Proposition 1:** Given the jurisdictional population, voting equilibrium exists and is the preferred choice of the median-income household.
\end{quote}

\(^6\) Using \((7)\), numeraire consumption is given by: \(x = y[1 - p_j^i z(p_j^i)]\). The linearity of numeraire consumption in \(y\) implies that all households will oppose tax rates that would lead anyone to have a negative numeraire. Hence, the issue of a negative numeraire is moot.

\(^7\) The acronyms are \textit{slope rising in income} and \textit{slope declining in income}.
Formal proof of Proposition 1 is in the literature and is omitted (see, e.g., Epple, Romer, and Filimon, 1993). Briefly, consider a preferred choice of the median-income household satisfying (3) – (4). Single-crossing implies that any other feasible choices would be opposed by at least half the population, including either all those with higher or with lower income (depending on the nature of the alternative and the “direction” of single crossing). A corollary of Proposition 1 is that voting equilibrium is unique generically if the voting population has a unique median income as arises in all the equilibria we examine.

Let \( y_m^j \) denote the median income in jurisdiction \( j \), assuming it is unique. The equilibrium tax in jurisdiction \( j \) then satisfies:

\[
\max_{s.t. (3)-(4)} V(p, g_j; y_m^j);
\]

(11)

given \( p \) and housing consumption that satisfy (2) and (7)-(8).

Now consider the first-stage household choices of jurisdictions and the implications for the full (three-stage) equilibrium. There are two types of equilibria that can arise. Our interest is in Tiebout-type equilibria with differences among jurisdictions in levels of provision of the public good and with at least some households having strict preference for their choice of jurisdiction. Assume \( g_i \neq g_j \), for all jurisdictions \( i \neq j \). As discussed below, such equilibria cannot arise in our model if \( \rho = 0 \) so assume otherwise. Key properties of such equilibria are summarized in Proposition 2.

Proposition 2: Tiebout equilibria with jurisdictions numbered such that \( g_1 < g_2 < \ldots < g_J \):

a. Have ascending bundles: \( p'_1 < p'_2 < \ldots < p'_J \).
b. Are income stratified: If household with income $y_1$ resides in higher-numbered jurisdiction than household with income $y_2$ and SRI (SDI) holds, then $y_1 \geq (\leq) y_2$ with equality for at most one income level.

c. Exhibit boundary indifference and strict preference for non-boundary households: A household with income level $y_j^i$, $j=1,2,\ldots,J-1$, exists who is indifferent between residing in jurisdiction $j$ and $j+1$. All other households strictly prefer their residential choice.

Remarks on Proposition 2:

1. Again, formal proof of these results is in the literature (see, e.g., Epple, Romer, and Filimon, 1993). It is, however, straightforward to confirm these results using that households will choose to reside where

$$\hat{V}(p_j^i, g_j; y)=[\alpha_x (y - p_j^i h_d (p_j^i, y))^\rho + \alpha_h (h_d (p_j^i, y))^\rho + \alpha_g g_j^i]^{1/\rho}$$

is maximized while anticipating the equilibrium values of $(p_j^i, g_j)$. Part a is necessary to have all jurisdictions populated. The same single-crossing property that applies to $V(\cdot)$ applies to $\hat{V}(\cdot)$ by application of the Envelope Theorem, implying the income stratification in Part b (see the previous footnote). Continuity of the utility function along with single crossing implies Part c.

2. Tiebout equilibria are computed below. Figure 1 provides a two-jurisdictional example, assuming preferences satisfy SRI, with the subscript 2 indicating the richer jurisdiction. The government budget constraints (GBC’s) show the $(p_i, g)$ pairs feasible in each community (from (3) – (4)), and the equilibrium $(p_i, g)$ values occur at tangencies of

---

8 $\hat{V}(\cdot)$ differs from $V(\cdot)$ in that housing consumption is endogenously determined by $(p_j^i, y)$ in the former and fixed in the latter. $\hat{V}(\cdot)$ is relevant to the jurisdictional choice because housing consumption is not yet committed when residence is chosen. But the indifference curves for $\hat{V}(\cdot)$ and $V(\cdot)$ are identical because housing consumption is chosen to maximize utility.
the pivotal voters’ indifference curves to the GBC’s as shown by the bundles $B_1$ and $B_2$.\footnote{From (3) and (4), one can confirm that the GBC has slope equal to the inverse of the per capita housing consumption in the jurisdiction, keeping in mind that the jurisdictional population and housing consumption are fixed along it. Richer jurisdictions will then normally have flatter GBC’s as in the example.}

An indifference curve of the boundary household is drawn as well. When choosing residences in the first stage, households anticipate the bundles $B_1$ and $B_2$ in the respective communities. Those with income above (below) $y_b^1$ have steeper (flatter) indifference curves than $y_b^1$ through $B_1$ ($B_2$) and will choose to reside in jurisdiction 2 (1). (Keep in mind that the indifference curves that determine voting preferences are the same as those that choose residential preference.)

3. Existence of Tiebout equilibrium is not guaranteed\footnote{Restrictions on preferences and technology sufficient for existence in the model with myopic voting are developed in Epple, Romer, and Flimon (1993).}, but is not unusual. We show existence for realistic parameter values computationally below. The underlying forces relevant to existence are as follows. If Tiebout equilibrium arises, then it is income stratified and richer jurisdictions have greater demand for housing. Assuming the same housing supplies in each jurisdiction, then richer communities are likely to provide more of the public good at relatively high gross housing prices than poorer communities (see the previous footnote). It is then likely that the median preferred choices will have higher housing price and public good provision in the richer communities: bundles are likely to ascend with income. The issue is whether such choices are consistent with residential preferences. In the case of SRI, this is not unusual, since higher income types have greater willingness to bear a higher housing price for more of the public good. In the case of SDI, however, bundles that ascend with income are inconsistent with residential choices. For Tiebout equilibrium to arise, the voting preferences must be strong enough
to offset the character of the GBC’s, leading to bundles that descend with income for there to be a chance for equilibrium. The latter is not likely. In the case where \( \rho = 0 \), preferences over \((p_t, g)\) are the same, and Tiebout equilibrium will not arise generically. Tiebout equilibrium would require that all households are indifferent to living in richer or poorer communities (that must arise in such an equilibrium), which will not arise generically in the space of income distributions.

4. Multiplicity of Tiebout equilibria can arise if housing supplies differ across jurisdictions. For example, with two jurisdictions having different land areas and thus housing supplies, either might be the poor jurisdiction. We assume in our computational model that jurisdictions have the same land areas and thus housing supplies, except for one larger jurisdiction, the “central city,” where the poorest segment reside. Hence, we focus on one Tiebout equilibrium, this intended to capture the reality of metropolitan areas in the U.S.

5. Efficiency implications are discussed below.

The mathematical conditions that Tiebout equilibrium adds to the other equilibrium conditions (i.e, to (2) – (8) and (11)) are:

\[
\tilde{V}(p_t, g_j; y_{b^j}) = V(p_t^{j+1}, g_j; y_{b^j}), \quad j = 1, 2, ..., J - 1; \quad (12)
\]

and, defining \( y_{b^j} = 0 \) and \( y_{b^j}^l = \infty \),

\[
\begin{align*}
\text{if SRI and numbering such that } g_1 < g_2 < ... < g_j, \quad f_j &= \begin{cases} 
  f \text{ for } y \in [y_{b^j+1}, y_{b^j}) \\
  0 \text{ otherwise}
\end{cases} \\
\text{if SDI and numbering such that } g_1 > g_2 > ... > g_j, \quad f_j &= \begin{cases} 
  f \text{ for } y \in [y_{b^j+1}, y_{b^j}) \\
  0 \text{ otherwise}
\end{cases}
\end{align*} \quad (13a)
\]

\[
\begin{align*}
\end{align*} \quad (13b)
\]
Equation (12) is the boundary indifference condition. Equations (13a) and (13b) correspond to the two types of income stratification that might arise in a Tiebout equilibrium. Hence, (2) – (8) and (11) – (13) describe a Tiebout equilibrium.

Non-stratified equilibrium generally exists in the model as well. Suppose, for example, that each jurisdiction has the same housing supply. Suppose, further, that households choose jurisdictions in the first stage such that \( f_j = f/J \) for all \( y \). Then the continuation equilibrium values are the same; the jurisdictions are clones. In turn, the initial residential choices are equilibrium ones since the (atomistic) households are indifferent to their community. These non-Tiebout equilibria do not require the same housing supplies; initial residence choices can be adjusted so that the same \((p, g)\) values arise in each jurisdiction. There are also mixed equilibria generally where proper subsets of jurisdictions are clones, these acting like one jurisdiction in a stratified equilibrium. These equilibria are unstable (see, e.g., Fernandez and Rogerson, 199?). Whether or not there is a compelling argument that the (full) Tiebout equilibrium will arise, this paper is concerned with comparing it to the centralized alternative.

The comparison centralized equilibrium assumes the metropolitan area is one jurisdiction, with housing supply that is the usual aggregation of the jurisdictional housing supplies in the non-centralized case. Equilibrium is determined analogously to above, but with no alternative jurisdictions to choose from in the first stage and with one vote of the entire population for the tax rate, followed by consumption and provision of the public good. From above, it follows that centralized equilibrium exists and is unique. Obviously, no matching of preferences to public goods arises in the centralized case. Our interest is in the welfare comparison of the centralized equilibrium to the Tiebout
equilibrium when the latter exists. We should note that the centralized equilibrium values correspond to those in the de-centralized non-stratified (clone) equilibrium discussed in the previous paragraph, so one can interpret the comparison this way as well.

c. Comparing Welfare. We treat the centralized equilibrium as the status quo and use (the negative of) aggregate compensating variation associated with the Tiebout equilibrium as our welfare measure. Let \( U^c(y) \) denote utility of household with income \( y \) in the centralized equilibrium and \( U^T(y) \) utility in Tiebout equilibrium.\(^{11} \) Let \( v(y) \) denote compensating variation, defined in \( U^c(y) = U^T(y+v) \). Let \( CV = \int_0^\infty v(y)f(y)dy \) denote aggregate household compensating variation. Let \( R^c = \sum_{j=1}^I \int_0^{p^c_j} H_j(p)dp \) denote housing rents in the centralized equilibrium, where \( p^c_j \) denotes the net housing price. Let \( R^T = \sum_{j=1}^I \int_0^{p^T_j} H_j(p)dp \) denote housing rents in the Tiebout equilibrium. Compensating variation of the absentee landlords is given by: \( R^c - R^T \). We report \( W^T = -[CV + R^c - R^T] \) as our welfare measure, while also reporting aggregate consumer welfare (\(-CV\)). The negative of compensating variation is reported just so a positive value indicates a gain from Tiebout sorting.

We know a priori that the Tiebout equilibrium is not efficient. If households choose residences followed by efficient public good provision satisfying the Samuelsonian condition financed by a head tax, then equilibrium would be (Pareto) efficient. Such an allocation would maximize welfare relative to the centralized equilibrium measured analogously (i.e., using the negative of aggregate CV). These

\(^{11} \) If Tiebout equilibrium does not exist, then the degenerate “Tiebout equilibrium” corresponds to the centralized equilibrium and \( U^c(y) \equiv U^T(y) \).
results are shown in the appendix. In the Tiebout equilibrium we study, use of a property tax to finance public provision is inefficient. Likewise, majority choice of the level of provision is generally inefficient. These inefficiencies further imply that externalities arise in the individual choice of residences.\textsuperscript{12} We know, then, that if we calculate welfare analogously in going from the centralized equilibrium to the efficient head-tax equilibrium, denoted by $W^H$, that $W^H > 0$ and $W^H > W^T$. In spite of the latter inequality, we perceive a strong belief among economists that $W^T > 0$ is to be expected: Some aggregate welfare gains will arise from the equilibrium matching of households to relatively desired public good levels.\textsuperscript{13} In fact, we will see that this belief is not justified. We show below that $W^T < 0$ when $W^H$ is substantial.

3. Results

a. Calibration of the Model. The number of jurisdictions and the parameters of the MA income distribution and housing supply and utility functions must be calibrated. We assume five local jurisdictions in the MA – a large city and four smaller suburbs that have equal area. The total land supply in the MA is normalized to 1. The city has 40% of the total land area and each of the suburbs has 15%. The conditions for income stratification are satisfied in all computational results reported below. We assume that the city is the poorest jurisdiction. The jurisdictions are numbered from poorest to richest: hence, $L_1 = .4$, and $L_2 = L_3 = L_4 = L_5 = .15$, where $L_j$ equals community j’s land share.

\textsuperscript{12} The character of these externalities is detailed below.

\textsuperscript{13} Our perception of the consensus belief is that $v(y)$ will be positive for low income types (i.e., there will be a welfare loss for them), but $v(y)$ will be negative and offsetting for higher income types. While we find $v(y)$ is decreasing as expected, the offset does not occur.
The distribution of MA income is calibrated using data from the 1999 American Housing Survey (AHS).\textsuperscript{14} Median income reported by the AHS is $36,942. Using data for the 14 income classes reported by the AHS, we estimate mean income to be $54,710. These values and the assumption that the income distribution is lognormal imply $\ln y \sim \mathcal{N}(0.886, 10.52)$.

The remaining parameter values are $\rho, \alpha_x, \alpha_h,$ and $\alpha_g$ from the utility function (1) and $\gamma$ and $w$ from the housing supply function (2). The baseline model’s calibrated values are summarized in Table 1. The calibration is based on a single jurisdictional equilibrium for simplicity. First, we set $\alpha_x = 1$, a normalization. While less obvious, $w$ is also a “free parameter,” which we also then set equal to 1. To see this, note from (2) that the housing supply function for the MA is:

$$H_s = \left( w \right)^{\gamma - 1} \left( p_s \right)^{\frac{1 - \gamma}{\gamma}} \left( 1 - \gamma \right)^{\frac{1 - \gamma}{\gamma}}$$

and this is the only place that $w$ appears in the model. For any $\gamma$, changing $w$ is equivalent to changing the units of measurement of housing. No equilibrium values relevant to utilities then vary with $w$.

The parameter $\gamma$ equals the share of land inputs in housing in our model. Based on the empirical evidence (see the discussion in Epple and Romer, 1991), we set $\gamma = \frac{1}{4}$. Note from (2) that this implies a housing supply elasticity equal to 3.

The values of $\alpha_g, \alpha_h,$ and $\rho$ are set so that in the single jurisdictional equilibrium the median voter chooses $t = .35$, the net-of-tax expenditure share on housing equals .20, and the price elasticity of housing is very close to -1. A $t = .35$ implies a tax rate on

\textsuperscript{14} http://www.census.gov/hhes/www/housing/ahs/99dtcht/tab2-12.html
property value that is realistic, on the order of 2.5% to 3.0%. The expenditure share on housing of .20 is in the range of values estimated in the literature (see Hanushek and Quigly (1980) and Rapoport (1997)). Likewise, the housing market literature indicates a price elasticity close to -1. The implied values of $\alpha_g$ and $\alpha_h$ are, respectively, 0.094 and .356. We set $\rho = -.01$, which implies a price elasticity of housing demand equal to -.993, while also implying SRI and existence of a Tiebout equilibrium.

Table 1: Parameter Values

<table>
<thead>
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<th>Parameters</th>
<th>Baseline Model</th>
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</thead>
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<tr>
<td>$\alpha_x$</td>
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<tr>
<td>$\alpha_h$</td>
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<tr>
<td>$\alpha_g$</td>
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<tr>
<td>$w$</td>
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</table>

b. Baseline Findings.

Table 2 summarizes the findings in the baseline specification, with positive results in the upper panel and normative results in the lower panel. Column 2 of the upper panel

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15 Observed property tax rates are expressed as a percent of property value. In our model, rates are expressed as a percentage of annual implicit rent. Employing the approach of Poterba (1992), Calabrese and Epple (2006) conclude that tax rates on annualized implicit rents can be converted to rates on property values using a conversion rate on the order of 7% to 9%. Thus, our annualized rate of .35 translates to a tax rate on property value on the order of 2.5% to 3%, which is the order of magnitude of observed property tax rates.

16 Cites.

17 A $\rho = 0$ implies a Cobb-Douglas utility function and a price elasticity of demand for housing exactly equal to -1. As discussed above, however, a Tiebout equilibrium does not arise in this case.
shows key values in the Tiebout equilibrium and column 1 corresponding values in the centralized equilibrium where the MA is one jurisdiction. Ignore the other columns for the moment. The Tiebout equilibrium is income stratified, supported by ascending housing prices, although the property tax rates vary little and are very close to that in the centralized equilibrium. Because these tax rates apply to substantially different housing expenditures, the public good levels vary substantially.

The lower panel shows the welfare effects. Only the poor and very rich are better off in the Tiebout equilibrium, with 95% worse off. On average consumers are worse off, with an average compensating variation of $41. The absentee land owners experience a negligible welfare loss. Column 5 reports values in the efficient allocation discussed above.\footnote{As shown in the appendix, the efficient allocation is that with residential choice given efficient head taxes are set in each jurisdiction. This allocation maximizes the welfare gain as we measure it (i.e., using CV) if household income is adjusted for CV (i.e., equals $y + v(y)$). We actually calculate the efficient allocation using $y$ as household income and then calculate the welfare gain in this allocation relative to the property-tax equilibrium. We do this because it is much simpler to compute and because we know the welfare gain will differ trivially from the maximum one (as in Willig, 197(?)). The “efficient allocation” we use to calculate the potential welfare gain is then slightly away from that which would maximize the gain on the Paretian frontier.} Although 74% of households are worse off in this allocation, households experience an average welfare gain of $726 and land owners an average gain of $711. Hence, the environment is one where Tiebout sorting could lead to substantial welfare gains on average, yet these not only fail to be realized in property tax equilibrium but are reversed.
**Table 2**

<table>
<thead>
<tr>
<th>Baseline Model</th>
<th>Property Tax</th>
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<th>Efficient Allocation</th>
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<td>One Jurisdiction</td>
<td>Multiple Jurisdictions</td>
<td>Multiple Jurisdictions</td>
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<tr>
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<td>$12.93</td>
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<td>$13.02</td>
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<tr>
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<td>$17.53</td>
<td>$13.74</td>
<td></td>
</tr>
<tr>
<td>P4 =</td>
<td>$18.43</td>
<td>$13.78</td>
<td>$17.50</td>
<td>$13.72</td>
<td></td>
</tr>
<tr>
<td>P5 =</td>
<td>$22.61</td>
<td>$13.78</td>
<td>$17.34</td>
<td>$13.61</td>
<td></td>
</tr>
<tr>
<td>Y1 =</td>
<td>$28,589</td>
<td>$56,409</td>
<td>$57,950</td>
<td>$57,950</td>
<td></td>
</tr>
<tr>
<td>Y2 =</td>
<td>$39,990</td>
<td>$82,073</td>
<td>$84,067</td>
<td>$84,067</td>
<td></td>
</tr>
<tr>
<td>Y3 =</td>
<td>$56,931</td>
<td>$119,902</td>
<td>$122,768</td>
<td>$122,768</td>
<td></td>
</tr>
<tr>
<td>Y4 =</td>
<td>$89,598</td>
<td>$192,823</td>
<td>$197,811</td>
<td>$197,811</td>
<td></td>
</tr>
<tr>
<td>Median Income J1 =</td>
<td>$36,942</td>
<td>$17,140</td>
<td>$25,741</td>
<td>$26,076</td>
<td></td>
</tr>
<tr>
<td>Median Income J2 =</td>
<td>$33,866</td>
<td>$67,131</td>
<td>$68,840</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Income J3 =</td>
<td>$47,475</td>
<td>$97,029</td>
<td>$99,320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Income J4 =</td>
<td>$69,929</td>
<td>$144,849</td>
<td>$148,279</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Income J5 =</td>
<td>$128,816</td>
<td>$249,542</td>
<td>$255,332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N1 =</td>
<td>39%</td>
<td>68%</td>
<td>69%</td>
<td>69%</td>
<td></td>
</tr>
<tr>
<td>N2 =</td>
<td>15%</td>
<td>13%</td>
<td>13%</td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td>N3 =</td>
<td>15%</td>
<td>9%</td>
<td>9%</td>
<td>9%</td>
<td></td>
</tr>
<tr>
<td>N4 =</td>
<td>15%</td>
<td>6%</td>
<td>6%</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>N5 =</td>
<td>16%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>T1 =</td>
<td>35%</td>
<td>35.33%</td>
<td>35.17%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T2 =</td>
<td>35.24%</td>
<td>35.17%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T3 =</td>
<td>35.20%</td>
<td>35.17%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T4 =</td>
<td>35.14%</td>
<td>35.16%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T5 =</td>
<td>34.96%</td>
<td>35.11%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1 =</td>
<td>$3,830</td>
<td>$1,195</td>
<td>$1,691</td>
<td>$1,952</td>
<td>$1,829</td>
</tr>
<tr>
<td>G2 =</td>
<td>$2,390</td>
<td>$4,410</td>
<td>$4,887</td>
<td>$4,569</td>
<td></td>
</tr>
<tr>
<td>G3 =</td>
<td>$3,359</td>
<td>$6,374</td>
<td>$7,071</td>
<td>$6,612</td>
<td></td>
</tr>
<tr>
<td>G4 =</td>
<td>$4,988</td>
<td>$9,516</td>
<td>$10,679</td>
<td>$9,987</td>
<td></td>
</tr>
<tr>
<td>G5 =</td>
<td>$10,987</td>
<td>$16,393</td>
<td>$20,665</td>
<td>$17,922</td>
<td></td>
</tr>
</tbody>
</table>

**Distributional and Welfare Results**

Interval of income made worse off

<table>
<thead>
<tr>
<th></th>
<th>Low bound</th>
<th>$0</th>
<th>$0</th>
<th>$0</th>
<th>$0</th>
</tr>
</thead>
<tbody>
<tr>
<td>High bound</td>
<td>$349,500</td>
<td>$66,500</td>
<td>$57,500</td>
<td>$65,500</td>
<td></td>
</tr>
</tbody>
</table>

% of pop. made worse off

<table>
<thead>
<tr>
<th></th>
<th>95%</th>
<th>75%</th>
<th>69%</th>
<th>74%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate per capita CV</td>
<td>-41</td>
<td>714</td>
<td>1158</td>
<td>726</td>
</tr>
<tr>
<td>Change in Rents</td>
<td>-0.22</td>
<td>721</td>
<td>-3.17</td>
<td>711</td>
</tr>
<tr>
<td>CV + Change in Rents</td>
<td>-41.68</td>
<td>1434.82</td>
<td>1154.37</td>
<td>1437.13</td>
</tr>
</tbody>
</table>

*The P’s are net housing prices; the Y’s minimum incomes in the jurisdictions; the N’s are the percentage populations; the T’s are the property tax rates; and the G’s are public good expenditures.*
To parcel the welfare losses that arise in Tiebout equilibrium, we calculate two other allocations. Three inefficiencies arise in Tiebout equilibrium. First, property taxation distorts housing consumption with the usual deadweight loss. Second, majority choice of the tax rate reflects the preference of the median-income household in a jurisdiction, which generally differs from the choice that would maximize average welfare. Third, externalities arise in household choice of jurisdiction, which we theoretically characterize in a moment. We will show that these household choice externalities are the primary source of welfare loss.

The second, majority voting inefficiency, is generally believed to be small in these models. To verify this here, we compute multi-jurisdictional equilibrium with majority choice of a head tax. Equilibrium is determined precisely as in the property-tax model, but voting is over a local head tax that fully finances the local public good. Versions of Propositions 1 and 2 apply to this variation of the Tiebout sorting model. Values for this equilibrium are shown in column 3 of Table 2. The head taxes are, of course, equal to the levels of public good provision. Comparing column 3 to column 5, one sees that the allocation is very close to the efficient allocation. The welfare gain relative to the single-jurisdictional equilibrium is 99.8% of the potential welfare gain from sorting.

The welfare loss in the Tiebout equilibrium is then largely attributable to property taxation and household jurisdictional choice externalities rather than voting bias. To

---

19 If median income were equal to mean income and if the (indirect) utility function were linear, then the preference of the median-income household would maximize average welfare. But neither of these conditions is satisfied. These biases are well known.

20 The ascending bundles property trivially regards the head tax, not the housing price. Since the head tax is a deterrent to moving into jurisdiction, it is theoretically possible that housing prices could decline with the level of the public good.
delineate these effects, we *assign* households to jurisdictions as arises in the efficient allocation, but then they vote for a local property tax to finance the public good. Hence, this allocation essentially removes externalities from household choice of jurisdiction, while retaining the property tax distortion (as well as the small voting bias). This is not an equilibrium allocation because some households would prefer to move. The associated values are reported in column 4 of Table 2. We see that most of the potential welfare gain from efficient sorting arises in this allocation; about 80%.

Given its apparent importance in explaining the welfare loss in the Tiebout property-tax equilibrium, the household choice externality warrants further investigation. We derive a theoretical description of this externality in Part B of the appendix, with a summary provided here. We solve the second-best social planner’s problem where the planner maximizes a social welfare function with given weights $\omega(y) > 0$ on household $y$’s utility (and with weight $\omega_R > 0$ on the absentee property owners’ rents); and where the planner (i) assigns households to jurisdictions; (ii) makes a lump-sum income transfer of $r(y)$ to households with income $y$; (iii) sets local property taxes to finance local public good provision; and (iv) the local housing markets clear. In the solution to the planner’s problem, household with income $y$ is assigned to the jurisdiction $j$ that has the highest value of:

$$j \text{ such that } \max_j \left[ \omega(y)V(p^j_i, g_j; y) - \lambda_j(g_j - \tau_j(y)\theta_j) \right];$$

where $\tau_j(y) = \frac{t_j p^j_i h(y) (p^j_i, y + r(y))/(1 + t_j)}{\theta_j}$ is the household’s property tax payment; $\theta_j \in (0,1)$, increases from 0 to 1 as the elasticity of housing supply increases from 0 to $\infty$; and $\lambda_j > 0$ is the multiplier on the jurisdictional budget constraint for provision of the public good. Because a household would, if

---

21 By varying the weighting in the planner’s objective one maps out the (second-best) Pareto Efficient allocations. For some weights, there would be no income transfer, i.e., $r(y)$ would equal 0 for all types. This allocation is most naturally compared to the equilibrium allocation.
permitted, choose the jurisdiction that maximizes \( V(\cdot) \), we define the “jurisdictional choice externality (JCE)” for household \( y \) at jurisdiction \( j \) as:

\[
JCE_j(y) \equiv -\lambda_j(g_j - \tau_j(y)\theta_j).
\] (14)

Letting \( \varepsilon_s \in (0, \infty) \) denote the housing supply elasticity in jurisdiction \( j \), we show in the appendix that:

(a) \( JCE_j(y) \to -\lambda_j g_j \) as \( \varepsilon_s \to 0 \); \( JCE_j(y) \to -\lambda_j (g_j - \tau_j(y)) \) as \( \varepsilon_s \to \infty \); and

(b) \( JCE_j(y) \) is negative for all households in jurisdiction \( j \) with housing demand below the mean there. Consider the case where jurisdictions have very low housing supply elasticity. Then, for given housing demand, the property tax is largely absorbed by property owners in a lower supply price of housing. A household moving into jurisdiction \( j \) would then pay close to zero taxes, implying no pricing of the congestion effect on consumption of the public good. Here JCE is approximated by \(-\lambda_j g_j \). If households choose jurisdictions, those with higher \( g \)’s would then attract too many residents. At the other extreme, as housing supply elasticities tend toward infinity, property taxes are largely passed along to residents and JCE approaches \(-\lambda_j (g_j - \tau_j(y)) \). Since \( g_j \) equals the tax payment of household with average housing demand, households with lower than average demand would have a negative externality, and higher \( g \) jurisdictions would tend to attract too many relatively poorer households in Tiebout property-tax equilibrium. The case with interior housing supply elasticity lies in between.\(^{22}\)

Because property taxation is chosen somewhat inefficiently by voting in the Tiebout property tax equilibrium, the externality measure in (14) is an approximation (see

\(^{22}\) The elasticity of housing demand then also plays a role. See the appendix.)
the appendix). But we have seen that the voting bias is small. Referring to Table 2, we see that the equilibrium populations of the richer jurisdictions are substantially higher and the income levels substantially lower than in the efficient allocation. The fundamental explanation for the welfare loss in Tiebout property tax equilibrium is that the resulting sorting of households is inefficient; it’s not stratified enough! While we are in a second-best economy so that “anything can happen,” we, nevertheless, find this very surprising. Given property taxation, the model indicates that the degree of Tiebout sorting is crucial for welfare gains to be realized. While it is well known that Tiebout sorting is not good for poorer types, that it will sometimes be also bad on average makes it difficult to support.

c. Robustness. To examine robustness, we vary the equilibrium concept with respect to the nature of assumed voter beliefs and key parameters. We focus on the former first. One alternative in the literature to “myopic voting” assumes that, given jurisdictional populations, households anticipate effects in the housing market when voting over the property tax rate. This specification is consistent with the following timing of choices. First, households commit to a jurisdiction, but do not yet purchase a house. Second, they vote in their jurisdiction over a property tax. Last, given the tax rate and jurisdictional population, the housing market clears and the local public good level is established with local government budget balance (and numeraire consumption results). Housing prices are established in the last stage as well. Of course, households anticipate all variables determined in later stages. We refer to this as the “moderate myopia” case.

---

23 The specification suffers from the criticism that the initial commitment to a jurisdiction is simply assumed.
A third case in the literature is referred to as “utility taking,” which has households anticipate migration between jurisdictions when voting. First, households choose an initial jurisdiction where they will vote. They then vote over the local property tax taking as given the equilibrium utility levels obtainable in all jurisdictions other than their own, anticipating in- and out-migration to and from their own jurisdiction whenever such would provide higher utility. Housing market clearance and local government budget balance are satisfied given the property tax, and jurisdictional choices are utility maximizing given all equilibrium values. In the utility-taking equilibrium studied, the initial residence choices correspond to the final residence choices, consistent with equilibrium since households anticipate equilibrium values. While no one actually migrates in equilibrium, the possibility of moving between jurisdictions has substantial effects on equilibrium.24

Stratified multi-jurisdictional equilibria exist in the model in both of these alternative specifications as well. Table 3 summarizes the positive and normative properties of these equilibria for the same parameters as in Tables 1 and 2. The first column of Table 3 shows values for the single-jurisdictional equilibrium, which is the same under moderate myopia and utility-taking since the possibility of migration disappears with one jurisdiction. While households now anticipate absorption of property taxes by land owners when voting since housing markets clear later, the results are virtually identical to those in column 1 of Table 2 where voting takes place after houses

---

24 The utility-taking equilibrium does not correspond to the more appealing subgame-perfect Nash equilibrium where initial jurisdictions are chosen followed by simultaneous voting in jurisdictions over property-tax rates with then migration. In the utility-taking equilibrium, voters anticipate all the effects of migration on their own jurisdiction, but hold constant utilities, not just property taxes, in other jurisdictions. In the related Nash equilibrium, voters would need to anticipate the effects of moving across jurisdictions on all equilibrium values in all jurisdictions (holding constant property tax rates). Computing the Nash equilibrium would be very difficult in a five jurisdiction model. Hopefully, the simpler utility-taking alternative is not a bad approximation.
Table 3: Alternative Voting Specifications

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>$17.092</td>
<td>$12.66</td>
<td>$10.23</td>
</tr>
<tr>
<td>P2</td>
<td>$15.17</td>
<td>$14.05</td>
<td>$13.75</td>
</tr>
<tr>
<td>P3</td>
<td>$16.58</td>
<td>$17.01</td>
<td>$13.74</td>
</tr>
<tr>
<td>P4</td>
<td>$18.38</td>
<td>$20.38</td>
<td>$13.72</td>
</tr>
<tr>
<td>P5</td>
<td>$22.56</td>
<td>$26.22</td>
<td>$13.61</td>
</tr>
<tr>
<td>Y1</td>
<td>$28.587</td>
<td>$24.496</td>
<td>$57.950</td>
</tr>
<tr>
<td>Y2</td>
<td>$39.987</td>
<td>$35.891</td>
<td>$84.067</td>
</tr>
<tr>
<td>Y3</td>
<td>$56.927</td>
<td>$54.136</td>
<td>$122.768</td>
</tr>
<tr>
<td>Y4</td>
<td>$89.593</td>
<td>$89.158</td>
<td>$197.811</td>
</tr>
<tr>
<td>Median Income J1</td>
<td>$36,942</td>
<td>$17,139</td>
<td>$15,344</td>
</tr>
<tr>
<td>Median Income J2</td>
<td>$33,863</td>
<td>$29,801</td>
<td>$43,873</td>
</tr>
<tr>
<td>Median Income J3</td>
<td>$47,472</td>
<td>$69,925</td>
<td>$67,801</td>
</tr>
<tr>
<td>Median Income J4</td>
<td>$128,810</td>
<td>$128,297</td>
<td>$128,297</td>
</tr>
<tr>
<td>N1</td>
<td>39%</td>
<td>32%</td>
<td>69%</td>
</tr>
<tr>
<td>N2</td>
<td>15%</td>
<td>17%</td>
<td>13%</td>
</tr>
<tr>
<td>N3</td>
<td>15%</td>
<td>18%</td>
<td>9%</td>
</tr>
<tr>
<td>N4</td>
<td>15%</td>
<td>17%</td>
<td>6%</td>
</tr>
<tr>
<td>N5</td>
<td>16%</td>
<td>16%</td>
<td>3%</td>
</tr>
<tr>
<td>T1</td>
<td>34.57%</td>
<td>34.82%</td>
<td>12.41%</td>
</tr>
<tr>
<td>T2</td>
<td>34.75%</td>
<td>22.70%</td>
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</tr>
<tr>
<td>T3</td>
<td>34.72%</td>
<td>35.07%</td>
<td></td>
</tr>
<tr>
<td>T4</td>
<td>34.68%</td>
<td>50.07%</td>
<td></td>
</tr>
<tr>
<td>T5</td>
<td>34.55%</td>
<td>64.09%</td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>$3,795</td>
<td>$1,182</td>
<td>$447</td>
</tr>
<tr>
<td>G2</td>
<td>$2,366</td>
<td>$1,493</td>
<td>$4,569</td>
</tr>
<tr>
<td>G3</td>
<td>$3,325</td>
<td>$3,102</td>
<td>$6,612</td>
</tr>
<tr>
<td>G4</td>
<td>$4,939</td>
<td>$6,228</td>
<td>$9,987</td>
</tr>
<tr>
<td>G5</td>
<td>$10,889</td>
<td>$16,524</td>
<td>$17,922</td>
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</table>

**Distributional and Welfare Results**

Interval of income made worse off

<table>
<thead>
<tr>
<th></th>
<th>Low bound</th>
<th>$8,500</th>
<th>$0</th>
<th>$0</th>
</tr>
</thead>
<tbody>
<tr>
<td>High bound</td>
<td>$349,500</td>
<td>infinity</td>
<td>$65,500</td>
<td></td>
</tr>
</tbody>
</table>

% of pop. made worse off

|                      | 95%       | 100%   | 74%  |

Aggregate per capita CV

|                      | -41       | -677   | 726  |

Change in Rents

|                      | 0.31      | -215   | 703  |

CV + Change in Rents

|                      | -41.12    | -892   | 1428.57 |

*Parameters are as in Tables 1 and 2.
have been purchased. Likewise, the multi-jurisdictional equilibrium with moderate myopia is virtually identical to that in the baseline case with multiple jurisdictions; compare the second columns in Tables 2 and 3. Again, then, a per capita welfare loss arises in going to the Tiebout sorting equilibrium, differing by less than one dollar between the baseline and moderate myopia voting specifications. Table 3 also shows the efficient allocation. This allocation is precisely as in Table 2 since the parameters are the same, while the potential welfare gains relative to the single-jurisdictional counterpart vary slightly because the single-jurisdiction counterparts vary slightly.

Multi-jurisdictional equilibrium under utility-taking, summarized in the column 3 of Table 3, differs more substantially from the baseline Tiebout equilibrium. The property-tax rate ascends steeply as jurisdictions get wealthier in the utility-taking case, while virtually constant in the moderate myopia and baseline cases. In the utility-taking case, the pivotal voter in poorer jurisdictions lowers the tax rate in an effort to keep in richer households. In richer jurisdictions, the pivotal voter increases the tax rate to drive out poorer households knowing that richer households are reluctant to leave. In the richest jurisdiction, the richest types have no viable alternatives! The public good levels rise very steeply in this case. Remarkably, virtually everyone is worse off in the Tiebout equilibrium, and the per capita welfare loss is substantially higher than in the other cases. We can conclude that it is not the voter myopia specification that underlies the welfare loss we find.

Now consider robustness with respect to the parameters. Recalling the discussion of Table 1, there are four parameters that might be varied, $\alpha_g$, $\alpha_h$, $\gamma$, and $\rho$. We return to

---

25 Those with income very near zero are actually better off, but the poorest 1% of the population are on average worse off.
the case of myopic voting and vary these parameters individually from the baseline values. Table 4A reports the welfare effects for variations in $\rho$, with the effects on all equilibrium values reported in the appendix. Table 4B likewise reports welfare effects for variations in $\gamma$. For each parameter variation, the welfare effects relative to the single jurisdiction benchmark and the potential welfare gain from the efficient allocation are presented.

Table 4A: Sensitivity with Respect to $\rho$

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Multiple Jurisdiction Equilibrium</th>
<th>Efficient Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = -.05$</td>
<td>Low bound $39,500$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\rho = -.05$</td>
<td>High bound $68,500$</td>
<td>$66,500$</td>
</tr>
<tr>
<td>$\rho = -.1$</td>
<td>Low bound $23%$</td>
<td>$75%$</td>
</tr>
<tr>
<td>$\rho = -.1$</td>
<td>High bound $89$</td>
<td>$813$</td>
</tr>
<tr>
<td>$\rho = -.1$</td>
<td>Change in Rents $-1.87$</td>
<td>$681$</td>
</tr>
<tr>
<td>$\rho = -.1$</td>
<td>CV + Change in Rents $86.79$</td>
<td>$1493.50$</td>
</tr>
</tbody>
</table>

Table 4B: Sensitivity with Respect to $\gamma$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Multiple Jurisdiction Equilibrium</th>
<th>Efficient Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = .2$</td>
<td>Low bound $0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\gamma = .2$</td>
<td>High bound Infinity</td>
<td>$62,500$</td>
</tr>
<tr>
<td>$\gamma = .3$</td>
<td>Low bound $100%$</td>
<td>$72%$</td>
</tr>
<tr>
<td>$\gamma = .3$</td>
<td>High bound $-390$</td>
<td>$888$</td>
</tr>
<tr>
<td>$\gamma = .3$</td>
<td>Change in Rents $0.49$</td>
<td>$570$</td>
</tr>
<tr>
<td>$\gamma = .3$</td>
<td>CV + Change in Rents $-389.39$</td>
<td>$1458.66$</td>
</tr>
</tbody>
</table>

We have also varied the parameters under the alternative specifications of voting. The welfare effects essentially combine those we find from the two types of variations and thus are not reported. These are available on request.

---

26 We have also varied the parameters under the alternative specifications of voting. The welfare effects essentially combine those we find from the two types of variations and thus are not reported. These are available on request.
We see in Table 4A that the potential welfare gain from sorting increases as $\rho$ declines. This is because the elasticity of substitution of goods is declining, so that better matching of consumption to preferences has more value. While the welfare loss we have identified disappears as $\rho$ declines, the per capita gain from Tiebout sorting under property taxation is small and the proportion of the potential gain is below 17%.

We examine a lower and higher value of $\gamma$ relative to the baseline calibration in Table 4B. Recall that a lower value of $\gamma$ corresponds to a lower share of land inputs in housing production and thus a higher housing supply elasticity. While the potential welfare gain from sorting increases with lower $\gamma$, the welfare loss from the Tiebout property tax equilibrium increases. The main effect appears to be that the residential choice externality worsens because housing prices react less to changes in demand. The reverse applies if $\gamma$ is increased, and we do find a per capita welfare gain from Tiebout sorting with property taxation. Only 13% of the potential gain is realized, however.

Other sensitivity analysis …

4. Concluding Remarks

Inequalities in the local public finance of schooling have lead to a revolution in education policy in much of the U.S. Few economist would challenge the notion that such Tiebout sorting had lowered the welfare of many, especially the poor. However, distributional issues aside, we perceive that most economists understand the Tiebout process to be efficiency enhancing. While the presence of inefficiencies in local property tax equilibria is understood, we know of no research that quantifies the net effects of such allocations. In pursuing such an analysis here, we have found that these inefficiencies are
substantial and overturn potential per capita welfare gains in a standard model that has been calibrated using the best empirical evidence. It is a bit shocking that the analysis here runs counter to our basic intuition concerning the Tiebout process.

The finding that per capita welfare losses arise in a reasonable model has led us to investigate the main source of the inefficiency. We find that the externality in choice of residence is the primary source of loss. It is almost as surprising as the finding of a net loss that the mobility central to the Tiebout argument underlies the inefficiency.

Our finding might help to explain the prevalence of residential zoning restrictions. In Calabrese, Epple, and Romano (2007), we pursue a theoretical and quantitative analysis of residential zoning that supports Hamilton’s argument: Zoning serves as a substitute for head taxation. We show that local public choice of a zoning restriction on housing quality combined with a property tax closely mimics head taxation and almost all potential Tiebout welfare gains are realized.

This paper does not, of course, refute Tiebout’s argument. Rather, it tells a cautionary tale about applying first-best arguments in a second-best environment. Moreover, our model is very Spartan. It is a three-good model with no peer effects or variation in preferences other than deriving from variation in income. We think that it is of interest to explore further the quantification of the welfare effects of local public goods equilibria.
References
Appendix

A. An Efficient Benchmark. We have shown that Tiebout sorting that is supported by public choice of property taxation can lead to aggregate welfare losses relative to the single jurisdiction equilibrium. Here we find an efficient benchmark for comparing these losses. The benchmark is the welfare gain relative to the single jurisdiction equilibrium, measured analogously (using aggregate CV), that would arise from efficient household sorting and efficient choice of (non-distorting) head taxes in each jurisdiction. We compute this welfare gain in our model, thus demonstrating the potential for gains from residential sorting and providing quantification of those potential gains. This analysis also serves to confirm that, if local choice of the public good is by head tax and determined efficiently, then equilibrium residential choice would be efficient. That is, there would then be no externalities in residential choice.

The efficient allocation minimizes the aggregate compensating variation relative to the centralized property-tax equilibrium.\footnote{This is a Pareto Efficient allocation as discussed further below.} Let a superscript c indicate a value in the centralized equilibrium. Let \( h_d(p^c,y) \) denote the ordinary demand for housing, where \( p^c \) is the gross or consumer price of housing.\footnote{We have written housing demand as independent of public good provision, as arises with a CES utility function for example. The results below do not depend on this independence.} Then utility in the centralized equilibrium is:

\[
U^c(y) = U(y - p^c h_d(y, p^c), h_d(y, p^c), g^c),
\]

(A.1)

where we use that every household faces the same housing price and public-good supply in this equilibrium.\footnote{Housing-market clearance in each jurisdiction, the voting equilibrium conditions, and balanced budget conditions are necessary to describe fully the equilibrium. For our purposes here, we need not write out these conditions.} In the efficient allocation, let \( T_j \) denote the head tax in jurisdiction \( j \) and \( p_j \) housing price there. Since there is no property taxation, the supplier and
consumer housing price are the same in the efficient allocation (where a market-clearing allocation in each housing market is assumed but is also efficient). Let \( v_j(y) \) denote household compensating variation if household with income \( y \) is assigned to jurisdiction \( j \) in the efficient allocation, which satisfies:

\[
U(y - T_j + v_j - p_j h_d(p_j, y - T_j + v_j), h_d(p_j, y - T_j + v_j), g_j) = U^c(y). \tag{A.2}
\]

The planner assigns households to jurisdictions in the efficient allocation. Let \( \alpha_j(y) \in [0,1] \) denote the proportion of households with income \( y \) assigned to jurisdiction \( j \). Then aggregate household compensating variation is given by:

\[
CV = \sum_{j=1}^{J} \int_{0}^{\infty} \alpha_j(y)f(y)v_j(y)dy. \tag{A.3}
\]

We include the compensating variation of absentee landlords in the planner’s objective function as well, to avoid incentives to redistribute rents from them to households. Their compensating variation is the negative of the change in housing rents from those in the centralized equilibrium. Let \( R^c \) denote housing rents in the latter equilibrium. The planner’s problem is:

\[
\text{MIN}_{\alpha_j(y), T_j} \quad CV + R^c - \sum_{j=1}^{J} \int_{0}^{\rho_j} H_j^1(p)dp; \tag{A.4}
\]

subject to:

\[
U(y - T_j + v_j - p_j h_d(p_j, y - T_j + v_j), h_d(p_j, y - T_j + v_j), g_j) = U^c(y) \text{ for all } y; \tag{A.5}
\]

\[
\alpha_j(y) \geq 0 \text{ for all } j \text{ and } y; \tag{A.6}
\]

\[
\sum_{j=1}^{J} \alpha_j(y) = 1 \text{ for all } y; \tag{A.7}
\]

\[
T_j = g_j \text{ for all } j; \tag{A.8}
\]
\[ \int_0^\infty \alpha_j(y) f(y) h_d(p_j, y - T_j + v_j) dy = H_s^e(p_j) \text{ for all } j. \] (A.9)

Constraint (A.5) is just (A.2) restated, hence determines \( v_j(y) \). Constraints (A.6) and (A.7) ensure that \( \alpha_j(y) \in [0,1] \) and that every household is assigned. Constraint (A.8) is the jurisdictional balanced budget requirement, given head taxes and a perfectly congested public good. Finally, (A.9) is housing market clearance in each jurisdiction.

Using (A.3), substituting for \( T_j \) everywhere from (A.8), and ignoring (A.6) for the moment, form the Lagrange function:

\[
L = \sum_{j=1}^{J} \int_0^\infty \alpha_j f \cdot [v_j - \lambda_j(y)(U^e - U^c)] dy + R_j^e - \sum_{j=1}^{J} \int_0^\infty H_s^e(p) dp \\
+ \eta(y) \left[ \sum_{j=1}^{J} \alpha_j(y) - 1 \right] + \sum_{j=1}^{J} \tau_j \left[ \int_0^\infty \alpha_j f h_s^e dy - H_s^e \right];
\] (A.10)

where we have suppressed arguments, a superscript \( e \) indicates the function is evaluated at the planner’s allocation, and \( (\lambda_j(y), \eta(y), \tau_j) \) are Lagrange multipliers. The first-order conditions are:

for all \( y \) and \( j \), \( L_{\alpha_j} = [f v_j + \eta(y) + f \tau_j h_s^e] \alpha_j \leq 0 \), with equality if \( \alpha_j > 0 \); \( ^{30} \) (A.11)

\[
\text{for all } y \text{ and } j, L_{v_j} = \alpha_j f [1 - \lambda_j(y) U^e_x + \tau_j \frac{\partial h_s^e}{\partial y}] = 0; \quad (A.12)
\]

\[
\text{for all } j, L_{\lambda_j} = \int \alpha_j f \left[ \lambda_j(y)(U^e_x - U^c_x) - \tau_j \frac{\partial h_s^e}{\partial x} \right] dy = 0; \quad (A.13)
\]

\[
\text{for all } j, L_{\tau_j} = \int \alpha_j f \lambda_j U^e_x h_d^e dy - H_s^e(p_j) + \tau_j \left[ \int \alpha_j f \frac{\partial h_s^e}{\partial p_j} dy - H_s^e \right] = 0. \quad (A.14)
\]

From (A.9), (A.12), and (A.14), we obtain

\[ \tau_j = 0 \text{ and } \lambda_j(y) = 1/U^e_x. \] (A.15)

\(^{30}\) This condition takes into account constraint (A.6).
Substituting (A.15) into (A.13) yields the standard Samuelsonian conditions for the provision of the local public goods. Using (A.15), from (A.7) and (A.11), we then have the result that:

\[
\text{if } v_k(y) = \min_{j=1,2,...,J} v_j(y) \text{ is unique, then:} \\
\alpha_k(y) = 1 \text{ and } \alpha_j(y) = 0 \text{ for } j \neq k. 
\]

(A.16)

If there is not a unique minimal \(v_k\) in (A.16), then the optimal assignment will generally split household types \(y\) among jurisdictions.

Remarks:

1. An allocation that solves problem (A.4) is Pareto Efficient. To confirm this, suppose that a Pareto Improvement is possible beginning in a solution to (A.4). It would be implied that if we compute the values of \(v_j(y)\) and the housing rents in the latter allocation, that some values would be lower and none higher, a contradiction.

2. One can interpret \(v_j(y)\) as the negative of the household’s reservation price for living in jurisdiction \(j\) where the alternative is the centralized equilibrium. If the public good is a normal good, as in the model in the text, then surplus from higher provision levels will rise with income and an income-stratified allocation will be efficient. Moreover, this efficient allocation would be an equilibrium allocation if local public goods were efficiently determined (i.e., satisfy the Samuelsonian conditions) and financed by a head tax and if households have endowed income \(y + v_j(y)\). The latter follows since no household would prefer to move to another jurisdiction or this would imply that \(v_j\) could be lowered.\(^{31}\) A Tiebout allocation is an efficient allocation with head taxes if the tax is determined efficiently. The departure from efficiency in our model is then a consequence

\(^{31}\) There would be no externalities from moving since housing is efficiently priced and the household would pay for his provision of the public good.
of property taxation and public choice of the level of the tax. One way to parcel these
effects is to examine compare aggregate compensating variation in the equilibrium
allocation with property taxation, in the equilibrium allocation with a head tax (chosen by
voting), and in the efficient allocation.

B. The Household Residential Choice Externality. In this part of the appendix we
analyze constrained social efficiency and examine externalities in household choices of
jurisdictions. The approach is to solve the social planner’s problem and determine
whether it is consistent with household jurisdictional choice. We refer to household with
income y as “household y.” Let \( \omega(y) > 0 \) denote the planner’s weight on utility in the
social welfare function on household y, \( \omega_R > 0 \) the same for absentee property owners,
\( r(y) \) the planner’s transfer of income to household y, and \( R \) the transfer to property
owners.\(^{32}\) Let:

\[
V(p^i, r(y) + T_i, g_i; y) \equiv \text{Max}_h U(y + r(y) - T_i - p^i h, h, g_i);
\]

(B.1)

denote household y’s indirect utility function in community i, allowing a head tax \( T_i \) and
using here \( p^i \) to denote the gross price of housing in community i. Let \( h_d(p, y + r(y) - T) \)
denote housing demand, which we assume for simplicity is independent of \( g \).\(^{33}\) We study
versions of the following planner’s problem:

\(^{32}\) We assume absentee property owners have quasi-linear utility function and the social planner treats all of
them equivalently.

\(^{33}\) The analysis is easily extended to allow dependence of housing demand on \( g \). We discuss below when
this matters.
Max \[ \sum_{i=1}^{J} \left\{ \int y \omega(y) V(p^i, r(y) + T_i, g_i; y) \alpha_i(y) f(y) dy + \omega_R \left( R \int \frac{p^i}{1+t_i} H_i(z) dz \right) \right\} \]  
\text{s.t.} \quad R + \int y r(y)f(y)dy = 0;  
\int y h_d(p^i, y + r(y) - T_i) \alpha_i(y) f(y) dy = H^i_y(p^i / (1 + t_i)), i = 1, 2, ..., J;  
T_i \int y \alpha_i(y) f(y) dy + \frac{t_i p^i}{1 + t_i} H^i_y(p^i / (1 + t_i)) = g_i \int y \alpha_i(y) f(y) dy, i = 1, 2, ..., J;  
\alpha_i(y) \in [0,1] \text{ and } \sum_{i=1}^{J} \alpha_i(y) = 1 \forall y; 

where \( \int y \) indicates integration over the support of \( y \). In the planner’s problem, \( \alpha_i(y) \) is an assignment function that indicates the proportion of households \( y \) assigned to community \( i \). The problem allows both head and property taxation. It is a second-best social maximization problem because it requires housing market clearance in each community, government budget balance within each community and takes jurisdictional boundaries (hence community housing supplies) as given.\(^34\) These restrictions are because we compare the solution to the market allocation that satisfies these requirements.

A solution to the problem is constrained Pareto Efficient, constrained by the latter restrictions. As the social weights \( (\omega(y), \omega_R) \) are varied alternative (constrained) Pareto Efficient allocations are determined. If the constrained utility possibilities set is convex, then all Pareto Efficient allocations are a solution to the problem for some set of weights.\(^35\) Note, too, that \( r(y) = R = 0 \) will arise in the solution to the planner’s problem.

\(^{34}\) Which constraints actually matter, however, depend on which version of the planner’s problem is considered.

\(^{35}\) If the constrained utilities possibilities set is not convex, then one can still find all Pareto Efficient allocations as extrema of the planner’s problem. Some solutions would be local minima of the problem but would satisfy the same (first-order) conditions we derive below.
for some weights \( (\omega(y), \omega_R) \), which is the case most naturally compared to the market equilibrium allocation.

To solve the planner’s problem, write the Lagrangian function:

\[
L = \sum_{i=1}^{J} \left\{ \int_y \omega V^i(\cdot) \alpha_i fdy + \omega_R \left( R/I + \int_0^{p^i/(1+t_i)} H^i_i dz \right) \right\} + \sum_{i=1}^{J} \lambda_i \left( [T_i - g_i] \right) \int_y \alpha_i fdy + \frac{t_i p^i}{1+t_i} H^i_i \left( p^i/(1+t_i) \right) + \sum_{i=1}^{J} \eta_i \left[ \int_y h_d(p^i, y + r(y) - T_i) \alpha_i fdy - H^i_i \left( p^i/(1+t_i) \right) \right] + \Omega \left[ R + \int_y r(y) fdy \right]; \tag{B.7}
\]

where \( \lambda_i, \eta_i, \) and \( \Omega \) are multipliers, \( V^i \) has arguments corresponding to community \( i \), and constraint (B.6) is taken account of below. The first-order condition on \( (r(y), R) \) can be written:

\[
-\Omega = \sum_{i=1}^{J} \omega U^i_i \alpha_i + \sum_{i=1}^{J} \eta_i \frac{\partial h^i_i}{\partial y} \alpha_i = \omega_R \forall y; \tag{B.8}
\]

where \( U^i_i \) is the partial derivative of \( U \) with respect to its first argument and the superscript indicates evaluation of the function at community \( i \) values. (We continue to use such notation below.) Let:

\[
MSV_i(y) \equiv L_{\alpha_f} = \omega V^i + \lambda_i \left[ T_i - g_i \right] + \eta_i h^i_d \tag{B.9-1}
\]

denote the marginal social value of assigning a measure \( \alpha_f(y) \) of household type \( y \) to community \( i \), which equals the first variation in the Lagrangian with respect to type \( y \).\(^{36}\)

Using this notation and taking account of (B.6), the optimal admission criterion can be written\(^{37}\):

\[^{36}\] This is scaled by \( f(y) \) just to be comparable across types.

\[^{37}\] If the middle line of (B.9-2) characterizes the solution for a household \( y \), then the summation constraint in (B.6) comes into play. However, we will focus on cases where this does not characterize the optimum as discussed below.
\( \alpha_i(y) = 0 \) \( \in [0,1] \) as MSV \( \langle y \rangle \) \( \leq \) \( \max_{j \in i} \) MSV \( \langle y \rangle \) \( \forall y \). \hfill (B9-2)

To write out the remaining first-order conditions, let:

\[
N_i \equiv \int y \alpha_i(y)f(y)dy \quad \text{and} \quad \varepsilon_i \equiv \frac{H_i'}{H_i (1 + t)} \frac{p_i}{p}
\]

denote respectively the number of residents of community \( i \) and the elasticity of housing supply. We have:

\[
L_{\varepsilon_i} = 0 \rightarrow -\omega_i + \lambda_i (1 - t \varepsilon_i^i) + \frac{1 + t}{p} \eta \varepsilon_i^i = 0; \hfill (B.11)
\]

\[
L_{t} = 0 \rightarrow -\int y \omega U_i^i \alpha_i fdy + \lambda_i N_i - \eta_i \int y \frac{\partial h_i}{\partial y} \alpha_i fdy = 0; \hfill (B.12)
\]

\[
L_{\varepsilon_i} = 0 \rightarrow \int y \omega U_i^i \alpha_i fdy - \lambda_i N_i = 0; \hfill (B.13)
\]

and

\[
L_{p} = 0 \rightarrow \frac{1 + t}{H_i} \left[ \eta \int y \frac{\partial h_i}{\partial p} \alpha_i fdy - \int y \omega U_i^i h_i^i \alpha_i fdy \right] + t \lambda_i (1 + \varepsilon_i^i) - \frac{\eta_i (1 + t) \varepsilon_i^i}{p} + \omega_i = 0. \hfill (B.14)
\]

We restrict attention to cases where it is optimal to have differentiated communities. This conforms to cases such that \( \alpha_i(y) = 1 \) for some community \( i \) for a.e. household (see (B.9-1) and (B.9-2)). We refer to such allocations as “differentiated.”

The alternative has homogeneous communities. Whether differentiation is optimal depends on the utility weights in the social welfare function. Essentially we want to examine whether the equilibrium allocation \emph{with differentiation} is associated with localized externalities in community choice.
First we confirm what is very intuitive: The social optimum will have no property taxation, just head taxes. More to our purposes, unilateral household choice would be consistent with the efficient allocation. We will then go on to examine the “more constrained” problem where taxes are suboptimal.

**Proposition B1.** In an efficient differentiated allocation: (i) \( t_i = \eta_i = 0 \) and \( T_i = g_i \); (ii) \( g_i \) satisfies the Samuelsonian condition; and (iii) households are assigned to the community where \( V_i \) is at a maximum.

**Proof of Proposition B1.** (i) First we show that \( t_i = \eta_i = 0 \). From (B.8) and that the allocation is differentiated,

\[
\omega U_i + \eta_i (\partial h^i / \partial y) = \omega R \text{ for all households } y \text{ assigned to community } i. \tag{B.15}
\]

Multiply through (B.15) by \( \alpha_i \) and integrate to obtain:

\[
\int_y \omega U_i \alpha_i f dy + \eta_i \int_y \frac{\partial h^i}{\partial y} \alpha_i f dy = N_i \omega R. \tag{B.16}
\]

Then (B.16) and (B.12) imply:

\[
\lambda_i = \omega R. \tag{B.17}
\]

Also (B.17) and (B.11) imply:

\[
t_i \omega R = \frac{\eta_i (1 + t_i)}{p_i}. \tag{B.18}
\]

Since \( \omega R > 0 \), if \( t_i = 0 \), then \( \eta_i = 0 \) and the reverse. Now we show that \( t_i \neq 0 \) implies a contradiction. Multiply through (B.15) by \( h^i \alpha_i f \) and integrate to obtain:

\[
\int_y \omega U_i h^i \alpha_i f dy = \omega R H_i - \eta_i \int_y \frac{\partial h^i}{\partial y} h^i \alpha_i f dy; \tag{B.19}
\]
where we have substituted the housing market clearance condition ((B.4)). Now substitute from (B.17), (B.18), and (B.19) into (B.14) to get:

\[
\eta_i \left\{ \frac{1 + t_i}{H_i} \left( \int_y \frac{\partial h_d^i}{\partial p^i} \alpha_i f dy + \int_y \frac{\partial h_d^i}{\partial y} h_d^i \alpha_i f dy - \frac{1 + t_i}{t_i p^i} H_i \right) + \frac{1 + t_i}{p^i} (1 + \epsilon^i_s) - \frac{1 + t_i}{t_i p^i} \epsilon^i_s + \frac{1 + t_i}{t_i p^i} \right\} = 0.
\]

This simplifies to:

\[
\eta_i \left\{ \int_y \left( \frac{\partial h_d^i}{\partial p^i} + \frac{\partial h_d^i}{\partial y} h_d^i \right) \alpha_i f dy \right\} = 0.
\]

(B.20)

The term in parentheses in the integrand in (B.20) is the slope of the compensated demand for housing and is then negative. Hence, the integral term is negative, implying \(\eta_i = 0\). This contradicts (B.18), so it must be that \(t_i = \eta_i = 0\).

Since \(t_i = 0\), \(T_i = g_i\) by local budget balance (i.e., (B.5)).

(ii) Using \(\eta_i = 0\), substitute from (B.12) into (B.13). Then use that \(\omega U_i^i\) equals a constant from (B.15) to obtain the Samuelsonian condition for a congested public good:

\[
\int_y \frac{U_i^i}{U_i^i} \alpha_i f dy = N_i.
\]

(B.21)

(iii) Using the results in part (i), (B.9-1) and (B.9-2) imply that a household is optimally assigned to the community where \(V_i\) is maximized.

Remarks:

1. The main interpretation is that if a community uses head taxation to provide the local public good optimally, then household choice of communities would be socially optimal. Likewise, competitive provision of housing is efficient. The non-distorted price of housing and the head tax efficiently price access to communities. There are no externalities in community choice.
2. These results hold as well if housing demand depends on $g_i$, as is straightforward to confirm.

3. The fact that the multiplier ($\eta_i$) on the housing market clearance condition (B.4) equals 0 is crucial to the proof and may not be intuitive. By the usual “Envelope Theorem argument” applied to the planner’s problem, this may seem to suggest that exogenously increasing a community’s housing stock would not be welfare improving. This seeming paradox is resolved by noting that such an increase in the housing stock in community $i$ would show up three places in the Lagrangian, and in fact social welfare increases with the housing stock at rate $\omega_R p^1$ at the optimum. Related to this, the constraint (B.4) should be interpreted as a constraint on housing prices, not on the supply of housing.

To determine the character of jurisdiction choice externalities in the property tax equilibrium, we now examine the planner’s problem assuming head taxation is not allowed. Set $T_i = 0$ everywhere above and drop the first-order condition describing the efficient choice of $T_i$, i.e., (B.12). With $T_i = 0$, the other first-order conditions remain valid.\(^{38}\) Of course, $t_i$ will be positive here. We are assuming that $t_i$ is optimally chosen by the planner, but will also discuss later the alternative where $t_i$ is also suboptimal. Household choice of a jurisdiction would now be associated with an externality, and its character is the focus. With reference to (B.9-1)-(B.9-2), the value of the “jurisdictional choice externality (JCE)” by household $y$ of jurisdiction $i$ is given by:

$$
\text{JCE}_i(y) \equiv -\lambda_i g_i + \eta_i h_a(p_i, y + r(y)).
$$

(B.22)

To convey the main results, we introduce a bit more notation. Let $h_c(\cdot)$ denote a household’s compensated demand function for housing. Let:

\(^{38}\) We continue to study cases with differentiated allocations.
\[ \tau_i(y) \equiv \frac{t_i p^i h_i(p^i, y + r(y))}{(1 + t_i)}; \quad (B.23) \]

and

\[ \theta_i \equiv \frac{(1 + t_i) \varepsilon^i}{(1 + t_i) \varepsilon^i - \frac{p^i}{H^i} \int_y \frac{\partial h^i}{\partial p} \alpha fdy}. \quad (B.24) \]

Observe that \( \tau_i \) is household \( y \)'s tax payment in jurisdiction \( i \), and \( \theta_i \in [0,1) \) where the integral term in the denominator of \( \theta_i \) is a weighted average of households’ compensated demand elasticities. The main result here is:

**Proposition B2.** (i) The jurisdictional choice externality in the planner’s solution is:

\[ JCE_i(y) = -\lambda_i [g_i - \tau_i(y) \theta_i]; \quad (B.25) \]

with

\[ \lambda_i = \int_y \omega U^i \alpha_f dy \frac{N_i}{> 0}. \quad (B.26) \]

(ii) \( JCE_i(y) \to -\lambda_i g_i \) as \( \varepsilon^i \to 0 \); \( CCE_i(y) \to -\lambda_i (g_i - \tau_i(y)) \) as \( \varepsilon^i \to \infty \).

(iii) \( JCE_i(y) \) is negative for all households in community \( i \) with housing demand below the mean.

**Proof of Proposition A2.** (i) Substitute from (B.11) and (B.19) into (B.14) to obtain:

\[ \eta_i = -\lambda_i \frac{t_i p^i \varepsilon^i}{\frac{p^i}{H^i} \int_y \frac{\partial h^i}{\partial p} \alpha fdy - (1 + t_i) \varepsilon^i}. \quad (B.27) \]

Substituting (B.23), (B.24), and (B.27) into (B.22) yields (B.25). Expression (B.26) follows immediately from (B.13) and the value is obviously positive.
(ii) These results follow trivially from (B.25) and the definition of $\theta_i$ (i.e., (B.24)).

(iii) This follows from (B.25) since $\theta_i < 1$ and $g_i$ equals the tax payment of the household in community $i$ with average housing consumption.

Remarks:

1. The main implication is that an equilibrium allocation with efficient property tax would have too many households choosing jurisdictions with high $g$’s, especially poorer households. The most extreme case (ironically) has housing supply elasticities equal to 0. For given housing demand, the entire tax is, of course, absorbed by property owners; and there is no distortion in the housing market. But there is not pricing of the congestion externality from consumption of the (perfectly congested) public good. Capitalization of higher $g$ in housing prices would induce those with relatively higher values of $g$ to choose higher $g$ jurisdictions, but this implicit pricing is incomplete because it is substantially absorbed by property owners. As supply elasticity rises, the implicit pricing of the congestion cost increases, but those with lower housing demand pay less and thus would crowd richer jurisdictions. The equilibrium model in the paper does not have efficient choice of property tax due to majority choice, but this distortion is typically of second order. The theoretical distortion identified here – poorer types crowding richer jurisdictions – we find to be key to the welfare losses from Tiebout sorting with property taxation.

2. Unless housing supply elasticity equals 0, note from (B.27) that the multiplier ($\eta$) on the housing market clearance condition is no longer 0, but positive. This is because the gross housing price inefficiently deters housing consumption and this alone inefficiently deters moving into a jurisdiction. This effect is weighed, however, against the congestion

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39 Household community choice would be efficient if the local public good were not congested.
externality in JCE (which we have seen to “typically” be the dominant effect). If the tax \( t_i \) is inefficient, one finds that\(^{40}\):

\[
\eta_i = \frac{t \left[ \int_y \omega U^i_y h^i_q \alpha_i fdy - (1 + \varepsilon^i_y) \frac{H^i}{N_i} \int_y \omega U^i_y \alpha_i fdy \right]}{\int_y \frac{\partial h^i_c}{\partial p} \alpha_i fdy + t \int_y \frac{\partial h^i_u}{\partial p} \alpha_i fdy - \frac{(1 + t_i)\varepsilon^i H^i}{p^i}}.
\]

Now \( \eta_i \) can be positive or negative. This is because \( g_i \) might be over-provided (given property taxation) and limiting housing consumption would reduce this distortion.

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\(^{40}\) This is found by solving the planner’s problem with \( t_i \) exogenous, hence suppressing condition (B.11). We continue to assume that \( T_i \) must be 0.