Public-Private Input Substitution and Growth in a Two-Stage Education System

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Abstract

The paper studies the interaction between public and private inputs in a two-stage education framework (K-12 and tertiary education). We find that an increase in the overall educational public spending crowds out the total level of private contributions and increases the share of total private resources that households devote to K-12 education. Given a fixed level of public funding, a higher share of K-12 public funding prompts households to spend more on education overall and in the same time, to allocate a higher share of their investments towards higher education. In terms of optimal policies, our results suggest that the share of public spending devoted to K-12 should be high irrespective of the size of the public budget. We also find that higher income taxes produce a reallocation of private resources towards the first educational stage. Consequently, the optimal fraction of public budget allocated to higher education is higher in economies with higher taxes.

JEL classification: H23, H4, H52, I2

Keywords: public inputs, two-stage education, parental transfers, human capital accumulation

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1 Introduction

A highly educated labor force is closely associated with increased innovation and hence, instrumental for economic growth. Since the education process is hierarchical in nature, the basic education stages are prerequisites for subsequent skills accumulation. Moreover, since both the wealth of nations and the individual opportunities depend on the quality and the quantity of human capital available, governments and households are equally interested in education. This brings up a twofold question: First, how are resources allocated across different stages? Second, how do private and public education expenditures interact, in the aggregate and at each stage?

Data on education spending in OECD countries (see Figure 1 below) reveals significant variations both in the size and the structure of spending across the primary and secondary (K-12) and tertiary (college) education. Aghion et al. (2005) mention the sizeable difference between investment in higher education in the European Union (1.1% of its total annual gross domestic product) versus the United States (3%). In the US the total public spending on education increased on average by 3.6% annually between 1983-2004 with K-12 public spending representing around 70% of the total educational budget. The European Union has been rethinking the role of higher education as part of the Lisbon Agenda, a more comprehensive strategy to boost competitiveness and employment (see Jacobs and van der Ploeg (2005)), with a view of increasing funding of tertiary education.

Households’ reaction to public policy in education can give rise to unintended effects depending on the degree of substitutability between private and public inputs in the production of human capital. When a two stage education system is considered, the interaction between these two types of resources can be quite complex. A closer look at the case of US versus EU reveals that public spending on education as a fraction of GDP is approximately the same, while private spending is three times higher in the US than in the EU. Moreover, the allocation of resources across different education stages seems to differ widely, from private resources going almost exclusively to tertiary education in the US versus primary and secondary education in European countries.

Table 1: Size and structure of education spending in EU and US. Source: OECD, WDI, Eurostat

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>Private Expenditure</th>
<th>Public Expenditure</th>
<th>Total</th>
<th>Av. Growth</th>
</tr>
</thead>
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<tr>
<td></td>
<td>%GDP</td>
<td>K-12 Tertiary Total</td>
<td>K-12 Tertiary Total</td>
<td>2000-2005</td>
<td></td>
</tr>
<tr>
<td>EU Average</td>
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<td>0.10 0.61</td>
<td>4.01</td>
<td>5.01</td>
<td>5.62</td>
</tr>
<tr>
<td>United States</td>
<td>0.36</td>
<td>1.49 1.85</td>
<td>3.91</td>
<td>5.35</td>
<td>7.20</td>
</tr>
</tbody>
</table>

The differences in growth rates across developed economies have motivated a lot of debate on the role of education in fostering innovation and growth. Aghion et al. (2005), Sapir et al. (2003), Krueger and Lindahl (2001), Romer (2000) are just few of the studies that address the relationship between human capital investment and growth. In particular,

1, 2, Aghion et al. (2005) provide extensive empirical support to this claim.

2Source: U.S. Census Bureau, The 2007 Statistical Abstract
Aghion et al. (2005) use data for 50 US states to show that economies closer to the technological frontier benefit more from investments in the advanced stages of education. While their result has important policy implications, we take the question to the next level and ask how governments can optimally allocate resources across the education process and how households respond to these allocations. Previous literature has studied the effects of changing the mix of public education expenditures across K-12 and tertiary education. However, with a few exceptions, the main concern is either the public or the private spending allocation across the stages, with only limited attention given to the interaction between the two.

Driskill and Horowitz (2002) were the first to investigate the investment in hierarchical human capital in a dynamic setting. Blankenau (2005) studies the optimal allocation of public spending across the two stages of the educational process, but he does not take into account the interaction between households behavior and the government policies, looking only at how the tuition in tertiary education is split between government subsidies and private contributions (college loans). Su (2004) considers the case of private transfers as an extension; nonetheless these transfers enter the parent’s utility function in a simplified way which basically does not alter the main outcomes of the public transfers. However, the interaction between public and private decision has the potential of affecting both the size and structure of the aggregate education spending since public allocations alter the return of tertiary education relative to K-12 education, prompting an adjustment in the pattern of private spending. Kaganovich (2005) models the interaction between private and public education spending.
spending in a mixed system of access to higher education (recently developed in Eastern Europe) featuring a two-track admission process: one that is tuition-free and based on academic performance and the second that charges full tuition, offered to less prepared students. Consequently, households need to allocate their resources between pre-college preparation and/or college cost. There, the focus is on distributional issues, while this paper studies the aggregate outcomes in a general equilibrium framework.

In addition to the papers mentioned above, there is an abundant economic growth literature where public and private expenditures are inputs in a single stage human capital production, either as a time cost or as real inputs, possibly, but not necessarily, tuition (Glomm and Ravikumar [1992], Eckstein and Zilcha [1994], Glomm and Ravikumar [1998], Kaganovich and Zilcha [1999], Brauninger and Vidal [2000], Cassou and Lansing [2003], Bearse et al. [2005]).

This paper contributes to the literature by providing a very general, yet simple model of hierarchical education that incorporates households’ optimal decisions with respect to enrollment, overall education spending and its allocation across stages. The model is used to analyze how the size and the structure of the public spending affect the optimal household decisions and consequently, the aggregate economy. In contrast to Blankenau [2005], private education inputs arise due to intergenerational altruistic transfers, not by self-financing. We also study how the growth maximizing government policies affect private spending level and composition. Depending on the size of the government budget for education, we find the optimal shares allocated by households at each stage and describe how they vary with the total level of private spending on education.

An observation is in order. We analyze education provision in a very aggregated economy with a representative agent. Besides offering analytical tractability, this enables us to circumvent modelling the sheer variety of education providers, and their particular production functions. Thus, we consider that education quality at each stage is produced by an aggregated technology encompassing the entire educational system. We do not distinguish between public and private ownership of these institutions or firms, nor do we account for the mix of funds they use. Consequently, we use the terms spending and inputs interchangeably.

The degree of substitutability between private and public inputs in the various stages of education is of central importance to the issue analyzed. The literature has not reached a consensus on this matter. Thinking of these inputs exclusively as monetary transfers makes a strong case for perfect substitution. However, welfare and education public policies can have an impact, if any, at the early stages of child development rather than later. If human capital investment is considered to include adequate child health care and nutrition, public spending might be a complement or a substitute to private spending for children from different income families. Moreover, the public spending can be used in a variety of ways, from vouchers to public schools, subsidies to private schools or coordination and curricula improvement, each having potentially different effects on private resources devoted to education. Moreover, during K-12 education most students live with their parents who support room and board expenditures while in the same time providing the so called "within the family" education (Nordblom [2003]). This strengthens the argument for complementarity.
during early education stages. Also, if the public spending in education (both K-12 and college level) is dominant or at least sizeable (as it is the case in most developed countries), private preparation might be viewed as an enhancer, rather than a substitute for public spending.

Houtenville and Conway (2001) bring micro-level evidence in favor of substitutability at K-12 level. Kim (2001) finds that poor parents invest less in the child’s human capital when public schooling increases, while rich parents instead increase their investment. At the aggregate level, Psacharopoulos and Patrinos (2004) show that social returns of primary education are significant in both developed and developing countries. The implied externalities suggest that public inputs might be good complements to private inputs in the production of primary and secondary education. Similar evidence is less compelling for tertiary education, as reviewed by Jacobs and van der Ploeg (2005). However, potential externalities in research and development, activities in which tertiary education plays an important part, might suggest less than perfect substitutability between private and public inputs at this stage too.

Summing up, it seems that the elasticity of substitution varies with the level of schooling, from low elasticities at early stages to high elasticity at later stages. The economic literature concerned with educational spending has used various specifications: Kaganovich and Zilcha (1999) use a Cobb-Douglas specification, implying an input elasticity of substitution equal to one. In Blankenau (2005), public and private inputs are complements in the human capital production function at the first stage and perfect substitutes in the production of tertiary education. Glomm and Kaganovich (2003) use an additive specification to model the relationship between education provision and social security. In light of the arguments presented above, we consider a more general specification that allows for a constant elasticity of substitution (CES), similar to Bearse et al. (2005).

The paper is structured as follows. Section 2 presents the model. Section 3 contains the equilibrium analysis. Analytical results along a balanced growth path are derived for the case of perfect input substitutability in the higher education production. In Section 4 we study the effects of changes in the policy parameters on private resource allocations across the two educational stages and the growth rate of the economy. Section 5 relaxes the assumption of perfect substitution between private and public inputs. Section 6 looks at the optimal (growth-maximizing) policies when government takes into account households’ decision rules as functions of public policies parameters using numerical results. Section 7 examines the tax level effects on the structure of both public and private education funding. Section 8 concludes. All proofs are in the Appendix A.

2 The Model

The economy is populated by a large number of identical households. The household consists of a parent and a measure one of ex-ante identical children, so that the population is constant. Each agent lives two periods, called youth and old age. Young agents (children) get educated in a two-stage schooling system. When the agents are old, they give birth to offsprings, earn a wage specific to the human capital acquired in the first period,
consume and make an educational transfer to their children. The size of each generation is normalized to one.

2.1 Households

2.1.1 Child’s educational problem

Education is provided in two steps. The first step is mandatory and corresponds to the primary and secondary education years (henceforth K-12 education). The second stage is optional and corresponds to tertiary education. Also, to accomplish it, people must give up an exogenous fraction \( n_2 \) of their adult working time. Since in most countries, education stages are quite standardized in duration, we set \( n_2 \) exogenously. The advanced stage training is pursued only by a part of the population, \( \Pi_t \in (0, 1) \). Thus, in equilibrium, a fraction of the young people will go to college and use the first stage education to increase their human capital. This is the skilled labor supply. Remaining workers will provide unskilled labor.

Similar to Blankenau (2004), each young agent has perfect foresight and chooses an education strategy taking the strategy of others as given, such that in equilibrium their behavior is consistent with the aggregate outcomes. Thus, a young agent at time \( t \) chooses her probability of attending college \( \pi_t \in [0, 1] \), taking as given the proportion of college graduates in her generation \( \Pi_t \in (0, 1) \), market wages and public policies such that to be indifferent between the two choices. Consequently, the equilibrium \( \pi_t \in [0, 1] \) is given by the following indifference condition:

\[
\pi_t = \begin{cases} 
0 & \text{if } (1 - n_2) w_{2t+1} h_{2t} < w_{1t+1} h_{1t} \\
\in (0, 1) & \text{if } (1 - n_2) w_{2t+1} h_{2t} = w_{1t+1} h_{1t} \\
1 & \text{if } (1 - n_2) w_{2t+1} h_{2t} > w_{1t+1} h_{1t}
\end{cases},
\]

where \( w_{1t+1} \) and \( w_{2t+1} \) are the next period unskilled and skilled wage per efficiency unit, \( h_{1t} \) and \( h_{2t} \) the human capital accumulated in the first and second stage, respectively.

The human capital in the first stage of education is produced using both privately and publicly provided inputs. Since the focus of this paper is on the interaction between public and private spending, we do not consider other factors that play a role in human capital production, such as the stock of human capital accumulated by the previous generation or parental time. However, including these factors does not alter the main conclusions of the paper. The human capital acquired in the first stage by a young agent at time \( t \) is given by the following production function:

\[
h_{1t} = B_1 \left[ \rho e_{1t}^{-\gamma_1} + (1 - \rho) b_{1t}^{-\gamma_1} \right]^{-\frac{1}{\gamma_1}},
\]

where \( e_{1t} \) and \( b_{1t} \) are the public and private inputs per student, \( \gamma_1 \in [-1, +\infty) \), \( 0 < \rho < 1 \) and \( B_1 > 0 \).

The human capital of a college educated agent is then produced according to:

\[
h_{2t} = B_2 h_{1t}^{\alpha} \left[ e_{2t}^{-\gamma_2} + b_{2t}^{-\gamma_2} \right]^{\frac{1-\alpha}{\gamma_2}},
\]

where
where \( e_{2t} \) and \( b_{2t} \) are the public and private inputs per student, \( \gamma_2 \in [-1, +\infty) \), \( 0 < \theta < 1 \) and \( B_2 > 0 \). The human capital accumulated in the first stage \( (h_{1t}) \) is an essential input in the production of tertiary education, by the very nature of the educational process. We explore the strength of this complementarity in Appendix B by extending the production function in \( (3) \) to a CES specification. The coefficient \( \theta \) shows how important knowledge acquired during K-12 studies is for success in tertiary education. If \( \gamma_2 = -1 \), public and private inputs are perfect substitutes. In line with the discussion in the introduction, we expect private and public inputs to be better (potentially perfect) substitutes in tertiary education compared to earlier stages of education.

**Assumption 1.** \( -1 \leq \gamma_2 \leq \gamma_1 \).

To keep the model tractable, we abstract from the interaction between the child and parent in financing the cost of college, by assuming all private inputs are provided by parents.

In order to analytically derive the main results and policy implications of the model, we first focus on particular cases of human capital production functions. We initially assume a Cobb-Douglas education production function instead of the more general CES form. This implies \( \gamma_1 = 0 \) and

\[
h_{1t} = B_1 e_{1t}^{\rho} b_{1t}^{1-\rho}
\]  

(4)

This specification has been frequently used in the literature studying human capital accumulation (see for example Blankenau (2005), Su (2004)). Furthermore, for the higher education production function, we first assume perfect elasticity of substitution, or \( \gamma_2 = -1 \). Thus, similarly to Glomm and Kaganovich (2003), we have:

\[
h_{2t} = B_2 h_{1t}^{\theta} [e_{2t} + b_{2t}]^{1-\theta},
\]

(5)

We use this special case as a starting point in order to illustrate the mechanisms at work in the model. We relax the assumption of \( \gamma_1 = 0 \) and \( \gamma_2 = -1 \) in Section 5 where we solve the model numerically for the general case and study how different assumptions about the elasticity of substitution affect the results. Then we proceed to analyzing the optimal education policies in this more general setup.

2.1.2 Parent’s educational spending decisions

The parents allocate a portion of their income to educational transfers. The overall amount of parental resources spent on education depends on income and stems from an altruistic motive. Ex-ante, each offspring decides to attend college with probability \( \pi_t \). Given perfect foresight, this corresponds to an ex-post measure \( \pi_t = \Pi_t \) of college educated offsprings in each household. Parents will allocate the transfers across the two stages in order to maximize the expected utility derived from the human capital accumulated by their children.\footnote{This is the ex-ante interpretation. Ex-post, this corresponds to a parent that derives utility from each type of human capital of the offspring, weighted by the measure of children that acquire it.}
Denote the share of household’s disposable income allocated for educational purposes $\Delta_t$ and the fraction dedicated to K-12 education as $\psi_t$. Then the amount spent at each stage is given by:

\begin{align}
 b_{1t} &= \psi_t \Delta_t Y_t (1 - \tau), \\
 b_{2t} &= (1 - \psi_t) \Delta_t Y_t (1 - \tau),
\end{align}

where $\tau$ is the constant income tax rate.

Formally, the parent’s problem is

\begin{equation}
 \max_{(c_t, b_{1t}, b_{2t})} U = \ln c_t + \mu \left[ (1 - \pi_t) \ln h_{1t} + \pi_t \ln h_{2t} \right] \quad \text{s.t.} \quad (4), (5), (6) \text{ and } (7)
\end{equation}

where $c_t$ represents parent’s consumption, $w_t$ is the wage per efficiency unit at time $t$ and $h_{t-1}$ the parental human capital accumulated in the previous period. The wage income is taxed at a constant rate $\tau$. The parameter $\mu \geq 0$ captures the utility weight the parent attaches to children’s human capital. Recall that agents have the same lifetime income ex-post as given by the indifference condition (1).

The parent needs to choose jointly the level of education spending as a fraction of disposable income ($\Delta_t$) and the fraction of resources devoted to each educational stage ($\psi_t, 1 - \psi_t$). We use the budget constraint, (6) and (7) to rewrite the parent’s problem at time $t$:

\begin{equation}
 \max_{(\Delta_t, \psi_t)} U = \ln [(1 - \tau)(1 - \Delta_t)w_t h_{t-1}] + \mu \left[ (1 - \pi_t) \ln h_{1t}(\Delta_t, \psi_t) + \pi_t \ln h_{2t}(\Delta_t, \psi_t) \right] \quad \text{s.t.} \quad (4) \text{ and } (5)
\end{equation}

Taking the first order conditions with respect to ($\Delta_t, \psi_t$) yields the following expressions:

\begin{align}
 (\Delta_t) : & \quad \frac{-1}{1 - \Delta_t} + \mu \left[ (1 - \pi_t) \frac{1}{h_{1t}} \frac{\partial h_{1t}}{\partial \Delta_t} + \pi_t \frac{1}{h_{2t}} \frac{\partial h_{2t}}{\partial \Delta_t} \right] \leq 0, \quad = 0 \text{ if } \Delta_t \in (0, 1) \\
 (\psi_t) : & \quad (1 - \pi_t) \frac{1}{h_{1t}} \frac{\partial h_{1t}}{\partial \psi_t} + \pi_t \frac{1}{h_{2t}} \frac{\partial h_{2t}}{\partial \psi_t} \geq 0, \\
 & \quad = 0 \text{ if } \psi_t \in (0, 1), < 0 \text{ if } \psi_t = 0, > 0 \text{ if } \psi_t = 1
\end{align}
2.2 Production sector

A representative firm uses the two types of human capital to produce the final output:

\[ Y_t = A H_{1t-1}^{\gamma} H_{2t-1}^{1-\gamma} \]  \hspace{1cm} (11)

where \( A \) is the total factor productivity and \( 0 < \gamma < 1 \).

The total supply of unskilled and skilled labor, respectively are given by the following expressions:

\[ H_{1t-1} = (1 - \Pi_{t-1}) h_{1t-1}, \]  \hspace{1cm} (12)

\[ H_{2t-1} = (1 - \Pi_{t-1}) h_{2t-1}. \]  \hspace{1cm} (13)

2.3 Government

The government taxes income at a constant rate \( \tau \) and uses a fraction of the tax revenues to provide public inputs in both stages of the education process. Denote the share of income that goes to public education as \( g, g < \tau \). Then,

\[ g Y_t = E_t = e_{1t} + \Pi_t e_{2t}. \]  \hspace{1cm} (14)

If total public spending at K-12 level is a fraction \( \phi_t \) of the total education budget and the first stage of education is compulsory, the per capita allocation for K-12 education is just \( \phi_t g \). Similarly, the per capita public spending in tertiary education is \( \frac{1 - \phi_t}{\Pi_t} g \), since only a measure \( \Pi_t \) of the population decides to attend college. Thus,

\[ e_{1t} = \phi_t E_t = \phi_t g Y_t, \]  \hspace{1cm} (15)

\[ e_{2t} = \frac{1 - \phi_t}{\Pi_t} E_t = \frac{1 - \phi_t}{\Pi_t} g Y_t. \]  \hspace{1cm} (16)

3 Equilibrium analysis

We first define a competitive equilibrium in an economy where policies \( \{g, \phi_t\} \) are exogenous.

Nickell and Bell (1995) use a CES output production function in skilled and unskilled labor to study the demand for unskilled labor in OECD countries. They report an elasticity greater than one. Manacorda and Petrongo (1999) however, find that for OECD countries the hypothesis of a Cobb-Douglas production function cannot be rejected.

\(^3\)Given the paper focuses on analyzing the interaction between private and public provision of education, we do not model the usage of the remaining revenues \( \tau - g \). This share can be thought of as a waste or unproductive government consumption.
Definition 1. An equilibrium with exogenous public policies is defined as a triplet \( \{\pi_t, \Delta_t, \psi_t\} \) such that, at the beginning of each time period \( t \):

1. The young agent chooses the probability to attend college \( \pi_t \) such that to satisfy (8), given the aggregate probability \( \Pi_t \), wages \( w_{1t} \) and \( w_{2t} \), parental allocations \( b_{1t}, b_{2t} \) and governmental outlays \( e_{1t}, e_{2t} \) for education across the two stages;
2. Individual and aggregate decisions are consistent: \( \pi_t = \Pi_t \);
3. The parent chooses \( \{\Delta_t, \psi_t\} \) to solve (8);
4. The firms pay competitive market wages;
5. The government budget constraints (14), (15), and (16) hold.

3.1 Optimal enrollment

To find the equilibrium enrollment \( \Pi_t \), we use (11). The production function satisfies the Inada condition with respect to both inputs. This guarantees that in equilibrium \( \pi_t \) is strictly between zero and one. A corner solution of zero (one) would drive the wage for skilled (unskilled) labor to positive infinity, which in turn would make the choice for \( \pi_t \) unsustainable. The equilibrium probability to attend college is derived from the indifference condition:

\[
(1 - n_2)w_{2t+1}h_{2t} = w_{1t+1}h_{1t}. \tag{17}
\]

Plugging in the competitive wages

\[
w_{1t+1} = \gamma \frac{Y_{t+1}}{H_{1t}}, \quad w_{2t+1} = (1 - \gamma) \frac{Y_{t+1}}{H_{2t}},
\]

and using the supply definitions (12) and (13), (17) yields a constant enrollment\(^6\)

\[
\Pi_t = \Pi = 1 - \gamma, \forall t > 0. \tag{18}
\]

Consequently, the fraction of skilled people equals the fraction of the skilled labor wage bill in total income. Estimates of the unskilled labor share in income (\( \gamma \)) are in the range of 0.6-0.7. This implies an enrollment of around 30%, which is in line with numbers reported for the OECD countries.

3.2 The balanced growth path

We look for a balanced growth path equilibrium, where the output and both types of human capital grow at a common, constant rate \( g_y \) and the shares \( \{\Delta_t, \psi_t, \phi_t\} \) are constant.

Plugging the expressions for \( e_{1t}, e_{2t}, b_{1t} \) and \( b_{2t} \) into (2) and (5) and using the fact that agents are homogenous within the skill group (i.e. \( h_{1t} \) and \( h_{2t} \) are the same for all agents),

\(^6\)Cobb-Douglas production allows us to concentrate on the substitution between public and private spending in a simple setup. In an extension, we allow for a more general CES production function, such that enrollment depends on the relative supply of human capital. However, the main results of the paper hold.
we get the following expressions for human capital accumulated in the first and second stage, respectively:

\[ h_{1t} = B_1(\phi g)^{\rho}[\psi \Delta (1 - \tau)]^{1-\rho}Y_t \]  

(19)

\[ h_{2t} = B_2 \left\{ (\phi g)^{\rho}[\psi \Delta (1 - \tau)]^{1-\rho} \right\}^\theta \frac{1}{\Pi^{1-\theta}} \times [\left(1 - \phi \right) g + (1 - \psi) \Delta (1 - \tau)]^{1-\theta} Y_t \]  

(20)

Combining (11), (12), (13) and using \( t_1 = t \), we get

\[ Y_t = A(1 - \Pi)^\gamma[\Pi (1 - n_2)]^{1-\gamma}h_{1t-1}^{1-\gamma}h_{2t-1}^{1-\gamma}, \]  

(21)

Using (19) and (20) in (11) we get the growth rate of the economy:

\[ g_y = \frac{Y_t}{Y_{t-1}} = C \left\{ (\phi g)^{\rho}[\psi \Delta (1 - \tau)]^{1-\rho} \right\}^{\gamma+\theta(1-\gamma)} \times [\left(1 - \phi \right) g + (1 - \psi) \Delta (1 - \tau)]^{(1-\theta)(1-\gamma)}, \]

where \( C = AB_1^2B_2^{1-\gamma}(1 - \Pi)^\gamma\Pi^{\theta(1-\gamma)}(1 - n_2)^{1-\gamma}. \)

### 3.3 Equilibrium spending allocations

In the following we solve for household decisions as functions of public policy parameters on the balance growth path, so that \( \Delta_t = \Delta \) and \( \psi_t = \psi \). Proposition 2 provides the condition under which an interior solution for the fraction of the household’s education budget allocated to K-12 education \( \psi \) exists. All proofs are in the Appendix.

**Proposition 1.** The household’s problem yields a unique and interior solution \( \Delta \in (0, 1) \) for all \( \psi \in [0, 1] \).

**Proposition 2.** There exists a threshold \( \tilde{\Delta} = \frac{(1 - \rho)[1 - \Pi(1 - \theta)](1 - \phi)g}{\Pi(1 - \theta)(1 - \tau)} \) such that

\[ \psi = \frac{(1 - \rho)[1 - \Pi(1 - \theta)]}{1 - \rho[1 - \Pi(1 - \theta)]} \left\{ \frac{(1 - \phi)g}{\Delta(1 - \tau)} + 1 \right\} \in (0, 1) \text{ if } \Delta > \tilde{\Delta} \text{ and } \psi = 1 \text{ otherwise.} \]

In other words, the fraction of total private spending out of disposable income has to be sufficiently high for household to allocate resources for higher education. They are more likely to do so if they have a larger disposable income (\( \tau \) lower), or the total public spending in education \( g \) and the fraction of it devoted to higher education (\( 1 - \phi \)) are sufficiently low. Also, the importance of the private input in the human capital production in the first stage (\( 1 - \rho \)) decreases the likelihood of private investment in the second stage. The opposite holds for the elasticity of spending per student (\( 1 - \theta \)) in the production of human capital in the advanced stage.

The fraction of private resources allocated to the advanced education stage is increasing in the total amount of private contributions. This is an important intermediate result since it highlights the fact that all factors (including policy variables) that make the households richer will generate higher private spending in higher education.
Proposition 3. \( \mu > \frac{g(1-\phi)}{\Pi(1-\theta)(1-\tau) - g(1-\phi)(1-\rho)[1-\Pi(1-\theta)]} \) is a sufficient and necessary condition for \( \psi \in (0, 1) \). Then,

\[
\Delta_t = \Delta = \frac{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi)}{(1-\tau)\{1+\mu\{1-\rho[1-\Pi(1-\theta)]\}\}} \quad \text{and} \quad (22)
\]

\[
\psi_t = \psi = \frac{\mu(1-\rho)[1-\Pi(1-\theta)][1-\tau + g(1-\phi)]}{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi)}. \quad (23)
\]

Otherwise,

\[
\Delta_t = \frac{\mu(1-\rho)[1-\Pi(1-\theta)]}{1-\mu(1-\rho)[1-\Pi(1-\theta)]} \quad \text{and} \quad \psi_t = 1.
\]

Since public inputs enter the production of human capital in both stages, private resources are not essential, so the households will allocate private resources for tertiary education only if the altruism coefficient is strong enough, as suggested above.

Assuming interior solutions for \( \Delta \) and \( \psi \), we perform the following policy experiments: we change the generosity of total public spending on education \( g \), the public spending mix \( \phi \) and the tax rate \( \tau \) in order to analyze their effects on both \( \Delta \) and \( \psi \).

4 Policy experiments

Proposition 4. (A change in the generosity of public spending \( g \)) Assume \( \Delta, \psi \in (0, 1) \) and a fixed \( \phi \in (0, 1) \). Then \( \frac{\partial \Delta}{\partial g} < 0 \) and \( \frac{\partial \psi}{\partial g} > 0 \).

An increase in the total public budget for education (\( g \)) has a negative effect on the overall level of private spending on education. It also leads to a higher fraction of total private spending allocated to the first educational stage. Given the fraction \( \phi \) of public resources allocated to K-12 stage, a higher budget \( g \) implies more public spending in both stages. The complementarity from the first stage between private and public resources is dominated by the substitution effect in the second stage. Since the overall level of human capital increases with \( g \), its marginal utility decreases, hence \( \Delta \) goes down.

The second result indicates that at a given level of educational private spending, the share allocated to first stage education increases with \( g \) since higher public inputs increase the productivity of private inputs in K-12 but not in tertiary education. Thus agents substitute towards the more productive use of their resources.

Proposition 5. A change in the public spending mix (\( \phi \)). Assume \( \Delta, \psi \in (0, 1) \) and a fixed \( g \in (0, 1) \). Then \( \frac{\partial \Delta}{\partial \phi} > 0 \) and \( \frac{\partial \psi}{\partial \phi} < 0 \).

Assuming the level of public funding for education is fixed, a higher share of K-12 public funding generates an increase in total private spending in education and hence higher private contributions at both stages. Second, an increase in public K-12 spending prompts households to allocate a larger share of their investments towards higher education.
Propositions 4 and 5 are particularly important. Households respond to both the size and the composition of public education spending. In the context of policy making, governments have two alternative ways to expand higher education provision given a fixed level of public resources $g$. First, they could directly decrease $\phi$ or on the contrary, increase their participation in the basic stages and let households use more of their resources in the advanced stage.

Complementarity between public and private inputs in K-12 education is instrumental in obtaining this result. A higher share of K-12 public inputs produces two effects on $\psi$. Since $\phi$ goes up, $1 - \phi$ must go down, which implies that less public inputs are available in the second stage, since $g$ is fixed. Households will compensate this decrease by decreasing $\psi$. This is the direct effect. Second, an increase in $\phi$ raises the marginal productivity of private resources in the first stage. This would prompt an increase in $\Delta$ which in turn leads to a decrease in $\psi$. It can be seen that the effects on $\psi$ reinforce each other, such that in equilibrium $\psi$ will decrease. In the next section, we test the sensitivity of these results to changes in the parameters of the human capital production functions.

However, private investment in education depends on many other factors. In particular, notice that $\Delta$ also depends on the general tax level, $\tau$, so differences in taxation undoubtedly explain why private contributions vary a lot across economies. For example, total tax revenue share in GDP is 0.26 in US compared to around 0.4 in EU. Thus, it is useful to establish the following results.

**Proposition 6.** A change in the income tax rate ($\tau$). Assume $\Delta, \psi \in (0, 1)$. Then for given $\phi, g \in (0, 1)$, $\frac{\partial \Delta}{\partial \tau} < 0$ and $\frac{\partial \psi}{\partial \tau} > 0$.

Higher tax rates lead to a lower disposable income and consequently diminish the private incentives to invest in education at all stages. Interestingly, in the same time higher taxes produce a reallocation of private resources towards the first stage, where scarcer private resources yield higher human capital and hence higher utility.

## 5 The general model

The case $\gamma_1 = 0$ and $\gamma_2 = -1$ was considered for analytical tractability. However, a more general analysis seems to be warranted given the scant and ultimately inconclusive empirical evidence on the estimates of such elasticities for education production functions. While the education literature reviewed above does not imply any particular values for $\gamma_1$ and $\gamma_2$, it does suggest that substitution elasticity between private and public inputs is higher in the first stage compared to the second. In a somewhat related exercise, Clotfelter (1977) analyzes the role of substitution between private and public inputs in the production of law and order. He estimates a production function of security that is CES in private and public inputs. He finds an elasticity of 2.47, suggesting that even in the case of traditional "public goods" such as law and order, market underprovision, and hence the externalities associated with it, might be lower than expected.

We now generalize the model in two steps. First, we analyze the case when the inputs in the tertiary education stage are less than perfect substitutes, keeping $\gamma_1 = 0$. 

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Then, the production in the first education stage is allowed to take a more general CES form. We keep \( \gamma_2 \) in the range of good input substitutability \((0 \leq \gamma_2 \leq -1)\) while we restrict \( \gamma_1 \) to a symmetric range around zero, \((-0.5 \leq \gamma_1 \leq +0.5)\) such that to ensure our assumption \( \gamma_2 \leq \gamma_1 \) holds. Since no closed form solutions are available for \( \Delta \) and \( \psi \), we numerically solve the household problem at different values for \( \phi \) and \( g \).

The graphs in the first column in Figure 2 show the households allocations \( \Delta, \psi \) and the growth rate of the economy \( g_Y \) for \( \phi \in [0, 1] \) and \( g = 0.05 \), a value close to US data. The graphs in the second column display \( \Delta, \psi \) and \( g_Y \) when \( g \in [0, 0.1] \) and \( \phi = 0.7 \). We limit \( g \) to the empirically relevant range (most countries invest less than 5% of GDP in education while the highest spending among OECD countries, in Denmark, is around 9%).

In the model a period is approximately 30 years, so \( n_2 = 0.12 \) matches the average duration of tertiary education of about 4 years. The altruism parameter, \( \mu \), was set to 0.09, to obtain reasonably low values (under 0.05) for \( \Delta \), in line with OECD data. The tax rate \( \tau \) was set to 0.3. We normalize \( B_1 \) to 1 and set \( B_2 = 5 \) such that to insure that the human capital accumulated in the second stage exceeds the one in the first stage. However, the Cobb-Douglas production function implies a constant enrollment and skill premium, so that \( B_1 \) and \( B_2 \) do not play an instrumental role in the calibration. The total factor productivity \( A \) was chosen such that to obtain an annual growth rate of approximately 2% when \( \gamma_2 = -0.7 \) and \( \phi = 0.5 \) (\( g = 0.05 \), respectively ). This corresponds to a per period growth factor of 1.8. The share of unskilled labor, \( \gamma \), is 0.65. This gives an equilibrium enrollment of 0.35.

As it can be seen in the Figure 2, results established in the previous section hold. In particular, \( \Delta \) increases with \( \phi \) and decreases with \( g \) while \( \psi \) decreases with \( \phi \) and increases with \( g \). Note however, that in our model the education budget \( g \) is limited by the tax rate \( \tau \). Thus on the relevant ranges for \( g \) and \( \tau \), the economy is on the upward sloping side of the Laffer curve - where an increase in \( g \) increases the the growth rate.

The output growth rate behavior with respect to \( \phi \) deserves particular attention. Public resources substitution from tertiary toward K-12 education is facilitated by the different per capita effects of these resources. At the advanced stage, a given amount of public resources has a higher productivity than in the first stage since only a measure \( \Pi < 1 \) of people attend college. As previously explained, a higher share of public resources allocated to K-12 education induces more private educational inputs, which results in a higher overall share of educational investment in disposable income (higher \( \Delta \)), and hence more human capital of both types. This has a positive effect on growth rate. Moreover, note that higher growth can be achieved with the same \textit{level} of public resources \( g \) when more private inputs flow optimally toward higher education (\( \psi \) decreases). Also, as expected, the growth maximizing \( \phi \) is higher when inputs are better substitutes in tertiary education (lower values of \( \gamma_2 \)). The model offers an explanation for the high observed values for \( \phi \) (OECD countries use between 60% and 90% percent of their education budget in K-12 education).

In conclusion, allowing \( \gamma_2 \) to vary does not alter any of our analytical results. Since the input complementarity in the first stage is at the centre of the interactions described above, we now explore how changes in the strength of this complementarity affect our results. Based on the previous results, we fix \( \gamma_2 = -0.7 \) and allow \( \gamma_1 \) to take both negative
Figure 2: Exogenous policies when $\gamma_1 = 0$ and $\gamma_2 = \{-0.7, -0.5, -0.3\}$. In the top three panels $g = 0.05$. In the bottom panels $\phi = 0.7$.

and positive values around zero, which corresponds to the Cobb-Douglas case.

Graphs in Figure 3 follow the convention from the previous figure. When $\gamma_1 = -0.5$ the input elasticity of substitution is 2, while $\gamma_1 = +0.5$ implies an elasticity of substitution of $1/2$. Compared to the analytical results, established for $\gamma_1 = 0$, in the case $\gamma_1 = -0.5$, the functions $\Delta(\phi)$ and $\psi(g)$ become non-monotonic as the effects implied by input complementarity are weaker. Thus, the crowding out effect of an increase in $\phi$ dominates at low shares of K-12 spending in total public budget. However, in the empirically relevant ranges of $\phi$ and $g$ the previous results hold.

Similar conclusions can be drawn in the case where $\gamma_1 = 0.5$. Now, $\Delta$ increases first and than decreases with $g$. This result stems from the enhanced complementarity between private and public inputs in the first stage. At low values of $g$, the crowding out effect is dominated by the increase in marginal productivity of private resources, while the reverse is true for high values of $g$. The growth maximizing $\phi$ remains above 60% for all values of $\gamma_1$.

From the previous analysis, we can conclude that, in addition to the size of the public education budget, its structure across stages can also have substantial effects on economic
growth. From a positive point of view, the model offers an explanation of why countries generally decide to use the bulk of their public educational resources in K-12 public education provision. In the same time, this result is important from a normative perspective, as education reform in developed countries often calls for increased spending without devoting much attention to most beneficial way to use extra resources. To gain further insight about the latter issue, we turn our attention to optimal policies.

6 Optimal policies

In the following, we focus on the optimal allocation of public inputs across stages. Recall that due to constant returns in the production of output and human capital, in equilibrium, the model generates balanced growth in output, denoted by $g_y$. We assume the government’s objective is to maximize the steady-state growth rate of output by choosing the fraction of educational resources invested in K-12 education $\phi$, given the general tax level

Figure 3: Exogenous policies when $\gamma_2 = -0.7$ and $\gamma_1 = \{-0.5, 0, +0.5\}$. In the top three panels $g = 0.05$. In the bottom panels $\phi = 0.7$. 
and the size of the overall education budget $g$. Thus, the government takes into account the reaction function of households to public policies $\Delta(\phi)$ and $\psi(\phi)$ when it chooses $\phi$, while households take the public educational policies as given. Formally, the government problem is stated as follows:

$$\max_{\{\phi\}} g_y(\phi, g) \quad s.t. \quad g \gamma_1 = E_t = e_1 + \Pi_1 e_2, $$

given $\tau$, $g$, $\psi(\phi)$, $\Delta(\phi)$,

where $\psi(\phi)$ and $\Delta(\phi)$ solve (8)

We solve for the optimal $\phi$ numerically using the parametrization described in the previous section.

![Graphs showing K-12 share in public spending, total private spending, output growth rate, and K-12 share in private spending vs. $g$.]

Figure 4: Optimal size and structure of the public spending when $\gamma_2 = -0.7$ and $\gamma_1 = \{-0.5, 0, +0.5\}$.

\footnote{It makes sense to consider $g$ as fixed, since on the relevant range, higher $g$ always implies a higher growth rate. Moreover, the size of the public budget allocated to education reflects political preferences that are quasi fixed for a specific country and depend on many factors among which the degree of unionization in education, size of private education sector, level of development etc.}
Figure 4 shows the growth-maximizing $\phi$ and the corresponding households’ decisions, $\psi$ and $\Delta$, as well as the growth rate of the economy $g_Y$, for different values of $g$, $\gamma_1 = \{-0.5, 0, +0.5\}$ and $\gamma_2 = -0.7$.

As previously discussed, public spending is more productive in tertiary education than it is in K-12, due to the "per capita" effect. Thus, an extra dollar spent in tertiary education will be spread across a measure $\Pi < 1$ students rather than the entire population (of measure 1) if spent on K-12 education. Thus, everything else constant, at low levels of the education public budget, it is optimal to allocate all funds to the first educational stage. This result is in line with findings in Blankenau (2004) who finds that tertiary education is financed only if the total spending $g$ exceeds a certain threshold. However, if inputs are sufficiently substitutable in K-12 education, the optimal share of K-12 inputs does not decrease with $g$, but follows a non-monotonic trajectory. This is due to the crowding out effect that an increase in $g$ has on private education contributions. A secondary effect comes from households’ adjustment in the composition of spending across stages. At lower levels of $\Delta$, households rebalance their allocation of educational inputs which leads to a higher $\psi$. In turn, this makes public resources more productive in the first stage, hence the increase in $\phi$.

This suggests that public funding should be primarily channelled towards the primary and secondary stages. While an increase in $g$ has positive effects on the growth rate of output, allocating too many public resources to higher education will actually diminish this effect.

In all the previous experiments, the tax rate was assumed to be fixed. However, there is considerable heterogeneity, even among developed countries, with respect to the general tax level and this will have significant effects on the private incentives to invest in education across stages. We look at these effects in the next section.

7 Tax level effects

Higher taxes diminish the disposable income and hence the overall resources directed to education. However, higher tax levels do not necessarily imply higher public education spending as a fraction of total tax receipts. Public education spending in the EU is somewhat lower than in the US despite a higher tax level. On the other side, consider the case of the Scandinavian countries with higher taxes than the European average but also higher public spending on education. We explore the predictions of this model regarding the optimal public policy at different tax levels, assuming education spending $g$ is independent of the tax level $\tau$.

As one might expect, Figure 5 shows that higher taxes imply lower growth rates and lower overall private spending, at all levels of public spending $g$. It is however interesting to discuss the optimal K-12 shares in public and respectively private spending ($\phi$ and $\psi$). The optimal fraction of public budget allocated to higher education $(1 - \psi)$ is higher in economies with higher taxes. This happens because households’ incentives to invest in education respond to the tax level in two ways. First, they decrease their total spending. Second, they spend more on K-12 education, since, as we have shown in the previous
section, the returns to first stage spending are higher when $\Delta$ is low, which is the case here at all levels of $g$, due to the higher taxes.

![Graphs showing Total Private Spending, K12 share in Public Spending, K12 share in Private Spending, and Output growth rate.]

Figure 5: Optimal size and structure of the public spending. $\gamma_1 = -0.1$ and $\gamma_2 = -0.7$.

The model’s predictions fit well the data for OECD countries as all EU countries tend to have a higher fraction of private resources spent on K-12 education compared to the United States or Canada. In addition, high tax countries such as Denmark, Sweden and Finland spend more public resources on tertiary education compared to United States, United Kingdom or Australia. It is also apparent that total private spending varies inversely with the tax level.

8 Conclusion

Improving education quality figures high on political agendas everywhere. In particular, higher education competitiveness has been receiving a lot of attention during last decade. While more public funding is often touted as a panacea, the disincentives for private investment should be given careful consideration. This paper studies the interaction between public and private spending in a two-stage education framework where households optimally choose the total resources to be spent on education as well as their allocation across stages. The elasticity of substitution between private and public inputs at each stage is crucial for this analysis. While little is known on the degree of substitutability/complementarity between private and public inputs in K-12 versus tertiary education, our analysis shows that under very general conditions, the allocation of public inputs across education stages
can be growth-improving if the households’ response to policies is taken into account.

We present analytical results for the particular case when private and public inputs are unit elastic substitutes in the first stage and perfect substitutes in the production of human capital in the second stage and study the effects of changing both the level of the education budget and its structure on households’ education spending.

First, we find that overall public spending crowds out private contributions and increases the share of private spending on K-12 education. Second, assuming the level of public funding for education is fixed, a higher share of K-12 funding generates higher private contributions at both stages. These findings are robust to changes in the substitution elasticities in both stages.

Third, our results suggest that the optimal share of public spending devoted to K-12 should be high irrespective of the size of the public budget. This is consistent with observed values for OECD countries. We also find that countries with a high tax level will optimally direct a relatively higher fraction of public resources toward tertiary education while the households respond by focusing their spending on the first stage.

Education policies are very concerned with inequality and access to schooling. Our paper does not explicitly address this issue. However, the policy implications of the model are consistent with the usual top to bottom redistributive effect of increased public funding for basic education that is documented in the literature (for example Su (2004)). An extended analysis using a heterogenous agent framework would allow for a more detailed description of the interaction between stages (by incorporating an admission threshold at the later stage) or type of public spending (such as means-tested aid, merit-based subsidies).

References


Appendix A

Proof of Proposition 1. This is a direct consequence of logarithmic utility and Cobb-Douglas specification of the human capital production function. Substituting (2) and (3) in (9), the parent’s problem can be written as follows:

$$\max_{\{\Delta_t, \psi_t\}} U = \ln [Y_t(1-\tau)(1-\Delta_t)] + \mu \rho [1 - \Pi (1-\theta)] \ln (\phi g) + \mu (1-\rho) [1 - \Pi(1-\theta)] \times$$

$$\ln [\psi_t \Delta_t(1-\tau)] + \mu \Pi (1-\theta) \ln [(1-\phi)g + \Delta_t(1-\psi_t)(1-\tau)] + \mu \ln Y_t$$

We can easily see that

$$\lim_{\Delta_t \to 0} \frac{\partial U(\Delta_t, \psi_t)}{\partial \Delta_t} = -\infty \quad (25)$$

$$\lim_{\Delta_t \to 1} \frac{\partial U(\Delta_t, \psi_t)}{\partial \Delta_t} = +\infty \quad (26)$$

The function $\frac{\partial U(\Delta_t, \psi_t)}{\partial \Delta_t}$ is decreasing in $\Delta_t$. Together with (25) and (26) this implies the parent’s problem has a unique solution in the interval $(0, 1)$.

Proof of Proposition 2. The first order condition with respect to $\psi_t$ is:

$$f(\psi_t) = \mu (1-\rho) [1 - \Pi (1-\theta)] \frac{1}{\psi_t} - \mu \Pi(1-\theta) \frac{\Delta_t(1-\tau)}{(1-\phi)g + \Delta_t(1-\psi_t)(1-\tau)} \geq 0 \text{ with CS,}$$

$$< 0 \text{ if } \psi_t = 0, \quad > 0 \text{ if } \psi_t = 1 \quad (27)$$

Since $f(\psi_t)$ is a decreasing function of $\psi_t$ it has at most one solution. The solution is in the interval $(0, 1)$ if the following two conditions are satisfied:

$$\lim_{\psi_t \to 0} f(\psi_t) > 0 \quad (28)$$

$$\lim_{\psi_t \to 1} f(\psi_t) < 0 \quad (29)$$

We can see that the first condition holds since $\lim_{\psi_t \to 0} f(\psi_t) = \infty$. The second condition holds iff:

$$\mu (1-\rho) [1 - \Pi(1-\theta)] - \mu \Pi(1-\theta) \frac{\Delta_t(1-\tau)}{(1-\phi)g} < 0 \quad (30)$$

Consequently, there exists a threshold $\tilde{\Delta} = \frac{(1-\rho) [1 - \Pi(1-\theta)] (1-\phi)g}{\Pi(1-\theta)(1-\tau)}$ such that $\psi_t \in (0, 1)$ if $\Delta > \tilde{\Delta}$ and $\psi = 1$ otherwise. If $\psi_t$ is interior, it is the solution of $f(\psi_t) = 0$. Using (27) we get:

$$\psi_t = \frac{(1-\rho) [1 - \Pi(1-\theta)]}{1 - \rho [1 - \Pi(1-\theta)]} \left\{ \frac{(1-\phi)g}{\Delta_t(1-\tau) + 1} \right\}$$

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Proof of Proposition 3. a) Suppose $\Delta_t > \tilde{\Delta}$. The first order condition with respect to $\Delta_t$ is:

$$-rac{1}{1-\Delta_t} + \frac{\mu(1-\rho)(1-\phi)(1-\tau)}{\Delta_t} + \frac{\mu \Pi(1-\theta)(1-\psi_t)(1-\tau)}{(1-\phi)g + \Delta_t(1-\psi)(1-\tau)} = 0$$

(32)

Using (31) in (32) and rearranging we obtain the following solutions for $\Delta_t$ and $\psi_t$:

$$\Delta_t = \Delta = \frac{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi)}{(1-\tau)\{1+\mu\{1-\rho[1-\Pi(1-\theta)]\}\}}$$

and

$$\psi_t = \psi = \frac{\mu(1-\rho)[1-\Pi(1-\theta)] \{1-\tau + g(1-\phi)\}}{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi)}$$

(33)

(34)

The solution for $\Delta$ obtained above needs to satisfy $\Delta > \tilde{\Delta}$. This is the case when:

$$\frac{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi)}{\{1+\mu\{1-\rho[1-\Pi(1-\theta)]\}\}} > \frac{(1-\rho)[1-\Pi(1-\theta)](1-\phi)g}{\Pi(1-\theta)}$$

Rearranging the expression above yields:

$$\mu > \frac{g(1-\phi)}{\Pi(1-\theta)(1-\tau) - g(1-\phi)(1-\rho)[1-\Pi(1-\theta)]}$$

b) Suppose $\Delta > \tilde{\Delta}$. Using the fact that $\psi_t = 1$ in (32) we get:

$$\frac{\mu(1-\rho)[1-\Pi(1-\theta)]}{\Delta} = \frac{1}{1-\Delta}$$

Rearranging, we obtain:

$$\Delta = \frac{\mu(1-\rho)[1-\Pi(1-\theta)]}{1-\mu(1-\rho)[1-\Pi(1-\theta)]}$$

Proof of Proposition 4. From (33) we can easily see that $\frac{\partial \Delta}{\partial g} < 0$. From (34) we get:

$$\frac{\partial \psi}{\partial g} = \frac{\mu(1-\rho)(1-\tau)[1-\Pi(1-\theta)](1-\phi)\{\mu\{1-\rho[1-\Pi(1-\theta)]\} + 1\}}{\mu(1-\tau)\{1-\rho[1-\Pi(1-\theta)]\} - g(1-\phi))^2} > 0$$

Proof of Proposition 5. From (33) we can easily see that $\frac{\partial \Delta}{\partial \phi} > 0$. Using (31) we get:

$$\frac{\partial \psi}{\partial \phi} = -\frac{(1-\rho)[1-\Pi(1-\theta)]}{1-\rho[1-\Pi(1-\theta)]} \frac{g\Delta(1-\tau) + (1-\phi)g(1-\tau)}{[\Delta(1-\tau)]^2} \frac{\partial \Delta}{\partial \phi} < 0$$

Proof of Proposition 6. From (33) we obtain:

$$\frac{\partial \Delta}{\partial \tau} = \frac{-\mu g(1-\phi)[1-\Pi(1-\theta)]}{\{1-\tau\{1+\mu\{1-\rho[1-\Pi(1-\theta)]\}\}} < 0$$

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Using (34) yields:

\[
\frac{\partial \psi}{\partial \tau} = \frac{\mu g(1 - \phi)(1 - \rho)[1 - \Pi(1 - \theta)] \{1 + \mu \{1 - \rho[1 - \Pi(1 - \theta)]\}\}}{\mu(1 - \tau) \{1 - \rho[1 - \Pi(1 - \theta)]\} - g(1 - \phi)} > 0
\]

**Appendix B**

Here we analyze the role of complementarity of resources between stages by studying a more general human capital production function in the second stage, which is a CES in the amount of human capital accumulated in the second stage and the aggregate resources invested in the second stage:

\[
h_{2t} = \left\{ \theta h_{1t}^{-\lambda} + (1 - \theta) \left[ \rho e_{1t}^{\gamma_1} + (1 - \rho) \beta_{1t}^{\gamma_1} \right] \right\}^{-\frac{1}{\lambda}}
\]

where \( \lambda \in [-1, +\infty), \gamma_1 \in [-1, +\infty), 0 < \theta < 1, 0 < \rho < 1. \)

This complementarity is captured by the importance attached to the amount of human capital accumulated in the first stage, which can be interpreted as a "preparation effect". Thus, \( \lambda \) controls for the strength of this effect. For relatively low values of \( \lambda \) the human capital acquired in the first stage can be easily replaced by adding extra-resources in the second stage (for example, remedial classes). As \( \lambda \) increases, K-12 human capital is more instrumental in producing tertiary education.

In the top three panels \( g = 0.05 \). In the bottom panels \( \phi = 0.7 \).
We vary the degree of complementarity between the level of preparation achieved and the amount of resources invested. The qualitative results do not change as it can be seen from Figure 6. When the degree of complementarity is high, $\Delta$ becomes non-monotonic in $\phi$. The strong preparation effect in the second stage exacerbates the complementarity between the private and public inputs in the first stage, such that the total private contribution is decreasing for small values of $\phi$. In other words, in this situation fewer public resources invested in K-12 diminish the return of education in both stages and thus discourages the overall private spending. Notice that $\Psi$ decreases at a slower rate when $\lambda$ is high, as households allocate a higher share of their resources to insure adequate provision of K-12 education.