Private Investment in Higher Education: Comparing Alternative Funding Schemes*

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Summary. This paper uses an overlapping generations framework to analyze the implications of different financing regimes in the education sector for human capital formation and economic welfare. Agents privately invest in education after they have received a noisy information signal about their abilities. The incentives of the individuals to invest in education are determined by the financing regime under which the economy operates. The paper analyzes and compares three financing regimes. Under each regime, the payback obligation of an educational loan is contingent, to some extent, on an individual’s future income.

Keywords and Phrases: Higher education, funding regimes, human capital, welfare.

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1 Introduction

Ample empirical evidence shows that higher education is an important element in generating personal incomes and in promoting the economic performance of countries [see, Barro (1998), Bassinini and Scarpenta (2001), Restuccia and Urrutia (2004). Consistent with this finding, investment in the education sectors of the OECD countries has increased substantially during the second half of the twentieth century [Greenaway and Haynes (2003), Checchi (2006)]. Yet, the expansion of higher education has often collided with fiscal pressures, thereby creating a tendency to shift the financial burden of an expanding education sector away from public funding towards private funding.¹ In particular, some European countries have recently substituted various forms of income support transfers with programs based on student loans. On balance, this process has resulted in a significant decline of public funding per student.

The shift towards private funding of higher education is sometimes justified with reference to an egalitarian income distribution. On average, students have a better socio-economic background and, hence, higher income prospects than other members of society. Therefore, public subsidies of higher education constitute an implicit monetary transfer from the poor towards the more affluent individuals. This problematic aspect of public funding has been pointed out by Friedman in his famous contribution on *Capitalism and Freedom* in 1962. In addition, as long as frictionless financial markets (for private education financing) exist, individual abilities will be used efficiently in the production process under a private funding scheme. In reality, however, financial markets for education financing are imperfect and sometimes even non-existent. Young individuals often cannot provide sufficient collateral which would allow them to borrow against their future incomes. Therefore, a structural change away from public funding towards private funding requires the provision of suitable loan programs which remove financial barriers for the young generation to participate in the higher education system. Ideally, such loan programs would also provide some diversification of individual ability risks.

¹ Even in Russia 47% of the students in the higher education system must finance their tuition fees (which are significant), as well as other related costs from their own resources [see Kaganovich (2005)].
within a given cohort of agents.

Friedman (1955, 1962) was the first to raise this issue and to suggest income-contingent financing of students’ investments in higher education. More recently, various forms of integrating income-sensitive elements into financing schemes for higher education have been discussed in Barr and Crawford (1988) and in Greenaway and Haynes (2003). Lleras (2004) focuses more generally on strategies to implement income-contingent funding of higher education via the private sector.²

Some countries have already gained experience with income-contingent student loan programs. Australia led the way in 1989 and was followed by Ghana, Sweden, Chile, New Zealand and the UK. The Australian model was unique not only in making the payback obligations of education loans contingent on future individual incomes. It was also unique in using the existing tax authorities as a collection agency, which was unprecedented [see Lleras (2004)]. Recently the US have also introduced income-sensitive components into existing student loan repayment plans,³ and Israel is considering similar steps.

Our aim in this paper is to analyze the implications of different financing schemes for higher education in the framework of an overlapping generations model with endogenous human capital formation. Models of this type were used, for example, by Azariadis and Drazen (1990), Orazem and Tesfatsion (1997), Viaene and Zilcha (2002). Individuals are randomly endowed with innate abilities and, when young, they take out loans in order to finance investment in education. Investment decisions take place after agents have received noisy signals which are correlated to their random abilities (or random future incomes).

We distinguish between three financing regimes which specify the terms of repayment for education loans. Under the first regime, the government guarantees students unrestricted access to competitive credit markets. The government also guarantees enforcement of debt collection. The second regime links the repayment

²The design of student loan programs, repayment and debt default as well as some international experience has been discussed by Woodhall (1988). Albrecht and Ziderman (1993) provide evidence on loans collectibility and on the cost of such programs.

³The enactment of the Higher Education Reconciliation Act (HERA) of 2005 modifies the Direct Loan Program. It offers various forms of contingent repayment plans as well as income-sensitive repayments.
of a loan to an agent’s future income in a way which allows risk sharing across all
members in the same generation. The third regime also links the terms of repay-
ment to future income, but in a ‘narrower’ sense which supports sharing of ability
risks only among individuals in the same signal group. The second regime cannot be
decentralized in a competitive financial market setting, and it has major drawbacks
with regard to the efficiency of the human capital formation process. Therefore,
this regime serves mainly as a standard of comparison for the other two regimes.

As a main result we find that a repayment scheme that allows pooling of ability
risks within signal groups (third regime) stimulates investment in education and
economic growth compared with a funding scheme of competitive credit markets
(first regime). Also, the third regime generates higher welfare than the first regime.
The welfare comparison between the second and the third financing regimes can be
characterized in terms of technological and preference parameters.

The paper is organized as follows. We present our model and the above men-
tioned three financing regimes in Section 2. Section 3 examines the implications of
these financing schemes for investment in education and human capital accumula-
tion. Section 4 compares the welfare implications of these funding schemes. Section
5 concludes the paper. All proofs are relegated to the Appendix.

2 The Model

We consider an overlapping-generations economy with a single commodity, say,
physical capital, which can be consumed or invested in production. Individuals
live for three periods: the ‘youth period’, where each individual is supported by
parents. In this period, the agent takes out a loan and makes a capital investment
in education in order to acquire skills; the ‘middle period’, where individuals work,
earn labor income, consume and save. Labor income depends on each agent’s skills,
or human capital, which is assumed to be observable. Part of the labor income is
earmarked for the repayment of the loan. Finally, the ‘retirement period’ in which
individuals consume their total savings. There is no population growth and each
generation $G_t$ (i.e., all individuals born at date $t - 1$), $t = 0, 1, 2...$, consists of a
continuum of agents with (Lebesgue-) measure 1, say the interval $[0, 1]$. 


Our framework is characterized by heterogenous individuals in each generation, where heterogeneity is generated by random innate ability. While nature assigns abilities to individuals at birth, no individual knows exactly his own ability when, at young age, he invests privately in education. Therefore, the investment decision, \( x \), is made under uncertainty. In the next period, the agent learns his ability \( A \). We denote by \( \nu(A) \) the time-invariant density of agents with ability \( A \), where \( A \in \mathcal{A} = [A, \bar{A}] \subset \mathbb{R}_{++} \). From the perspective of a young individual, ability is random as it is the realization of a random variable with distribution \( \nu(\cdot) \). Yet, there is no aggregate uncertainty in the economy, i.e., the ex post distribution of abilities across the members of a generation is exactly \( \nu \). Our modeling approach follows the technique suggested in Feldman and Gilles [1985, Proposition 2], where uncertainty exists at the individual level but in the aggregate there is no uncertainty.

The production function of human capital is, in general, a complex function which depends on individual, family, and other parameters. We shall restrict the structure of the human capital formation process, in order to make our equilibrium comparative dynamics analytically manageable. We assume that the level of human capital, or skills, of an individual \( i \in G_t \), denoted by \( h^i_t \), depends on the (random) innate ability \( A^i \), the private investment in education \( x^i \in \mathbb{R}_+ \), and the average human capital of the older generation, denoted by \( H_{t-1} \) (which may represent the human capital of ‘teachers’). Namely,

\[
h^i_t = \varphi(A^i) g(x^i, H_{t-1}).
\]

Public investment in individual education, which is assumed to be the same for all agents, is included in the accumulation function, \( g \), through some implicit additive component. We make the following assumption about this process:

**Assumption 1** \( \varphi(A) \) is an increasing and differentiable function. \( g(x, H) \) is twice differentiable, strictly increasing and concave in the first argument, and satisfies \( \lim_{x \to 0} g'_1(x, H) = \infty \) for \( H > 0 \). Also \( g'_1(x, H) \) is non-decreasing in \( H \). Furthermore,

\[
K(x, H) := -\frac{g''_{11}(x, H)}{g'_1(x, H)}
\]

is non-increasing in \( x \), i.e., \( K'_1(x, H) \leq 0 \ \forall x, H \).
$K(x, H)$ is a measure of concavity (with respect to $x$) of the accumulation function $g$. By Assumption 1, this measure of concavity is decreasing in $x$ which implies that $g'_1(x, H)$ is convex in $x$. Thus, the marginal product of investment in education decreases at a declining rate. This restriction is satisfied by most functional forms commonly used in the literature to describe the transformation of educational investment into human capital formation.

Each agent chooses private investment in education after he has received a publicly observable signal $y \in Y \subset \mathbb{R}$. Within the group of agents with ability $A$, the signals are distributed according to the density $\nu_A(y)$. By assumption, the distribution of signals and abilities are correlated. Hence, the signal assigned to an agent can be used as a screening device for his unknown ability. Based on the screening information conveyed by the signal, the agent forms expectations about his ability in a Bayesian way. The distribution of signals received by agents in the same generation has the density $\mu(y) = \int_A \nu_A(y) \nu(A) \, dA$. Average ability of all agents who have received the signal $y$ is $\bar{\psi}(y) := E[\bar{\psi}(\tilde{A})|y] = \int_A \bar{\psi}(A) \nu_y(A) \, dA$, where $\nu_y(A)$ denotes the density of the conditional distribution of $A$ given the signal $y$.

In our model both the signals and the investments made by individuals in their education are publicly observable. We assume throughout the paper that the Monotone Likelihood Ratio Property (MLRP) holds, i.e., the signals are ordered in such a way that $y' \geq y$ implies that the posterior distribution of ability conditional on $y'$ dominates the posterior distribution of ability conditional on $y$ in the first-degree stochastic dominance. In this sense, higher signals are ‘good news’ [see, Milgrom (1981)].

Each young individual needs a loan in order to finance his investment in education. The terms of repayment are subject to government intervention. We shall consider three different forms of government intervention in the market for education loans:

1. **Regime I (Unrestricted Access to Credit Markets):** Under this regime the government guarantees each student unrestricted access to credit markets for funds needed to finance higher education. The government also guarantees enforcement of debt collection.
2. Regime II (Unrestricted Insurance of Loans): Under this regime the terms of repayment of a loan are linked to the realization of an individual’s future income (hence, linked to the realization of his human capital). This insurance arrangement pools the risks of all young agents in the same cohort who choose to invest in education. The governmental intervention includes releasing information about individual incomes, as well as guaranteeing the collection of debt.

3. Regime III (Restricted Insurance of Loans): Again, the terms of repayment are linked to random individual future incomes. Yet, the insurance arrangement pools the risks within each signal group (group of agents who have received the same signal) separately.

We shall study these three financing regimes separately, assuming that the same regime applies to all agents. Thus, in our economy only one regime prevails; in particular, students cannot choose between a loan in the credit market with un-contingent terms of repayment (Regime I) and a loan with contingent repayment (Regimes II or III). This assumption seems reasonable because the implementation of any regime requires some government intervention.\(^4\) Hence, the regimes do not emerge, and compete against each other, endogenously. Rather, they should be viewed as political choice variables. The implications of those political choice variables for the time path of aggregate human capital and welfare will be analyzed below.

Regime II serves as a benchmark in our analysis. In some European countries like Germany funding concepts for higher education in the spirit of Regime II have been discussed in the 70s. While this regime may have some appeal from the viewpoint of an egalitarian income distribution, it has major drawbacks in terms of inefficiencies for the human capital formation process. In addition, Regime II cannot be decentralized in a competitive financial markets setting because banks have an incentive to offer individuals with high signals loans on better terms than

\(^4\)This holds true even for Regime I. Given the evidence about borrowing constraints that students face in the financial markets (see, e.g., Galor and Zeira (1993)), some intervention by the government is needed to implement the regime.
individuals with low signals. Therefore, in this paper we treat Regime II mainly as a standard of comparison for the other two regimes.

The agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function
\[ U(c_1, c_2) = u_1(c_1) + u_2(c_2). \]
c_1 and c_2 denote consumption in the second and third period of life, respectively. In his first period of life each agent makes a capital investment in education, but he does not consume. The utility functions \( u_i : \mathbb{R}_+ \to \mathbb{R}, i = 1, 2 \), are strictly increasing and strictly concave.

In each period, competitive firms produce a commodity that can be used for consumption. The firms use physical capital, \( K \), and human capital, \( H \), as production factors. Physical capital fully depreciates in the production process. We describe the production process by an aggregate production function \( F(K, L) \), which exhibits constant returns to scale. In his ‘working period’ each agent \( i \) inelastically supplies \( l \) units of labor and, hence, his supply of human capital is \( lh \). Without loss of generality, we set \( l = 1 \). The production function has the following properties:

**Assumption 2** \( F(K, H) \) is concave, homogeneous of degree 1, and satisfies \( F_K > 0, F_H > 0, F_{KK} < 0, F_{HH} < 0 \).

Physical capital is internationally mobile while human capital is assumed to be immobile.\(^5\) This implies that the interest rate, \( \tilde{r}_t \), is exogenously given at each date (small country assumption). Having assumed full depreciation of physical capital in each period, marginal productivity of aggregate physical capital, \( K_t \), equals \( 1 + \tilde{r}_t \). Thus, given the aggregate stock of human capital at date \( t, H_t \), the stock of physical capital, \( K_t \), adjusts such that
\[ R_t := 1 + \tilde{r}_t = F_K(K_t, H_t) \quad t = 1, 2, 3, \cdots \]

\(^5\)This assumption is in line with some implications of the globalization process that we have witnessed in recent decades. While globalization has increased the international mobility of physical capital tremendously, movements of labor across international borders are still the exception rather than the rule.
is satisfied. This implies by Assumption 1, that $K_t/H_t$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate (price of one unit of human capital), $w_t = F_L(K_t/H_t, 1)$, is also determined once $\bar{r}_t$ is given.

### 2.1 Financing Regime I

Let us consider the decision problem that each $i \in G_t$ faces under Regime I, given $\bar{r}_t, w_t$, and $H_{t-1}$. At date $t - 1$, when ‘young’, this individual chooses investment in education, $x^i$, while his ability is still unknown. The investment decision will be based on the noisy information about the agent’s ability that is conveyed by the signal $y^i$. The investment, $x^i$, is financed through a standard loan contract which is signed at date $t - 1$, and which involves the obligation to pay back $R_t x^i$ in period $t$.

An optimal decision is taken in two consecutive steps. At date $t - 1$, after the signal $y^i$ has been observed, our agent $i \in G_t$ chooses an optimal level of investment in education, $x^i$, and signs the associated loan contract. When choosing the investment level, the agent perceives his ability to be randomly distributed according to $\nu_{y^i}(\cdot)$. Optimal savings, $s^i$, are chosen at date $t$ after ability, $A^i$, has been observed. At this time, $x^i$ (which has been chosen at date $t - 1$) is predetermined.

For given levels of $h^i, x^i, w_t, R_t$, and $R_{t+1}$ the optimal consumption and saving decision is determined by

$$\max_{c^i_1, c^i_2, s^i} \left[ u_1(c^i_1) + u_2(c^i_2) \right]$$

s.t. \hspace{1em} $c^i_1 = w_t h^i - R_t x^i - s^i$ \hspace{1em} (3)

$$c^i_2 = R_{t+1} s^i.$$ \hspace{1em} (4)

Optimal savings satisfy the necessary and sufficient first order condition

$$u'_1(w_t h^i - R_t x^i - s^i) = R_{t+1} u'_2(R_{t+1} s^i), \forall A^i.$$ \hspace{1em} (5)

The optimal level of investment in education is determined by (we mark random variables by a $\tilde{\cdot}$)

$$\max_{x^i} E \left[ u_1(\tilde{c}^i_1) + u_2(\tilde{c}^i_2) \middle| y^i \right]$$ \hspace{1em} (6)
\[ \begin{align*}
\text{s.t. } \hat{c}_1 &= w_t \hat{h}_i - R_t x_i - \hat{s}_i \\
\hat{c}_2 &= R_{t+1} \hat{s}_i, 
\end{align*} \]  

(7)  

(8)

where \( \hat{h}_i \) is given by equation (1) and \( \hat{s}_i \) satisfies equation (5). By the Envelope theorem and the strict concavity of the utility functions, this optimization problem has a unique solution determined by the first order condition

\[
E \left\{ \left[ w_t \varphi(\hat{A}) g'_1(x_i, H_{t-1}) - R_t \right] u'_1(\hat{c}_i) \bigg| y^i \right\} = 0. 
\]

(9)

At date \( t - 1 \), the members of \( G_t \) differ only by the signals they have received. Therefore, all individuals in the same signal group, \( G_t(y) \), choose the same investment level, denoted \( x_t(y) \). The net income (gross income net of repayment of the loan) in the working period of individuals in \( G_t(y) \) is

\[
I_t(A, y) = w_t \varphi(A) g\left(x_t(y), H_{t-1}\right) - R_t x_t(y). 
\]

(10)

The aggregate stock of human capital at date \( t \) can be expressed as

\[
H_t = \int_Y \tilde{\varphi}(y) g\left(x_t(y), H_{t-1}\right) \mu(y) \, dy. 
\]

(11)

Using (10) in (5), we may write optimal savings as \( s_t(I_t(A, y)) \). Optimal consumption levels in the second and third periods of life are denoted by \( c^1_t(I_t(A, y)) \) and \( c^2_t(I_t(A, y)) \), respectively. From (5) we derive \( s'_t(I_t) \in (0, 1) \). Equations (3) and (4) then imply \( c^1_t(I_t) \in (0, 1) \) and \( c^2_t(I_t) \in (0, R_{t+1}) \). Our economy starts at date 0 with given initial stocks of physical capital, \( K_0 \), and human capital, \( H_0 \). The dynamic equilibrium describes the time path of factor prices, savings and consumption profiles as well as the evolution of the individual human capital stocks which depend on the investments in education of the young generations.

**Definition 1** Given the international interest rates \( (r_t) \) and the initial stocks of human and physical capital \( H_0 \) and \( K_0 \), a competitive equilibrium consists of a sequence \( \{(c^1_t, c^2_t, s^i, x^i)_{i \in G_t}\}_{t=1}^{\infty} \), and a sequence of wages \( (w_t)_{t=1}^{\infty} \), such that:

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\( x_t(y) \) depends on \( H_{t-1} \). For notational convenience, we have chosen not to include \( H_{t-1} \) as an argument of the investment function. We shall apply this convention to all behavioral functions and maintain it when we turn to other financing regimes.
At each date \( t \), given \( r_t, H_{t-1}, \) and \( w_t \), the optimum for each \( i \in G_t \) in problems (2)-(4) and (6)-(8) is given by \((c_i^1, c_i^2, s_i, x_i)\).

The aggregate stocks of human capital, \( H_t, t = 1, 2, \ldots \), satisfy (11).

Wage rates are determined by \( w_t = F_L(K_t/H_t, 1), t = 1, 2, \ldots \).

Our comparative dynamics analysis assumes that competitive equilibria (under various regimes) start from the same initial stocks, \( K_0, H_0 \), and compares the allocations along these dynamic paths period by period. The above definition of equilibrium also applies (with minor and obvious modification), if the economy operates under one of the two financing schemes outlined below.

2.2 Financing Regime II

Next we analyze the behavior of young individuals when funds needed to finance investment in higher education take the form of ‘insured loans’. Assume that the payback obligation of a loan is linked to an individual’s future income: agents with higher incomes (i.e., higher abilities) have higher payback obligations.\(^7\) Clearly, such loan contracts provide insurance against uncertain income prospects which are due to random ability realizations. We shall consider a risk pooling program of education loans that includes all young individuals of a given generation and which requires no subsidization from the government. In particular, by assumption, the regular credit markets cannot be used for funding educational expenditures. Let \( \bar{\varphi} := \mathbb{E} y \bar{\varphi}(y) \). An agent \( i \) in \( G_t \) who receives a loan to finance investment \( x^i \) is obliged to pay back \( R_t x^i \bar{\varphi}(\bar{A}^i) / \bar{\varphi} \) in his working period, if his random income component turns out to be \( \varphi(A^i) \).

Proceeding as in Section 2.1, the necessary and sufficient conditions for optimal

\(^7\text{An example of income-dependent rate of interest on educational loans exists now at the US tax code: all interest payments related to student loans are tax deductible!} \)
savings and investment decisions are
\[ u_1' \left( w_t h^i - R_t x^i \frac{\varphi(A^i)}{\bar{\varphi}} - s^i \right) = R_{t+1} u_2'(R_{t+1} s^i), \quad \forall A^i \tag{12} \]
\[ \bar{\varphi} g_1'(x^i, H_{t-1}) = \frac{R_t}{w_t}. \tag{13} \]

(13) implies that all individuals will invest the same amount, regardless of the signal they have received, i.e., \( x^i = \hat{x}_t \forall i \in G_t \). Clearly, \( \hat{x}_t \) depends on \( H_{t-1} \) and, by our assumptions, it is nondecreasing in \( H_{t-1} \). Due to the ‘fair insurance’ arrangement provided under Financing Regime II, coupled with the risk aversion assumption, the optimal investment in education \( \hat{x}_t \) maximizes the expected lifetime net income prior to the revelation of the signal; namely, \( \hat{x}_t \) solves
\[ \max_x E \left\{ w_t \varphi(\hat{A}) g(x, H_{t-1}) - R_t x \frac{\varphi(\hat{A})}{\bar{\varphi}} \right\} \tag{14} \]
and, hence, it is independent of \( y \).

Net income in the working period of an agent in \( G_t \) with ability \( A \) is given by
\[ \hat{I}_t(A) = w_t \varphi(A) g(\hat{x}_t, H_{t-1}) - R_t \hat{x}_t \frac{\varphi(A)}{\bar{\varphi}}, \tag{15} \]
and the aggregate stock of human capital at date \( t \) is
\[ \hat{H}_t = \bar{\varphi} g(\hat{x}_t, H_{t-1}). \tag{16} \]

Using (15) in (12), we may write optimal savings as \( \hat{s}_t(\hat{I}_t(A)) \).

### 2.3 Financing Regime III

We finally consider a further class of ‘insured’ loan contracts which specify different terms of repayment for individuals in different signal groups. Again, the payback obligation of a loan is linked to an agent’s future income and, hence, his random ability, but the implied risk pooling is restricted to individuals in a given

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*To ease notation we simply write \( \hat{x}_t \) instead of \( \hat{x}_t(H_{t-1}) \) unless the dependency on \( H_{t-1} \) is explicitly needed.*
signal group. An agent \( i \) in \( G_t \) with signal \( y^i \) who receives a loan to finance investment in education \( x^i \) is obliged to pay back \( R_t x^i \frac{\varphi(A^i)}{\varphi(y)} \) in his working period, if his income turns out to be determined by \( A^i \). This program of education loans allows risk sharing on fair terms within each signal group, but does not provide risk sharing, or cross-subsidization, among different signal groups.\(^9\) As before, this income-linked loan program does not require any funding from the government: The agency providing the loans pays a gross interest rate \( R_t \) in the capital market which is just equal to the rate realized on total loans within each signal group, i.e.,

\[
\int_A R_t \frac{\varphi(A)}{\varphi(y)} \nu_y(A) \, dA = R_t.
\]

The necessary and sufficient conditions for optimal savings and investment decisions are

\[
w_1' \left( \frac{w_t h^i - R_t x^i \varphi(A^i)}{\varphi(y^i)} - s^i \right) = R_{t+1} u'_2(R_{t+1} s^i), \quad \forall A^i \tag{17}
\]

\[
\varphi(y^i) g'_1(x^i, H_{t-1}) = \frac{R_t}{w_t}, \quad \forall y^i. \tag{18}
\]

According to (18), optimal investment in education of agents in the signal group \( G_t(y) \) depends on the signal only via the term \( \varphi(y) \). We may, therefore, express individual investment as \( \tilde{x}_t(\varphi(y)) \). Again, our notation suppresses the dependence of investment on \( H_{t-1} \). From (18) we see that \( \tilde{x}_t(\varphi(y)) \) maximizes the expected conditional net income \( \tilde{\varphi}(y) w_t g(x, H_{t-1}) - R_t x \).

Since \( g(x, H) \) is concave in \( x \) and since \( \tilde{\varphi}(y) \) is increasing in \( y \) (due to MLRP), equation (18) implies

**Lemma 1** Optimal investment in education under Financing Regime III, \( \tilde{x}_t(\cdot) \), is increasing in the signal \( y \), and non-decreasing in \( H_{t-1} \).

Thus, good news (higher signal) stimulates investment in education. Net income in the working period of an agent in \( G_t \) with ability \( A \) is given by

\[
\tilde{I}_t(A, \varphi(y)) = w_t \varphi(A) g(\tilde{x}_t(\varphi(y)), H_{t-1}) - R_t \tilde{x}_t(\varphi(y)) \frac{\varphi(A)}{\varphi(y)} \tag{19}
\]

\(^9\)There exist real world examples where private fundings are based on grouping students either by universities (e.g., at Yale, Harvard, etc.) or by fields of career. Lleras (2004, p. 66) argues that such practice is justified because ‘grouping students by fields reflects similarity in the risks and the expected returns within the same group’.
and the aggregate stock of human capital at date $t$ is

$$\hat{H}_t = \int_Y \tilde{\varphi}(y) g\left(\tilde{x}_t(\tilde{\varphi}(y)), H_{t-1}\right) \mu(y) \, dy.$$  \hspace{1cm} (20)

Using (19) in (17), we may write optimal savings as $\tilde{s}_t\left(\tilde{I}_t(A, \tilde{\varphi}(y))\right)$.

## 3 Human Capital Accumulation

In this section we compare the implications of the three financing schemes of educational investment for the equilibrium accumulation of human capital. The financing schemes involve different degrees of risk sharing in the economy. It is well known from the literature that an investor may invest more funds into a risky project, if, due to effective risk sharing arrangements, he can insure part of the project risk on easy terms. On the other hand, more effective insurance mechanisms also have the potential of destroying incentives for some agents to properly invest in education. The role of the various financing schemes for investment in education and human capital accumulation therefore deserves close scrutiny.

**Proposition 1** In equilibrium,$^{10}$

(i) each agent chooses higher investment in education under Financing Regime III compared to Financing Regime I: $x_t(y) \leq \tilde{x}_t(\tilde{\varphi}(y))$ for all signals $y$;

(ii) the stock of human capital under Financing Regime III is larger than that under Financing Regime I: $\hat{H}_t \geq H_t$ for $t = 1, 2, \ldots$

This result demonstrates the critical role of risk pooling, which is restricted to signal groups. If such risk pooling takes place on conditionally fair terms, it enhances individual investment in education and, thereby, stimulates the formation of human capital, compared to non-insured funding via credit markets. Based on this finding, we are led to speculate that risk pooling on unconditionally fair terms may

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$^{10}$The inequalities in (i) and (ii) are strict, if the conditional distribution of $\varphi(\tilde{A})$ is non-degenerate for all signals $y$. 

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provide even stronger incentives for investment in education and human capital formation. As it turns out, this presumption is misleading. While average investment in education may (but need not) be higher under Regime II than under Regime III, the latter regime always generates higher levels of aggregate human capital. To derive these results, we introduce the concepts of ‘moderately decreasing concavity’ and ‘strongly decreasing concavity’. Let

\[
\hat{K}(x, H) := -\frac{g''_1(x, H)}{(g'_1(x, H))^2} \quad \left[= K(x, H)/g'_1(x, H)\right].
\]

\(K(\cdot)\) and \(\hat{K}(\cdot)\) are both (different) measures of concavity w.r.t. \(x\) for the accumulation function \(g(\cdot)\).

**Definition 2** Given the restrictions formulated in Assumption 1, the accumulation function \(g(x, H)\) exhibits

(i) moderately decreasing concavity, if \(\hat{K}(x, H)\) is increasing in \(x\).

(ii) strongly decreasing concavity, if \(\hat{K}(x, H)\) is decreasing in \(x\).

Note that ‘moderately decreasing concavity’ and ‘strongly decreasing concavity’ are mutually exclusive properties. Let aggregate investment in education at time \(t\) under Regime I be \(X_t := E[x_t(y)]\). For regimes II and III, aggregate investments \(\hat{X}_t\) and \(\check{X}_t\) are defined analogously.

**Proposition 2** In equilibrium,

(i) aggregate investment in education under Regime III is higher than under Regime II, i.e., \(\hat{X}_t \geq \check{X}_t\) for all \(t\), if the accumulation function \(g(x, H)\) exhibits strongly decreasing concavity;

(ii) \(\check{X}_t \geq \hat{X}_t\) holds for all \(t\), if \(g(x, H)\) is independent of \(H\) and exhibits moderately decreasing concavity.

This result is quite surprising because the better talented agents subsidize the less talented ones more heavily under Regime II, where all risks are pooled, than
under Regime III, where risks are pooled conditional on the signals. One might conjecture, therefore, that the incentives to invest in education are always stronger under the latter regime. In view of Proposition 2, this intuition is misleading, if the accumulation function exhibits moderately decreasing concavity and is independent of $H$. Such curvature implies (as compared to the case of strongly decreasing concavity) that the marginal return to investment decreases more rapidly. As a consequence, investment is concave, i.e., $x(\cdot)$ responds increasingly less sensitive to higher signals. In other words, if the marginal return to investment declines very fast, then it is not worthwhile to increase $x$ a lot in response to a higher signal. Yet, if the investment function is concave, then investment chosen at the average signal (which is the investment level under Regime II) is higher than average investment under Regime III.

Surprisingly, according to our next proposition, the financing regimes II and III can unambiguously be ranked with regard to their impact on human capital formation. Thus, in general, higher investment in education is neither necessary nor sufficient for higher economic growth.

**Proposition 3** The equilibrium aggregate human capital levels under Regime III are higher than those under Regime II at all dates: $\tilde{H}_t \geq \hat{H}_t$, for all $t$.

Thus, the Financing Regime III is more efficient than Regime II in terms of generating economic growth. Propositions 2 and 3 together imply that higher human capital formation does not necessarily require higher aggregate investment in education. Since marginal returns to investment depend on individual abilities, the distribution of individual investments across agents with different abilities affects the formation of human capital in the economy. In particular, a financing regime that encourages investments of highly talented agents and discourages investments of poorly talented agents may achieve high levels of aggregate human capital with relatively low levels of aggregate investment in education. In fact, if $g(\cdot)$ is independent of $H$, this happens under moderately decreasing concavity of the accumulation function, when we switch from Regime II to Regime III. Under Regime II, investment in education is high but uncorrelated to individual ability. Under Regime III, by contrast, the better talented agents tend to invest more aggressively than the
poorly talented agents. Since individual investments and abilities are better aligned (and, in this sense, the transformation of aggregate investment in education into aggregate human capital is more efficient) under Regime III than under Regime II, aggregate human capital levels are higher even though the economy as a whole may invest less in education.

3.1 Special Cases

In order to illustrate the results in this section we focus on two classes of accumulation functions. The first class is the family of CRRA functions, and the second class is the family of CARA functions.

Case 1: Let \( g(x, H) : \mathbb{R}_+^2 \to \mathbb{R} \) belong to the CRRA family, i.e.,

\[
g(x, H) = \frac{x^{1-\gamma}}{1-\gamma} H, \quad 0 < \gamma < 1.
\]  

(21)

Straightforward calculation shows that \( K'_1(x, H) \leq 0, \forall x, H \), i.e., the accumulation functions exhibit decreasing concavity. Furthermore, \( \hat{K}(x, H) = \gamma x^{\gamma-1}/H \) is strictly decreasing in \( x \), hence, the accumulation function exhibits strongly decreasing concavity. Based on our earlier results we conclude:

**Corollary 1** If \( g(x, H) \) belongs to the class of CRRA functions in (21), then

(i) aggregate investment is higher under Regime III than under Regime II: \( \hat{X}_t \geq \tilde{X}_t, \forall t; \)

(ii) the stock of human capital is higher under Regime III than under Regime II: \( \hat{H}_t \geq \tilde{H}_t, \forall t. \)

Case 2: Let \( g(x, H) \) belong to the CARA family, i.e.,

\[
g(x, H) = (1 - e^{-\gamma x})H, \quad \gamma > 0.
\]  

(22)

In this case, \( \hat{K}(x, H) = e^{\gamma x}/H, K(x) = \gamma \) and, hence, \( g(x, H) \) exhibits moderately decreasing concavity. Our earlier results then imply
Corollary 2 If \( g(x, H) \) belongs to the class of CARA functions in (22), then

(i) aggregate investment is lower under Regime III than under Regime II: \( \breve{X}_t \leq \hat{X}_t, \forall t; \)

(ii) the stock of human capital is higher under Regime III than under Regime II: 
\[ \breve{H}_t \geq \hat{H}_t, \forall t. \]

4 Welfare Implications

Our welfare analysis of the various financing regimes will be based on an ex-ante welfare concept. Note that all agents of the same generation are identical \textit{ex ante}, i.e., before their signals have realized. We therefore define economic welfare, \( W_t \), of generation \( G_t \) as the ex-ante expected utility of members of \( G_t \). A financing regime \( j \) will be ranked higher than a financing regime \( k \) \((j, k = I, II, III)\) if \textit{all} generations attain higher welfare under Regime \( j \) than under Regime \( k \).

Welfare of generation \( G_t \) under Regime I is defined by

\[ W_t := \int_Y V_t(y, H_{t-1}) \mu(y) \, dy \]

where

\[ V_t(y, H_{t-1}) := E \left[ u_1(c_t^1(\tilde{A}, y)) + u_2(c_t^2(I_t(\tilde{A}, y))) \right] \bigg| y . \]

\( V_t(y, H_{t-1}) \), the value function for generation \( G_t \), represents the conditional expected utility of a member of \( G_t \) with signal \( y \). Since \( g(x, H_{t-1}) \) is increasing in \( H_{t-1} \), the value function is also increasing in \( H_{t-1} \). The value functions and welfare levels of generation \( G_t \) under regimes II and III, \( \hat{V}_t(y, \hat{H}_{t-1}), \breve{V}_t(y, \breve{H}_{t-1}) \) and \( \hat{W}_t, \breve{W}_t \), are defined symmetrically. We say that, e.g., welfare is higher under Regime III than under Regime II, if \( \hat{W}_t \geq \breve{W}_t \) holds for all \( t \geq 1 \).

Proposition 4 In equilibrium, economic welfare is higher under Regime III than under Regime I.
Thus, under any political voting process, if it were to be conducted prior to the revelation of signals, the arrangement of Regime III, which provides conditionally insured financing of private investment in education, will prevail against a regime of pure credit markets. Regime III leads to higher welfare, because the individuals benefit from partial risk pooling. This positive impact on welfare is not counteracted by adverse incentive effects which might result from partial risk sharing: since risks are only shared within signal groups, the signal risk remains uninsured. Therefore, the incentive structure remains intact and all agents continue to take their signals into account when choosing investment in education.

Next we turn to a comparison of economic welfare under regimes II and III. Under Regime III aggregate human capital is accumulated more efficiently because agents take their signals into account when deciding about investment in education: agents with good signals who are, on average, better talented invest more than agents with bad signals. Under Regime II, by contrast, everybody invests the same amount regardless of the signal. On the other hand, Regime II provides better pooling of individual income risks than Regime III. To illustrate the interaction between economic efficiency and risk sharing we specialize our economy by choosing the following functional forms:

\[
\begin{align*}
  u_1(c) &= \frac{c^{1-\beta}}{1 - \beta}; \\
  u_2(c) &= \delta \frac{c^{1-\beta}}{1 - \beta}; \\
  g(x) &= \frac{x^{1-\gamma}}{1 - \gamma},
\end{align*}
\]

(23)

where \(\gamma \in (0, \frac{1}{2}) \) and \(1 \neq \beta > 0\). Note that the specification in (23) implies that \(g(\cdot)\) depends only on investment in education, but not on the human capital stock of the previous generation. The main implication of this simplification is that optimal individual investment in education no longer depends on the human capital stock of the earlier generation.

For some suitably chosen \(m \in (0, 1)\) optimal consumption under Regime III can
be stated as

\[ c^1_t = \frac{\varphi(A)\gamma(1 - m)}{1 - \gamma} w_t^{1/\gamma} \left( \frac{\bar{\varphi}(y)}{R_t} \right)^{1 - \frac{\gamma}{\gamma}} \]

\[ c^2_t = \frac{\varphi(A)R_{t+1}\gamma m}{1 - \gamma} w_t^{1/\gamma} \left( \frac{\bar{\varphi}(y)}{R_t} \right)^{1 - \frac{\gamma}{\gamma}}. \]

From the last two equations we derive the value function \( \tilde{V}_t(\cdot) \) which satisfies

\[ \frac{1 - \beta}{M_t} \tilde{V}_t(y) = E \left[ \varphi(\tilde{A})^{1 - \beta} \left| y \right| \left( \frac{\bar{\varphi}(y)}{R_t} \right)^{\frac{(1 - \gamma)(1 - \beta)}{\gamma}} \right], \quad (24) \]

where \( M_t := [\gamma w_t^{1/\gamma}/(1 - \gamma)]^{1 - \beta} [(1 - m)^{1 - \beta} + \delta(R_{t+1}m)^{1 - \beta}] \) is a positive constant.

Similarly, the value function for Regime II satisfies

\[ \frac{1 - \beta}{M_t} \hat{V}_t(y) = E \left[ \varphi(\tilde{A})^{1 - \beta} \left| y \right| \left( \frac{\bar{\varphi}(y)}{R_t} \right)^{\frac{(1 - \gamma)(1 - \beta)}{\gamma}} \right]. \quad (25) \]

**Proposition 5** Assume that the utility functions and the human capital formation function are of the type specified in (23).

(i) If the measure of relative risk aversion, \( \beta \), is larger than 1, then economic welfare under Regime II is higher than under Regime III.

(ii) If \( \beta \leq (1 - 2\gamma)/(1 - \gamma) \), then economic welfare under Regime II is lower than under Regime III.

In Section 3, we have seen that individual investments and individual abilities are better aligned and, therefore, the allocation of investment in education is more efficient under Regime III than under Regime II. Nevertheless, according to Proposition 5(i) all agents may be better off under Regime II. This result can be

\[ m \text{ represents the saving propensity out of net income, which is constant under the above specification.} \]
reconciled with economic intuition once we realize that economic welfare depends not only on the efficiency of the human capital accumulation process, but also on the equilibrium risk allocation. Under Regime II individual ability risks are better insured, while under Regime III investment is more efficiently transformed into human capital. According to Proposition 5 the former effect is dominant in terms of economic welfare, if the individuals are highly risk-averse; and the latter effect is dominant if individuals are moderately risk-averse, i.e., if the measure of relative risk aversion is sufficiently small.

Proposition 5 has been derived under the assumption that the capital formation function \( g(\cdot) \) is independent of \( H \). If, under a more general specification, \( g(\cdot) \) is an increasing function of \( H \), the welfare comparison between regimes II and III shifts in favor of Regime III: In view of Proposition 3, aggregate human capital levels are higher under Regime III than under Regime II. Therefore, since under any regime the value function of generation \( G_t \) is increasing in the human capital stock of \( G_{t-1} \), the second part of Proposition 5 remains valid (and can even be strengthened). By contrast, the first part of Proposition 5, which claims that welfare under Regime II can be higher than welfare under Regime III, may no longer hold, if the previous generation’s capital stock exerts a strong externality on capital formation in the current period.

5 Conclusion

The incentives of individuals to invest in higher education are affected by the financing scheme under which educational loans are available to them. In this paper we have analyzed and compared the implications of three different funding regimes. The regimes differ with regard to the terms of repayment of educational loans. In particular, the extent to which the payback obligations are contingent on the individuals’ future incomes plays a critical role. While all regimes are self-financing, i.e., they do not require government subsidies, some government intervention is necessary in order to make the funding mechanisms operative. The first regime works via competitive credit markets. The role of the government is to ensure that students have unrestricted access to those markets and to enforce debt collection.
The second regime pools the income risks of all agents in the same generation and treats them equally, i.e., it imposes the same income-dependent payback obligations on all individuals. This regime cannot be decentralized but must be implemented by the government. The third regime pools income risks within each signal group (partial risk pooling). All agents in the same signal group are treated equally, but individuals with good signals receive loans on more favorable (income-contingent) terms than agents with bad signals.

We have studied these three financing regimes under the assumption that the same regime applies to all agents. In particular, agents are not free to choose a repayment scheme that looks most attractive to them. This specification constitutes an important limitation for the generality of our model. Of course, in a more general setting several financing schemes might coexist at the same time so that in equilibrium agents self-select into different groups according to the repayment schemes they prefer. This possibility is excluded in our analysis and may be the topic of future research.

We found that aggregate investment in education and human capital stocks are higher under Regime III compared with Regime I, i.e., partial pooling of income risks stimulates economic growth. By contrast, unrestricted risk pooling causes efficiency losses: investment in education is more efficiently transformed into human capital under Regime III compared with Regime II. Finally, Regime III leads to higher welfare than Regime I. And the welfare comparison between regimes II and III depends on the individuals’ attitudes towards risk: Regime III generates higher (lower) welfare than Regime II, if the measure of relative risk aversion is sufficiently small (high). This result reflects the interaction of two mechanisms resulting from the fact that income risks are better pooled under Regime II, while the process which transforms educational investment into human capital is more efficient under Regime III.

The main purpose of our study was to compare Regime I (competitive credit markets) with Regime III (partial risk pooling), because these regimes are implementable in a decentralized setting. Our analysis yields a clear and unambiguous policy recommendation in favor of Regime III which generates higher growth as well as higher welfare than Regime I.
Our model also has implications with regard to income inequality under the various financing schemes for higher education. These implications have not been reported here. Based on a standard concept of income dispersion, it can be shown that, not surprisingly, Regime II leads to a more egalitarian income distribution than any of the two other regimes.

Appendix

Proof of Proposition 1: (i) Under scheme I, individuals have access to loans provided by the banks at the market interest rates $R_t$. For each given $y$ and fixed $H_{t-1}$ we have,

$$\text{Cov} \left[ (\varphi(\hat{A})|y), u'_t(c^1_t(\hat{A}, y)) \right] \leq 0. \quad (26)$$

The covariance in (26) is non-positive, since $c^1_t(A, y)$ and $\varphi(A)$ are both increasing in $A$. From equation (9) and equation (26) we derive $E[w_t g'_1(x_t(y), H_{t-1}) \varphi(\hat{A}) - R_t|y] \geq 0$ which implies

$$g'_1(x_t(y), H_{t-1})\varphi(y) \geq \frac{R_t}{w_t}.$$  \quad (27)

Combining (18) and (27), and making use of the concavity of $g(x, H)$ in $x$, we conclude that $x_t(y) \leq \hat{x}_t(\varphi(y))$.

(ii) The proof is by induction over time periods $t = 1, 2, \cdots$. Since $K_0, H_0$ are given at the outset, part (i) implies $\hat{H}_1 \geq H_1$. Assume $\hat{H}_{t'} \geq H_{t'}$ for all $t' \leq t$. Since, by assumption, $g'_1(x, H)$ is non-decreasing in $H$, $g'_1(x_{t+1}(y), \hat{H}_t)\varphi(y) \geq g'_1(x_{t+1}(y), H_t)\varphi(y) \geq \frac{R_{t+1}}{w_{t+1}}$ and $g'_1(\hat{x}_{t+1}(\varphi(y)), \hat{H}_{t+1})\varphi(y) = \frac{R_{t+1}}{w_{t+1}}$ are satisfied. Thus, $x_{t+1}(y) \leq \hat{x}_{t+1}(\varphi(y))$ holds for each individual in generation $G_{t+1}$ with signal $y$. Integrating over all signals yields $H_{t+1} \leq \hat{H}_{t+1}$. \hfill \blacksquare

Proof of Proposition 2: (i) Differentiating (18), we obtain

$$\frac{\partial \hat{x}_t(\varphi(y))}{\partial \varphi(y)} = \frac{w_t}{K(\hat{x}_t(\cdot), H_{t-1})} R_t.$$

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\( \hat{x}(\cdot) \) is increasing in \( \hat{\varphi}(y) \) according to (18). Since \( g(\cdot) \) exhibits strongly decreasing concavity, \( \hat{x}(\cdot) \) is convex in \( \hat{\varphi}(y) \). Now, (13) and (18) imply \( \hat{x} = \hat{x}(\hat{\varphi}) \). Proposition 3 claims (and proves independently of this proposition) that \( \hat{H}_{t-1} \geq \hat{H}_{t-1} \). Using this fact along with monotonicity of \( \hat{x} \) in \( \hat{H}_{t-1} \) and convexity in \( \hat{\varphi}(y) \), we conclude \( \hat{X}_t = \mathbb{E}\left[ \hat{x}_t(\hat{\varphi}(\hat{y}), \hat{H}_{t-1}) \right] \geq \hat{x}_t(\hat{\varphi}, \hat{H}_{t-1}) = \hat{x}_t(\hat{H}_{t-1}) = \hat{X}_t \). (28)

(ii) Under this restriction on the functional form of \( g(\cdot) \), \( \hat{x}(\cdot) \) is independent of \( \hat{H}_{t-1} \) and concave in \( \hat{\varphi}(y) \). The inequality signs in (28) are thus all reversed which proves the claim. □

**Proof of Proposition 3:** The proof consists of two steps.

(i) Let \( \bar{h}(z, H_{t-1}) := zg(\bar{x}(z), H_{t-1}) \). In a first step we show that \( \bar{h}(z, H_{t-1}) \) is convex in \( z \). Differentiating \( \bar{h}(\cdot) \) with respect to \( z \) and using equation (18), we get

\[
\bar{h}''_{11}(z, H_{t-1}) = \frac{R_t \bar{x}_t'(z)}{w_i z} \left[ 1 + \frac{\bar{x}_t''(z) z}{\bar{x}_t'(z)} \right].
\]

From (18) we calculate the elasticity of the investment function as

\[
\frac{\bar{x}_t''(z) z}{\bar{x}_t'(z)} = - \left( 1 + \frac{K'_1(\bar{x}_t(z), H_{t-1})}{[K(\bar{x}_t(z), H_{t-1})]^2} \right).
\]

Combining the last two equations we obtain

\[
\bar{h}''_{11}(z, H_{t-1}) = - \frac{K'_1(\bar{x}_t(z), H_{t-1})/z}{[K(\bar{x}_t(z), H_{t-1})]^2 K(\bar{x}_t(z), H_{t-1})}.
\]

By Assumption 1, \( K'_1(\cdot) \) is non-positive and, hence, \( \bar{h}(\cdot) \) is convex in \( z \).

(ii) Now we can prove the claim of the proposition by an induction argument. Assume \( \hat{H}_{t-1} \geq \hat{H}_{t-1} \) for \( t' \leq t \). We conclude that

\[
\hat{H}_t = \mathbb{E}\left[ \bar{h}(\bar{\varphi}(\bar{y}), \hat{H}_{t-1}) \right] \geq \bar{h}(\bar{\varphi}, \hat{H}_{t-1}) = \bar{\varphi}g(\bar{x}(\bar{\varphi}), \hat{H}_{t-1}) \geq \bar{\varphi}g(\hat{x}_t, \hat{H}_{t-1}) = \hat{H}_t,
\]

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where the first inequality follows from step (i), and the second inequality follows from the induction hypothesis in conjunction with Lemma 1.

\[ \text{Proof of Proposition 4:} \]

We show that \( \bar{V}(y, \hat{H}_{t-1}) \geq V_t(y, H_{t-1}) \) holds for all \( y \) and any fixed \( t \), from which the claim in the proposition follows immediately. From Proposition 1 we know that \( \hat{H}_{t-1} \geq H_{t-1} \). Therefore, since \( \bar{V}(\cdot) \) is increasing in the second argument, it is sufficient to show that \( \bar{V}(y, H_{t-1}) \geq V_t(y, H_{t-1}) \) \( \forall y \) is satisfied. Optimal consumption decisions under Regime I are given by

\[
c_t^1(A, y) = \left[ w_t g(x_t(y), H_{t-1}) \varphi(A) - s_t(I_t(A, y)) \right] - R_t x_t(y) \tag{29}
\]

\[
c_t^2(I_t(A, y)) = R_{t+1} s_t(I_t(A, y)),
\]

where net income \( I_t(\cdot) \) has been defined in (10). The value function is

\[
V_t(y, H_{t-1}) = E \left\{ u_1(c_t^1(\tilde{A}, y)) + u_2(c_t^2(I_t(\tilde{A}, y))) \middle| y \right\}.
\]

If we set \( \hat{H}_{t-1} = H_{t-1} \) (as argued above) and denote by \( \bar{s}_t(y) := E[s_t(I_t(\tilde{A}, y))] \) average savings conditional on the signal \( y \), then under Regime III the following \( \sim \)-allocation is admissible (but not necessarily optimal):

\[
\bar{x}_t(y) = x_t(y)
\]

\[
\bar{s}_t(A, y) = s_t(I_t(A, y)) \left[ 1 - \frac{R_t x_t(y)}{w_t \bar{g}(x_t(y), H_{t-1})} \right] + R_t x_t(y) \frac{\bar{s}_t(y)}{w_t \bar{g}(x_t(y), H_{t-1})}
\]

\[
\bar{c}_t^1(A, y) = \left[ 1 - \frac{R_t x_t(y)}{w_t \bar{g}(x_t(y), H_{t-1})} \right] \left[ w_t g(x_t(y), H_{t-1}) \varphi(A) - s_t(I_t(A, y)) \right] - R_t x_t(y) \frac{\bar{s}_t(y)}{w_t \bar{g}(x_t(y), H_{t-1})} \bar{\varphi}(y) \tag{30}
\]

\[
\bar{c}_t^2(A, y) = R_{t+1} \bar{s}_t(A, y)
\]

To complete the proof we show that the \( \sim \)-decision leads to higher expected utility conditional on \( y \) than the optimal decision under Regime 1. From (29) and (30)
it is immediate that \( E\{\tilde{c}_1(t, \tilde{A}, y)|y\} = E\{c_1(t, \tilde{A}, y)|y\} \). Also, \([w_t g(x_t(y), H_{t-1})\varphi(A) - s_t(I_t(A, y))]\) is increasing in \( A \) (see equation (5)). Thus, \( c_1(t, \tilde{A}, y) \) differs from \( \tilde{c}_1(t, \tilde{A}, y) \) by a mean preserving spread which implies \( E\{u_1(\tilde{c}_1(t, \tilde{A}, y))|y\} \geq E\{u_1(c_1(t, \tilde{A}, y))|y\} \). Similarly, \( E\{u_2(\tilde{c}_2(t, \tilde{A}, y))|y\} \geq E\{u_2(c_2(t, \tilde{A}, y))|y\} \) because \( s_t(I_t(A, y)) \) is a mean preserving spread of \( \tilde{s}_t(\tilde{A}, y) \). Thus we have shown that \( \tilde{V}_t(y, H_{t-1}) \geq V_t(y, H_{t-1}) \).

\[\blacksquare\]

Proof of Proposition 5: (i) For \( \beta > 1 \), \((\bar{\varphi}(y)/R_t)^{(1-\gamma)(1-\beta)/\gamma}\) is a convex function of \( \bar{\varphi}(y) \), which is positively correlated with \( E[\varphi(A)^{1-\beta}|y] \). The representations in (24) and (25) therefore imply the following assessment:

\[
\frac{1-\beta}{M_t} \tilde{W}_t = \frac{1-\beta}{M_t} E[\tilde{V}_t(\tilde{y})] > E[\varphi(A)^{1-\beta}] E\left[\left(\frac{\bar{\varphi}(\tilde{y})}{R_t}\right)^{\frac{(1-\gamma)(1-\beta)}{\gamma}}\right]
\]

\[
> E[\varphi(A)^{1-\beta}] \left(\frac{\bar{\varphi}}{R_t}\right)^{\frac{(1-\gamma)(1-\beta)}{\gamma}} = \frac{1-\beta}{M_t} E[\tilde{V}_t(\tilde{y})] = \frac{1-\beta}{M_t} \tilde{W}_t.
\]

Since \((1-\beta)\) is negative, \( \tilde{W}_t < \tilde{W}_t, t \geq 1 \), follows.

(ii) Under this specification, \((\bar{\varphi}(y)/R_t)^{(1-\gamma)(1-\beta)/\gamma}\) is again a convex function of \( \bar{\varphi}(y) \) which is positively correlated with \( E[\varphi(A)^{1-\beta}|y] \). The same assessment as under (i) therefore yields \( \tilde{W}_t > \tilde{W}_t \) since now \((1-\beta)\) is positive. \(\blacksquare\)

References


