Price Dispersion Within and Across Retailers at a Comparison Site

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Abstract. Price data from comparison sites are easy to obtain from the internet, providing ample opportunities to analyze theories on price dispersion empirically. We propose a generalized analytical model that generates heterogeneous mixed pricing strategies. Making seller service ratings public on comparison site leads to price difference because of vertical product differentiation. Furthermore, we show that each store plays a mixed pricing strategy, a result of classic papers in the field such as Baye and Morgan (2001) and Varian (1980). In addition, optimal advertising fee of comparison site is studied. One fee is shown to beat the combination of click-through fee and listing fee.

1. Introduction

Price dispersion in a homogenous product market has long been a research topic. Most of the development is theoretical where models with different assumptions on market structure and behaviors of market participants are shown to yield price dispersion as an equilibrium result. As internet gains popularity and online retailing matures, new data sources provide ample opportunities for empirical research on this field (Ellison and Ellison, 2005). Two characteristics emerge from those studies. First, comparison site is a major source of price data where a list of prices offered by different retailers is displayed with just a click of consumers who are planning to buy an identical product. Second, service quality of a retailer contributes to a significant source of price dispersion even for otherwise homogeneous physical products (Smith, 2002). Brynjolfsson and Smith (2000) find that branded retailers hold a $1.72 price advantage over generic retailers on books in head-to-head price comparison. They argue that consumers view store brand as a proxy for service quality and essentially consumers compare utilities provided by the service and product bundle.

This paper proposes a general analytical model in which price dispersion is a robust feature. Three sources of price dispersion are embodied in the model. First, consumers have different search costs. Like Varian (1980), consumers in our model either access a price list from the comparison site or not. In equilibrium, retailers employ a common mixed strategy that balances the incentive to maximize profits from consumers who are not informed of the lowest price and those who are. Second, the gatekeeper sets advertising fee so that price dispersion persists on the comparison site even when all consumers are

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2 This includes both consumer search models (Burdett and Judd, 1983) that assume consumers conduct fixed-sample search or sequential search for the locations and price of retailers and clearing house models (Varian, 1980) that assume retailers transmit prices to consumers who are either loyals and shoppers. Clearing house model is more suitable for data from online comparison sites. Not only retailers pay to list their prices but also online consumers are attracted to price comparison sites for free as compared to browsing through retailers’ product webpages for the information. Popularity of comparison site among buyers and sellers is documented Wikipedia, a popular online encyclopedia, which puts its annual growth rate at 30% to 50%.
Price dispersion within and across retailers at a comparison site

Informed of prices. Following Baye and Morgan (2001), we explicitly model the profit-maximizing incentive of the comparison site owner, the gatekeeper. Price dispersion is induced by the advertising fee set by the gatekeeper, the reason why consumers check and find the lowest price. The search behavior of consumers is the exact reason why retailers advertise on the site. When all consumers search the comparison site, firms switch on and off from advertising on the comparison site in response to the advertising fee. The randomness in posting prices online lessens the competition among retailers and hence, marginal cost pricing does not occur. Third, heterogeneity in service quality translates into price differences when consumers compare utilities offered by each retailer. Wildenbeest (2007) models this vertical product differentiation on a grocery market where consumers conduct a fixed-sample search. Although utilities offered are drawn from the same distribution, each retailer has its price distributions. We show that asymmetric pricing strategies stemming from service difference can be extended to clearing-house models. Price dispersion persists even when all consumers do not have access to the comparison site.

This generalization in price dispersion models can explain the contradictory empirical finding on mixed pricing strategy by Baylis and Perloff (2002). They find that price ranks of a retailer on comparison site change little over time, evidence against the mixed pricing strategy proposed by Varian (1980) that each firm is drawn from. However, many other studies do find support with Varian’s claim. Baye, Morgan and Scholten (2004b) examine four million daily price observations for 1000-best selling consumer electronics products listed at Shopper.com, a comparison site over 8 months. After controlling for retailer specific dummies and product specific dummies, they still find that 28% of price dispersion is unexplained. The same observation is made by Clemons, Hamm and Hitt (2002) who study online air tickets prices and Pan, Ratchford and Schanker (2002) on 6,739 price observations across 8 product categories. We show that the contradiction can be resolved when retailers differ in their non-degenerate pricing strategies, whose support depends on service quality. If prices of any two firms do not overlap, price ranks will remain the same over time. However, prices are dispersed for each firm because they are from a mixed strategy.

Two recent practices of comparison sites are considered in the paper. First, store service ratings are displayed together with prices on comparison sites. For example, the returned search result of digital camera Canon SD600 consist of a list of retailers. Each retailer’s webpage link, price, availability and service rating are displayed at the same time on the computer screen. Retailer service quality is evaluated either by the comparison site or by consumers with previous purchase experiences. Considering service quality is also an important factor in consumer purchase decisions, the simultaneous display of price and service rating make easy weighing those two factors. In return, by providing convenience and information, the comparison site attract and retain consumers. It provides validation for modelling service quality in the purchasing decisions of consumers. Second, major comparison sites charge click-through fee nowadays in place of listing fee in the late

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3. The top four positions on the initial returned result of PriceGrabber.com are occupied by "Featured Merchants" who make extra payments to the comparison site. However, the results can be easily sorted by price.

4. For example, Shopper.com and MySimon.com evaluate participating retailers from 1/2 to 5 stars based on four categories with detailed lists and price is not part of the evaluation. PriceGrabber.com solicits ratings from consumers who buy from the retailer and the majority of comments does not include price. Nextag.com combines user reviews into their evaluation program.
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Click-through fee is paid by retailer to the gatekeeper when a consumer clicks the retailer’s webpage link on the comparison site. In contrast, listing fee is a fixed fee from the retailer for unlimited appearance on the site for a time period. Click-through fee varies for different product categories. For example, PriceGrabber.com charges $0.35 per click for books, $0.75 for digital cameras and $1.00 for plasma and LCD televisions. We model the incentives of the gatekeeper when applying both click-through fee and listing fee. An interesting finding is that the gatekeeper uses only one fee although he can choose both. We show that click-through fees yields more profits to the gatekeeper than listing fee when the operation cost is assumed zero.

The structure of our model is from Baye and Morgan(2001) who study the comparison site that charges only listing fee. Baye, Morgan and Scholten(2004a) include loyal consumers, showing a unified treatment of clearing house models. Different from their papers, we incorporate the changes of the gatekeeper to generate heterogenous pricing and demonstrate one advertising fee is superior to the combination of click-through fee and listing fee. Furthermore, we show that click-through fees yields more profits to the gatekeeper than listing fee.

For vertical differentiation on the comparison site, we assume that retailers are endowed with fixed service quality levels in our study period. In other words, the time frame is short-run when retailers cannot adjust service quality. The maximum profit margin of every store is restricted to be the same. Consumers understand that higher service quality levels come with greater costs and they are willing to pay more. This assumption is somehow restrictive but is not unrealistic in a market where retailers have similar costs and name recognition among consumers.

The paper is organized in the following way. We lay out the model and introduce the notations in section II. Then, the optimal behaviors of consumers, firms and the gatekeeper are analyzed in section III, IV and V respectively. Section VI concludes and all the proofs of the propositions are put in the appendix, Section VII.

2. The Model

Let’s imagine there are N geographically separated towns in a homogenous product market and each town is served by a local store. Each consumer has unit demand for the product. Local consumers do not travel to another town to make purchase because transportation costs are prohibitively high. Without the internet, consumers are confined to their local stores for purchasing the product. When internet access is available, a proportion of consumers (we call them shoppers) checks out prices online. The remaining consumers (they are called loyal consumers) still buy from their local stores. The total number of shoppers is denoted as \( S \). In each town there are \( \frac{S}{N} \) shoppers and \( L \) local loyal consumers. Each local store is a monopoly when consumers do not have access to the internet. Internet breaks down the geographical isolation. Shoppers can check prices online without incurring transportation cost.

Consumers care about both product prices and store service qualities. When they make their purchasing decisions, they compare value of service quality, denoted as \( V \) minus product price, denoted as \( P \). That is, utility of an individual received from buying a product of firm \( i \) is denoted as \( V_i - P_i \). Consumers pick the firm that offers the highest utility available. Although the product itself is homogenous, the product bundle including store service is heterogenous. Stores are assumed to have the same marginal cost of

\(^5\)See the article titled "Price Comparison Service" on Wikipedia, an online encyclopedia, which is accessed on Oct.19,2007.
production while service quality are different, denoted by \( r_i \) for each firm. The maximal margin is restrained to be the same across all stores, which is denoted as \( X^6 \).

The gatekeeper controls the comparison site, which provides a platform for consumers to access the price and service information of firms. The site is offered for free to the consumers and is financed by advertising fees from firms. The gatekeeper charges advertising fees: listing fee \((\phi)\) and click-through fee \((t)\). Listing fee is paid for the right to appear on the comparison site while click-through fee is charged when an online consumer is directed to the firm’s online webpage.

As compared to the free comparison site service online, consumers need to pay a cost of \( \varepsilon \) to travel to the local store. The cost is sufficiently small so that consumers still obtain surplus at the monopoly price\(^7\). Even when consumers expect the monopoly price, they are still willing visit and buy from their local firms.

Firms choose to advertise or not on the comparison site, which publicizes both price and service rating information\(^8\). When firms advertise, they decide on the prices to charge to online shoppers and local loyal consumers. When firms do not advertise, they decide on prices to charge local consumers\(^9\). In fact, since consumers care both price and service quality, the competition of firms is in offering the highest utility level to win over online shoppers. When firms choose prices, what they have in mind is to offer the highest utility. The resulting utility setting strategy is a one-to-one mapping of the price strategy assuming constant profit margin across stores\(^10\).

If a firm advertises on comparison site, its product price is updated without additional fee. However, firms incur a non-negative fixed cost \( k \), which does not contribute to the revenue of the gatekeeper. The cost \( k \) can be understood in real-life practice. To advertise online, a firm has to set up a webpage with detailed product and service information first and maintain the webpage. Then, to update prices, a firm also prepares a data feed file which is acceptable to the gatekeeper. In other instances, firms pay to some third party business (e.g. LinkShare and Commission Junction) that consolidates the data feed files

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\(^6\) \( X \) is defined as \( P_{m_i} - r_i \) \( \forall i \) where \( i \) denotes the firm. Here, we can assume the marginal production cost equals zero without loss of generality. Or, \( r_i \) can be considered the sum of service cost and production cost for each firm.

\(^7\) Retangle unit demand is ruled out by the assumption but not the trapezoid unit demand. For example, \( q(p) = 1 \) if \( p \) is no greater than 1 and \( q(p) = 2 - p \) otherwise. The monopoly price the stores is 1 and consumer who buys one unit obtains \( \frac{1}{2} \) surplus. For small transportation cost \( \varepsilon \) less than \( \frac{1}{2} \), consumers are willing to travel to shop in local firms. In general, the surplus at monopoly price \( S(P_{m_i}) \), which is the same across firms and defined as \( \int_{P_{m_i}}^{p_{max}} q(p)dp \), is greater than \( \varepsilon \). Commitment of linear pricing from the firms to avoid a total shutdown of the market is needed, which is similar to the first-price-quote-for-free assumption in the fixed sample search models. Since consumer valuation is different across firms, the monopoly price varies by the amount of service cost.

\(^8\) In practice, store service is evaluated either by the comparison site or by consumers with previous purchase experiences at the stores. For example, Shopper.com and MySimon.com belong to the former category while PriceGrabber.com belongs to the latter one. It is possible that the ratings are biased. The gatekeeper may disclose less accurate information in exchange of more payments from firms, in which case we need a more elaborate model specifying the tradeoffs. This paper assumes service quality ratings reflect the truth.

\(^9\) We assume that firms do not price discriminate online and offline purchases.

\(^10\) Suppose firm \( i \) offers utility \( m_i \). On one hand, profit margin from each consumer is the maximum profit margin minus the surplus over the monopoly price, \( X - (m_i - S(P_{m_i})) \). Since \( S(P_{m_i}) \) is a positive constant specific to demand, we can redefine \( m_i \) and rewrite the above as \( X - m_i \). On the other hand, profit margin is naturally expressed as product price minus service cost \( P_i - r_i \). The equality \( X - m_i = P_i - r_i \) shows that stores choosing utilities is equivalent to choosing prices.

\(^11\) Since \( m_i \) is rewritten as \( X - m_i \), the lowest utility 0 in our later discussion yields surplus at monopoly price \( S(P_{m_i}) \), avoiding market inactivity.
from different retailers for comparison site. In return, firms receive feedback on the performance of online advertising (e.g. the webpage clicks received from the comparison site). Hence, it is reasonable that firms pay a fixed cost \( k \) when they advertise online\(^{12} \). Since it is a cost paid by an online advertising firm but cannot be gathered by the gatekeeper, we refer to it later as hassle cost, meaning leakage from the participants of the game. The specification is not restrictive because the model allows \( k = 0 \).

Timing and nature of the decisions by consumers, firms and comparison site are as follows. First, the gatekeeper announces listing fee and click-through fee \((\phi, t)\). Second, given the advertising fee, firms decide whether or not to advertise and what utility to offer. Third, consumers search and make purchase decisions.

The three-stage game is analyzed using backward induction. We start with the optimal behaviors of consumers given the firm’s advertising and pricing strategies. Then, given the consumers’ behavior rules and the advertising fee, we solve the firms’ profit maximization problem. Finally, the gatekeeper makes his decision on listing fee and click-through fee given the responses of consumers and firms.

3. Optimal Behaviors of Consumers

The optimal behaviors of consumers are summarized in the following proposition.

**Proposition 1.** Sequential rationality is achieved when: shoppers (a) first visit the comparison site and (b) buy from the firm that offers the highest utility level. (c) If no firm advertises on the site, consumers go to their local stores and buy the product. And, loyal consumers buy from their local stores without searching online.

We omit the proof here because the intuition is clear. The strategy of shoppers is weakly dominant and the proof closely mirrors that in Baye and Morgan (2001). Their model assumes homogenous store service qualities and hence, consumers only compare prices. In this paper, vertical differentiation in store services exists and consumers compare utilities they receive from purchasing at different stores.

4. Optimal Behaviors of Firms

Firms make decisions on advertising and offering utilities. Let’s define a symmetric equilibrium strategy of firms as a pair of advertising probability \( \alpha \) and utility offering distribution \( G(m) \). Suppose that \( N - 1 \) firms follow the same strategy, the question is will the remaining firm deviate from the posited strategy? If not, it is indeed an equilibrium. The expected profit of firms for such an equilibrium strategy is the sum of non-advertising profit and advertising profit weighted at their respective probabilities: \( (1 - \alpha)\pi^{NA}_i + \alpha\pi^A_i \).

Let’s take a look at \( \pi^{NA}_i \) and \( \pi^A_i \).

If a firm does not advertise, it optimally offers 0 utility, equivalently, charges the maximal margin \( X \) to consumers. Sales to its share of loyal consumers \( L \), are secure regardless of the actions of other firms. Only when all other firms do not advertise, sales to its share of shoppers \( \frac{S}{N} \), can be realised.

\[
\pi^{NA}_i = LX + (1 - \alpha)^{N-1}\frac{S}{N}X
\]  

\(^{12}\)The behaviors of the third party business (or is called the online affiliating network industry) are not modeled to keep the model simple but explain the essential features of the comparison site data. They charge firms instead of comparison sites for service.
If a firm does advertise to compete for all the shoppers $S$, its expected profits depend on advertising and utility offering decisions of the remaining $N - 1$ firms. Recall that utility and price are related to a firm’s profit margin such that $X - m = P_i - r_i$.

$$\pi_i^A = \sum_{j=0}^{N-1} \left( \begin{array}{c} N-1 \\ j \end{array} \right) \alpha^j(1-\alpha)^{(N-1)-j}G(m)^j \left[ S(X - m - t) \right]$$

$$L(X - m) - (\phi + k)$$

$$= L(X - m) + [1 - \alpha(1 - G(m))]^{N-1} \left[ S(X - m - t) \right] - (\phi + k)$$

(2)

An advertising firm has three parts in its profits: expected profits from shoppers, profits from loyal consumers and costs that include listing fee ($\phi$) and online operation costs ($k$). The summation term of the first line of the equation is the probability that $j$ firms advertise times the number of ways that this can happen among the $N - 1$ remaining firms. The probability that firm $i$ offers the highest utility is $G(m)^j$ compared with the other $j$ firms which also advertise online. Consumers who check prices on site will then be directed to the webpage of firm $i$ and buy from the firm. The gatekeeper collects a click-through fee $t$, for each online transaction and hence, the profit margin for each online buyer is $(X - m - t)$. In sum, the expected profits from shoppers are the probability of offering the highest utility and winning over all the shoppers times the size of the profits, the first term before the plus sign in equation 4. The second line follows directly from the Binomial Theorem.

$m = 0$ is the lower bound of the support of $G(m)$, which earns the same profit as any other value in the support of $G(m)$. In the profit expression below, the utility offering strategy drops out since $G(0) = 0$, a property of the atomless distribution.$^{13}$

$$\pi_i^A = \pi_i^A(0) = LX + (1 - \alpha)^{N-1}S(X - t) - (\phi + k)$$

(3)

We then compare $\pi_i^A$ and $\pi_i^{NA}$. Both expressions are a function of the endogenous variables: firms’ advertising probability $\alpha$, gatekeeper’s fee ($\phi, t$) and the exogenous variables.

$$\pi_i^A - \pi_i^{NA} = (1 - \alpha)^{N-1}S((1 - \frac{1}{N})X - t) - (\phi + k)$$

(4)

The above equation imposes a feasible range of advertising fee although the optimal fee is not solved until the next section. As we recall that the gatekeeper responds to firms’ strategies in the third stage, it will be in his interest to adjust $(\phi, t)$ such that the gain for a firm to advertise is non-negative. We assume naturally that the lower bounds of both $\phi$ and $t$ are zero. The two pricing instruments of the gatekeeper are substituties in generating profits. Let’s look at the upper bounds of the fees separately. For listing fee ($\phi$), the non-negative constraint on equation 4 yields $\phi \leq (1 - \alpha)^{N-1}S((1 - \frac{1}{N})X - t) - k$ where $\alpha$ is part of the sequential strategy of the firms. Regardless of the firms’ strategy, the maximal $\phi$ is bounded by $S((1 - \frac{1}{N})X - t) - k$. For the click-through fee ($t$), the gatekeeper cares only that some firm advertises because one firm will offer the highest utility attracting all shoppers. Even if all but one firm opt out of online advertising ($\alpha = 0$), the gatekeeper can charge the remaining firm the maximal click-through fee $(1 - \frac{1}{N})X$ to advertise.$^{14}$

13 See the proof of proposition 2 in the appendix.
14 See section 5 for a more detailed discussion of using listing fee and click-through fee.
15 In proposition 2, all-firms-advertise ($\alpha = 1$) is an equilibrium when only a click-through fee is charged.
Given the advertising fee \((\phi, t)\), we are interested in finding symmetric mixed strategies of firms. Equation 4 represents the gain of the remaining firm when advertising through the comparison site keeping constant strategies of the other firms. The gain must be zero such that the remaining firm is indifferent in his advertising choices for a symmetric mixed strategy. There are two cases concerning the solution \(\alpha\), which are discussed in propositions 2 and 3 respectively.

First, the difference is zero for any \(\alpha\). That is, when the gatekeeper’s fee is \(t = (1 - \frac{1}{N})X\) and \(\phi = 0\) in addition to exogeneous variable \(k = 0\) and \(m = 0\), expected profits \((1 - \alpha)\pi_i^{NA} + \alpha\pi_i^{A}\) are independent of advertising probability. Whichever advertising propensity of the \(N - 1\) firms may agree on, the remaining firm is indifferent in his advertising choices for a symmetric mixed strategy. There are two cases concerning the solution \(k\), which are discussed in propositions 2 and 3 respectively.

Proposition 2. Assume \(k = 0\), when the gatekeeper charges only a click-through fee, i.e. \(t = (1 - \frac{1}{N})X\) and \(\phi = 0\), there exists a continuum of the symmetric equilibria: If all firms advertise with \(\alpha\) probability, they offer utilities according to

\[
H(m) = 1 - \frac{1}{\alpha} \left(1 - \left(\frac{Lm + (1 - \alpha)^{N-1} S \frac{X}{N}}{S \left(\frac{X}{N} - m\right)}\right)^{\frac{1}{\alpha}}\right) \quad \text{where } m \in \left[0, (1 - (1 - \alpha)^{N-1}) \frac{S}{S + L \frac{N}{X}}\right]
\]

and \(E \pi_i^* = LX + (1 - \alpha)^{N-1} \frac{S}{N} X\).

In particular, the two pure strategies of advertising are:

(a) If all firms advertise \((\alpha = 1)\), they offer utilities according to \(\bar{H}(m) = \left(\frac{Lm}{S \left(\frac{X}{N} - m\right)}\right)^{\frac{1}{\alpha}}\) where \(m \in \left[0, \frac{S}{S + L \frac{N}{X}}\right]\) and earn \(E \pi_i^* = LX\). Consumers receive the highest utility.

(b) If all firms do not advertise \((\alpha = 0)\), they offer zero utility and earns \(E \pi_i^* = LX + \frac{S}{N} X\). Consumers receive the lowest utility.

Advertising fee specified in this case extracts the expected surplus from advertising (see equation 4). As a result, a slightly higher fee will put off all the firms, making not advertising a strictly dominant strategy. To see its effect on the firms, let’s first talk about the two pure strategies and then extend to the continuum of mixed advertising strategies. If all other firms stay away from the comparison site, the remaining firm will weakly preferred to follow suit rather than advertising because the fee collected by the gatekeeper is so high that no gain can be made by switching otherwise. On the other hand, if all other firms advertise, the firm will take the chance to compete on site hoping to win over the shoppers. The firm need not pay the click-through fee when it fails to be the highest-utility firm. It will not hurt to advertise. Hence, that all firms compete on the site is an equilibrium. For advertising probability between zero and one, similar argument applies: firms can do no better by deviation.

The expected profits of firms are a decreasing function of the advertising probability \(\alpha\). For example, in the all-firms-advertise equilibrium \((\alpha = 1)\), firm earns the lowest possible profit, resulting from the most intensive competition on the comparison site. To gauge the level of competition, we can look at the rough measure of utility dispersion, the difference between the upper and lower bounds. As \(\alpha\) increases, the upper bound becomes
larger, meaning firms compete in offering higher utility levels as advertising intensity goes up. A possible lower profit margin reduces the expected profits. In addition, the utility dispersion shrinks towards zero as the number of firms increases but expands away from zero as the proportion of shoppers increases. The explanation is that as the number of firms gets larger, the probability of a firm winning over shoppers decreases exponentially and hence, firms increasingly offer low utilities, leading to a decrease in utility dispersion. When \( N \) goes to infinity, utility dispersion disappears and all firms offer zero utility. If there is no service differentiation, price dispersion disappears altogether\(^{16}\). In the presence of store differentiation, across-store dispersion in pure strategies of multi-price equilibrium still persists. In addition, more shoppers increase potential profits from advertising on comparison site for firms, which allows deeper discount in competition.

The continuum of advertising strategy can be ranked in first order stochastic dominance in terms of utility dispersion and profits to the firms. The most dispersed and least profitable one is the all-firms-advertise equilibrium while the no-firm-advertise equilibrium is the opposite extreme. Equilibria in between vary smoothly from the one polar to the other.

The \( k > 0 \) case we are about to look at is that firms have a unique symmetric strategy in advertising and a corresponding utility offering strategy when \( k > 0 \). Again, let’s look at equation 4 that describes the profit difference between advertising and not advertising. It is zero for a unique \( \alpha \) when \( k > 0 \). Conditional on other firms’ strategies, such an \( \alpha \) will make the remaining firm indifferent between advertising or not. The \( k = 0 \) case can be perceived as a limiting case when managing online stores becomes more cost efficient over time. As the technological support for online retailing matures and firms gain plenty of experience in online operations, we will expect a decline of the hassle cost. In this sense, the \( k > 0 \) case is more general. The optimal behaviors of the firms are summarized in the following proposition:

**Proposition 3.** Assume \( k > 0 \), given the fees \((\phi, t)\) that the gatekeeper charges satisfy: \( 0 \leq \phi \leq S \left((1 - \frac{1}{N})X - t\right) - k \) and \( 0 \leq t < (1 - 1/N)X \), the optimal advertising and utility offering strategy of the firms in a symmetric equilibrium can be described as follows:

(a) Firms advertise with probability \( \alpha^* = 1 - \left[ \frac{\phi + k}{S((1 - \frac{1}{N})X - t)} \right]^{\frac{1}{1 - \frac{1}{N}}} \).

(b) When a firm advertises, it offers a utility level \( m \) according to the c.d.f. \( G(m) \) over \([0, \overline{m}]\), where \( 0 < \overline{m} < X \).

\[
G(m)^* = 1 - \frac{1}{\alpha^*} \left( 1 - \left[ \frac{(\phi + k)(1 - \frac{1}{N})X - t + Lm}{S(1 - \frac{1}{N})X - t} \right]^{\frac{1}{1 - \frac{1}{N}}} \right)
\]

and \( \overline{m} = \frac{X - t}{S + L} \left( S - \frac{\phi + k}{(1 - \frac{1}{N})X - t} \right) \) \( (5) \)

(c) When a firm does not advertise on the site, it offers the utility level 0.

(d) The expected profit for a firm is \( \pi^* = \frac{\phi + k}{(1 - \frac{1}{N})X - t} \frac{X}{N} + LX \).

A couple of differences are noted in proposition 3 in comparison with proposition 2.

\(^{16}\)Varian (1980) and Rosenthal (1980) have similar conclusions in the limiting case.
First, the symmetric double-mixed strategy is unique including advertising probability \( \alpha \) and the utility offering distribution that pairs with it. No equilibrium selection is needed from the gatekeeper’s perspective. The double-mixed strategy implies that firms do not always appear on the comparison site. Checking the firm-level data of comparison site, we can confirm that firms do appear on-and-off the site. An somewhat surprising result is that profits earn by firms are higher than the all-firms-advertise equilibrium when \( k = 0 \). Why profits are higher when the additional online advertising cost of firms is greater? Competition among firms and the gatekeeper’s fee practice help shed light on the question. Before the invention of internet, there is no competition and firms earn the highest profits by offering the minimum utility level. Later, firms are forced to increase utilities offered on the comparison site to attract online shoppers, which increases competition and lowers expected profits. Hassle cost \( (k) \) is a disincentive for firms to advertise online. To overcome the adverse effects, the comparison site offers low enough advertising fees to firms. The gatekeeper compensates firms more than the cost because every advertising firm pays \( k \) regardless of the competition outcome. When \( k = 0 \), the gatekeeper can use a click-through fee to extract all the surplus from the winning firm. However, it is impossible to get all firms advertise when \( k > 0 \). The positive hassle cost acts like a listing fee which is not collected by the gatekeeper. It creates uncertainty in advertising, meaning retailers do not all appear on the comparison site at the same time in the observed data. The extra expected profits earned by firms are in fact profits from local shoppers when no firm advertises online, which occurs with a positive probability.

Second, in the limit when \( N \) goes to infinity, utility dispersion does not disappear. It is easy to see that because the upper bound of the utility distribution does not converge to zero although the probability for a particular store to advertise is infinitely small.

**Proposition 4.** There is a one-to-one mapping from utility \( m_i \) to price \( p_i \). From the utility offering strategy \( H(m) \) in proposition 2 and \( G(m) \) in proposition 3, we can recover an individual firm’s pricing strategy \( F_i(p) = 1 - H(V_i - p) \) or \( F_i(p) = 1 - G(V_i - p) \) where \( V_i \) is the consumer valuation of the service quality of firm \( i \) and \( p \in [V_i - \Pi_i, V_i] \).

\( F_i(p) \) is a mixed pricing strategy, which generates price dispersion for firm \( i \). Moreover, each pricing distribution differs in supports. Simplest example is the no-firm-advertise equilibrium in proposition 2. Firms offer zero utility to consumers and hence, individual pricing strategy is reduced to its respective monopoly pricing at \( V_i \). If there is no service differentiation, the multiple-price equilibrium becomes the monopoly price described in Diamond(1971).

Different price distributions across firms arise from a common symmetric utility offering strategy. Supports shift forward and backward depending on the valuations of service qualities without changing the shape. Firms with higher \( V_i \) first order stochastically dominate those with lower \( V_i \). Put simply, firms with better service qualities will have higher average prices over time. However, they may offer better deals at times.

Price dispersion as a result of a common mixed pricing strategy is developed in a number of classic papers such as Varian(1980), Stahl(1989) and Baye and Morgan(2001). Adding to the already rich literature, this paper provides a way to introduce heterogeneity that generates asymmetric pricing strategies and explains why some stores price consistently high or low. A common mixed pricing strategy implies that price ranks of a firm at various times will be random. Contrary to the theory, Baylis and Perloff(2002) observe that price ranks change little over time by looking at the correlation table between price ranks and store identities. The explanation offered in this paper is that price ranks may
remain stagnant because of their service quality differentiation. Even the extreme case - no change in price ranks is possible. Markets with substantially different service qualities may imply the supports of asymmetric pricing strategies do not overlap. Still, prices charged by these stores differ from time to time because they price according to the common utility offering distribution. Even if the service differentiation is not big, a narrow interval of utility support will work to separate the distributions. Hence, to test whether a mixed strategy explains price dispersion, we will need to control for the store heterogeneity and use the residuals after regression instead of the pure price data. Indeed, Lach(2002) does so and finds support for a common mixed pricing strategy. He studies four products sold in 37 supermarkets for 48 months in Israel. The residual of the regression on price instead of price itself are used for analysis after controlling for the store effects and other observable heterogeneities. The residual, interpreted as prices of a homogenous product, behave consistently with the mixed pricing strategy \( F(p) = 1 - H(V - p) \) or \( 1 - G(V - p) \).

The idea and technique that firms compete in utility and pricing strategies are recovered from the one-to-one mapping of utility offering strategy are from Wildenbeest(2007). The model is a simultaneous two-stage game where firms do not advertise prices but consumers actively search for prices. He studies a fixed sample search model where consumers decide on and commit to the optimal number of firms they will search before observing price quotations. Like Burdett and Judd(1983), price dispersion arises because the proportion of consumers who search once is between 0 and 1. Instead of assuming one search cost, Wildenbeest(2007) estimate the underlying search cost distribution and correspondingly, the proportion of consumers who search once, twice and up to \( N \) times. Different from his setting, this paper studies a clearing house model where firms actively transmit prices to the consumers through advertising. Famous papers include Varian(1980) which is applicable to the traditional gatekeepers such as newspapers and magazines and Baye and Morgan(2001) which investigated the modern gatekeeper of the information market. The model is cut out for the online market. It describes two important changes brought by the internet. One is the substantially lower search costs of online consumers and the other is firms advertising efforts in competition. Comparison site serves as an information clearing house that allows online consumers to check out prices and firms to advertise prices. Price dispersion exists because it is costly for firms to advertise prices. We have shown that his technique can be extended in clearing house model to generate asymmetric pricing strategies.

We now show the relations between this model and the previous clearing house models.

**Proposition 5.** Varian(1980) is a special case of our proposition 2 and Baye and Morgan(2001) is a special case of proposition 3 after utility-to-price transformation described in proposition 4.

Varian’s two-stage game involves no advertising fee. Equivalent in interpretation, retailers have paid advertising fee, which is sunk when they set prices. The opposing force of price and sales are at work in generating the maximal profit, giving rise to price dispersion. In Baye and Morgan, prices are dispersed because firms pay listing fee to advertise on comparison site. This paper incorporates those elements and adds store heterogeneity.

5. **Optimal Behaviors of the Comparison Site**

The objective function of the gatekeeper is:

\[
\pi(\phi, t) = N \alpha \phi + St \left(1 - (1 - \alpha)^N\right)
\]
\( \alpha \) is the endogenous advertising propensity as specified in proposition 2 and in proposition 3 depending on \((\phi, t)\) and \(k\). The first term before the plus sign is the expected profits from listing fee, equal to the expected number of firms that advertise on the comparison site times the listing fee. The second term is profits from click-through fee, calculated as the size of shoppers times the click-through fee charged for each consumer and the probability that at least one firm advertise on the site. In the model, shoppers will buy from a firm that offers the highest utility. That firm pays \(St\) to the gatekeeper on top of listing fee, if any. From consumer behaviors summarized in proposition 1, sales will be made so long as there is one firm advertising on the site.

An important finding from solving the profit maximization is that the gatekeeper will optimally use either a listing fee or a click-through fee but not the combination of the two instruments\(^{17}\). Consistent with the theory, we see the evolution of the comparison site’s advertising fee practice in reality matches the prediction\(^{18}\). Hoffman and Novak (2000) observed that flat fees, the earliest Web advertising model were a dominant approach in internet pricing. An empirical study of Baye, Morgan and Scholten (2003) recorded that participating firms at Shopper.com, a comparison site paid monthly fees to list product prices. Nowadays, major comparison sites charge only click-through fee.

We now discuss briefly the different motives for \(\phi\) and \(t\) on attracting advertising, where the focus of the gatekeeper differs slightly. For a listing fee, each firm pays \(\phi\) and gets exposure on the comparison site whether or not online shoppers buy from them. To maximize his profits, the gatekeeper wants as many firms to advertise as possible. This objective leads to a low listing fee per firm because only one firm will capture the shoppers online while all firms pay the fee. On the other hand, for a click-through fee, a firm pays only when consumers are ready to buy. In our setting, consumers will buy even when one firm advertises. Hence, the sole concern of the gatekeeper is to lure one firm instead of the maximal number of firms. The gatekeeper looks for an optimal balance in the tradeoff of the fee \((\phi\) or \(t\)) and his respective incentives. The choice of \(\phi\) and \(t\) eventually depends on the change rate of the advertising probability functions in response to the fee changes. In the \(k = 0\) case, click-through fee \((t)\) can be shown optimal analytically and in the \(k > 0\) case, simulation results point to the same result.

**Proposition 6.** When hassle cost of advertising online is zero to firms, i.e. \(k = 0\), it is optimal for the gatekeeper to charge an optimal click-through fee \(t = (1 - \frac{1}{N})X\) under the all-firms-advertising equilibrium (see discussion in Proposition 2). In duopoly case when \(N = 2\), profits from listing fee \((\phi^*)\) are 50\% of click-through fee \((t^*)\). As \(N\) increases, the gap widens monotonically. When \(N = \infty\) in the extreme case, profits from listing fee \((\phi^*)\) are only 37\% of click-through fee \((t^*)\).

How do firms arrive at the all-advertising equilibrium in the subgame given the optimal click-through fee? One possible explanation is the gatekeeper may initially charge a strictly less than optimal click-through fee \((t^*)\) to jump start the market. In doing so,
the site successfully entices the firms to advertise. Then, the gatekeeper ratchets up the fee to the optimal level. Given all other firms advertise with the site, the remaining firm does not find it beneficial to do otherwise.

The advertising profits of the comparison site are at a maximal of \((1 - \frac{1}{N})SX\) because the click-through fee is the largest possible and there is no uncertainty in demand for advertising in this case. The superiority of click-through fee over listing fee can be shown analytically, which does not change with either the size of shoppers \((S)\) or maximal profit margin \((X)\) although the optimal fee and expected profits are affected.

**Proposition 7.** i) The gatekeeper finds it optimal to use only a click-through fee \(t^*\), or to use only listing fee \(\phi^*\) to maximize profits although the gatekeeper can choose a combination of the two pricing instruments.

ii) If click-through fee is optimal, the unique optimal \(t^*\) is between \([0, (1 - \frac{1}{N})X - \frac{k}{S}]\) and is equal to

\[
\arg \max \{\pi(0, t^*)\}
\]

iii) If listing fee is optimal, the unique optimal \(\phi^*\) is between \([0, S(1 - \frac{1}{N})X - k]\) and is equal to

\[
\arg \max \{\pi(\phi^*, 0)\}
\]

iv) As the size of shoppers \(S\) increases, \(t^*\) and \(\phi^*\) increases; As the maximal profit margin \(X\) increases (either because consumers value store services more highly or the costs for providing them are lower), \(t^*\) and \(\phi^*\) increases; As the hassle cost of advertising online \(k\) increases for firms, \(t^*\) and \(\phi^*\) decreases.

**Simulation Results:** Numerical simulation suggest that for \(N = 2\) to 50, the comparative statistics w.r.t. \(N\) are summarized in the following table:

<table>
<thead>
<tr>
<th>(\Delta t^*)</th>
<th>(\Delta \phi^*)</th>
<th>(\Delta \pi(t^*))</th>
<th>(\Delta \pi(\phi^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

v) Depending on exogenous parameters \((S, X, N, k)\), the monopoly gatekeeper charges click-through fee \(t^*\) when \(\pi(t^*) > \pi(\phi^*)\); and the gatekeeper charges listing fee \(\phi^*\) when \(\pi(\phi^*) > \pi(t^*)\).

**Simulation Results:** We cannot find any instance that listing fee is superior to click-through fee for various parameter specifications \((S, X, N, k)\).

Part i), ii) and iii) conveys the interesting result that the gatekeeper optimally uses only one fee although he has two options at discretion. Part iv) is the standard comparative statistics exercises with the exception of \(N\), which cannot be determined analytically. The results are intuitively. When there are more shoppers, the value of advertising via the comparison site increases and hence, the gatekeeper can charge a higher fee. The same is true with the increase in profit margin. However, when it inflicts higher costs for firms to go online, the gatekeeper needs to adjust downward the fee so that firms still favors advertising with a positive probability. The comparative statistics of \(N\) is presented in the simulation results. As the number of firms increases, the gatekeeper increases both fees and the resulting advertising probabilities decreases for an individual firm. However, the overall profits rise meaning the increase in unit fee more than offset the decline in advertising probability. Part v) is a natural conclusion of the profit-maximization problem.

\(^{19}\) The other parameters are specified as \(k = 0.01\), \(X = 1\) and \(S = 0.1\). Other values of \(k, X\) and \(S\) are also tried but signs remain unchanged.
that lacks a clear-cut analytical answer. Again, numerical stimulations with different values of the exogenous parameters are presented. Ceteris paribus, we vary $N$ as a first step from 2 to 50 and then $N = 100$. Finally, the limit case $N = 10^{10}$ is considered. Optimal $\phi^*$ and $t^*$ are solved numerically and we compare $\pi(\phi^*)$ and $\pi(t^*)$. Next, we change $k$, $S$ and $X$ one at a time while keeping other variables the same. We cannot find any instance that listing fee is superior and the result shares a similar pattern as proposition 6 when $k = 0$. For example, when all consumers search online and no loyal consumers remain, $k = 0.01$ and $X = 1$, the profits from listing fee $\pi(\phi^*)$ are 56% of those from click-through fee $\pi(t^*)$ for duopoly firms ($N = 2$). In the limit where $N = 10^{10}$, the ratio decreases to 44%.

Comparative statistics concerning $N$ are somewhat expected. As there are more firms in the market, the gatekeeper optimally charges a higher fee for advertising. Consequently, advertising probability decreases. The expected profits actually increases because the increase in fee outweights the decrease in the demand of advertising.

Why does click-through fee perform better than listing fee? We may be able to provide an explanation by examining the expected profits generated by the two instruments.

$$\pi(\phi^*) = N\alpha \phi^*$$

$$\pi(t^*) = St^* \left(1 - (1 - \alpha)^N\right)$$

When the gatekeeper uses only listing fee, profits depend on the expected number of firms that sign up with the service ($N\alpha$) and the listing fee ($\phi$). A higher listing fee set by the gatekeeper will lower $\alpha$, resulting in fewer firms on the comparison site. To find an optimal listing fee, the gatekeeper compares the tradeoff or the rate of change between the sales $N\alpha$ and the price $\phi$. For example, if a small rise in listing fee leads to a sharp decrease in the number of firms that advertise, fewer profits will be made than a slower fall in advertising firms. The second order condition shows that $N\alpha$ decreases with decreasing rate with respect to $\phi$. That is, as the gatekeeper increases listing fee, the expected number of firms falls a lot initially and the decrease tapers off. So, a low advertising fee that attracts more firms or a high listing fee that cuts deep in the pockets of a handful of firms may yield the same profits. The optimal solution is thus a moderate listing fee from a number of firms. By switching to click-through fee, the gatekeeper shifts its focus to the probability that at least one firm advertise. From every click of the shoppers, he will collect a fee. Now $S \left(1 - (1 - \alpha)^N\right)$ is the advertising demand and $t$ is the price. The second order condition reveals that the probability decreases very slowly and then falls steeply at the very end of the permissible range. The gatekeeper will be able to set a high click-through fee while keeping the at-least-one-firm-advertises probability comfortably high at the same time. If $\alpha$ is exogenous, the maximal possible profits from both fees within their constraints are the same. Weighing in the effects of an endogenous $\alpha$, the gatekeeper is more likely to use click-through fee because it manages to keep both sales and price high while listing fee can only keep both at moderate levels at best.

6. Conclusion

We consider an analytical model motivated by the changes in practices on comparison sites. Including service ratings of retailers allows us to derive tractable asymmetric pricing strategies for different stores, which contributes to the observed price dispersion across and within retailers. In addition, we show why comparison site uses only one fee, either
listing fee or click-through fee although the gatekeeper is allowed to use both. Click-through fee in our model is demonstrated to be more profitable, which is the common fee practice of major comparison sites.

Hassle cost \( k \), a fixed cost paid by the firm but not collected by the gatekeeper gives rise two equilibria in the model depending on its specification. For \( k = 0 \) case, which is not very realistic in reality, click-through fee is shown analytically to yield more profits than listing fee. The gatekeeper entices all firms to advertise and earns the highest possible profit. Price dispersion exists here because there are loyal consumers who do not access the internet. In fact, our model coincides with Varian(1980) that considers only the firms and consumers and advertising fee is sunk. For \( k > 0 \) case, click-through fee is shown numerically to be better. Firms do not always advertise on the comparison site which lowers the competition pressure. Price dispersion arises because of the advertising fee. The special case of our model here is Baye and Morgan(2001) when only listing fee is charged and service differences do not exist. The first case \( k = 0 \), we expect all firms advertise on the comparison site for all the observed time periods if the site acts optimally. However, when \( k > 0 \), we can check from one time period to another to see if firms do appear on and off the site. Limited support is found for the pattern in a comparison site dataset\(^2\).

We also make the distinction of price dispersion among retailers at a time point and that of one retailer over time. Our model allows for both price dispersions. A practical lesson in using comparison data is that the observed prices may not be suitable to use directly in estimation because they are not from a common distribution. However, the way to do it right is easy. After controlling for service quality differences, the residuals are from a non-degenerate distribution, the common mixed utility offering strategy in our model.

Further research will be to apply to the model on data collected from comparison sites. Some practical questions can be answered empirically. For example, 1. How much money value does consumers place in service quality rating and how is it different from market to market? 2. Whether firms play a mixed common utility offering strategy? 3. What are the estimates of the underlying parameters in the cumulative distribution function of price? A reduce-formed regression approach can be used for the first two questions while the third one can be estimated using a structural approach. The motivation for a generalized model is that price data are plenty but sales data are hard to come by. Using equilibrium conditions, parameters can be estimated with price data only. Hong and Shum(2006) shows that consumer search model of Burdett and Judd(1983) can be estimated using MEL. Moraga-Gonzalez and Wildenbeest(2007) uses MLE in fixed sample search model. It would be interesting to extend the estimation to clearing house models.

7. Appendix

Proof to Proposition 2. First, we claim that if firms advertise with the same probability \( \alpha \), the symmetric utility offering distribution will have no mass points over the support \([0, \bar{m}]\). The boundary of the distribution is determined in the following way. A utility zero is the lowest level that a consumer is willing to accept and firms have no incentive to push over that lower bound. \( \bar{m} \) is such that it earns the same profit as 0 that is consistent with the utility offering distribution. The claim can be substantiate by

\(^2\)The result is from Yang(2007) unpublished doctoral dissertation "Firms Heterogeneities, Click-through Fees and Pricing in Oligopoly: Theory and Evidence". Data are collected from Shopping.com. There are 10,755 prices by 127 firms in 39 product categories.
there will be no gap in the distribution and 2. the distribution does not have a mass point. For the first point, suppose \( m_1 < \hat{m} < m_2 \), where no firm chooses utility levels between \( m_1 \) and \( m_2 \). If a firm chooses \( m_2 \), we can show that \( \hat{m} \) is a better choice. If \( m_2 \) is the highest utility offered, \( \hat{m} \) clearly is. If \( m_2 \) is not the highest utility, \( \hat{m} \) is not as well. Hence, \( \hat{m} \) earns more profits from a higher profit margin than \( m_2 \). Profitable deviation shows the equilibrium distribution contains no gap. As for the second point, on the open half \([0, \overline{m}]\), suppose the distribution does have a mass point. Then stores will have positive probability for a tie, in which case sales to the shoppers are equally shared among the stores who offer the highest utility levels. However, it is not an equilibrium because positive gain arises from deviation. A store can shift the positive probability ahead by \( \epsilon \). Although it loses an amount \( \epsilon \) in the profit margin of each consumer, the firm gains a positive fixed amount of profits by winning the sales of all the shoppers of size \( S \). If the mass point is on \( \overline{m} \), the maximal utility offered from the sell side, a firm will earn higher profit by transferring the mass from an \( \epsilon \) neighborhood below \( \overline{m} \) to 0. The gain is a fixed positive higher profit margin (\( \overline{m} - \epsilon \)) from the loyal consumers and the loss is \( \epsilon \) probability that the other firms do not charge a positive probability of \( \overline{m} \). That concludes the proof to the claim. The lines of arguments are from Varian(1980) and Baye, Kovenock and Vries(1990), where the former studies symmetric equilibria and the latter asymmetric ones in Varian’s model.

Next, we demonstrate the derivation of the utility offering strategies for a given \( \alpha \). The expected profits of firm \( i \) are \((1 - \alpha)\pi_i^{NA} + \alpha \pi_i^A = (1 - \alpha) (LX + (1 - \alpha)^{N-1} \frac{N}{X} X) + \alpha (L(X - m) + S(\frac{X}{N} - m)) (1 - \alpha (1 - H(m)))^{N-1})\). When \( m = 0 \), the equation yields expected profits \( LX + (1 - \alpha)^{N-1} \frac{N}{X} X \). \( H(m) \) and the upper bound \( \overline{m} \) are obtained by the property that every utility as zero in the support earns the same profits. Clearly, the expressions of pure strategies in advertising and not advertising are the extreme cases of \( \alpha \). It is straightforward that the above strategies constitute symmetric equilibria. Given \( N - 1 \) firms adopt the pair of choices \((\alpha, H(m))\), firm \( i \) earns the same profits from advertising or not. The same holds true for any \( \alpha \) between 0 and 1.

**Proof to Proposition 3.** As we have proved in proposition 2, the utility offering distribution is atomless in the support for a common advertising probability. Let’s solve for the unique \( \alpha \). Straightforward from equation 4, a unique \( \alpha \) is obtained21. In order for \( G(m)^* \) to be part of the symmetric equilibrium, the expected profits should be constant for all the utilities in the support of \( G(m)^* \). Then, equating equation 4 and 1 we will be able to get \( G(m)^* \) and \( \overline{m} \).

We can also verify that \( G(m) \) is an atomless c.d.f. and no firms can gain by choosing a utility choice outside the interval. From the expression of \( G(m) \), it is clear that \( G(0) = 0 \) and \( G(\overline{m}) = 1 \). Besides, \( G(m) \) increases with \( m \) continuously in the support. The strategy space in utility offering is \([0, X]\). If a firm offers utility \( m' \) so that \( \overline{m} < m' < X \), its profits \( \pi(m')^A = L(X - m') + S((X - m') - t) - (\phi + k) \) are strictly lower than \( \pi(\overline{m})^A = L(X - \overline{m}) + S((X - \overline{m}) - t) - (\phi + k) \). Hence, the solution to the problem is unique and indeed an equilibrium.

---

21 It is easy to see that there is no pure strategy in advertising in the constraint specified above. When firm \( i \) advertises, the expected profits are \( \pi(0)^A = L \times X - (\phi + k) \). When it does not advertise, \( \pi(0)^{NA} = L \times X \). Clearly, because \( (\phi + k) \) is positive, advertising is dominated by not advertising when the other firms advertise. Now, suppose all firms but \( i \) choose not to advertise on the site. Firm \( i \)'s expected profits are \( \pi(0)^{NA} = L \times X + \frac{S}{N} \). Compared with its expected profits from advertising, \( \pi(0)^A = L \times X + S \times (X - t) - (\phi + k) \). Combined with the restriction of \( \phi, t, \) and \( k \), advertising is a dominant strategy when all other firms do not. In sum, there is no pure strategy in advertising in a symmetric equilibrium.
Proof to Proposition 4. Although stores are differentiated in service levels \( V_i \) and costs \( r_i \), maximal profit margin \( X \) is the same across stores, i.e. \( X = V_i - r_i \) \( \forall i \). From \( P_i - r_i = X - m_i \), utility offering strategies can be mapped to pricing strategies by change of variable: \( F_i(p) = \Pr(P_i \leq p) = \Pr(V_i - m_i \leq p) = \Pr(m_i \geq V_i - p) = 1 - G(V_i - p) \).
The support of \( p \) directly changes the constraint in the support of \( m \). As a result, firms with different service quality levels will have their own pricing strategies, which differ in intervals of supports. To ensure that price is nonnegative, \( \min(V_1, \ldots , V_N) \) should be greater than \( m \).

Proof to Proposition 5. (a) Varian’s result: Take \( \alpha = 1 \) in proposition 2. Assuming no service quality or cost differentiation, the pricing distribution is then \( 1 - H(V - p) \). Recall that \( V - r = X \). From proposition 2, we know that \( \frac{\phi}{N} \) is the maximal profit margin for a firm because the comparison site takes away \((1 - \frac{1}{N})X \) through click-through fee upon a successful transaction of the shoppers. As Varian studies a two-stage game without consideration of the gatekeeper, \( V - r = \frac{X}{N} \) in the tranformation instead. Then, if we take \( \alpha = 1 \), \( L \) to be uninformed consumers \( \frac{L}{n} \) and \( r \) to be \( m \), we will have Varian’s result.

(b) Baye and Morgan’s result: Service qualities are assumed to be the same denoted as \( V \) and cost \( r \). Take \( L = 0 \), \( k = 0 \) and click-through fee \( t = 0 \), mixed utility offering strategy in proposition 3 after transformation is \( F(p) = 1 - G(V - p) = \frac{1}{\alpha} \left( 1 - \frac{\phi}{S(1 - \frac{\phi}{p(r)})} \right)^{\frac{1}{\alpha - 1}} \) and \( \alpha = 1 - \left( \frac{\phi}{S(1 - \frac{\phi}{p(r)})} \right)^{\frac{1}{\alpha - 1}} \). The lower bound becomes \( V - X \left( 1 - \frac{\phi}{1 - \frac{\phi}{p(r)}} \right) \) and the upper bound is \( V \). Since \( X = V - r \), \( p_0 = \frac{N}{N - 1} \phi + r \) and where \( p \in [p_0, V] \), which is Baye and Morgan’s proposition 3.

Proof to Proposition 6. Among the continuum of equilibria described in proposition 2, the all-firms-advertise equilibrium is the most profitable for the gatekeeper and hence we assume it is picked in the subgame. Solve for the constrained profit maximization problem and we obtain the solution forms in explicit forms below.

\[
\phi^* = \left( 1 - \frac{1}{N} \right)^{N-1} \left( 1 - \frac{1}{N} \right)XS \\
\pi(\phi^*) \text{ is the same as } \phi^*
\]

and

\[
t^* = \left( 1 - \frac{1}{N} \right)X \\
\pi(t^*) = \left( 1 - \frac{1}{N} \right)XS
\]

Profits from an optimal listing fee are always a fraction of profits of click-through fee. The fraction is \( (1 - \frac{1}{N})^{N-1} \) if we take the ratio of \( \pi(\phi^*) \) and \( \pi(t^*) \). The term is a decreasing function of \( N \). When \( N = 2 \), the ratio is 50% and when \( N = \infty \), the ratio converges to \( \frac{1}{e} \), which is roughly 37%. Clearly, click-through fee is superior to listing fee in the all-firms-advertising equilibrium when hassle cost to online advertising is zero. In sum, when \( k = 0 \), the gatekeeper charges the optimal click-through fee \( t^* = \left( 1 - \frac{1}{N} \right)X \) and \( \phi = 0 \), the specific advertising fee that generates the subgame perfect equilibria of firms in proposition 2.

Proof of Corner Solution in Gatekeeper’s Profit Maximization Problem. For values \( 0 \leq \phi \leq S \left( (1 - 1/N)X - t \right) - k \) and \( 0 \leq t \) that ensure \( 0 \leq \alpha \leq 1 \), the set of \( \phi \) and \( t \) is a compact simplex. The existence of a maximal solution is guaranteed because the objective function is continuous in the compact set. An initial check on first order conditions will tell us that they are hard to solve explicitly.
\[ \frac{\partial \pi(\phi, t)}{\partial \phi} : (\frac{\phi + k}{s((1 - \frac{1}{N})X - t)})^{\frac{N}{N-1}} \left( St \frac{\phi + k}{(1 - \frac{1}{N})X - t} + (N - 1)k + \phi N \right) = (N - 1)(\phi + k). \]

\[ \frac{\partial \pi(\phi, t)}{\partial t} : (\frac{\phi + k}{s((1 - \frac{1}{N})X - t)})^{\frac{N}{N-1}} \left( (N^2 S X + S X + 1) \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} + N^2 \phi \right) = N^2 S X + S X + 1 - N^2 S t. \]

Let’s look at the Hessian matrix.

\[ \frac{\partial^2 \pi(\phi, t)}{\partial \phi \partial t} = \frac{N St}{(1 - \frac{1}{N})X - t} \left( \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} \right)^{\frac{N}{N-1}} + (2k(N-1)+\phi N)(\frac{\phi + k}{s((1 - \frac{1}{N})X - t)})^{\frac{N}{N-1}} \]

\[ \frac{\partial^2 \pi(\phi, t)}{\partial \phi^2} = \frac{N k(N-1)+\phi N}{s((1 - \frac{1}{N})X - t)} \left( \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} \right)^{\frac{N}{N-1}} + S(N-1)^2 + N t \left( \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} \right)^{\frac{N}{N-1}} \]

\[ \frac{\partial^2 \pi(\phi, t)}{\partial t^2} = \frac{N^2 S X(N-1)^2 + S t N}{(1 - \frac{1}{N})X - t} \left( \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} \right)^{\frac{N}{N-1}} + \phi N^2 (\frac{\phi + k}{s((1 - \frac{1}{N})X - t)})^{\frac{N}{N-1}} \]

\[ \frac{\partial^2 \pi(\phi, t)}{\partial \phi^2} - \frac{\partial^2 \pi(\phi, t)}{\partial t^2} = \frac{S X(N-1)}{s((1 - \frac{1}{N})X - t)} \left( \frac{\phi + k}{s((1 - \frac{1}{N})X - t)} \right)^{\frac{N}{N-1}} - k N (\frac{\phi + k}{s((1 - \frac{1}{N})X - t)})^{\frac{N}{N-1}} \]

Globally, \( \frac{\partial^2 \pi(\phi, t)}{\partial \phi^2} \) and \( \frac{\partial^2 \pi(\phi, t)}{\partial t^2} \) are negative. However, \( \frac{\partial^2 \pi(\phi, t)}{\partial \phi^2} - \frac{\partial^2 \pi(\phi, t)}{\partial t^2} \) is negative, violating the semi-negative definite matrix conditions. In other words, any \( \phi \) and \( t \) combination that is determined by the f.o.c. will not be local maxima. Henceforth, the solution will be in the corner, which are possible along the three sides of the simplex. Furthermore, the indefiniteness of the Hessian matrix holds for any possible parameter values of \( S, N, X \) and \( k \).

We can rule out the longest side of the right triangle because along the hypotenuse, firms have zero probability to advertise and hence, no profit can be collected to the gatekeeper. This means the gatekeeper will either charge click-through fee or charge listing fee, but not the combination of the two. Since the second order condition for \( \phi \) and \( t \) are strictly negative, the corner solutions will be unique, that is, the candidates are either a unique click-through fee and a unique listing fee.

REFERENCES


Price Dispersion Within and Across Retailers at a Comparison Site


