Heterogeneous Households, Real Rigidity, and Estimated Duration of Price Contract in a Sticky-Price DSGE Model

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Abstract

This paper introduces heterogeneous households into an otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model with Calvo-style sticky goods prices. Labor skills are industry specific. The households are heterogeneous because each household possesses a labor skill specialized exclusively for a certain industry and asset markets are incomplete. Then a household’s consumption depends positively on its labor income and the industry output, which leads to a smaller wage elasticity of industry labor supply due to wealth effect relative to the case in which the households are homogeneous in consumption. Firms then adjust their prices by a smaller amount than when the asset markets were complete because marginal costs become more sensitive to their pricing decisions. Hence I show that the model with heterogeneous households is characterized by a greater degree of "endogenous stickiness" or "real rigidity" relative to the standard representative household model. The household heterogeneity therefore provides a new explanation for large and persistent real effects of monetary shocks as well as inertial aggregate inflation even when firms update their prices frequently. I then estimate the durations of price contracts implied by the sticky-price DSGE models. The model with the heterogeneous households is consistent with empirical evidence in terms of not only mean but also sectoral durations of price contracts. I also show that recognizing the different degrees of price stickiness across sectors is also crucial when one estimates the model-implied durations and makes inferences.

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1. Introduction

The monetary Dynamic Stochastic General Equilibrium (DSGE) models with nominal rigidity have become an important tool for macroeconomic analysis. Not only they still provide theoretical insights but also they fit macroeconomic time series well, so that economists can employ them for various quantitative analyses as well as for forecasting.

On the other hand, there is a large body of literatures in macroeconomics that study how aggregate dynamics would change when one steps out of the commonly assumed representative agent framework.¹ Mostly the literatures on this subject investigate the question in flexible price environments. Surprisingly, up to my knowledge, there are few works that document implications of heterogeneous households in the framework of the sticky-price DSGE models.² Perhaps the only exception would be a set of literatures in the New Open Economy Macroeconomics (NOEM), the open economy version of the sticky-price DSGE models. They often assume that there are two types of households (e.g. Home vs. Foreign households) with incomplete financial markets across countries. They however still assume there is a representative household within a country.

In this paper, I fill this gap. I introduce heterogeneous households due to incomplete asset markets into the otherwise prototype sticky-price DSGE model. What I mean by the prototype sticky-price DSGE model is the baseline linearized model that consists of the three equations (IS, Phillips curve, and monetary policy equation) that one would find in a textbook. I then document quantitative as well as qualitative differences between the two versions of the sticky price model, which I refer to as the representative household and heterogeneous household models respectively (or the RH and HH models for short).

To be more specific, I show how the household heterogeneity could generate a greater degree of "endogenous stickiness" or "real rigidity." For a given degree of nominal friction, adjustment of aggregate price level in response to a shock is slower in the heterogeneous household model relative to the standard representative household model. Therefore, the household heterogeneity provides a new explanation to why monetary policy shocks have large and persistent real effects as well as why inflation is so inertial although many firms seem to update prices frequently.

Consider a standard New Keynesian (NK) model with Calvo-style sticky goods prices, for instance the one presented in Woodford (2003, chapter 3). The model has already embraced a rich set of heterogeneities. There is a continuum of industries that produce different types of products in the unit interval. More recently, it has become quite standard that labor skills are industry-specific so that there is also a continuum of labor types and that wage rates are potentially different across the industries. A usual assumption however is that there is a representative household that alone supplies all the different types of labor services. Equivalently, one often assumes that there are infinitely many households, each of which possesses a differentiated

¹This research agenda is relatively young, but is growing rapidly. Some important contributions include Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998). For international macro model, see Heathcote and Perri (2002).

²In this paper, I use the two terms, "sticky-price DSGE model" and "New Keynesian DSGE model" interchangeably.
skill and thus is a supplier of one type of labor service. However, with presence of complete set of state contingent assets, the households are homogeneous in asset holdings and consumption. Hence the model economy becomes identical to the economy with a representative household who supplies every type of labor.

In this paper, I relax the complete market assumption. Since the households are employed in different industries, their labor incomes would be different one another. Trading riskless bonds is the only way to buffer the idiosyncratic income risks. Then one important prediction of the \( \mathcal{H} \) model is that each household’s consumption depends positively on its labor income and thus on the wage rate and output of the industry in which the household is employed. This feature of the model makes the wage elasticity of labor supply smaller due to wealth effect: for instance, when industry wage rate is high, household’s consumption level is also high, and consequently the household has less incentive to supply labor.

To see how the wealth effect on labor supply influences a firm’s pricing decision, let us consider firms in an industry hit by a shock that reduces the firms’ marginal costs (a positive productivity shock for instance). The firms then have an incentive to lower their prices. Lowering the prices would induce more demands for their products. To produce more, the firms would demand more labor hours, which would shift the industry labor demand curve out. It would raise the industry wage rate and also the firms’ marginal costs, which might offsets the initial decrease in their marginal costs. However, the later increase in marginal costs must be larger when the asset markets are incomplete because the industry labor supply curve is steeper due to the wealth effect. The firms therefore reduce their prices by a lesser amount than they would do if the asset markets were complete.

Being motivated by the theoretical finding documented above, I estimate the two versions of sticky-price models, the \( \mathcal{RH} \) and \( \mathcal{HH} \) models, to conduct a quantitative analysis. The focus in this paper is on frequency of price changes and duration of price contracts implied by the estimated models.

Estimating the duration of price contract implied by the models is a reasonable way to quantify the degree of endogenous stickiness or real rigidity. It is also motivated by much of the recent empirical study based on micro-level data that shows the duration of price contracts, on average, is less than 2 quarters. This number is much smaller than what many macroeconomists have thought. More importantly, sticky-price DSGE models are not consistent with the new empirical evidence in the sense that the durations estimated using the structural DSGE models are often longer than 2 quarters. For example, Rabanal and Rubio-Ramirez (2005) estimate a baseline NK DSGE model employing a Bayesian method, and find that the average duration of price contracts is either 4.49 or 7.71 quarters depending on sample period of the data that the model is fitted to. My own estimation in this paper also shows that the model-implied duration is longer than a year in the representative agent framework. This is because the sticky-price DSGE models need a large degree of nominal rigidity in order to produce persistent dynamics. The inconsistency is perhaps summarized best by Altig et al. (2004):

"Macroeconomic and microeconomic data paint conflicting pictures of price behavior. Macro-
economic data suggest that inflation is inertial. Microeconomic data indicate that firms change prices frequently."

The paper presented here investigates the consequences of the newly discovered source of real rigidity, "heterogeneous households", for the estimated frequency of price changes and duration of price contract within the prototype NK framework. The intention is to see whether the prototype model, when the complete market assumption is relaxed, can explain empirical frequency of price changes documented by recent works such as Bills and Klenow (2004) and Nakamura and Steinsson (2006). In estimating the frequency and duration, I use a likelihood-based Bayesian method. The estimation method allows me to compute the marginal likelihoods of the data, so that I can also compare the two versions of the baseline sticky-price model based on how they fit to the U.S. data.

For a benchmark, I first estimate the models under the assumption that every firm in the economy must show an identical frequency of price adjustments, which I would call the representative frequency of the economy. This has been a conventional approach taken by existing literatures. I then relax the special assumption and consider the case where the model economies consist of multiple sectors with potentially different degrees of price stickiness as in Carvalho (2006).

There are more than one reason to consider the case of heterogeneous frequencies. First, the empirical studies show that durations of price contracts or frequencies of price changes are different across sectors. Thus it might be interesting to see if the durations implied by sticky-price DSGE models are consistent with the empirical evidences in terms of not only a mean or representative duration, which has been the focus in the existing literatures, but also sectoral durations. Second, even if we are only interested in a mean or representative duration of the entire economy as in the existing literatures, the mean duration estimated allowing the firms to have different frequencies of price changes might be different from the one estimated imposing the restriction that every firm should have an identical frequency from the beginning. Indeed the empirical literatures, such as Bills and Klenow (2004) and Nakamura and Steinsson (2006), first estimate sectoral frequencies and then make inference for the representative duration and frequency by taking weighted mean or median of the estimated sectoral frequencies. Therefore, estimating "multiple-sector-sticky-price DSGE models" would better mimic the estimation procedure employed in those literatures. Third, there is a growing interest in how disaggregated economic variables respond differently to various economic shocks, especially a monetary shock (e.g. Boivin et al., 2007). It also might be interesting to study whether shocks from certain sectors have a disproportionate impact on aggregate fluctuations relative to shocks from some other sectors. Therefore, it is still worthwhile to estimate the multiple-sector-sticky-price DSGE model even when one does not have much interest in the model-implied durations.

The model considered here is necessarily more complicated because there is now a continuum of heterogeneous households in addition to the continuum of the firms. In principle, the distributions of consumptions as well as of asset holdings across the households should affect aggregate equilibrium dynamics, and therefore the evolution of the distributions should be computed. It
is thus not possible to solve the model without resorting to an approximation scheme. I follow the same approximation strategy commonly employed in the NK literatures. First, I assume the time-dependent pricing as in Calvo (1983) and Yun (1996) rather than state-dependent pricing. Second, I take a log-linear approximation of the model.

With those two schemes and also the symmetric structure of the model, one does not have to compute the time path of the distributions in the single-sector case where the degrees of price stickiness (i.e. the probability of firms’ updating price each period) are identical across the firms. Also the forms of the equations that characterize the equilibrium conditions are identical between the $RH$ and $HH$ models. It however does not mean that the assumption of heterogeneous households does not affect the aggregate equilibrium dynamics in a linear approximation because the mapping between the deep structural parameters and the parameters in the equilibrium conditions are different between the two models. Another way to put it is that, up to first order approximation, the heterogeneous household model is observationally equivalent to the representative household model in the sense that the two models are statistically indistinguishable if no restriction is imposed on the structural parameters. Roughly speaking, the $RH$ model with a low frequency of price adjustment is identical to the $HH$ model with a high frequency. They are however different in economic sense, in that if I use identical structural parameter values, the implied dynamics are different between the two models. On the other hand, in the multiple-sector cases where I allow different frequencies of price adjustments across the firms, the evolutions of cross-sector distributions as well as the aggregate variables do affect aggregate equilibrium dynamics because it is no longer possible to utilize the symmetry properties.

The main quantitative findings are summarized as follows: First, under the assumption that every firm in the economy faces an identical frequency of price changes, the posterior mean of duration of price contract is between 1.51 and 3.81 quarters in the $HH$ model depending on the degree of financial market friction, while it is 4.65 quarters in the $RH$ model. Thus the $HH$ model is more consistent with the empirical evidence of price stickiness. Put it differently, in a conventional single-sector framework, the heterogeneous household model can potentially account for persistent aggregate dynamics without imposing an implausibly large degree of nominal rigidity, while the representative household model under consideration cannot.

Second, allowing heterogeneous nominal rigidities across the sectors has a non-trivial implication for inference of representative duration of price contracts. The duration implied by weighted average of estimated sectoral frequencies is 1.74 quarters in the $RH$ model and 1.39 quarters in the $HH$ model. Therefore even the $RH$ model is consistent with the view that the duration should be less than 2 quarters on average.

Third, the estimated sectoral durations in both the $HH$ and $RH$ models are quite consistent with the empirical sectoral durations estimated based on micro-level data without using any structural model. I consider this finding somewhat surprising. The structural sticky-price DSGE models under consideration (even in the multiple-sector case) are highly stylized with lots of strong restrictions. Yet the models still can predict the plausible cross-sector distribution of
frequency of price changes while fitting the major U.S. time series data. The \( \mathcal{HH} \) model still seems to explain the mean and sectoral durations of price contracts better than the \( \mathcal{RH} \) model. However, the differences between the two models in terms of model-implied durations of price contracts are not as large as in the single-sector case.

Finally, the marginal likelihood of the \( \mathcal{HH} \) model is larger than that of the \( \mathcal{RH} \) model in the multiple-sector case. The ratio of the likelihood of the \( \mathcal{HH} \) model to that of the \( \mathcal{RH} \) model is \( e^{43.8} \). This implies that the heterogeneous household model is better at explaining the joint dynamics of aggregate and sectoral U.S. time series data. The magnitude of the likelihood ratio is relatively large. It indicates the difference in model fits between the two models is statistically significant, and one can reject the \( \mathcal{RH} \) model in favor of the \( \mathcal{HH} \) model.

Let me state differences as well as similarities between some existing literatures and the current paper. First, the paper presented here belongs to a large set of literatures that study sources of real rigidities. Some of the earlier works include Ball and Romer (1990), Kimball (1995), and Basu (1995). Chari et al. (2000) also have stressed importance of endogenous stickiness. The current work, however, is the first to present how heterogeneous households due to incomplete markets could enhance the endogenous stickiness. It, thus, provides a new and additional source of real rigidities.

Second, more recently there are papers that examine the consequences of various sources of real rigidities for the estimated frequency of price change and the estimated duration of price contract. Some important works include Altig et al. (2004), Eichenbaum and Fisher (2007), and Woodford (2005). Besides the very source of real rigidity, the current paper is different from them in estimation strategy. I employ the likelihood-based Bayesian method, which allows utilizing the entire predictions from general equilibrium of the models. Eichenbaum and Fisher (2007) uses only one of the entire equilibrium conditions, the Phillips curve, and estimate the duration of price contracts using GMM. Altig et al. (2004) choose the parameters that minimize the distance between the model and VAR based impulse responses. Another key difference is that I allow multiple sectors with potentially different degree of price stickiness, which turns out to have non-trivial consequences for inference of durations of price contracts.

Third, the current paper is related to Carvalho (2006), in that the both explicitly consider the possibility that nominal rigidities are different across the sectors. My approach is, however, different because he calibrates the sectoral frequencies in his model to the empirical cross-sector distribution of the frequencies from Bills and Klenow (2004) as if the empirical distribution were consistent with the NK model. My intention on the other hand is to examine if the frequencies implied by the structural macro-models are consistent with the empirical frequencies based on micro studies, and thus provide such a calibration with a justification.

Forth, the current paper is a part of a growing body of literatures that estimate sticky-price DSGE models using the likelihood-based Bayesian method.\(^3\) Previous literatures however pay little attention to the model-implied duration of price contracts. In addition, up to my

\(^3\)For recent contributions, see Smet and Wouters (2003), Rabanal and Rubio-Ramirez (2005), Del Negro and Schorfheide (2004), Lubik and Schorfheide (2005), and many other papers cited in An and Schorfheide (2006).
knowledge, this paper is one of the first attempts to estimate the DSGE model with multiple sectors as well as heterogeneous households.

The rest of the paper is organized as follows. In the next section, I present a simple static model. The intention is to present the main theoretical results explicitly and thus to show that the theoretical predictions are not subject to my approximation scheme, especially linearization. In the following section, I introduce a NK DSGE model with Calvo-style sticky goods prices with heterogeneous households as well as a representative household. The NK DSGE model will be the basis for the quantitative experiments in later sections. Section 4 defines the equilibrium of the model economies. Section 5 details derivations and properties of the NK Phillips curve under heterogeneous households and show how asset market frictions increase the degree of real rigidity. In section 6, I discuss about estimation and evaluation of the model economies. Section 7 summarizes the results and concludes.

2. Static Model

I present a simple static model in this section. A main advantage of the simple model is that I can show the results analytically. On the other hand, the model cannot be used for any serious quantitative analysis. A full-blown DSGE model for that purpose is presented in the next section.4

Suppose there is a continuum of industries in the economy indexed by \( i \in [0, 1] \), each of which produces a distinguished type of product \( Y(i) \). In each industry \( i \), there is a representative firm called type-\( i \) firm. The differentiated products \( \{Y(i)\}_{i \in [0, 1]} \) are used to produce final consumption good \( Y \), through a CES technology given by

\[
Y = \left( \int_0^1 Y(i)^{\frac{\theta-1}{\theta}} di \right)^{-\frac{1}{\theta-1}}. \tag{2.1}
\]

The corresponding price index \( P \) for the final consumption good is

\[
P = \left( \int_0^1 P(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \tag{2.2}
\]

where \( P(i) \) is the price of type-\( i \) product. The optimal demand for each type of good is then given by

\[
Y(i) = \left( \frac{P(i)}{P} \right)^{-\theta} Y. \tag{2.3}
\]

Each firm has a linear production technology:

\[
Y(i) = H(i), \tag{2.4}
\]

where \( H(i) \) denotes type-\( i \) labor which can produce only type-\( i \) product \( Y(i) \) but cannot produce

\[4\]Those who have little interest in analytical results can skip this section and go to section 3 without loss of continuity.
the other types of goods. Therefore labor skill is industry-specific. The type-\(i\) firm chooses its price \(P(i)\) to maximize the profit:

\[
\Pi(i) = P(i)Y(i) - W(i)H(i),
\]

subject to the demand function (2.3) given above. I let \(W(i)\) denote the competitive wage rate in industry \(i\). The first order condition then can be obtained as

\[
\frac{P(i)}{P} = \delta \frac{W(i)}{P},
\]

where \(\delta = \theta/(\theta - 1)\) is the firm’s desired mark-up.

In each industry \(i\), there is a representative household called type-\(i\) household. The type-\(i\) household possesses a labor skill specialized exclusively for industry \(i\), and thus supplies labor services to type-\(i\) firm. The household chooses consumption level \(C(i)\) and labor hours \(H(i)\) to maximize utility:

\[
\log C(i) - H(i),
\]

subject to budget constraint:

\[
PC(i) = W(i)H(i) + \Pi(i) + K(i),
\]

\[
K(i) = \lambda \int_{z \neq i} P(z)Y(z)dz.
\]

In addition to labor income \(W(i)H(i)\), I assume that type-\(i\) household receives profit from industry \(i\) for simplicity.\(^5\) \(K(i)\) represents a pre-arranged financial portfolio that is exogenous in this model. Introducing \(K(i)\) allows me to nest both complete and incomplete market economies within a single framework. Even though there is no market for state-contingent assets, the pre-arranged financial contracts, when \(\lambda = 1\), always guarantee that the households receive the incomes they would obtain if the asset markets were complete. Thus, the economy is effectively under complete markets when \(\lambda = 1\), while the asset markets are effectively incomplete when \(\lambda < 1\). The household’s first order condition is given by

\[
C(i) = \frac{W(i)}{P}.
\]

The model is completed by adding the quantity equation:

\[
PY = M,
\]

where \(M\) is the government policy variable.

Given exogenous \(\{M\}\), the equilibrium of the model economy is characterized by allocations at industry level,

\[
\{C(i), Y(i), H(i), P(i), W(i)\}_{i \in [0,1]}.
\]

\(^5\)This assumption is not necessary to show the results. But it makes the exposition simpler. In the next section, I consider a more general case.
and two aggregate variables: output and price level,

\[ \{Y, P\}, \]

which satisfy the followings:

1. **Definitions of the aggregates:** (2.1) and (2.2),

2. **Firms’ optimality conditions and production functions:** (2.3), (2.4) and (2.5),

3. **Households’ budget constraints and optimality conditions:** (2.6) and (2.7),

4. **Quantity equation:** (2.8)

5. **Market clearing condition:** \( \int_0^1 C(i) \, di = Y. \)

As a benchmark case, I first consider the economy where prices are completely flexible in that every firm observes the exogenous policy variable \( M \) before setting prices. It turns out that there exists a unique symmetric equilibrium regardless of asset market completeness. It is because there is no idiosyncratic shock in the model. In the absence of idiosyncratic shocks, firms charge the same prices and produce the same amount of outputs. Consequently, the households’ incomes are symmetric and there is no need for state-contingent asset markets.

**Proposition 2.1.** If prices are flexible in the sense that firms can adjust the prices in the event of changes in the exogenous variable \( M \), then there exists a unique symmetric equilibrium in which \( P(i) = P \) and \( Y(i) = C(i) = Y \) for all \( i \in [0,1] \). Moreover, money is neutral. The equilibrium output \( Y \) is determined independently from \( M \). The output and price level are explicitly given by

\[ Y = \frac{1}{\delta} \quad \text{and} \quad P = \delta M. \]

**Proof.** See the section C in appendix. ■

In the absence of idiosyncratic shocks, therefore, the asset market incompleteness by itself does not affect equilibrium outcomes when prices are flexible. I show this is no longer the case when not every firm adjusts its price in response to a change in economic environment.

Let us consider the simplest case. Suppose that the government announces a certain level of \( M = \bar{M} \). Some firms believe the announcement and set their prices accordingly, while the other firms wait until they observe actual \( M \). This amounts to assume that the firms, which set the prices before realization of \( M \), form a subjective probability distribution of \( M \) that places entire mass on \( \bar{M} \) after the announcement. It is may not be the most realistic or elegant way to model firms’ belief. It however helps us to see the main results more explicitly. The common price chosen by those firms, which believe \( M \) would be equal to \( \bar{M} \) with probability one, must be \( \delta \bar{M} \). If the government indeed keeps its promise, then the remaining firms also choose the price of \( \delta \bar{M} \) and the equilibrium output would be equal to \( \delta^{-1} \). That is if there were no surprise in monetary policy, the aggregate output would be equal to the flexible-price level of output.
When the government does not keep the promise, however, money is no longer neutral because of the prices that are predetermined. The degree of non-neutrality hence would depend on how much those remaining prices would respond to a change in \( M \). I show that the responsiveness of the remaining prices is smaller, and thus the monetary non-neutrality is larger, when the asset markets are incomplete.

After some algebra, it can be shown that the optimal price \( P^* \), that is common to all the firms who set the prices after realization of \( M \), must satisfy:

\[
P^* = (\lambda \delta M + (1 - \lambda) \delta \left( \frac{P^*}{P} \right)^{1-\theta} M). \tag{2.9}
\]

The equation (2.9) implicitly determines the equilibrium level of \( P^* \) given \( \{M, P\} \). The response of \( P^* \) to a change in \( M \) is characterized by the sum of the two coefficients \((i)\) and \((ii)\) on \( M \) in (2.9). The second coefficient \((ii)\) is however decreasing in \( P^* \) given \( P \), so it dampens the response of \( P^* \) to a change in \( M \). This provides an idea of why monetary shock might be less neutral under incomplete markets (i.e. when \( \lambda < 1 \)).

Under complete asset markets (i.e. when \( \lambda = 1 \)), \( P^* = \delta M \), and from (2.2) I can obtain the price level:

\[
P_C = \left( n [\delta M]^{1-\theta} + (1 - n) [\delta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}}.
\]

\( P_C \) denotes the price level when the asset markets are complete, and \( n \) is the measure of the firms that set the prices after observing \( M \). Under incomplete markets, however, \( \lambda \) is less than 1. Let us consider a special case in which \( \lambda = 0 \). Then the price level \( P_{IC} \) under incomplete markets must be given by

\[
P_{IC} = \left( n \left[ \frac{1}{\theta} [\delta M]^{\frac{\theta}{1-\theta}} P_{IC}^{\frac{\theta}{1-\theta}} \right]^{1-\theta} + (1 - n) [\delta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}}.
\]

Looking at the equation above, one might notice that it is hard to obtain an explicit solution for the price level as a function of the exogenous variables \( M \) and \( \bar{M} \) only, even in this simple special case. It is still possible, however, to show that the price level under incomplete markets does not adjust as much as when the asset markets are complete.

**Proposition 2.2.** Let \( P_F \) and \( Y_F \) denote price level and output that would be realized when the prices were completely flexible (i.e. \( Y_F = 1/\delta \) and \( P_F = \delta M \)). Also let \( Y_C \) and \( Y_{IC} \) denote the aggregate output under complete markets and under incomplete markets respectively. If \( M > \bar{M} \), then \( P_{IC} < P_C < P_F \) and \( Y_{IC} > Y_C > Y_F \), and vice versa.

**Proof.** See the section C in appendix. ■

**Proposition 2.3.** Let \( Y_1 \) denote a common level of outputs produced by the group of the firms which set the prices before realization of \( M \), and \( Y_2 \) is the level of output produced by the other group. Then \(|Y_1 - Y_2|\) are smaller under incomplete markets than under complete markets.
Proof. This is a direct implication of the Proposition 2.2.

The two propositions show that asset market incompleteness has not only an aggregate but also a distributional implication. Clearly, the arguments made in this section are not limited to the case of a monetary policy shock. For instance, the same arguments could be made for a productivity shock if an exogenous productivity were added to the model. In the next section, I investigate how much the results obtained here would influence dynamics in a more quantitatively oriented model.

3. New Keynesian Model

I incorporate the ideas explored in the previous section into the prototype NK DSGE model. Some of the model settings are similar to the previous toy model. I however describe every detail for completeness.

3.1. Households

In this subsection, I describe households’ decision problems and optimality conditions. There is a continuum of industries indexed by $i \in [0,1]$, each of which produces a different type of product. Each industry $i$ is represented by one firm called type-$i$ firm. Different labor skills are required to produce different types of product, that is labor skill is industry-specific. In each industry $i$, there is a representative household called type-$i$ household. The type-$i$ household possesses a labor skill specialized exclusively for industry $i$, and thus supplies labor service to type-$i$ firm. To summarize, each industry is composed of one representative household and one representative firm.

The economy can be divided into finite number of sectors indexed by $j \in \{1, 2, \cdots, J\}$, and the sectors are characterized by potentially different degree of nominal rigidities $\{\alpha_j\}_{j=1}^J$. I let $I_j$ denote the space for sector $j$. The elements of the collection $\{I_j\}_{j=1}^J$ are disjoint sets whose union is the unit interval, that is, $\bigcup_{j=1}^J I_j = [0,1]$. The measure of sector-$j$ is the length of the interval $I_j$ and is denoted by $n_j$, with $\sum_{j=1}^J n_j = 1$. One important special case is when the degrees of nominal rigidities are identical across the sectors (i.e. $\alpha_j = \alpha \forall j$). The model then would be identical to the standard single-sector model often seen in textbooks (see chapter 3 of Woodford (2003) for example).

Type-$i$ household seeks to maximize a discounted sum of utilities of the form
\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[ \log C_{j,t}(i) - \Xi_t \frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi} \right] \right),
\]
where $C_{j,t}(i)$ denotes type-$i$ household’s consumption and $H_{j,t}(i)$ denotes the hours of labor services supplied to industry $i$ in sector $j$. $\Gamma_t$ and $\Xi_t$ are intertemporal preference shocks. They

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\[\text{Note that, in general, more than one industries make up one sector unless the measure of the sector is equal to zero.}\]
serve as aggregate demand and aggregate supply shocks respectively.

The household’s flow budget constraint is given by

\[ C_{j,t}(i) + \frac{B_{j,t}(i)}{P_t} + \text{Cost}(B_{j,t}(i)) = \frac{R_{t-1}B_{j,t-1}(i)}{P_t} + \tau_t + \frac{W_{j,t}(i)H_{j,t}(i)}{P_t} + \frac{K_{j,t}(i)}{P_t}, \tag{3.1} \]

where \( \tau_t \) denotes the government net transfer, \( R_t \) denotes gross nominal interest, and \( W_{j,t}(i) \) is the competitive wage rate in industry \( i \).

The households do not trade state-contingent assets. Instead, they are allowed to borrow and lend one another by trading riskless nominal bonds (IOUs). \( B_{j,t}(i) \) denotes holding of the bond by type-\( i \) household at time \( t \). I assume a quadratic ”borrowing-lending-cost”:

\[ \text{Cost}(B_{j,t}(i)) \equiv \tilde{\epsilon} \left( \frac{B_{j,t}(i)}{P_t Y_t} \right)^2 \]

to make the model stationary with \( \tilde{\epsilon} > 0 \). With the borrowing-lending cost, every household holds zero net borrowing in the steady state. One can consider this cost term is completely ad-hoc, with its only role being to conveniently give the model stationarity.\(^7\)

It is however well known that there are some important frictions in the private bond markets. Heaton and Lucas (1996) extensively discuss about these frictions and employ a similar form of borrowing-lending cost in their model to investigate the effects of various financial market frictions on asset pricing and the equity premium. For instance, there is a substantial spread between the lending and borrowing rates, reflecting the fact that lenders should pay a monitoring cost to prevent defaults. There also could be a borrowing constraint that makes it difficult for the households to borrow a large amount due to physical and/or mental costs from a lender’s side. I consider the ”borrowing-lending-cost” term as a proxy that provides me with a convenient shortcut to impose those frictions in the bond market, while avoiding making the model too complicated. Thus the private bond trading is only \textit{ex-post} riskless in the sense that it is riskless but only after the market agents pay the participation costs.

This borrowing-lending cost should be distinguished from a \textit{trading cost} which might occur when there is a change in bond holding, \( \Delta B_{j,t}(i) \). As noted by Heaton and Lucas, while the trading cost might be important in the stock markets (for instance one has to pay commissions to a financial agency when buying or selling stocks), the borrowing-lending cost is more relevant in private bond markets.

The parameter \( \tilde{\epsilon} \) controls the magnitude of the cost. With a positive value of \( \tilde{\epsilon} \), households find it harder to borrow larger amount and at the same time are more reluctant to lend larger amount. The two limiting cases would be when \( \tilde{\epsilon} \) goes to zero and to infinity. When \( \tilde{\epsilon} \) is close to zero, a household borrows or lends frictionlessly against future income to smooth consumption. On the other hand, if \( \tilde{\epsilon} \) is sufficiently large, households decide not to trade bonds.

\( K_{j,t}(i) \) denotes type-\( i \) household’s capital income from the firms’ profits. Since the firms are monopolistically competitive, they make positive profits that should be distributed to house-\(^7\)

\(^{7}\)See Schmitt-Grohe and Uribe (2003) for further discussions.
holds. There are two extreme cases. One extreme is the case in which each household receives the entire profits from the industry it is employed in, but has zero shares of the other industries. Although this case may not be far from reality for some households, it may be too extreme to capture the reality.

The other extreme would be the case in which each household holds equal shares of the profits of all the firms in the economy. In that case, the economy’s entire profits would be equally distributed across the households. This may be equally unrealistic. There are studies which document that a substantial portion of population does not participate in the stock markets and that even the stockholders somewhat irrationally own a disproportionately large number of stocks of companies in the industry in which they are employed.

Therefore I instead allow for all the intermediate cases. With this regard, I define $K_{j,t}(i)$ as

$$K_{j,t}(i) = \chi \left( \sum_{k=1}^{J} \int_{I_k} \Pi_k(z) dz \right) + (1 - \chi) \Pi_{j,t}(i), \quad 0 \leq \chi \leq 1. \quad (3.2)$$

Note that the two extreme cases are when $\chi = 0$ and $\chi = 1$. One problem is that the parameter $\chi$ in principle must be different across the households. It is therefore difficult to decide what value of $\chi$ would be appropriate in representing the entire economy, so that it can be used in macroeconomic models. I use $\chi = 1$ as a benchmark number, believing that a stockholder’s portfolio should be more or less diversified. The parameter $\chi$ is to be estimated along with the other parameters in a later section.

Note that the characteristics of the market incompleteness are exogenous in that I have just assumed them instead of deriving them. This paper does not intend to explain why $\chi$ might be less than one, nor study why households do not write state-contingent financial contracts on their labor incomes. While those are interesting issues, I focus on the question of how the assumed market incompleteness would affect the aggregate dynamics relative to the complete market case. With this regard, the way I model the market incompleteness is just good enough for the purpose of this paper. In the end, I only need to consider a pair of parameters $(\bar{\epsilon}, \chi)$ to measure the asset market efficiency.

The wage in each industry is fully flexible and both the household and firm in the industry take the wage as given. The household’s first order conditions are then given by

$$1 + 2\epsilon \frac{B_{j,t}(i)}{P_t Y_t} = \beta R_t \left[ \frac{\Gamma_{t+1}}{\Gamma_t} \left( \frac{C_{j,t}(i)}{C_{j,t+1}(i)} \right) \left( \frac{P_t}{P_{t+1}} \right) \right], \quad (3.3)$$

$$\left( \frac{H_{j,t}(i)}{\Xi_t} \right)^{\varphi} C_{j,t}(i) = \frac{W_{j,t}(i)}{P_t}. \quad (3.4)$$

In the RH model, there is a representative household who simultaneously supplies every type of labor service. The household maximizes the discounted sum of utilities:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[ \log C_t - \Xi_t^{-\varphi} \sum_{j=1}^{J} \int_{I_j} \frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi} \right] \right),$$

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subject to budget constraint:

\[ C_t + \frac{B_t}{P_t} = R_{t-1} B_{t-1} + \tau_t + \sum_{j=1}^J \int_{I_j} W_{j,t}(i) H_{j,t}(i) \frac{d\Pi_{j,t}(i)}{P_t} + \sum_{j=1}^J \int_{I_j} \Pi_{j,t}(i) \frac{d\Pi_{j,t}(i)}{P_t}. \]

After imposing the market clearing condition, \( C_t = Y_t \), the first order conditions are given by

\[ 1 = \beta (1 + i_t) E_t \left[ \left( \frac{\Gamma_{t+1}}{\Gamma_t} \right) \left( \frac{Y_t}{Y_{t+1}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right], \]

\[ \left( \frac{H_{j,t}(i)}{\Xi_t} \right)^{\varphi} Y_t = \frac{W_{j,t}(i)}{P_t}. \]

Comparison between the last equation and the equation (3.4) gives an idea of how the household heterogeneity leads to a greater degree of real rigidity. In the HH model, type-i household’s consumption \( C_{j,t}(i) \) depends positively on real wage \( \frac{W_{j,t}(i)}{P_t} \) as well as labor hour \( H_{j,t}(i) \), which makes the wage elasticity of labor supply smaller. In consequence, type-i firm’s marginal cost gets more sensitive to a change in the firm’s production. On the other hand, there is no such channel in the RH model since each industry is so small that industry wage rate \( \frac{W_{j,t}(i)}{P_t} \) or labor hour \( H_{j,t}(i) \) do not affect directly the aggregate output \( Y_t \).

### 3.2. Firms

I describe firms’ behaviors in this section. The final consumption good \( Y_t \), is produced by perfectly competitive firms using the sectoral intermediate outputs, \( \{Y_{j,t}\}_{j=1}^J \) with a CES production technology:

\[ Y_t = \left( \sum_{j=1}^J (n_j D_{j,t}^R)^{1/\eta} Y_{j,t}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}, \tag{3.5} \]

where \( \eta \) is the elasticity of substitution among the sectoral outputs and \( D_{j,t}^R \) is given by

\[ D_{j,t}^R = \frac{D_{j,t}}{D_t}. \]

It can be interpreted as a sector-specific demand shock relative to \( D_t \), a measure of strength of the overall demand. I assume that \( E \left( D_{j,t}^R \right) = 1 \). Note that the final consumption good is normalized so that it has a measure 1. On the other hand, each sectoral output, \( Y_{j,t} \) has a time varying weight \( n_j D_{j,t}^R \). To be consistent, the stochastic weights should sum to be 1 for every time period, that is \( \sum_{j=1}^J n_j D_{j,t}^R = 1 \). Hence it must be that \( D_t = \sum_{j=1}^J n_j D_{j,t} \). The appropriate price index for the final consumption good is found as the minimum cost that should be paid by the firms for producing one unit of the consumption good and is given by

\[ P_t = \left( \sum_{j=1}^J (n_j D_{j,t}^R)^{P_{1-t}^{1-\eta}} \right)^{1/(1-\eta)} \tag{3.6}. \]
Given the aggregate consumption good, $Y_t$, and the price levels, $P_{j,t}$ and $P_t$, the optimal demand for the intermediate goods would be the one to minimize the total expenditure $P_t Y_t$. The demands for intermediate goods are then can be obtained as

$$ Y_{j,t} = n_j D_{j,t}^{R} \left( \frac{P_{j,t}}{P_t} \right)^{-\eta} Y_t \quad \forall j. \quad (3.7) $$

Like the final consumption good, each of the intermediate sectoral goods $\{Y_{j,t}\}_{j=1}^J$, is also an aggregate of the goods $\{Y_{j,t}(i)\}_{i \in \mathcal{I}_j}$ that are produced by the firms in sector $j$, and is given by

$$ Y_{j,t} = \left( \frac{1}{n_j} \int_{\mathcal{I}_j} Y_{j,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad \forall j, \quad (3.8) $$

where $\theta$ is the elasticity of substitution among different types of goods and is larger than one. The corresponding price indexes for intermediate sectoral goods are given by

$$ P_{j,t} = \left( \frac{1}{n_j} \int_{\mathcal{I}_j} P_{j,t}(i)^{1-\theta} di \right)^{1/(1-\theta)} \quad \forall j. \quad (3.9) $$

Given $Y_{j,t}$, the optimal demand for type-$i$ good $Y_{j,t}(i)$ would be

$$ Y_{j,t}(i) = \frac{1}{n_j} \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} Y_{j,t}. \quad (3.10) $$

Type-$i$ firm’s production function is given by

$$ Y_{j,t}(i) = A_{j,t} H_{j,t}(i), \quad (3.11) $$

where $A_{j,t}$ is an exogenous sector-specific productivity. The profit function is given by

\[
\Pi_{j,t}(i) = P_{j,t}(i) Y_{j,t}(i) - W_{j,t}(i) H_{j,t}(i) = \left\{ P_{j,t}(i) - \frac{W_{j,t}(i)}{P_t A_{j,t}} \right\} D_{j,t}^{R} Y_t \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} \left( \frac{P_{j,t}}{P_t} \right)^{-\eta}.
\]

I model the nominal rigidity following Calvo (1983) and Yun (1996). The firms in sector $j$ can adjust their prices with probability $1 - \alpha_j$ each period. Since only the fraction $1 - \alpha_j$ of all the prices in that sector is set anew while the remaining fraction $\alpha_j$ of prices is carried over from the previous period, the sectoral price level $P_{j,t}$ should satisfy the following:

$$ P_{j,t} = \left[ \frac{1}{n_j} \int_{I_j^*} P_{j,t}^r(i)^{1-\theta} di + \frac{1}{n_j} \int_{I_j-I_j^*} P_{j,t-1}(i)^{1-\theta} di \right]^{1/(1-\theta)} $$

$$ = \left[ \frac{1}{n_j} \int_{I_j^*} P_{j,t}^r(i)^{1-\theta} di + \alpha_j P_{j,t-1}^{1-\theta} \right]^{1/(1-\theta)}, \quad (3.12) $$

where $P_{j,t}^r(i)$ is an optimal price chosen by type-$i$ firm when $i \in I_j^*$. The set $I_j^* \subset I_j$, whose
measure is $n_j (1 - \alpha_j)$, is a randomly chosen subset in which the firms get a chance to update their prices.

Those firms that adjust their prices at time $t$ maximize expected discounted profits:

$$\max_{p_{i,t}(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \alpha^k_j q_{j,t,t+k}(i) \tilde{\Pi}_{j,t+k}(i),$$

where $q_{j,t,t+k}(i)$ is a type-$i$ firm’s real stochastic discount factor between time $t$ and $t+k$, and $\tilde{\Pi}_{j,t+k}(i)$ is the real profit at time $t+k$ under the condition that price has not been updated since time $t$.$^8$ Note that when the asset markets are incomplete and firms are owned by more than one household, there is no unique way to determine a firm’s stochastic discount factor. Each shareholder would like to have the firm’s manager to use its own stochastic discount factor in maximizing the expected discounted profits. Therefore there is a conflict of interests among the shareholders. I here assume that the manager maximizes the weighted sum of different objective functions among the shareholders. Then type-$i$ firm’s discount factor would be

$$q_{j,t,t+k}(i) = \beta^k \left[ (1 - \chi) \left( \frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right) + \chi \left( \sum_{l=1}^{J} \int_{t}^{t+k} \left( \frac{\Gamma_t C_{l,t}(z)}{\Gamma_{t+k} C_{l,t+k}(z)} \right) dz \right) \right].$$

Alternatively one could assume that a household who has the largest voting right gets to choose the firm’s discount factor. In that case, the discount factor of type-$i$ firm would be $\beta^k \left( \frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right)$. It also could be the case that a firm discounts its future profits in a non-state-contingent way using the real interest rates. In that case, $q_{j,t,t+k}(i)$ would no longer be firm-specific and is instead given by $q_{j,t,t+k}(i) = \prod_{l=0}^{k} R_{t+l}^{-1} \frac{P_{i,l+1}}{P_{i,l+1}}$. However, the alternative choices of the discount factor do not make any difference quantitatively since only the steady state value of the discount factors enters the equilibrium conditions in the first order approximation. The steady state level of the shadow value of a dollar is identical across the households and it is equal to the steady state value of risk free real interest rate.$^9$

The first order condition of type-$i$ firm is given by$^{10}$

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} \alpha^k_j q_{j,t,t+k}(i) D^R_{j,t+k} Y_{t+k} \left( \frac{P^*(i)}{P_{j,t+k}} \right)^{-\theta} \left( \frac{P_{j,t+k}}{P_{l+k}} \right)^{-\eta} \left( \frac{P^*(i)}{P_{l+k}} \right)^{-1} \times \left\{ \left( \frac{P^*(i)}{P_{l+k}} \right) - \left( \frac{\theta}{\theta - 1} \right) MC_{j,t+k}(i) \right\},$$

$^8\tilde{\Pi}_{j,t+k}(i) = \frac{P^*(i)}{P_{l+k}} D^R_{j,t+k} Y_{t+k} \left( \frac{P^*(i)}{P_{l+k}} \right)^{-\theta} \left( \frac{P_{l+k}}{P_{l+k}} \right)^{-\eta} \left( \frac{P^*(i)}{P_{l+k}} \right)^{-1} \left( \frac{P_{l+k}}{P_{l+k}} \right)^{-\eta}$

$^9$Pescatori (2006) has also made a similar argument.

$^{10}$By looking at type-$i$ firm’s stochastic discount factor, one might think that the firm can manipulate its discount factor by influencing $C_{j,t}(i)$. But it is not the case in this model. Just like each firm takes the industry wage as given, it also takes the discount factor as given. Recall that type-$i$ firm and type-$i$ household is only a representative of infinite number of firms and households that share the same labor market. Therefore type-$i$ firm’s stochastic discount factor should in fact be interpreted as "industry-$i$ stochastic discount factor."
where $MC_{j,t+k}(i) = \frac{W_{j,t+k}(i)}{A_{j,t+k}P_{j,t+k}}$ is type-$i$ firm’s real marginal cost at time $t+k$. The optimal prices chosen at time $t$, $\left\{ P^*_{j,t}(i) \right\}_{i \in I^*_j}$ that satisfy the first order condition (3.13) characterize dynamics of the sectoral price level $P_{j,t}$ through (3.12). The aggregate price level is then determined by the price aggregate, $P_t = \left( \sum_{j=1}^{J} \left( n_j D^R_{j,t} \right) P_{j,t}^{1-n} \right)^{\frac{1}{1-n}}$.

3.3. Government

The government budget constraint is

$$\frac{B_t - R_{t-1}B_{t-1}}{P_t} + \sum_{j=1}^{J} \int_{I^*_j} \text{Cost}(B_{j,t}(i)) \, di = \tau_t + G_t,$$  \hspace{1cm} (3.14)

where $B_t$ is government bond supply and $G_t$ is government expenditure at time $t$. The government collects the borrowing-lending costs and redistributes them to the households as transfers.

Monetary policy is characterized by a Taylor rule:

$$R_t = \beta^{-1}R_{t-1}^{\phi} \left[ \left( \frac{P_t}{P_{t-1}} \right)^{\phi_y} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right]^{(1-\rho_y)} \exp(\mu_t). \hspace{1cm} (3.15)$$

I assume a simple Ricardian fiscal policy:

$$G_t = 0, \quad B_t = 0, \hspace{1cm} (3.16)$$

Even if this type of fiscal policy assumption is non-trivial as pointed out by Leeper (1992) and Sims (1994), I made that assumption for a direct comparison with many existing NK models as well as for simplicity.

4. Sticky Price Equilibrium

The equilibrium of the economy is characterized by within-sector resource and price allocations:

$$\left\{ \{C_{j,t}(i), Y_{j,t}(i), H_{j,t}(i), B_{j,t}(i), P_{j,t}(i), W_{j,t}(i), K_{j,t}(i)\}_{i,j} \right\}_{t=0}^{\infty},$$

cross-sector allocations of outputs and prices:

$$\left\{ \{Y_{j,t}, P_{j,t}\}_{j} \right\}_{t=0}^{\infty},$$

and three aggregate variables: output, price level, and nominal interest rate:

$$\{Y_t, P_t, R_t\}_{t=0}^{\infty}$$

that satisfy the households’ optimality conditions (3.3) and (3.4), the households’ budget constraints (3.1) and (3.2), the firms’ optimality conditions (3.7), (3.10), and (3.13), the government budget constraint (3.14), the monetary and fiscal policies (3.15) and (3.16), the CES aggregates
(3.5), (3.6), (3.8), and (3.9), and finally the market clearing conditions:

\[
\sum_{j=1}^{J} \int_{\mathcal{I}_j} C_{j,t}(i)di = Y_t, \quad \sum_{j=1}^{J} \int_{\mathcal{I}_j} B_{j,t}(i)di = 0
\]
given \(B_{j,-1}(i) = 0\), \(\forall i \in [0, 1]\).

Let me introduce some additional notations. In what follows, \(B_{j,t}\) denotes a sectoral bond holding, \(\int_{\mathcal{I}_j} B_{j,t}(i)di\), and \(C_{j,t}\) denotes a sectoral consumption, \(\int_{\mathcal{I}_j} C_{j,t}(i)di\). For any generic variable \(X_t\), I define \(X_{j,t}^R(i)\) and \(X_{j,t}^R\) as

\[
X_{j,t}^R(i) = \frac{X_{j,t}(i)}{n_j}, \quad X_{j,t}^R = \frac{n_j^{-1}X_{j,t}}{X_t}.
\]

I log-linearize the equilibrium conditions around a symmetric steady state to solve the model. I use lowercase letter \(x_t\) to denote the percentage deviation from the steady state \(X\) (i.e. \(x_t = \ln X_t - \ln X\)). There is one exception with \(b_{j,t} = \frac{B_{j,t} - B}{Y}\) denoting the deviation of nominal bond holdings from their steady-state level \(B = 0\), measured as a percentage of steady state nominal output.

5. The Generalized New Keynesian Phillips Curve

In this section, I derive the generalized Phillips curve and document some of its properties. Those who are not interested in detailed derivation of the Phillips curve can go directly to Proposition 5.3 without loss of continuity.

5.1. Derivation of the Generalized Phillips Curve

One important difference from the standard representative household model in constructing Phillips curve is that optimal prices chosen at a given time are not identical across the firms but firm-specific. This is because a firm’s optimal price is a function of its worker’s consumption, and the consumption then depends on \(b_{j,t-1}(i)\), the bond holding carried over from the previous period. This feature of the model creates a complication similar to one that arises when the capital used by firms is firm-specific. One cannot derive the Phillips curve with the conventional method in this case, and should instead use the undetermined coefficient method developed by Woodford (2005).

Since the dynamics of relative consumption and asset holding play important roles in firms’ pricing decisions, I first present a household’s optimality conditions. Log-linearizing a household’s Euler equation and budget constraint and then expressing them in terms of the relative consumption, relative bond holding, and relative price would yield

\[
\begin{align*}
\mathcal{C}_{j,t}(i) & = E_t \left[ \mathcal{C}_{j,t+1}(i) \right] + 2\epsilon b_{j,t}^R(i) \quad (5.1) \\
\mathcal{C}_{j,t}^R(i) & = -\psi_1 b_{j,t}^R(i) + \beta^{-1} \psi_1 b_{j,t-1}^R(i) - \psi_2 p_{j,t}^R(i) \quad (5.2)
\end{align*}
\]
where
\[ \psi_1 \equiv \left[ 1 - \chi \left( \frac{\theta - 1}{\theta} \right) \right]^{-1}, \quad \psi_2 \equiv (\theta - 1) \{ \chi (1 + \varphi) + (1 - \chi) \} \psi_1, \]
and \( \epsilon \equiv \xi / PY. \) \(^{11}\) Plugging (5.2) into (5.1), I can substitute out type-\( i \) household’s relative consumption \( c_{R_j}^j(i) \), which gives an equation that describes the dynamics of the household’s relative bond holding given relative price:

\[
E_t \left[ b_{j,t+1}^R(i) + \left( \beta^{-1} - 1 - \frac{2 \epsilon}{\psi_1} \right) b_{j,t}^R(i) + \beta^{-1} b_{j,t-1}^R(i) \right] = \frac{\psi_2}{\psi_1} E_t \left[ p_{j,t+1}^R(i) + p_{j,t}^R(i) \right] \quad (5.3)
\]

Turning to firm side, the log-linearized first order condition of a firm that sets its price at time \( t \) must be

\[
\hat{E}_t^i \sum_{k=0}^{\infty} (\alpha j \beta)^k \left\{ p_{j,t+k}^R - p_{t+k} \right\} = \hat{E}_t^i \sum_{k=0}^{\infty} (\alpha j \beta)^k mc_{j,t+k}(i). \quad (5.4)
\]

The expectation operator \( \hat{E}_t^i \) must be distinguished from \( E_t \) as emphasized by Woodford (2005). \( \hat{E}_t^i \) is type-\( i \) firm’s expectation at time \( t \) on the condition that its own price is not revised for the entire future since time \( t \). Because the size of the households and firms are so small that they cannot affect aggregate or sectoral level variables, distinguishing the two expectation operators would be important only for industry level variables. The equation (5.4) can be written as

\[
\sum_{k=0}^{\infty} (\alpha j \beta)^k \hat{E}_t^i \left[ p_{j,t+k}^R(i) \right] = \hat{E}_t^i \sum_{k=0}^{\infty} (\alpha j \beta)^k \left\{ c_{R_{j,t+k}}^j(i) + (\varphi + \eta^{-1}) y_{j,t+k}^R + c_{R_{j,t+k}}^j + (1 + \varphi)^2 y_{t+k} - (1 + \varphi) a_{j,t+k} - \varphi \xi_{t+k} - \eta^{-1} d_{j,t+k}^R \right\}.
\]

For further analysis, it is convenient to substitute out the relative consumption employing the following equation:

\[
\hat{E}_t^i \left[ c_{R_{j,t+k}}^j(i) \right] = -\psi_1 \hat{E}_t^i \left[ b_{j,t+k}^R(i) \right] + \beta^{-1} \psi_1 \hat{E}_t^i \left[ b_{j,t+k-1}^R(i) \right] - \psi_2 \hat{E}_t^i \left[ p_{j,t+k}^R(i) \right],
\]

which is implied by (5.2). Then I can write the firm’s first order condition presented above as

\[
p_{j,t}^R(i) = \left( 1 - \frac{\alpha j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha j \beta)^k E_t \left[ V_{j,t+k} \right] + \sum_{k=1}^{\infty} (\alpha j \beta)^k E_t \left[ \pi_{j,t+k} \right]
\]

\[ -\psi_1 (1 - \alpha_j) \left( 1 - \frac{\alpha j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha j \beta)^k \hat{E}_t^i \left[ b_{j,t+k}^R(i) \right] + \beta^{-1} \psi_1 \left( 1 - \frac{\alpha j \beta}{1 + \varphi \theta + \psi_2} \right) b_{j,t-1}^R(i), \quad (5.5)
\]

where

\[ V_{j,t} \equiv (1 + \varphi) y_t + (\varphi + \eta^{-1}) y_{j,t}^R + c_{j,t}^R - (1 + \varphi) a_{j,t} - \varphi \xi_t - \eta^{-1} d_{j,t}^R \]

\(^{11}\)Recall that \( x_{j,t}^R(i) \) denotes a percentage deviation of \( X_{j,t}^R \) from its steady state (which is equal to zero). Therefore it must be that

\[ c_{j,t}^R(i) = c_{j,t} - c_{j,t} \]
\[ b_{j,t}^R(i) = b_{j,t} - b_{j,t} \]
\[ p_{j,t}^R(i) = p_{j,t} - p_{j,t} \]
is the common factor across all the firms within a sector. I have replaced \( \hat{E}_t^i \left[ p_{j,t+k}^R(i) \right] \) by \( p_{j,t}^R(i) - \sum_{s=1}^k E_t \pi_{j,t+s} \) in (5.5). Note that I have used the operator \( E_t \) in the first two summations on the right hand side of (5.5) instead of \( \hat{E}_t^i \) since those terms are independent of type-\( i \) firm’s pricing decision.

Finally, the expected value of the firm’s next-period price must be weighted average of current price and next-period optimal price:

\[
E_t \left[ p_{j,t+1}^R(i) \right] = \alpha_j \left[ p_{j,t}^R(i) - E_t \pi_{j,t+1} \right] + (1 - \alpha_j) E_t \left[ p_{j,t+1}^*(i) \right].
\]  

(5.6)

The three equations (5.3), (5.5), and (5.6) characterize the dynamics of industry level variables \( f_{Rj,t}(i), p_{Rj,t}(i) \), given the time path of the aggregate and sectoral level variables \( V_{j,t}, \pi_{j,t} \). The system of the linear difference equations is, however, hard to solve in practice.

Following Woodford (2005)’s strategy, I take an educated guess and verify it later. From equation (5.3), I posit that the time path of relative bond holding follows

\[
b_{j,t}(i) = \delta b_{j,t-1}(i) + \nu p_{j,t}^R(i),
\]

(5.7)

where \( \delta \) and \( \nu \) are some functions of the model parameters. From (5.5) and (5.7), it then follows that a firm’s optimal price should have the form:

\[
p_{j,t}^*(i) = p_{j,t}^R + \lambda p_{j,t-1}^R(i),
\]

(5.8)

where \( \lambda \) is again a function of the parameters and \( p_{j,t}^R \) denotes the common component of optimal prices of the firms who set prices anew in sector \( j \), which is a function of the aggregate and sectoral variables. The set of parameters \( \{ \lambda, \delta, \nu \} \) and the common component \( p_{j,t}^R \) are still to be determined.

First step would be substituting (5.8) into (5.6). I then obtain:

\[
E_t \left[ p_{j,t+1}^R(i) \right] = \alpha_j p_{j,t}^R(i) + \lambda(1 - \alpha_j) b_{j,t}^R(i)
\]

(5.9)

I can express \( \lambda \) and \( \nu \) as a function of \( \delta \) using the fact that (5.7) must satisfy (5.3) combined with (5.9), which is true if and only if \( \{ \lambda, \delta, \nu \} \) satisfy

\[
\nu = \frac{(1 - \alpha_j) \psi_2 \delta}{\alpha_j \psi_1 \delta - \beta^{-1} \psi_1} \quad \lambda = \frac{\beta^{-1} - \alpha_j \delta}{(1 - \alpha_j) \psi_2} \left[ \frac{2\epsilon}{\beta^{-1} \delta - \psi_1 (1 - \delta)} - \psi_1 (1 - \delta) \right].
\]

(5.10)

(5.11)

One more relation is needed to determine \( \{ \lambda, \delta, \nu \} \) and the firm’s first order condition (5.5) provides the additional relation. From (5.7), \( \hat{E}_t^i \left[ b_{j,t+k}^R(i) \right] \) can be expressed as

\[
\hat{E}_t^i \left[ b_{j,t+k}^R(i) \right] = \delta \hat{E}_t^i \left[ b_{j,t+k-1}^R(i) \right] + \nu \hat{E}_t^i \left[ p_{j,t+k}^R(i) \right] = \delta \hat{E}_t^i \left[ b_{j,t+k}^R(i) \right] + \nu \left[ p_{j,t}^*(i) - \sum_{s=1}^k E_t \pi_{j,t+s} \right].
\]

20
The equation above implies that
\[ \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t \left[ b_{j,t+k}^R (i) \right] = \left( \frac{\delta}{1 - \delta \alpha_j \beta} \right) b_{j,t-1}^R (i) \]
\[ + \frac{\nu}{(1 - \alpha_j \beta) (1 - \delta \alpha_j \beta)} \left[ p_{j,t}^R (i) - \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t [\pi_{j,t+k}] \right] \]

Plugging this expression into the firm’s first order condition (5.5), I obtain:
\[ \Psi p_{j,t}^R (i) = \left( \frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t [V_{j,t+k}] + \Psi \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t [\pi_{j,t+k}] + \Phi \theta_{j,t-1}^R (i), \quad (5.12) \]

where
\[ \Psi \equiv 1 - \frac{\psi_2 (1 - \alpha_j)^2 \delta}{(1 + \varphi \theta + \psi_2) (1 - \alpha_j \beta) (\beta^{-1} - \alpha_j \delta)} \]
\[ \Phi \equiv \frac{\psi_1 (1 - \alpha_j \beta) (\beta^{-1} - \delta)}{(1 + \varphi \theta + \psi_2) (1 - \alpha_j \beta)}. \]

Therefore, by comparing (5.12) and (5.8), it must be true that
\[ p_{j,t}^R = \Psi^{-1} \left( \frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t [V_{j,t+k}] + \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t [\pi_{j,t+k}], \quad (5.13) \]

and
\[ \Psi \lambda = \Phi. \quad (5.14) \]

Then the system of equations (5.10), (5.11), and (5.14) jointly determine the undetermined coefficients \{\lambda, \delta, \nu\} if a solution exists. Note that the system is nonlinear in \{\lambda, \delta, \nu\}, and thus there could be more than one solution. Following Woodford (2005), I only consider a solution that would make the joint dynamics of relative price and relative bond holding be convergent so that the means and the variances remain bounded. I can rewrite equation (5.7) and (5.9) as the following system:
\[ \begin{pmatrix} E_t \left[ p_{j,t+1}^R (i) \right] \\ b_{j,t}^R (i) \end{pmatrix} = \begin{pmatrix} \alpha_j + (1 - \alpha_j) \lambda \nu & (1 - \alpha_j) \lambda \delta \\ \nu & \delta \end{pmatrix} \begin{pmatrix} p_{j,t}^R (i) \\ b_{j,t-1}^R (i) \end{pmatrix}. \quad (5.15) \]

The system is stable if and only if the eigenvalues of the coefficient matrix are inside the unit circle.

**Lemma 5.1.** If \( \alpha_j \beta^{-1} \leq 1 \), then the system (5.15) is stable if and only if \( 0 < \delta < \beta^{-1} \).

**Proof.** See the technical appendix. The technical appendix will be posted on the author’s webpage soon. ■

Therefore, in what follows, I focus only on the numerical values of \( \delta \) on the interval \((0, \beta^{-1})\), and \( \alpha_j \) on \((0, \beta)\). A following question at this point might be if there exists such \{\lambda, \delta, \nu\} that
satisfies the conditions (5.10), (5.11), and (5.14) as well as the stability condition, \( 0 < \delta < \beta^{-1} \). The following lemma shows that there indeed exists a unique set of \( \{ \lambda, \delta, v \} \) as long as \( \epsilon \) is positive.

**Lemma 5.2.** There exists a unique set of \( \{ \lambda, \delta, v \} \) that satisfies (5.10), (5.11), (5.14), and \( 0 < \delta < \beta^{-1} \) if \( \epsilon > 0 \).

**Proof.** See the technical appendix. ■

Once I find the solution for \( \{ \lambda, \delta, v \} \), I can construct the generalized NK Phillips curve by combining (5.12) that determines a firm’s relative optimal price \( p_{j,t}^R(i) \) and (3.12) that determines dynamics of sectoral price level \( p_{j,t} \). Log-linearizing (3.12) yields

\[
p_{j,t} = \frac{1}{n_j} \int_{T_j} p_{j,t}^R(i) di - \alpha_j p_{j,t-1}.
\]

Substituting (5.8) into the equation above, one obtains

\[
\alpha_j \pi_{j,t} = \frac{1}{n_j} \int_{T_j} (p_{j,t}^R + \lambda b_{j,t-1}^R(i)) di,
\]

implying

\[
p_{j,t}^R = \frac{\alpha_j}{1 - \alpha_j} \pi_{j,t},
\]

because \( \int_{T_j} b_{j,t-1}^R(i) di = 0 \) always holds due to the assumption of time-dependent pricing. With that assumption, one does not have to keep track of distribution of the households’ wealth to study aggregate dynamics. Substituting (5.16) into (5.13), I obtain

\[
\pi_{j,t} = g(\alpha_j) V_{j,t} + \beta E_t [\pi_{j,t+1}],
\]

where

\[
g(\alpha_j) = \begin{cases} 
\frac{(1 - \alpha_j \beta)(1 - \alpha_j)}{\alpha_j} q(\alpha_j, \epsilon, \chi) 
\end{cases}
\]

and

\[
q(\alpha_j, \epsilon, \chi) = \frac{(1 - \alpha_j \beta \delta)(\beta^{-1} - \alpha_j \delta)}{(1 + \varphi \theta + \psi_2)(1 - \alpha_j \beta \delta)(\beta^{-1} - \alpha_j \delta) - \psi_2(1 - \alpha_j)^2 \delta}.
\]

I summarize the results I have obtained so far in the proposition 5.3.

**Proposition 5.3.** Suppose the economy consists of multiple sectors indexed by \( j = 1, 2, \ldots, J \). In each sector \( j \), there is a continuum of firms whose prices are sticky in the sense of Calvo, with the probability of price adjustment in each period is given by \( 1 - \alpha_j \). Then, for each sector \( j \), the rate of inflation \( \pi_{j,t} \equiv p_{j,t} - p_{j,t-1} \) can be described by the following sectoral Phillips curve:

\[
\pi_{j,t} = \beta E_t [\pi_{j,t+1}] + g(\alpha_j) \left[ (1 + \varphi) y_t + (\varphi + \eta^{-1}) y_{j,t}^R + \zeta_{j,t}^R \right] - \zeta_{j,t},
\]

where \( g(\alpha_j) \) is given by (5.18) and \( \{ \lambda, \delta, v \} \) satisfy (5.10), (5.11), (5.14), and \( 0 < \delta < \beta^{-1} \). \( \zeta_{j,t} \) is a linear combination of exogenous processes. Consequently, the Phillips curve for the aggregate
inflation $\pi_t$ is given by

$$
\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t + \Theta_{c,t} + \Theta_{y,t} - \zeta_t,
$$

(5.21)

where

$$
\Theta_{c,t} = \sum_{j=1}^{J} n_j g(\alpha_j) c^{R}_{j,t}, \quad \Theta_{y,t} = (\varphi + \eta^{-1}) \sum_{j=1}^{J} n_j g(\alpha_j) y^{R}_{j,t},
$$

$$
\kappa \equiv (1 + \varphi) \sum_{j=1}^{J} n_j g(\alpha_j), \quad \zeta_t \equiv \sum_{j=1}^{J} n_j \xi_{j,t}.
$$

The aggregate Phillips curve (5.21) is obtained simply by taking weighted sum of the sectoral inflations. The exogenous process $\zeta_{j,t}$ is given by $g(\alpha_j) \left[ (1 + \varphi) a_{j,t} + \varphi \xi_{t} + \eta^{-1} d^{R}_{j,t} \right]$. From (5.21), one can see that the household heterogeneity generates a larger degree of real rigidity in two ways: (i) by reducing the slope of the Phillips curve $\kappa$ for a given degree of nominal rigidities $\{\alpha_j\}$, and (ii) by adding an endogenous shifter $\Theta_{c,t}$ to the Phillips curve. The endogenous shifters $\Theta_{c,t}$ and $\Theta_{y,t}$ are relevant only when the economy has multiple sectors with heterogeneous price stickiness. The shifters would disappear if the frequencies of price adjustments were identical across the firms in the economy. Even in that case, however, the slope of the Phillips curve is smaller in the $\mathcal{HH}$ model relative to the $\mathcal{RH}$ model.

5.2. The Slope of the Phillips Curves

The major determinant of the slope is the function $g(\cdot)$, which is convex and decreasing in the measure of nominal rigidity $\alpha$. The function is a product of two components:

$$
g(\alpha) \equiv \left\{ \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha} \right\} \times q(\alpha, \chi, \epsilon)
$$

The first term, $\frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha}$, is the common component across all different versions of the NK models. It is therefore $q(\alpha, \chi, \epsilon)$ that makes a difference and the second component is often considered as a measure of real rigidity. When $q$ is small, the real rigidity is larger.

The expressions for the real rigidity function $q(\cdot)$ are different in the $\mathcal{HH}$ and $\mathcal{RH}$ model and are given by

$$
q^{\mathcal{RH}}(\alpha, \chi, \epsilon) = q^{\mathcal{RH}} \equiv \frac{1}{1 + \varphi \theta} : \mathcal{RH} \text{ model}
$$

$$
q^{\mathcal{HH}}(\alpha, \chi, \epsilon) \equiv \left[ \frac{(1 - \alpha \beta) (\beta^{-1} - \alpha \delta)}{(1 + \varphi \theta + \psi_2) (1 - \alpha \beta) (\beta^{-1} - \alpha \delta) - \psi_2 (1 - \alpha)^2 \delta} \right] : \mathcal{HH} \text{ model}
$$

With a little algebra, it can be shown that $q^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ is always smaller than $q^{\mathcal{RH}}$, and thus the $\mathcal{HH}$ model is characterized by a larger degree of real rigidity. Not surprisingly, a set of parameters $(\chi, \epsilon)$ that controls asset market frictions plays a role in determining the degree of real rigidity when the households are heterogeneous.
Figure B.1 shows a contour map of $q_{HH(\alpha, \chi, \epsilon)}$. In addition, Figure B.2 and B.3 plot $q_{HH}$ and $q_{RH}$, while varying $\epsilon$ and $\chi$ respectively, for some alternative values for $\alpha$. Some observations are worth mentioning.

First, Figure B.1 shows that $q_{HH} < q_{RH}$ for all possible values of $(\chi, \epsilon)$: For a fixed nominal rigidity $\alpha$, the slope of Phillips curve is smaller in the $HH$ model, and consequently the inflation would be less responsive to economic shocks.

Second, $q_{HH}$ is decreasing in $\epsilon$, which is quite intuitive because it implies that the heterogeneous household model is characterized by a larger degree of real rigidity when there is a larger friction in lending and borrowing. Also, $q_{HH}$ is convex in $\epsilon$, so that even a small value of $\epsilon$ can reduce the slope of Phillips curve substantially.

Third, $q_{HH}$ is also decreasing in $\chi$. This is not quite as intuitive as the previous two observations because a larger value of $\chi$ implies that the capital incomes across the households are more symmetric. Recall that when $\chi = 0$, type-$i$ household receives the profits exclusively from type-$i$ firm. Thus the household’s total income would simply be the firm’s revenue and would be independent of the industry wage rate. Consequently, the wage rate the household would face does not directly affect the household’s consumption choice. On the other hand, labor income is an important part of the household’s total income when $\chi = 1$. Consequently, the household consumption choice would be sensitive to the market wage. If that is the case, the wage elasticity of labor supply would be small, which can be seen from (3.4). Then a firm’s changing price and thus production would have a bigger implication on its marginal cost. When $\chi$ is small, however, the effect through this channel is also small, and thus the model is characterized by a smaller degree of real rigidity.

5.3. The Endogenous Shifters of the Phillips Curves

I now turn to the effects of the endogenous shifters $\{\Theta_{c,t}, \Theta_{y,t}\}$ on real rigidities. The effect of $\Theta_{y,t}$ is well documented by Carvalho (2006), and only $\Theta_{c,t}$ is a new addition due to market incompleteness. Note that $\Theta_{c,t}$ and $\Theta_{y,t}$ are weighted sums of the relative sectoral consumption gaps $c_{j,t}^R (= c_{j,t} - y_t)$ and of the relative sectoral output gaps $y_{j,t}^R (= y_{j,t} - y_t)$ respectively. Since the function $g(\alpha)$ is decreasing and convex in $\alpha$, disproportionately larger weights are placed on the flexible sectors. As a result, both $\Theta_{c,t}$ and $\Theta_{y,t}$ have positive values when the economy is hit by a shock that makes the firms to find it optimal to reduce their price as they reoptimizes, and vice versa. Note that by solving the Phillips curve (5.21) forward, the inflation can be written as a weighted sum of expected future values of the shifters and the output gap:

$$\pi_t = \sum_{k=0}^{\infty} \beta^k E_t \left[ \kappa y_{t+k} + \Theta_{c,t+k} + \Theta_{y,t+k} - \zeta_{t+k} \right].$$

With a contractionary shock, output falls below its natural level of output. Inflation, however, does not fall as much because $\Theta_{c,t}$ and $\Theta_{y,t}$ are expected to rise for a time being after the shock. These endogenous Phillips curve shifters make the model "stickier" by making the response of
the price level and of the inflation more sluggish.

6. Estimation and Model Comparisons

This section provides a detailed explanation about estimation procedures and evaluates the two model economies, the RH and HH models, based on the estimated duration of price contracts implied by the models.

6.1. Single Sector

As mentioned at the beginning of the paper, there are two reasons for the interest in duration of price contracts implied by the models. First, it is a way to quantify the real rigidity. If a shorter duration is implied by a model, the model is characterized by a larger degree of real rigidity, so that it can account for persistent dynamics with a smaller nominal rigidity. Second, it is documented that estimated model-implied durations are often implausibly large or equivalently that the baseline sticky-price DSGE model often cannot explain the empirical frequency of price changes.

Following the more traditional approach in NK literatures, I first estimate both the RH and HH models assuming $\alpha_j = \bar{\alpha}$, that is all the firms in the economy face the same probability of updating prices each period. In the following section, I consider a more complicated, yet more interesting case in which nominal rigidities are different across the sectors.

There are two alternative ways to view the single-sector case. One can think that the economy is literally composed of one sector (i.e. $J = 1$). The other way to view the single-sector economy would be that the economy consists of multiple sectors (i.e. $J > 1$), yet the degree of nominal rigidities are homogeneous across the sectors. It can be shown that the two different views give the same log-linearized equilibrium conditions.

As mentioned in a previous section, in the single-sector case, the Phillips curve is reduced to have the conventional form with no endogenous shifters. The only difference from the standard Phillips curve, when the households are heterogeneous, is the expression for the slope. Also, we do not need to keep track of the distributions of households’ consumption and wealth to compute the equilibrium dynamics of the aggregate variables. Then the system to of the equations be estimated would look much like the standard NK model:

\begin{align}
  y_t &= E_t[y_{t+1}] - (r_t - E_t[\pi_{t+1}]) + (\gamma_t - E_t[\gamma_{t+1}]) \\
  \pi_t &= \beta E_t[\pi_{t+1}] + \kappa y_t - \zeta_t \\
  r_t &= \mu_m r_{t-1} + (1 - \mu_m) \{ \phi_x \pi_t + \phi_g y_t \} + \mu_t,
\end{align}

where $\kappa = (1 + \varphi) g(\bar{\alpha})$. Recall that the function $g(\cdot)$ is model-specific because of the different real rigidity functions $q(\cdot)$ from (5.22), but other than that, the equilibrium conditions look identical between the RH and HH models.
To close the model, I assume independent AR(1) processes for the exogenous variables. Specifically I assume:

\[
\begin{pmatrix}
\mu_t \\
\alpha_t \\
\gamma_t \\
\xi_t
\end{pmatrix} = \begin{pmatrix}
\rho_\mu & 0 & 0 & 0 \\
0 & \rho_\alpha & 0 & 0 \\
0 & 0 & \rho_\gamma & 0 \\
0 & 0 & 0 & \rho_\xi
\end{pmatrix} \begin{pmatrix}
\mu_{t-1} \\
\alpha_{t-1} \\
\gamma_{t-1} \\
\xi_{t-1}
\end{pmatrix} + \begin{pmatrix}
\sigma_\mu & 0 & 0 & 0 \\
0 & \sigma_\alpha & 0 & 0 \\
0 & 0 & \sigma_\gamma & 0 \\
0 & 0 & 0 & \sigma_\xi
\end{pmatrix} \begin{pmatrix}
\varepsilon_{\mu,t} \\
\varepsilon_{\alpha,t} \\
\varepsilon_{\gamma,t} \\
\varepsilon_{\xi,t}
\end{pmatrix},
\]

where \( \varepsilon_t \equiv (\varepsilon_{\mu,t} \ varepsilon_{\alpha,t} \ varepsilon_{\gamma,t} \ varepsilon_{\xi,t})' \) is i.i.d \( N(0_4, I_4) \). The only exception is placed on monetary shock \( \mu_t \). Since I have included interest rate smoothing term in Taylor rule (6.3), I assume that monetary policy shock \( \mu_t \) is i.i.d, and thus set \( \rho_\mu \) to be zero.

The system of linear equations (6.1), (6.2) and (6.3) completely characterizes the joint distribution of \( \{\Delta y_t, \pi_t, r_t\}_{t=0}^T \), where \( \Delta y_t \) is the growth rate of output, \( \pi_t \) is inflation and \( r_t \) is nominal interest. The empirical counterparts are the growth rate of real GDP, the growth rate of GDP deflator, and federal fund rates from 1954:Q3 to 2006:Q4. I use quarterly time series. The real GDP is calculated by dividing nominal GDP by GDP deflator. Every time series is demeaned when the models are fitted to the data.

Note that since both the productivity shock \( \alpha_t \) and the preference shock \( \xi_t \) enter into the Phillips curve only and not elsewhere, the data cannot distinguish between the two.\(^{12}\) Both of them serve as a Phillips curve shifter. One possible solution would be to ignore \( \xi_t \) from the beginning so that the Phillips curve is shifted only by \( \alpha_t \).\(^{13}\) I instead add a measure of hours worked \( h_t \) in the observable data set. Since the aggregate hour \( H_t \) is not originally defined by the model, I naturally define \( H_t \) as

\[
H_t \equiv \sum_{j=1}^{J} \int_{I_j} H_{j,t}(i) \, di,
\]

the sum of hours worked by all the households in the economy. It then follows that \( y_t = \alpha_t + h_t \) in the first order approximation and thus \( \alpha_t \) can be identified as the average labor productivity. I include \( \Delta h_t \) the quarterly growth rate of hours in the set of observables. In constructing the time series for \( \Delta h_t \), the non-farm business sector hour is used. Since the size of the model economies has been normalized to one, I divide the real GDP and the hours by the total civilian non-institutional population over age of 16.

To summarize, the set of observables includes the growth rate of the per-capita real GDP, the growth rate of the GDP deflator (i.e. inflation), the federal fund rates and the growth

\(^{12}\)Recall that \( \xi_t \) is a linear function of \( \alpha_t \) and \( \xi_t \). The exogenous variable \( \alpha_t \) is a weighted mean of sectoral productivity (i.e. \( \alpha_t = \sum n_j \alpha_{j,t} \)). Since there is only one sector, it is equivalent to the aggregate productivity.

\(^{13}\)Many previous studies, including Ireland (2004), document that it is important to include a shock, often called "cost-push shock" in the Phillips curve, in addition to the productivity shock, to improve the model fit. In the current models, the preference shock \( \xi_t \) plays a role of the cost-push shock although economic interpretation might be different. Without \( \xi_t \), one can no longer interpret \( \alpha_t \) as a productivity shock since it captures every factors that might shift the Phillips curve. When I estimated the models without \( \xi_t \) in the Phillips curve, the estimated process of \( \alpha_t \) was too volatile so that it is inconsistent with the estimated or calibrated productivity processes from previous studies.
rate of per-capita non-farm business sector hours, and the corresponding counterparts from the structural models are given by $X_t = \{\Delta y_t, \pi_t, r_t, \Delta h_t\}_{t=0}^T$.

As mentioned above, I employ a Bayesian method to characterize posterior distributions of the structural parameters $\omega = (\alpha, \phi_\pi, \phi_\gamma, \rho_\mu, \rho_\xi, \rho_\gamma, \sigma_\mu, \sigma_\sigma, \sigma_\xi)'$. Let me briefly mention how one would obtain the posterior distribution of $\omega$. Given the data set $X_T$, the DSGE models give the likelihood function $f(X_T|\omega)$. Then the posterior distribution of $\omega$ is determined by Bayes theorem:

$$f(\omega|X_T) = \frac{f(X_T|\omega)f(\omega)}{f(X_T)} = \int f(X_T|\omega)f(\omega)d\omega,$$

where $f(\omega)$ is prior distribution, which reflects modeler’s prior belief about the structural parameters before estimation.

Note that some parameters such as $(\theta, \epsilon, \chi, \varphi)$ are not identified in the current system. I fix them at the benchmark values as a default and then try some variations. I also fix the discount factor $\beta$ to be 0.99 throughout this paper. I set $\theta$ to be 6 so that a firm’s mark-up is 20 percent. I set $\varphi$ to be 1, which implies the Frisch elasticity of labor supply is 1. There is no consensus on appropriate values for $\epsilon$, the parameter that controls the degree of friction in bond trading. As a benchmark, I set $\epsilon = 0.1$. I however also consider the cases in which $\epsilon$ has smaller and larger values to see how different degrees of bond market friction affect the estimated duration of price contract. As mentioned in section 3 the benchmark value for $\chi$ is 1, but I also consider the other extreme case where $\chi = 0$.

The prior and posterior distributions for the remaining parameters are summarized in Table 1. The prior distributions are mostly standard. Perhaps one departure would be that instead of putting a larger weight on a certain range of $\alpha$, I assume a flat prior for $\alpha$. Therefore the model has complete freedom to choose any frequency of price changes that makes itself to fit to the time series best.

Once the posterior distribution of $\alpha$ is obtained, I construct a probability distribution of the duration of price contract $D$, employing the relation:

$$D = -1/\log \alpha.$$

Table 2 presents the posterior means of $\alpha$ and $D$ under the RH model and also under the HH model with different sets of values for $(\epsilon, \chi)$.

In the RH model, the posterior mean of the duration is 4.65 quarters, with 3.32 and 6.80 being the lower and the upper bounds of 95% highest posterior density region (HPD). If the true duration is indeed less than 2 quarters, it is reasonable to reject the representative household model on the basis of the estimated duration.

In the HH model, the posterior mean is about 2.29 quarters and the 95% HPD interval goes from 1.74 to 3.12 quarters when $\epsilon = 0.1$ and $\chi = 1$. Even a smaller value of $\epsilon$ can reduce the implied duration substantially. When $\epsilon = 0.01$ (and $\chi = 1$), the estimated duration of price contract is about 2.96 quarters, and it is only about 60% of the estimated duration.
implied by the RH model. This result is somewhat expected because one can see that the slope of Phillips curve is highly convex in ε from Figure B.2. Overall, Table 2 suggests that introducing heterogeneous households with some financial market frictions can "potentially" decrease the estimated duration to the point where the model-implied duration is consistent with the empirical evidence. Of course, an important caveat is that at this moment we do not have a good sense about what would be reasonable values for (ε, χ) that can be used for sticky-price DSGE models.

The two models are different in terms of implied durations or in terms of the nominal rigidity required to explain persistent aggregate dynamics. On the other hand, the two economies are observationally equivalent. The only difference between the two models in the equilibrium conditions is the form of the function g(·). Therefore, if we allow α to vary freely, which was the case here since I have imposed a flat prior for α, the optimization algorithm would just pick the best α to fit the data. The values of α chosen would be different in the RH and HH models, but the implied value for κ, the slope of the Phillips curve, would be same. Put it differently, unless we do not impose any restriction on α, the data cannot tell a difference between the RH and HH models, and consequently the maximized posterior densities are identical.

Note that the baseline NK model is abstract from investment decision for simplicity. In order to be more consistent with the construction of the models, I also use the Personal Consumption Expenditure (PCE) as a measure of aggregate output and PCE price index as a measure of aggregate price level. The estimated durations are almost the same as those reported in Table 2.

6.2. Multiple Sectors

I now relax the restriction that every firm in the economy should have an identical frequency of price adjustment, so that I can make inference on sectoral in-frequencies and durations:

\{α_1, α_2, α_3, \ldots, α_J\},
\{D_1, D_2, D_3, \ldots, D_J\}.

Derivations of log-linear first-order approximations of the equilibrium conditions are detailed in the technical appendix.\(^{14}\) For here, I present the following \((4 + 4 \times J)\) equations that determine the equilibrium path for \(\{y_t, π_t, r_t, h_t, \{c^R_{j,t}, y^R_{j,t}, b^R_{j,t}, π_{j,t}\}_{j=1}^J\}_{t=0}^\infty\):

\begin{align*}
    r_t &= ρ_m^t r_{t-1} + (1 - ρ_m) \{φ_π π_t + φ_y y_t\} + μ_t \quad (6.5) \\
    y_t &= E_t[y_{t+1}] - (r_t - E_t[π_{t+1}]) + (γ_t - E_t[γ_{t+1}]) \quad (6.6) \\
    π_t &= β E_t[π_{t+1}] + κ y_t + \left\{\sum_{j} n_j g(α_j)c^R_{j,t}\right\} + \left\{\left(φ + η^{-1}\right)\sum_{j} n_j g(α_j)y^R_{j,t}\right\} - ζ_t \quad (6.7) \\
    y_t &= \left(\sum_{j} n_j a_{j,t}\right) + h_t \quad (6.8)
\end{align*}

\(^{14}\)The technical appendix will include detailed procedures of log-linear approximations of the models and the proofs of the lemmas. It will be posted on the author’s website very soon.
The main difference from the single-sector case is that the cross-sector distributions should now be included in the set of state variables, and hence I have to know the time path of \( \{ c_{j,t}^R, y_{j,t}^R, b_{j,t}^R, \pi_{j,t} \} \) along with \( \{ \eta_j \} \) to determine the dynamics of aggregate variables, \( \{ y_t, \pi_t, r_t, h_t \} \).

The \( \mathcal{HH} \) model differs from the \( \mathcal{RH} \) model at two aspects. First, as in the single-sector case, the function \( g(\cdot) \) have different forms. As shown before, \( g(\cdot) \) has a smaller value in the \( \mathcal{HH} \) model for a given value of \( \alpha \). Second, in the \( \mathcal{RH} \) model, the equation (6.9) and (6.10) are not parts of equilibrium conditions, so only (6.5)-(6.8), (6.11) and (6.12) constitute the equilibrium conditions in the \( \mathcal{RH} \) model.

Before discussing the estimation procedure, I present impulse responses in Figure B.4-B.6 in order to provide ideas of how the model economies work.\(^{15}\) The impulse responses confirm my theoretical results shown in section 2. The aggregate price level and thus the aggregate inflation respond less to aggregate shocks in the \( \mathcal{HH} \) model than in the \( \mathcal{RH} \) model as shown in the second column of Figure B.4. The \( \mathcal{HH} \) model generates stronger co-movements among the sectoral outputs in response to all kinds of shocks, regardless of aggregate or sectoral shocks as shown in the first columns of Figure B.5 and B.6. The stronger co-movements are due to the wealth effects as discussed in a previous section. Sectoral inflations respond in the direction of accommodating the stronger output co-movements. In the \( \mathcal{HH} \) model, therefore, the sectoral inflations also tend to move together in response to aggregate shocks, but not necessarily to sectoral shocks. To see this, let me suppose a certain sector is hit by a positive demand shock (see the second row in Figure B.6). Then the firms in that sector would optimally increase their productions and prices. The households in that sector would become relatively wealthier (and the relative consumptions increase in the \( \mathcal{HH} \) model) and therefore would have a less incentive to supply labors due to the wealth effect, which would make the firms to produce less, charging even higher prices, than they would produce in the absence of the wealth effect. Consequently, the co-movements among the sectoral inflations are not stronger but weaker in response to a sectoral demand shock in the \( \mathcal{HH} \) model than in the \( \mathcal{RH} \) model.

The sectors could have different meanings depending on how one wants to disaggregate the economy. They could be individual goods such as banana or laptop computer, or could be more aggregate, such as fruit or electronic device. The Bureau of Economic Analysis (BEA) divide

\(^{15}\) A set of benchmark parameters, not estimated parameters, are used for plotting the impulse responses.
PCE into 13 broad categories. The 13 categories are then further divided into smaller groups. It would be ideal to work with more disaggregated data. In this paper, however, I estimate the 13-sector model for a computational issue. Accordingly, the measure of aggregate output and aggregate price index are PCE and PCE price index instead of GDP and GDP deflator. Table 3 presents the 13 broad categories in PCE. The sectoral weights are the expenditure weights averaged over the time period of 1954:Q3-2006:Q4.

The sectoral level data as well as aggregate data are used in estimating the models. I include in the set of observables the growth rate of sectoral outputs and of sectoral price indices, with their counterparts from the DSGE models being \( \{ \Delta y_{j,t}, \pi_{j,t} \} \). I use the same data for \( \{ r_t, \Delta h_t \} \) as in the previous section. Note that from the definition of the CES aggregates, we can obtain the two accounting identities:

\[
\Delta y_t = n_1 \Delta y_{1,t} + n_2 \Delta y_{2,t} + \cdots + n_J \Delta y_{J,t}
\]

\[
\pi_t = n_1 \pi_{1,t} + n_2 \pi_{2,t} + \cdots + n_J \pi_{J,t}.
\]

The series for \( \Delta y_t \) and \( \pi_t \) are redundant once we include all the sectoral outputs and inflations. Thus, only the time series for \( \{ r_t, \Delta h_t, \Delta y_{1,t}, \Delta y_{2,t}, \cdots, \Delta y_{13,t}, \pi_{1,t}, \pi_{2,t}, \cdots, \pi_{13,t} \} \) are used in estimating the models. The series run from 1954:Q3 through 2006:Q4.

The sectoral demand and supply shocks are assumed to follow independent AR(1) processes:

\[
a_{j,t} = \rho_{a,j} a_{j,t-1} + \sigma_{a,j} \varepsilon_{a,j,t}; \quad \varepsilon_{a,j,t} \overset{i.i.d}{\sim} N(0,1)
\]

\[
d_{j,t} = \rho_{d,j} d_{j,t-1} + \sigma_{d,j} \varepsilon_{d,j,t}; \quad \varepsilon_{d,j,t} \overset{i.i.d}{\sim} N(0,1)
\]

The prior distributions for the additional parameters are presented in Table 4. The sectoral weights \( \{ n_j \} \) are calibrated to the values reported in Table 3.

The posterior means and 95% HPD of the model parameters are presented in Table 5s. The means and 95% HPD of the durations of price contracts are presented in Table 6 through Table 8. Table 7 is constructed in the following way. For the mean duration of the economy, I first obtain the posterior distribution of \( \bar{\alpha} = \sum_{j=1}^{13} n_j \alpha_j \), the weighted mean of in-frequencies of price changes. Once I obtain the posterior distribution of \( \bar{\alpha} \), the posterior distribution of mean duration is obtained employing the relation \( \bar{D} = -1/\log(\bar{\alpha}) \). The durations for durable, non-durable, and service sectors are computed in a similar way.\(^{16}\)

To see if the model-implied durations are well matched with empirical durations from micro data, I also report estimated durations from Bills and Klenow (2004) (BK henceforth), which are denoted by \( D_{BK} \).\(^{17}\) To be consistent, they are computed in the same way as above, that is I first take \( \bar{\alpha} \), the weighted mean of the sectoral in-frequencies reported in BK, and then compute

\(^{16}\)For instance, the posterior distribution of the durable-sector duration can be obtained by taking posterior draws of \(-1/\log(\bar{\alpha})\), where

\[
\bar{\alpha} = \frac{\sum_{j=1}^{3} n_j \alpha_j}{\sum_{j=1}^{3} n_j}.
\]

\(^{17}\)\( D_{BK} \) (eis) denotes the estimated durations, excluding observations with item substitutions.
the corresponding duration by $\hat{D} = -1/\log(\hat{\alpha})$. Figure B.8 is a graphical representation of Table 7.

BK and Nakamura and Steinsson (2006) (NS henceforth) use the consumption categories constructed by the Bureau of Labor Statistics as the sectors. Those categories are not exactly matched with the NIPA classifications. However there are a few categories that are comparable. I report them in Table 8. Note that the reported durations from BK and NS in Table 8 are the ones estimated including observations with temporary sales. There is no wide agreement if the temporary sales should be included for macroeconomic analyses. Therefore, $D_{BK}$ and $D_{NS}$ in Table 8, the durations from BK and NS, would be one of the most conservative criteria.

Some important conclusions can be drawn from Table 7 and 8. First, allowing different degrees of price stickiness across the sectors has non-trivial implications for inferences of durations and frequencies. This is true even when our interests are solely on a representative frequency/duration, not sectoral frequencies/durations. The mean duration implied by the $\mathcal{RH}$ model is 1.74 quarters and thus even the $\mathcal{RH}$ model is consistent with the view that the duration should be less than 2 quarters on average. Second, the $\mathcal{HH}$ model is very consistent with empirical evidence in terms of not only the mean but also the sectoral durations. However, the $\mathcal{RH}$ model is also reasonably good, if not as good as the $\mathcal{HH}$ model, in that dimension.\footnote{One caveat is that the models still over-estimate the durations of price contract for durable sector.}

Let me provide some explanations for why not only the $\mathcal{HH}$ model but also the $\mathcal{RH}$ model behave reasonably well. First of all, sectoral inflations are far less persistent than aggregate inflation as shown in Figure B.9. The autocorrelation function of aggregate inflation is represented by dotted black line, and those of sectoral inflations are represented by solid lines. Therefore it may not be so surprising that estimated sectoral durations are not large. But it is still true that the less-persistent sectoral inflations do not necessarily imply the estimated not-so-large model-implied durations should be matched with the empirical sectoral durations.

How about the aggregate inflation? The aggregate inflation is very persistent. Don’t we need a large nominal rigidity to account for persistent aggregate dynamics? This question is already answered by Carvalho (2006). He has documented that allowing heterogeneous frequencies gives rise to two effects, "frequency composition effect" and "strategic interaction effect", which endogenously increase the persistence of aggregate variables. Thus there is no need for a large degree of nominal rigidity at the sectoral level to account for persistent aggregate dynamics.
Figure B.10 presents autocorrelation of aggregate and sectoral inflations implied by the models with estimated parameters (the parameters are set to be their posterior modes). It shows that both models are able to explain joint behaviors of persistent aggregate inflation and less persistent sectoral inflations. The estimation scheme that I employ here utilizes the whole equilibrium conditions and thus is able to capture the two effects predicted by the DSGE models when the degrees of price stickiness are heterogeneous across the sectors.

Another reason for smaller difference between the RH and HH models relative to the single-sector case is that a response of sectoral inflation to a sectoral demand shock is smaller in the RH model (see Figure B.6) due to weaker co-movements among the sectoral outputs. Therefore unlike the other shocks, the sectoral demand shocks generate more persistent dynamics in the RH model than in the HH model.

6.3. Some Additional Observations for Multiple-Sector Economies

Unlike the single-sector cases, the RH and HH models are not observationally equivalent with respect to the observables. The log marginal likelihoods of the two models are given as

\[ \log f_{RH}(X^T) = -8335.2 \]
\[ \log f_{HH}(X^T) = -8291.4, \]

and consequently the posterior odd ratio (or Bayes factor) is given by

\[ \frac{f_{HH}(X^T)}{f_{RH}(X^T)} = \exp(43.8). \]

This implies that the heterogeneous household model is better at explaining the joint dynamics of aggregate and sectoral U.S. time series data. The magnitude of the posterior odd ratio is relatively large. It indicates the difference between the two models is statistically significant, and one can reject the RH model in favor of the HH model. However, the difference may not be economically significant. Investigating if the HH model is systematically better in any economic sense and studying what feature of the HH model is responsible for the improved fit might be potentially important but it is beyond the scope of this paper. I leave that as a future research.

There are some common features of the estimated multiple-sector DSGE models worth mentioning. First, the sectoral shocks seem to be more volatile than aggregate shocks on average. Second, many sectoral shocks are as persistent as the aggregate shocks. Third, there is a negative correlation between the volatilities of sectoral demand shocks and the sectoral durations of price contracts. This implies that firms in the sectors with more volatile demand shocks tend to adjust prices more frequently. All three features may not be surprising, if not expected.

On the other hand, there is a positive correlation between the volatilities of sectoral supply shocks and the sectoral durations of price contracts, implying that firms in the sectors with less volatile supply shocks tend to change prices more frequently, which is somewhat counter-intuitive. This result might reflect a failure of baseline sticky-price DSGE models, or it might
tell us a truth. It certainly deserves a further investigation in a near future.

7. Conclusion

This paper studies the endogenous stickiness due to heterogeneous households and its consequence for the estimated duration of price contracts in a standard sticky-price DSGE framework.

Introducing incomplete asset markets can increase the degree of real rigidity by reducing the wage elasticity of labor supply due to wealth effect.

In a conventional single-sector framework, I show that the heterogeneous household model can potentially account for persistence aggregate dynamics without imposing implausibly large degree of nominal rigidity, while the representative household model implies a large expected duration of price contract that is at odds with micro-level evidences.

I however also show that the conclusion made above could be misleading because it is based on the conventional (yet strong) assumption that every sector in the economy has an identical frequency of price adjustment, and the assumption misses some important theoretical predictions of the more realistic multiple-sector-sticky-price DSGE models. In the multiple-sector framework, the representative household model is also consistent with the empirical durations of price contracts. However, even in that case, it is still true that the additional source of real rigidity provided by the household heterogeneity helps the model in explaining the empirical frequency of price changes.

Overall, the baseline sticky-price DSGE models can explain the empirical cross-sector distribution of frequency of price changes surprisingly well while fitting the U.S. time series data.
References


### A. Tables

<table>
<thead>
<tr>
<th>prior distribution</th>
<th>prior mean</th>
<th>prior std</th>
<th>posterior mean &amp; 95% HPD</th>
</tr>
</thead>
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</tr>
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<td>0.0701 [0.0363, 0.1111]</td>
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</tr>
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<td>0.25 see Table 2</td>
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Table 2. Posterior Distribution of $\alpha$ and $D$ (Single-Sector)

<table>
<thead>
<tr>
<th>$\mathcal{RH}$</th>
<th>$\alpha$, In-Frequency</th>
<th>$D$, Duration (quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 0.1, \chi = 1$)</td>
<td>$0.8065$</td>
<td>[4.65, 6.80]</td>
</tr>
<tr>
<td>benchmark</td>
<td>[0.7393, 0.8632]</td>
<td>[3.32, 6.80]</td>
</tr>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 0.01, \chi = 1$)</td>
<td>$0.6464$</td>
<td>[2.29, 3.12]</td>
</tr>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 1, \chi = 1$)</td>
<td>$0.6464$</td>
<td>[2.29, 3.12]</td>
</tr>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 10, \chi = 1$)</td>
<td>$0.5530$</td>
<td>[1.69, 3.47]</td>
</tr>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 0.01, \chi = 0$)</td>
<td>$0.5155$</td>
<td>[1.51, 3.45]</td>
</tr>
<tr>
<td>$\mathcal{HH}$ ($\epsilon = 1, \chi = 0$)</td>
<td>$0.7699$</td>
<td>[3.81, 5.68]</td>
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<tr>
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<td>$0.7497$</td>
<td>[3.47, 5.56]</td>
</tr>
</tbody>
</table>

For the benchmark cases (the first two rows), the posterior means and 95% HPDs are presented. For the other cases, only the posterior means are presented.
Table 3. Sectors and Weights

<table>
<thead>
<tr>
<th>$j$</th>
<th>Categories</th>
<th>Weights ($n_j$)</th>
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<tr>
<td>1</td>
<td>Motor vehicles and parts</td>
<td>4.91</td>
</tr>
<tr>
<td>2</td>
<td>Furniture and household equipment</td>
<td>2.52</td>
</tr>
<tr>
<td>3</td>
<td>Other durable goods</td>
<td>1.71</td>
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<tr>
<td>4</td>
<td>Food</td>
<td>18.94</td>
</tr>
<tr>
<td>5</td>
<td>Clothing and shoes</td>
<td>3.69</td>
</tr>
<tr>
<td>6</td>
<td>Gasoline, fuel oil, and other energy</td>
<td>4.21</td>
</tr>
<tr>
<td></td>
<td>goods</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Other nondurable goods</td>
<td>7.96</td>
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<tr>
<td>8</td>
<td>Housing</td>
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<tr>
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<td>Household operation</td>
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<td>Transportation</td>
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<tr>
<td>11</td>
<td>Medical care</td>
<td>14.37</td>
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<tr>
<td>12</td>
<td>Recreation</td>
<td>2.91</td>
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<tr>
<td>13</td>
<td>Other services</td>
<td>12.77</td>
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<tr>
<td></td>
<td>Total</td>
<td>100%</td>
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Table 4. Prior Distributions (Multiple Sectors)

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<th>prior std</th>
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<td>$\alpha_j \ (j=1,2,\ldots,13)$</td>
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<td>0.25</td>
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<td>$\rho_{a,j} \ (j=1,2,\ldots,13)$</td>
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<tr>
<td>$\rho_{\delta,j} \ (j=1,2,\ldots,13)$</td>
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</tr>
<tr>
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<td>0.2</td>
</tr>
<tr>
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<td>Gamma</td>
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<td>0.03</td>
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Table 5-2. Prior & Posterior Distributions (Multiple Sectors)
(Sectoral Supply Shocks, Persistence)

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<th>Sector</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>Prior Std</th>
<th>RH Economy Posterior Mean &amp; 95% HPD</th>
<th>HH Economy Posterior Mean &amp; 95% HPD</th>
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<tr>
<td>( \rho_{a,1} )</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>( 0.8682, [0.8155, 0.9211] )</td>
<td>( 0.9156, [0.8764, 0.9540] )</td>
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<tr>
<td>( \rho_{a,2} )</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>( 0.9835, [0.9674, 0.9951] )</td>
<td>( 0.9881, [0.9781, 0.9972] )</td>
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<tr>
<td>( \rho_{a,3} )</td>
<td>Beta</td>
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<td>0.2</td>
<td>( 0.8354, [0.7665, 0.9070] )</td>
<td>( 0.8732, [0.8073, 0.9341] )</td>
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<tr>
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<td>0.2</td>
<td>( 0.9086, [0.8731, 0.9396] )</td>
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<td>0.2</td>
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<td>( 0.8263, [0.7631, 0.8864] )</td>
</tr>
<tr>
<td>( \rho_{a,6} )</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
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<td>( 0.9821, [0.9630, 0.9947] )</td>
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<td>( \rho_{a,9} )</td>
<td>Beta</td>
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<td>0.2</td>
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<td>( 0.8891, [0.8370, 0.9380] )</td>
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<td>Beta</td>
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<td>( 0.9733, [0.9519, 0.9927] )</td>
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<tr>
<td>( \rho_{a,12} )</td>
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<td>0.6</td>
<td>0.2</td>
<td>( 0.8893, [0.8351, 0.9457] )</td>
<td>( 0.9112, [0.8658, 0.9690] )</td>
</tr>
<tr>
<td>( \rho_{a,13} )</td>
<td>Beta</td>
<td>0.6</td>
<td>0.2</td>
<td>( 0.8398, [0.7831, 0.8892] )</td>
<td>( 0.8543, [0.8101, 0.8949] )</td>
</tr>
</tbody>
</table>
Table 5-3. Prior & Posterior Distributions (Multiple Sectors)
(Sectoral Demand Shocks, Persistence)

<table>
<thead>
<tr>
<th></th>
<th>prior distribution</th>
<th>prior mean</th>
<th>prior std</th>
<th>posterior mean $&amp;$95% HPD</th>
<th>posterior mean $&amp;$95% HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{d,1}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.7177 [0.6573, 0.7819]</td>
<td>0.6668 [0.6053, 0.7241]</td>
</tr>
<tr>
<td>$\rho_{d,2}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9753 [0.9556, 0.9933]</td>
<td>0.9608 [0.9327, 0.9855]</td>
</tr>
<tr>
<td>$\rho_{d,3}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9240 [0.8856, 0.9596]</td>
<td>0.9275 [0.8939, 0.9588]</td>
</tr>
<tr>
<td>$\rho_{d,4}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9758 [0.9537, 0.9937]</td>
<td>0.9834 [0.9675, 0.9959]</td>
</tr>
<tr>
<td>$\rho_{d,5}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.8712 [0.8156, 0.9229]</td>
<td>0.9799 [0.9596, 0.9965]</td>
</tr>
<tr>
<td>$\rho_{d,6}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9760 [0.9561, 0.9934]</td>
<td>0.9828 [0.9644, 0.9961]</td>
</tr>
<tr>
<td>$\rho_{d,7}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9871 [0.9763, 0.9967]</td>
<td>0.9862 [0.9766, 0.9964]</td>
</tr>
<tr>
<td>$\rho_{d,8}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9776 [0.9556, 0.9957]</td>
<td>0.9751 [0.9414, 0.9939]</td>
</tr>
<tr>
<td>$\rho_{d,9}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9442 [0.9127, 0.9769]</td>
<td>0.9619 [0.9358, 0.9854]</td>
</tr>
<tr>
<td>$\rho_{d,10}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9466 [0.9018, 0.9817]</td>
<td>0.9367 [0.8896, 0.9766]</td>
</tr>
<tr>
<td>$\rho_{d,11}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9908 [0.9816, 0.9975]</td>
<td>0.9865 [0.9703, 0.9967]</td>
</tr>
<tr>
<td>$\rho_{d,12}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9685 [0.9478, 0.9878]</td>
<td>0.9649 [0.9367, 0.9861]</td>
</tr>
<tr>
<td>$\rho_{d,13}$</td>
<td>$Beta$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9420 [0.9103, 0.9728]</td>
<td>0.9040 [0.8517, 0.9543]</td>
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Table 5-4. Prior & Posterior Distributions (Multiple Sectors)
(Sectoral Supply Shocks, Volatility)

<table>
<thead>
<tr>
<th>( \sigma_{a,i} )</th>
<th>prior distribution</th>
<th>prior mean</th>
<th>prior std</th>
<th>posterior mean &amp; 95% HPD</th>
<th>posterior mean &amp; 95% HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{a,1} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>5.9218 [4.9913, 7.0129]</td>
<td>6.9584 [6.0268, 7.9933]</td>
</tr>
<tr>
<td>( \sigma_{a,2} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>4.3581 [3.2713, 5.8067]</td>
<td>4.4731 [3.5550, 5.5924]</td>
</tr>
<tr>
<td>( \sigma_{a,3} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>8.9397 [6.3958, 11.6060]</td>
<td>9.2567 [6.8493, 13.1550]</td>
</tr>
<tr>
<td>( \sigma_{a,4} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>1.8594 [1.5873, 2.1313]</td>
<td>1.6962 [1.4690, 2.0037]</td>
</tr>
<tr>
<td>( \sigma_{a,5} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>5.3915 [4.1478, 6.8178]</td>
<td>5.0128 [4.0896, 6.2935]</td>
</tr>
<tr>
<td>( \sigma_{a,6} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>3.2209 [2.8756, 3.6376]</td>
<td>1.9048 [1.7545, 2.0786]</td>
</tr>
<tr>
<td>( \sigma_{a,7} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>2.8096 [2.1465, 3.6233]</td>
<td>2.3937 [1.8841, 2.9691]</td>
</tr>
<tr>
<td>( \sigma_{a,8} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>2.0141 [1.6857, 2.5392]</td>
<td>1.8656 [1.6229, 2.1155]</td>
</tr>
<tr>
<td>( \sigma_{a,9} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>3.2699 [2.5499, 4.1379]</td>
<td>3.6611 [2.9248, 4.3750]</td>
</tr>
<tr>
<td>( \sigma_{a,10} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>1.2322 [1.1138, 1.3573]</td>
<td>1.2955 [1.1786, 1.4270]</td>
</tr>
<tr>
<td>( \sigma_{a,11} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>2.0037 [1.6736, 2.3966]</td>
<td>2.0924 [1.8156, 2.4322]</td>
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<tr>
<td>( \sigma_{a,12} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>5.1175 [3.5623, 7.5629]</td>
<td>5.1796 [4.0211, 6.6388]</td>
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<tr>
<td>( \sigma_{a,13} )</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>2.2275 [1.8899, 2.6483]</td>
<td>2.2759 [1.9704, 2.6387]</td>
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<tr>
<td>$\sigma_{d_1}$</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>RH economy</td>
<td>6.2365</td>
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<td>&amp;95% HPD</td>
<td>[5.7256, 6.8163]</td>
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<td>$\sigma_{d_2}$</td>
<td>Inverse Gamma</td>
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<td>3</td>
<td>RH economy</td>
<td>1.4525</td>
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<td>&amp;95% HPD</td>
<td>[1.3399, 1.5678]</td>
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<td>$\sigma_{d_3}$</td>
<td>Inverse Gamma</td>
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<td>3</td>
<td>RH economy</td>
<td>2.1954</td>
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<td>&amp;95% HPD</td>
<td>[2.0034, 2.3916]</td>
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<tr>
<td>$\sigma_{d_4}$</td>
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<td>RH economy</td>
<td>0.6440</td>
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<td>&amp;95% HPD</td>
<td>[0.5812, 0.7078]</td>
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<td>3</td>
<td>RH economy</td>
<td>1.0735</td>
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<td>&amp;95% HPD</td>
<td>[0.9873, 1.1632]</td>
</tr>
<tr>
<td>$\sigma_{d_6}$</td>
<td>Inverse Gamma</td>
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<td>3</td>
<td>RH economy</td>
<td>3.3325</td>
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<td>&amp;95% HPD</td>
<td>[2.9423, 3.7816]</td>
</tr>
<tr>
<td>$\sigma_{d_7}$</td>
<td>Inverse Gamma</td>
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<td>RH economy</td>
<td>0.7561</td>
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<td>&amp;95% HPD</td>
<td>[0.6785, 0.8468]</td>
</tr>
<tr>
<td>$\sigma_{d_8}$</td>
<td>Inverse Gamma</td>
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<td>3</td>
<td>RH economy</td>
<td>0.5444</td>
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<td>&amp;95% HPD</td>
<td>[0.4870, 0.5987]</td>
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<tr>
<td>$\sigma_{d_9}$</td>
<td>Inverse Gamma</td>
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<td>3</td>
<td>RH economy</td>
<td>1.4792</td>
</tr>
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<td>&amp;95% HPD</td>
<td>[1.3667, 1.5960]</td>
</tr>
<tr>
<td>$\sigma_{d_{10}}$</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>RH economy</td>
<td>2.0892</td>
</tr>
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<td>&amp;95% HPD</td>
<td>[1.9064, 2.2694]</td>
</tr>
<tr>
<td>$\sigma_{d_{11}}$</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>RH economy</td>
<td>0.9742</td>
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<td>&amp;95% HPD</td>
<td>[0.9004, 1.0499]</td>
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<tr>
<td>$\sigma_{d_{12}}$</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>RH economy</td>
<td>1.4949</td>
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<td>&amp;95% HPD</td>
<td>[1.3759, 1.6245]</td>
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<tr>
<td>$\sigma_{d_{13}}$</td>
<td>Inverse Gamma</td>
<td>3</td>
<td>3</td>
<td>RH economy</td>
<td>1.1212</td>
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<td>&amp;95% HPD</td>
<td>[1.0135, 1.2236]</td>
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</tbody>
</table>
Table 5-6. Prior & Posterior Distributions (Multiple Sectors)
(Sectoral Nominal Rigidities)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>95% HPD</th>
<th>Posterior Mean</th>
<th>95% HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uniform (0, 1)</td>
<td>0.6334</td>
<td>[0.6034, 0.6629]</td>
<td>0.6334</td>
<td>[0.6034, 0.6629]</td>
</tr>
<tr>
<td>2</td>
<td>Uniform (0, 1)</td>
<td>0.8076</td>
<td>[0.7675, 0.8466]</td>
<td>0.8076</td>
<td>[0.7675, 0.8466]</td>
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<tr>
<td>3</td>
<td>Uniform (0, 1)</td>
<td>0.7712</td>
<td>[0.7407, 0.8012]</td>
<td>0.7712</td>
<td>[0.7407, 0.8012]</td>
</tr>
<tr>
<td>4</td>
<td>Uniform (0, 1)</td>
<td>0.4465</td>
<td>[0.3933, 0.4947]</td>
<td>0.4465</td>
<td>[0.3933, 0.4947]</td>
</tr>
<tr>
<td>5</td>
<td>Uniform (0, 1)</td>
<td>0.6795</td>
<td>[0.6379, 0.7229]</td>
<td>0.6795</td>
<td>[0.6379, 0.7229]</td>
</tr>
<tr>
<td>6</td>
<td>Uniform (0, 1)</td>
<td>0.0132</td>
<td>[0.0010, 0.0349]</td>
<td>0.0132</td>
<td>[0.0010, 0.0349]</td>
</tr>
<tr>
<td>7</td>
<td>Uniform (0, 1)</td>
<td>0.7065</td>
<td>[0.6576, 0.7496]</td>
<td>0.7065</td>
<td>[0.6576, 0.7496]</td>
</tr>
<tr>
<td>8</td>
<td>Uniform (0, 1)</td>
<td>0.7408</td>
<td>[0.7036, 0.7748]</td>
<td>0.7408</td>
<td>[0.7036, 0.7748]</td>
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<td>9</td>
<td>Uniform (0, 1)</td>
<td>0.5830</td>
<td>[0.5349, 0.6333]</td>
<td>0.5830</td>
<td>[0.5349, 0.6333]</td>
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<tr>
<td>10</td>
<td>Uniform (0, 1)</td>
<td>0.0278</td>
<td>[0.0106, 0.0486]</td>
<td>0.0278</td>
<td>[0.0106, 0.0486]</td>
</tr>
<tr>
<td>11</td>
<td>Uniform (0, 1)</td>
<td>0.6486</td>
<td>[0.6097, 0.6912]</td>
<td>0.6486</td>
<td>[0.6097, 0.6912]</td>
</tr>
<tr>
<td>12</td>
<td>Uniform (0, 1)</td>
<td>0.6981</td>
<td>[0.6401, 0.7486]</td>
<td>0.6981</td>
<td>[0.6401, 0.7486]</td>
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<tr>
<td>13</td>
<td>Uniform (0, 1)</td>
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<td>[0.3985, 0.4941]</td>
<td>0.4475</td>
<td>[0.3985, 0.4941]</td>
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<tr>
<td>Categories</td>
<td>$D^\mathcal{RH}$</td>
<td>$D^\mathcal{HH}$</td>
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<td>------------------------------------------------</td>
<td>------------------</td>
<td>------------------</td>
<td></td>
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</tr>
<tr>
<td>1 Motor vehicles and parts</td>
<td>$2.23 \text{ Q}$</td>
<td>$1.96 \text{ Q}$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>[1.98, 2.51]</td>
<td>[1.74, 2.19]</td>
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<tr>
<td>2 Furniture and household equipment</td>
<td>$5.26 \text{ Q}$</td>
<td>$3.70 \text{ Q}$</td>
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<td>[3.81, 7.01]</td>
<td>[2.86, 4.63]</td>
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</tr>
<tr>
<td>3 Other durable goods</td>
<td>$4.00 \text{ Q}$</td>
<td>$3.31 \text{ Q}$</td>
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<td>[3.42, 4.63]</td>
<td>[2.82, 3.88]</td>
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<td>4 Food</td>
<td>$1.26 \text{ Q}$</td>
<td>$0.98 \text{ Q}$</td>
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<td>[1.08, 1.46]</td>
<td>[0.84, 1.13]</td>
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<tr>
<td>5 Clothing and shoes</td>
<td>$2.82 \text{ Q}$</td>
<td>$1.98 \text{ Q}$</td>
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<td>[2.30, 3.38]</td>
<td>[1.66, 2.34]</td>
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<tr>
<td>6 Gasoline, fuel oil, and other energy goods</td>
<td>$0.23 \text{ Q}$</td>
<td>$0.19 \text{ Q}$</td>
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<td>[0.15, 0.31]</td>
<td>[0.12, 0.22]</td>
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<tr>
<td>7 Other nondurable goods</td>
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<td>$2.09 \text{ Q}$</td>
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<td>[2.45, 3.48]</td>
<td>[1.69, 2.56]</td>
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<tr>
<td>8 Housing</td>
<td>$3.48 \text{ Q}$</td>
<td>$2.42 \text{ Q}$</td>
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<td>9 Household operation</td>
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<tr>
<td></td>
<td>[2.32, 3.49]</td>
<td>[1.90, 2.85]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 Other services</td>
<td>$1.27 \text{ Q}$</td>
<td>$1.06 \text{ Q}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.11, 1.44]</td>
<td>[0.94, 1.19]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$D^\mathcal{RH}$, durations of price contracts implied by the $\mathcal{RH}$ model.

$D^\mathcal{HH}$, durations of price contracts implied by the $\mathcal{HH}$ model.
Table 7.

<table>
<thead>
<tr>
<th></th>
<th>$D^{RH}$</th>
<th>$D^{HH}$</th>
<th>$D^{BK}$</th>
<th>$D^{BK}$ (eis)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>1.74 Q</td>
<td>1.39 Q</td>
<td>1.10 Q</td>
<td>1.24 Q</td>
</tr>
<tr>
<td></td>
<td>[1.61, 1.88]</td>
<td>[1.30, 1.53]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>-Durable</strong></td>
<td>3.01 Q</td>
<td>2.49 Q</td>
<td>0.94 Q</td>
<td>1.24 Q</td>
</tr>
<tr>
<td></td>
<td>[2.68, 3.31]</td>
<td>[2.24, 2.78]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>-Nondurable</strong></td>
<td>1.38 Q</td>
<td>1.07 Q</td>
<td>0.94 Q</td>
<td>1.04 Q</td>
</tr>
<tr>
<td></td>
<td>[1.25, 1.53]</td>
<td>[0.98, 1.19]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>-Service</strong></td>
<td>1.87 Q</td>
<td>1.53 Q</td>
<td>1.44 Q</td>
<td>1.55 Q</td>
</tr>
<tr>
<td></td>
<td>[1.71, 2.04]</td>
<td>[1.40, 1.67]</td>
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<td></td>
</tr>
</tbody>
</table>

Table 8.

<table>
<thead>
<tr>
<th>Categories</th>
<th>$D^{RH}$</th>
<th>$D^{HH}$</th>
<th>$D^{BK}$</th>
<th>$D^{NS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Furniture and household equipment</td>
<td>5.26 Q</td>
<td>3.70 Q</td>
<td>1.09 Q*</td>
<td>1.5 Q*</td>
</tr>
<tr>
<td></td>
<td>[3.81, 7.01]</td>
<td>[2.86, 4.63]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Food</td>
<td>1.26 Q</td>
<td>0.98 Q</td>
<td>1.14 Q</td>
<td>0.81 Q**</td>
</tr>
<tr>
<td></td>
<td>[1.08, 1.46]</td>
<td>[0.84, 1.13]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Clothing and shoes</td>
<td>2.82 Q</td>
<td>1.98 Q</td>
<td>0.97 Q</td>
<td>0.93 Q</td>
</tr>
<tr>
<td></td>
<td>[2.30, 3.38]</td>
<td>[1.66, 2.34]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Gasoline, fuel oil, and other energy</td>
<td>0.23 Q</td>
<td>0.19 Q</td>
<td>0.17 Q***</td>
<td></td>
</tr>
<tr>
<td>goods</td>
<td>[0.15, 0.31]</td>
<td>[0.12, 0.22]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Transportation</td>
<td>0.28 Q</td>
<td>0.37 Q</td>
<td>0.67 Q</td>
<td>1.33 Q</td>
</tr>
<tr>
<td></td>
<td>[0.22, 0.33]</td>
<td>[0.34, 0.41]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 Medical care</td>
<td>2.39 Q</td>
<td>1.88 Q</td>
<td>3.38 Q</td>
<td>4.44 Q</td>
</tr>
<tr>
<td></td>
<td>[2.02, 2.78]</td>
<td>[1.64, 2.14]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 Recreation</td>
<td>2.87 Q</td>
<td>2.33 Q</td>
<td>2.78 Q</td>
<td>2.26 Q</td>
</tr>
<tr>
<td></td>
<td>[2.32, 3.49]</td>
<td>[1.90, 2.85]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Home Furnishing

** Weighted average of processed and unprocessed food.

*** Vehicle Fuel

$D^{RH}$, durations of price contracts implied by the $\mathcal{RH}$ model.

$D^{HH}$, durations of price contracts implied by the $\mathcal{HH}$ model.

$D^{BK}$, durations of price contracts reported in Bills and Klenow (2004).

$D^{NS}$, durations of price contracts reported in Nakamura and Steinsson (2006).
Figure B.1: $\frac{q^{HC}(\alpha,\chi,\epsilon)}{q^{HC}}$ : Ratio of the measure of real rigidities. ($\alpha$ is fixed at 0.5)
Figure B.2: $q^{\text{HH}}(\alpha, \epsilon, \chi = 1)$ & $q^{\text{RH}}$: dotted black line represents $q^{\text{RH}}$ and the other three curves represent $q^{\text{HH}}$ for $\alpha = 0.1$, 0.5, and 0.9, while $\chi$ is fixed at 1.

Figure B.3: $q^{\text{HH}}(\alpha, \epsilon = 0.1, \chi)$ & $q^{\text{RH}}$: dotted black line represents $q^{\text{RH}}$ and the other three curves represent $q^{\text{HH}}$ for $\alpha = 0.1$, 0.5, and 0.9, while $\epsilon$ is fixed at 0.1.
Figure B.4: Impulse Responses of Output, Inflation, and Interest to aggregate shocks. $\mathcal{HH}$ economy is represented by red solid curves and $\mathcal{RH}$ economy is represented by blue dotted curves.
Figure B.5: Impulse Responses of Sectoral Outputs and Sectoral Inflations to monetary, aggregate demand and aggregate supply shocks. \( \mathcal{RH} \) model is represented by red solid curves and \( \mathcal{RH} \) model is represented by blue dotted curves.
Figure B.6: Impulse Responses of Sectoral Outputs and Sectoral Inflations to a sectoral supply shock and a sectoral demand shock. HH economy is represented by red curves and RH economy is represented by blue curves.
Figure B.7: Posterior distribution of $\alpha$ and $D$ in single-sector economies.

Figure B.8: Posterior distribution of $D$ in multiple-sector economies.
Figure B.9: Autocorrelations of aggregate and sectoral inflations estimated from data. (Autocorrelation functions on the vertical axis and lags on the horizontal axis).

Figure B.10: Model implied autocorrelations of aggregate and sectoral inflations.
C. Proofs

C.1. Proof of the proposition 2.1

The equilibrium conditions can be reduced to

\[
\frac{P(i)}{P} = \delta C(i), \quad \text{(C.1)}
\]

\[
C(i) = \lambda Y + (1 - \lambda) \left( \frac{P(i)}{P} \right)^{1-\theta} Y, \quad \text{(C.2)}
\]

\[
P = \left( \int_0^1 P(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad \text{(C.3)}
\]

\[
M = PY. \quad \text{(C.4)}
\]

Combining (C.1) and (C.2) gives

\[
P_R(i) = \lambda \delta Y + (1 - \lambda) \delta Y P_R(i)^{1-\theta}, \quad \text{(C.5)}
\]

where

\[
P_R(i) = \frac{P(i)}{P}.
\]

Note that (C.5) should hold for all \(i\). Thus for any arbitrary \(i_1\) and \(i_2\) in \([0, 1]\), it must be true that

\[
P_R(i_1) - P_R(i_2) = (1 - \lambda) \delta Y \left\{ \left( \frac{1}{P_R(i_1)} \right)^{\theta-1} - \left( \frac{1}{P_R(i_2)} \right)^{\theta-1} \right\}. \quad \text{(C.6)}
\]

Note that both \((1 - \lambda) \delta Y\) and \((\theta - 1)\) are positive. Therefore, it is not possible that either \(P_R(i_1) > P_R(i_2)\) or \(P_R(i_1) < P_R(i_2)\) while satisfying (C.6). The only case consistent with the equation (C.6) is \(P_R(i_1) = P_R(i_2)\). Therefore \(P(i_1) = P(i_2)\). Then it should be that \(P(i) = P\), \(\forall i \in [0, 1]\) from (C.3) and that \(C(i) = Y\) from (C.2). Finally from (C.1) and (C.4), \(Y = 1/\delta\) and \(P = \delta M\).

C.2. Proof of the proposition 2.2

Let \(M > \bar{M}\). If \(P_{IC} \geq \delta M\), then we have

\[
P_{IC} = \left( n \left[ \delta M \right]^{\frac{1}{2}} P_{IC}^{\theta-1} \right)^{1-\theta} + (1 - n) \left[ \delta \bar{M} \right]^{1-\theta} \left( \frac{1}{1-\theta} \right)
\]

\[
\leq \left( n P_{IC}^{1-\theta} + (1 - n) \left[ \delta \bar{M} \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}
\]

\[
< \left( n P_{IC}^{1-\theta} + (1 - n) P_{IC}^{1-\theta} \right)^{\frac{1}{1-\theta}}, \quad \left( \therefore \bar{M} < M \leq \frac{P_{IC}}{\delta} \right)
\]

\[
= P_{IC}.
\]
This cannot be true. Therefore it must be that $P_{IC} < \delta M$. Then we have

$$P_{IC} = \left( n \left[ \left( \delta M \right)^{\frac{1}{2}} P_{IC}^{\frac{1}{2}} \right]^{1-\theta} + (1 - n) \left[ \delta M \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

$$< \left( n \left[ (\delta M)^{\frac{1}{2}} [\delta M]^{\frac{1}{2}} \right]^{1-\theta} + (1 - n) \left[ \delta M \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

$$= \left( n [\delta M]^{1-\theta} + (1 - n) [\delta M]^{1-\theta} \right)^{\frac{1}{1-\theta}} = P_C$$

$$< \left( n [\delta M]^{1-\theta} + (1 - n) [\delta M]^{1-\theta} \right)^{\frac{1}{1-\theta}} = \delta M = P_F.$$

Therefore, it has been shown that

$$P_{IC} < P_C < P_F.$$

From $Y = M/P$, it is also true that

$$Y_{IC} > Y_C > Y_F.$$