Price Dispersion and Price Rigidity in Online Markets:

Theory and Evidence

(Dissertation Proposal)

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Summary

The first chapter models the equilibrium behavior of firms and an information gatekeeper in a duopoly market where firms have asymmetric numbers of loyal consumers. In the theoretical model, there are two consumer segments; one segment views the products as homogenous and the other exhibits loyalty to a particular firm. The duopolists engage in costly price advertising to reach consumers. I show that, in equilibrium, the firm with more loyal consumers advertises less frequently due to its higher opportunity cost of advertising. Also, because of the larger base of loyal consumers, this firm prices more aggressively whenever it advertises its price, since it requires a smaller margin to offset the cost of advertising. This result is significantly different from Narasimhan (1988), where firms advertise with probability one because doing so is costless. Finally, I show that the model converges to a duopoly version of the symmetric Baye and Morgan (2001) model when the number of loyal consumers enjoyed by the two firms is identical.

The second chapter examines an n-firm model of oligopoly pricing in online markets with heterogeneous marginal costs, costly price adjustments, and temporal cost shocks. The equilibrium of the model exhibits three characteristics: (1) spatial price dispersion, in which the price rankings of different firms on the market are fixed and thus some firms persistently charge higher prices than other firms; (2) temporal price dispersion, in which the price charged by a particular firm changes from time-to-time; and (3) price rigidity, which refers to the fact that the firms do not immediately adjust prices in response to cost shocks. I propose a method to structurally
estimate the model via a simulated maximum likelihood approach that requires only pricing data. This approach yields estimates of parameters, including price adjustment costs and the price sensitivity of consumers. Monte-Carlo results reveal that the proposed approach yields accurate and efficient estimates when the price adjustment frequencies generated by the true parameters are not too low. Additionally, the simulations suggest that when the initial values are randomly chosen, the numerical method must be iterated multiple times in order to recover reliable estimates of the parameters. Further research is proposed to explore the possibility of enriching the model by allowing heterogeneity in price adjustment costs, as well as dynamic trends in prices.

The third chapter empirically examines online firms’ price adjustment behavior by exploiting a dataset on digital camera prices obtained from pricegrabber.com. The data provides evidence that is consistent with the existence of positive price adjustment costs in online markets. I find that (1) the frequency of price adjustments is relatively low at 7%, which implies that price adjustment costs prevent firms from responding immediately to cost shocks or other changes in market conditions; and (2) less popular and more expensive products have lower price adjustment frequencies. These reduced-form results lead me to structurally estimate a model to obtain estimates of price adjustment costs and the price sensitivities of consumers using the methods developed in the second chapter. As proposed future research, specification tests will be conducted to formally test for marginal cost and price adjustment heterogeneities.
Information Gatekeeper with Asymmetric Firms *†

Chenguang Li‡; Lan Zhang§

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Abstract

Asymmetry exists among the firms selling homogeneous product in online market. This paper provides a theoretical study on the equilibrium behavior of the firms and the information gatekeeper in a duopoly market with asymmetric sizes of loyal consumers. In equilibrium, the firm with larger number of loyal consumers tends to advertise less frequently but charges a lower average price whenever advertising. And the optimal advertising fee charged by the information gatekeeper depends on the degree of asymmetry of the market. As the market becomes more asymmetric, the gatekeeper charges a higher advertising fee.

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1 Introduction

The boom of e-commerce in the past decade has dramatically changed the ways in which consumers obtain information and purchase products. The firms, in the mean time, have been adjusting their advertising and retailing strategies. This provides opportunities for the price comparison service to burgeon and develop into a business of significant scale. According to E-consultancy, in the UK, the price comparison services made revenues between 120 to 140 Million Euros in 2005. Nowadays, for a popular product, there are usually dozens of firms that list their prices on the price comparison websites (will be also referred to as “information gatekeepers ”). The continuous increasing of the price comparison business makes the economic analysis of information gatekeeper models more and more important.

Baye and Morgan (2001)’s seminal paper is the first to investigate the market equilibrium when consumers, firms and the information gatekeeper make their own decisions to maximize their own utility/profit. In their model, the firms can list their price information at the information gatekeeper’s site so that they can have an opportunity to attract consumers outside their local markets. In equilibrium, to maximize profit, the information gatekeeper charges a positive listing fee to induce the price dispersion in the market. This result explains the persistent price dispersion even in the information era, when the cost of obtaining price information is small.

The firms in Baye and Morgan model are symmetric in the sense that they have the same number of local customers. In the real world, however, firms are asymmetric. Even if the firms are competing with a homogeneous product, they differ significantly in their service quality, website design and reputation. As a result, the firms build their own loyal consumer groups of different sizes. This asymmetry in the size of loyal consumer groups has an significant impact on the pricing and advertising decisions of the firms.

Some casual observations of the price comparison websites indicate that asymmetry widely exists in the pricing and advertising strategies of the firms. First, some firms list their prices significantly less frequently than others. Second, some firms list lower prices more frequently than others on the price comparison websites. For example, we examine the price listing of a single product (SanDisk 2 GB Secure Digital Card) at Shopper.com, the pricing lists were recorded 19 times in the period from 10/21/2006 to 1/7/2007. Figure 1 shows the listed price for Dell and Buy.com. Over the period, Dell lists its price 16 times while buy.com only lists 8 times. And whenever buy.com lists its price, it charges a lower price than Dell. This is just one example of the asymmetries in pricing and advertising that is ubiquitous on the pricing comparison sites.

Therefore, to characterize the asymmetric behavior of firms, studies on asymmetric models are necessary. However, the complexity introduced by asymmetry makes the models difficult to solve even for a duopoly market. In the literature, there are limited research on asymmetric models and most of them are focused only on pricing strategy of the firms. Narasimhan(1988) analyzes a duopoly pricing model with asymmetry in the sizes of loyal consumers. The equilibrium behavior of the duopoly turns out to be determined by the characteristics of the shoppers. If the shoppers are extremely price sensitive and only buy products from the firm offering the lowest price, the equilibrium price distribution of the firm with less loyal consumers is first-degree stochastic dominated by that of the firm with more loyal consumers. Baye, Kovenock and de Vries (1992) generalize Narasimhan’s model to N firms case and find that if firms have different sizes of loyal consumers, then two of the firms with the least loyal consumers will continuously randomize over some price interval and all other firms set prices equal to reservation price with probability one. However, in above models, there is no role for the information gatekeeper,
or in other words, the cost of advertising is zero. And “little is known about the general clearinghouse model with asymmetric consumers.”[2]

Our paper contributes to the literature by studying a duopoly clearinghouse model with consumer asymmetries and positive listing fees. The model gives insights on the asymmetries in firms’ advertising frequencies and pricing strategies observed on the price comparison sites. The model is based on the clearinghouse model in Baye and Morgan(2001). There are two types of consumers in the model, loyal consumers and comparison shoppers. The two firms are asymmetric in their numbers of endowed loyal consumers.\footnote{There are other types of asymmetries, such in costs, service qualities, but we will focus on the asymmetry in the size of the loyal consumers.} Shoppers first consult price comparison website and buy from the firm offering the lowest price. If there is no price listed on the website, they will randomly choose from one of the firms. Loyal consumers, however, buy directly from their own preferred firm. The information gatekeeper charges a fixed fee from the firm that lists price on its website. And firms make pricing and advertising decisions given the fixed fee.

We find that there are two sets of equilibria of firms’ behavior, each corresponds to a certain range of listing fee charged by the information gatekeeper. The firm with more loyal consumers will advertise less frequently, but whenever it decides to advertise, it charges lower prices more frequently than the firm that has less loyal consumers. This pricing behavior of the firms significantly differs from that when advertising fee is zero, which is predicted in Narasimhan(1988). The intuition for this result is as follows. For the firm with more loyal consumers, the opportunity cost of advertising and charge a price lower than $r$ is higher. Therefore, in equilibrium, it will advertise less frequently. Also because of the larger number of loyals, when the firm decides to advertise, it needs smaller profit margin to offset the cost of advertising. Therefore, it gives higher discount when it advertises. In short, in an economy where firms can choose both advertising and pricing strategy, the firm with larger market share tends to use a less aggressive advertising strategy to protect the profits from its own loyal customers.

We also find that as the market becomes more asymmetric (the difference of the number of loyal customers increases), the gatekeeper charges a higher listing fee since the firm with less loyals is willing to pay more to

![Graph showing the listed price of SanDisk 2 GB Secure Digital Card of Buy.com and Dell on Shopper.com]
advertise as the market gets more asymmetric.

The rest of the paper is organized as follows. Section 2 lays out the setup of the model. Section 3 solves the firm’s equilibrium strategy given fixed advertising fee. Then the optimal fee for the gatekeeper is analyzed in section 4. Section 5 concludes.

2 Model Setup

There are two price-setting firms \((i = 1, 2)\) competing in a homogeneous market. Firms have unlimited capacity to supply this product at a constant marginal cost, \(m\), which is assumed to be zero without loss of generality. This market is served by a price information gatekeeper, who provides an information portal for the firms to advertise their price for a fixed fee \(\phi\).\(^2\)

There is a continuum of consumers with size normalized to one, each having a unit demand up to a reservation price \(r\). We assume that there are two types of consumers, loyals and shoppers. The loyals will only purchase from their favorite firm given the price charged by the firm does not exceed \(r\). The shoppers, however, will consult the information gatekeeper to compare the price charged by different firms, and purchase from the firm charging the lowest price, given that it does not exceed \(r\). If there is no firm listing at the information gatekeeper, or the price listed are the same for both firms, the shopper will randomly choose a firm and purchase from there, given that the price does not exceed \(r\).\(^3\) Let the size of the loyals for each firm be \(L_1\) and \(L_2\), respectively. To introduce asymmetric structure into the model, we assume \(L_1 \geq L_2\) without loss of generality. And let \(S\) denote the size of shoppers, which equals to \(1 - L_1 - L_2\). The firms are not allowed to price discriminate between different consumers.

There are a few things regarding the assumption of the model that we would like to discuss. First, the unit demand assumption is made for computational simplicity. A more general assumption would be a downward sloping demand schedule \(q(p)\). The firms strategy under these two assumptions, however, are quite similar. Under both setups, the firm’s profit in their loyal market is maximized at certain price \((r\) for the unit demand case and the monopoly price for the downward sloping demand case). To gain the business from the shoppers, they must sacrifice the profit in the loyal market price by charging a lower price. Second, our model is a generalized model in the sense that it will converge to existing models with parameters take extreme values. When \(L_1 = L_2\), our model converges to Baye and Morgan (2001) with \(n = 2\) and unit demand assumption. When \(\phi = 0\), our model converges to Narasimhan(1988) with advertising options.

The market runs in three stages. First, the information gatekeeper decides its advertising fee, \(\phi\). Second, the firms make their pricing decisions and advertising decisions simultaneously. Finally, the consumers make their purchasing decisions according to the price and their type. Since the consumer’s behavior is exogenously assumed, we will first focus on the decisions of the firms.

\(^2\) There are different business models for online pricing comparison website, retailers can either pay a fixed fee, a click-through fee or a fee when a purchase is completed. We are focusing on the fixed fee case in our model.

\(^3\) This is a reasonable assumption, since in equilibrium, if a firm does not list price on the price comparison site, it will always charge \(r\). As a result, there is no incentive for consumers to do sequential search among the firms.
3  Equilibrium Analysis of the Firms

Given the advertising fee, $\phi$, there are two decisions that the firms need to make: whether to list the price information at the gatekeeper, and what price to charge. Neither of the decisions is trivial. For advertising, the firm needs to weigh the potential profit gain of advertising against the cost of advertising fee. For pricing, there is a trade off between extracting more profit per sale and attracting more sales from the shoppers. We prove, by construction, the existence of a mixed strategy equilibrium in this subgame.

To accommodate for the mixed strategy equilibrium, let $\alpha_i$ stand for the probability that firm $i$ will advertise. And let $F_i(p)$ be the price distribution that firm $i$ will draw from if it decides to advertise. Clearly, pure strategy for advertising will be characterized by $\alpha_i = 0$ or $1$, and pure strategy for pricing will be characterized by a degenerated price distribution function.

To analyze the potential gain of advertising, we first consider the strategy and expected profit of a firm when it does not advertise. Given the assumptions, if the firm does not advertise, as long as it charges price that does not exceed $r$, it can get sales from its own loyals, and half of the shoppers if the other firm does not advertise either. In other words, the firm can not change its demand by changing price if it does not advertise. Obviously, the firm’s optimal pricing strategy when it does not advertise is to charge $r$. And the expected profit that it can get is:

$$E\pi_{i}^{NA} = r[L_i + \frac{1}{2}(1 - \alpha_j)S]. \quad (1)$$

On the other hand, the expected profit when the firm advertises and charges price $p$ is:

$$E\pi_{i}^{A}(p) = p[L_i + [(1 - \alpha_j) + \alpha_j(1 - F_j(p))]S] - \phi, \quad (2)$$

where $F_j$ is the price distribution of the other firm. Basically, the equation says that if the firm advertises and charges price $p$, it will gets sales from its own loyals $L_i$, and all the shoppers if the other firm does not advertise (with probability $1 - \alpha_j$), or the other firm advertises but charges a higher price than $p$ (with probability $\alpha_j(1 - F_j(p))$). And it has to pay the advertising fee $\phi$.

The level of the advertising fee plays a critical role in the decision of the firms. When $\phi > \frac{rS}{2}$, neither firm advertises is a Nash Equilibrium, since the cost of advertising, $\phi$ is greater than the maximum possible gain in revenue the firm can get by deviating from not advertising, which is $\frac{rS}{2}$. When $\phi = 0$, then firm $i$ is indifferent between not advertising and advertising with charging $r$ if the other firm does not have mass point at $r$. This fact makes the existence of multiple equilibria possible. We will discuss this in detail in later part of the paper. For the following analysis, we will focus on the range of $0 < \phi \leq \frac{rS}{2}$.

At this point, it is useful to introduce $p_{i}$, which is defined as the minimum price that firm $i$ will ever consider charging when it advertises. In another words, pricing at $p_{i}$, firm $i$’s maximum possible profit equals to the profit when it does not advertise. Therefore, we have:

$$r[L_i + \frac{1}{2}(1 - \alpha_j)S] = p_{i}(L_i + S) - \phi, \quad i = 1, 2 \quad (3)$$

It can be shown that when $\alpha_i = 1$, we have $p_1 \geq p_2$.

Under the specified range of $\phi$, it turns out that there does not exist an equilibrium with pure pricing strategies when the firms advertise. And there does not exist an equilibrium with both firm having pure strategies in advertising. The results are given in Lemma 1 and Lemma 2.
Lemma 1 In equilibrium, if $0 < \alpha_i \leq 1$, i.e., both firm advertise with positive probability, then neither firm uses pure strategy in pricing when it decides to advertise.

Proof. We first prove that it is impossible for both firm to use pure strategy in price whenever advertising. Assume otherwise, then firm $i$ will charge $p_i \in [p_i, r]$ and firm $j$ will charge $p_j \in [p_j, r]$. If $p_i = p_j$, then we argue that firm $j$ can increase its profit by deviating and charge $p_j - \delta$, where $\delta$ is a small positive number. We have $E\pi_j^A(p_j) = p_j[L_j + (1 - \alpha_i)S + \frac{\alpha_i S}{2}] - \phi$ and $E\pi_j^A(p_j - \delta) = (p_j - \delta)(L_j + S) - \phi$. Therefore,

$$E\pi_j^A(p_j - \delta) - E\pi_j^A(p_j) = \frac{p_j \alpha_i S}{2} - \delta(L_j + S). \tag{4}$$

Since $\alpha_i > 0$ by assumption, and $\delta$ can be arbitrarily small, we have $E\pi_j^A(p_j - \delta) - E\pi_j^A(p_j) > 0$, which means when $p_i = p_j$, the pure strategy can not be an equilibrium.

If $p_i \neq p_j$, assume $p_i < p_j$ without loss of generality, then firm $i$ will be better off by charging $p_j - \delta > p_i$ instead of charging $p_i$. To see this, notice when $p_i < p_j$ the expected profit for firm $i$, $E\pi_i^A(p_i) = p_j(L_j + S) - \phi$, is a strictly increasing function in $p_i$. Therefore, firm $i$ can always increase its expected profit by increase its price when $p_i$ is strictly lower than $p_j$.

It is also impossible for one firm to have pure strategy and the other firm to have mixed strategy in equilibrium. If firm $i$ charge $p_i$ whenever it advertises, then $E\pi_j(p_j)$ is a strictly increasing function on $[p_j, p_i]$ and on $(p_i, r]$, respectively. Therefore, there does not exist two prices that are both best responses of firm $j$ to $p_i$. Consequently, firm $j$ does not have mixed strategy in equilibrium.

Summarizing from the above, it is impossible for any firm to have pure pricing strategy when both firm advertise with positive probability. ■

Lemma 2 When $0 < \phi \leq \frac{S}{2r}$, there does not exist an equilibrium such that (1) either firm does not advertise, (2) both firms advertise with probability one.

Proof. First, if firm $i$ does not advertise, it is always optimal for firm $j$ to advertise with probability one and charge $r$ given $\phi \leq \frac{S}{2r}$. Then, firm $i$ can increase its profit by advertising with probability one and undercut $r$ by a small amount $\delta$. Since $\phi < Sr$, $\phi < S(r - \delta)$ for an arbitrarily small $\delta$. Therefore, in equilibrium, both firm have to advertise with positive probability.

Second, if both firm advertise with probability one, by Lemma 1, there is no pure strategy in pricing. However, if both firm apply mixed strategies in pricing, then according to Narasimhan (1988), the price distribution of at least one of the firms will have positive density at $r$, and the other firm will not have positive mass at $r$. So for the firm that have positive density at $r$, advertising and charging $r$ will not generate any more sale than not advertising and charging $r$, but will incur a positive fixed advertising cost. In other words, advertising and charging $r$ is dominated strategy and can not be part of the mixed strategy equilibrium. Therefore, both firm advertise with probability one can not be an equilibrium. ■

Lemma 1 and Lemma 2 together rule out the existence of pure strategy equilibria in pricing when firms advertise. And they also narrow down the possible combinations of advertising strategy to two cases: (1) both firm mix in advertising, (2) one firm mixes in advertising and the other firm advertises with probability one.

The following analysis investigate the equilibrium strategies of the firms.
If either firm is mixing in price, it must be true that $E\pi^A_i(p)$ is a constant for any price in the domain of its price distribution. Also, if the firm is mixing also in advertising, it must be true that $E\pi^{NA}_i = E\pi^A_i(p)$, i.e., the expected profit of advertising equal to the expected profit of not advertising.

The relative size of $p_i$ turns out to be very critical in analyzing the equilibrium behavior of the firms. Notice that in equilibrium, if $p_i > p_j$, then firm $j$ can not mix in advertising. By definition, we have $E\pi^{NA}_j = p_j(L_j + S) - \phi = E\pi^A_j(p_i)$. However, since firm $i$ will not charge any price below $p_i$, firm $j$ can strictly increase its price to $p_i$ and still sell to all the shoppers, which gives a higher profit than $\pi^A_j(p_i)$. In other words, not advertising is a strictly dominated strategy for firm $j$ if $p_i > p_j$.

Given this insight, there are three possible cases for mixed strategy equilibrium to exist:

1. $p_1 > p_2$, $0 < \alpha_1 < 1$, $\alpha_2 = 1$.
2. $p_1 < p_2$, $\alpha_1 = 1$, $0 < \alpha_2 < 1$.
3. $p_1 = p_2$, $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$.

It is straightforward to show that case 2 is not possible. And case 1 and case 3 are both equilibria for the firms’ strategies, each corresponding to a different range of the advertising fee $\phi$. The details of equilibrium strategies of the firms are described in Proposition 3.

**Proposition 3** If the gatekeeper sets the listing fee $\phi$, and firms make optimal pricing and advertising decisions, then there exists an asymmetric Nash equilibrium which is dependent on the level of $\phi$:

1. (Range 1) If $0 \leq \phi < rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$:
   - Firm 2 will advertise its price with probability one and firm 1 will advertise its price with probability
     \[
     \alpha_1 = \frac{L_2 + S}{L_1 + S} (1 - \frac{\phi}{rS}).
     \] (5)
   - When a firm chooses to advertise, its will charge a price from the distribution that is characterized by the following c.d.f.:
     \[
     F_1(p) = \frac{p(L_1 + S) - (rL_1 + \phi)}{pS(1 - \frac{\phi}{rS})}, \text{ on } [p, r], \] (6)
     \[
     F_2(p) = 1 - \frac{L_1(r - p) + \phi}{pS}, \text{ on } [p, r], \] (7)
     where $p = \frac{rL_1 + \phi}{L_1 + S}$. And if a firm chooses not to advertise, it will charge $r$.
   - And the profits for the two firms are:
     \[
     E\pi_1 = rL_1, \] (8)
     \[
     E\pi_2 = \frac{rL_1(L_2 + S) + \phi(L_2 - L_1)}{(L_1 + S)}. \] (9)

2. (Range 2) If $rs \frac{L_1 - L_2}{1 + L_1 - 2L_2} \leq \phi \leq \frac{rS}{2}$, then
Both firms will have mixed strategies in advertising, and the probabilities are:

\[ \alpha_1 = \frac{rS - 2\phi}{rS}, \]  
\[ \alpha_2 = \frac{(rS - 2\phi)(2L_1 + S - L_2)}{rS(L_2 + S)}, \]  

When a firm chooses to advertise, its will charge price from the distribution that is characterized by the following c.d.f.:

\[ F_1(p) = \frac{1}{\alpha_1} (1 - \frac{rL_2 - pL_2 + \frac{1}{2}rS(1 - \alpha_1) + \phi}{pS}), \text{ on } [p, r], \]  
\[ F_2(p) = \frac{1}{\alpha_2} (1 - \frac{rL_1 - pL_1 + \frac{1}{2}rS(1 - \alpha_2) + \phi}{pS}), \text{ on } [p, r], \]  

where, \( p = \frac{rL_2 + 2\phi}{L_2 + S} \). And when a firm chooses not to advertise, it will charge \( r \).

And the profits for the two firms are:

\[ E\pi_1 = \frac{rL_2(L_1 + S) + \phi(2L_1 + S - L_2)}{L_2 + S}, \]  
\[ E\pi_2 = rL_2 + \phi. \]

Proof. See Appendix.

Figure 2 plots the price distributions \( F_i(p) \) for the two firms for certain values of the parameters. In both ranges of \( \phi \), firm 2’s price distributions have positive mass at the reservation price \( r \). According to the analysis given in Narasimhan(88), firm 1’s price distribution will have zero probability density at \( r \). The reason is when firm 2 has mass point at \( r \), for firm 1, charging \( r \) is strictly dominated by charging \( r - \delta \), where \( \delta \) is an arbitrarily small number.

The primary purpose of this paper is to examine the effect of asymmetry on the equilibrium strategies of the firms, or to be more specific, the differences in strategies for firms with different numbers of loyals. In both ranges of \( \phi \), we have \( \alpha_1 \leq \alpha_2 \) and \( F_1(p) \geq F_2(p) \), i.e., the firm with more loyals will advertise less frequently. But whenever it decides to advertise, it is more likely to give a bigger discount off the reservation price. The intuition for this result is as follows. Since firm 1 has more loyal consumers, its opportunity cost of advertising and charge price lower than \( r \) is higher. Therefore, in equilibrium, firm 1 will advertise less frequently. Also because of the larger number of loyals, when firm 1 decides to advertise, it needs smaller profit margin to offset the cost of advertising. Therefore, it gives higher discount whenever it advertises.

Since our model is a generalized version of Narasimhan(1988), and Baye and Morgan (2001) with \( n = 2 \). It is interesting to see whether our equilibrium results will converge to the special cases when the parameters take special values. First, when \( L_1 = L_2 = L \), the equilibrium will always in range 2 since the threshold of advertising fee that separates the two ranges becomes zero. And we have

\[ \alpha_1 = \alpha_2 = \alpha = \frac{rS - 2\phi}{rS}, \]  
\[ F_1(p) = F_2(p) = \frac{1}{\alpha} (1 - \frac{(r - p) - \frac{1}{2}rS}{pS}), \text{ on } [p, r], \]  

\[ r = \frac{rS - 2\phi}{rS}. \]
Figure 2: Price Distribution of two firms \((r = 1, S = 0.5, \theta = 1/3, \phi = 0.1 \text{ (range 1) or 0.2 (range 2)})\)

\[
p = \frac{r(1 - S) + 4\phi}{1 + S}, \quad \text{and}
\]

\[
E\pi = \phi + rL.
\]

These results are consistent with Baye and Morgan (2001), with \(n = 2\) and unit demand assumptions.

Second, when \(\phi = 0\), the equilibrium will always be in range 1 of the equilibrium. And we have:

\[
\alpha_1 = \frac{L_2 + S}{L_1 + S},
\]

\[
F_1(p) = \frac{pL_1 + pS - rL_1}{pS} \text{ on } [p, r],
\]

\[
F_2(p) = 1 - \frac{L_1(r - p)}{pS} \text{ on } [p, r],
\]

where \(p = \frac{rL_1}{L_1 + S}\). On the face, the equilibrium that is characterized here is different from Narasimhan(1988), which does not allow for the possibility of not advertising. So in the equilibrium described in Narasimhan(1988), we have \(\alpha_1 = 1\) and \(F_1(p) = 1 + \frac{L_2}{S} - \frac{L_1(r(L_2 + S))}{S(pL_1 + S)}\). However, these two seemingly different equilibria generate the same profits for the two firms. When the advertising fee is zero, we have \(F_2(r) = 1\), which means that firm 2 no longer has mass point at the reservation price. Then for firm 1, advertising and charging \(r\) is the same as not advertising and charging \(r\), since in either case, firm 1 is selling only to its loyals to earn a profit of \(rL_1\).

From another perspective, firm 1’s total pricing distribution (taking into account both the advertising case and non-advertising case) is:

\[
\alpha_1 F_1(p) = 1 - \frac{L_2}{S} - \frac{L_1(r(L_2 + S))}{S(pL_1 + S)}.
\]
which matches the pricing distribution given by Narasimhan(1988). Actually, any convex combination of the equilibrium that is the special case of Proposition 3 and the equilibrium that is characterized in Narasimhan(1988) is an equilibrium in our model. And all of the equilibria generate the same profits for both firms.

To study the effect of the parameters $(\phi, r, \theta, S)$ on the equilibrium behavior of the firms, we compute the comparative statics of $\alpha_i$ and $F_i(p)$ w.r.t. the parameters. Here $\theta = L_1 - L_2$ is a measure of the degree of asymmetry of the market. Basically, the loyal and shopper sizes of the market can be fully characterized by the pair $(S, \theta)$. The comparative statics results are summarized in Table 1. We also include the comparative statics for the symmetric case for comparison purpose.

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<td>$+$</td>
</tr>
<tr>
<td>$F_i$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

+:positive effect, -: negative effect, o: no effect
N/A: not applicable

Table 1: Comparative Statics

We are able to calculate analytically the comparative statics for all the variables with respect to the four parameters except for $F_2(p)$ in range 2. Most of the comparative statics results are consistent with economic intuitions. As the advertising fee $\phi$ increases, the advertising frequency reduces (except $\alpha_2$ in range 1, which is constant at one) due to the higher cost of advertising, and when the firm advertises, the average price charged by the firms increases to cover the higher cost. As the reservation price $r$ increases, firms will advertise more often since the potential gain by advertising is higher. Also, the raise of the reservation price allows the firms to charge a higher average price in the equilibrium. The increase of the number of shoppers, $S$, will increase the incentive of the firms to competing for the shoppers. So in equilibrium, the probabilities to advertise increase, and the amounts of discounts are larger for both firms.

The above comparative statics are all consistent with the results for the symmetric case. A nice feature of the model is that the asymmetric structure of our model enables us to study the effect of the degree of asymmetry, $\theta$ on the equilibrium strategies of the firms. As $\theta$ increases, we see a decrease of advertising probability for firm 1 in range 1 and an increase of advertising probability for firm 2 in range 2. The reason is that the increased degree of asymmetry reduces the incentive of the larger firm to advertise while raise the incentive of the smaller firm to advertise. A less intuitive result is for the price distributions. As $\theta$ increases, in range 1, firm 1 will shift its price distribution to the right; in range 2, however, firm 1 will shift its price distribution to the left. The reason for this result is as follows. The direct effect of increase of the degree of asymmetry will make firm 1 charge a higher price and firm 2 charge a lower price. However, if firm 2 reduces its price, it will have an indirect effect on firm 1 to cause firm 1 to reduces the price in equilibrium. The final results of the comparative statics will be dependent on which effect has a larger magnitude. In range 1, the direct effect on firm 1 is larger than the indirect effect so firm 1 will raise its average price. In range 2, the direct effect on firm 1 is smaller than the indirect effect so
firm 2 will lower its price.

4 Information Gatekeeper’s Optimal Fee Decision

Foreseeing the firm’s behavior given the fixed advertising fee, $\phi$, the gatekeeper will set the value of $\phi$ to maximize its own profit in the equilibrium. The gatekeeper has a fixed setup cost, $K$, which is assumed to be zero without loss of generality. Then the expected profit the gatekeeper get is:

$$E\pi_G = (\alpha_1 + \alpha_2)\phi.$$  \hfill (24)

While choosing the optimal advertising fee, the gatekeeper faces the trade-off between profit per advertising activity and the frequency of advertising. Higher fee will increase the profit per advertising activity but decrease the frequency of advertising. Proposition 4 describes the gatekeeper’s best strategy.

**Proposition 4** Assume the gatekeeper can set the advertising fee $\phi \in [0, rS_2]$ to maximize its profit. Then at equilibrium, the optimal fee of the gatekeeper is:

1. When $3L_1 \leq 1 + 2L_2$,

   $$\phi^* = \frac{rS}{4}.$$  \hfill (25)

   The market will be within range 2 as described in proposition 3. And the expected profit of the gatekeeper is:

   $$E\pi_G = \frac{1}{4} rS \frac{L_1 + S}{L_2 + S}.$$  \hfill (26)

2. When $3L_1 > 1 + 2L_2$,

   $$\phi^* = \frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2}.$$  \hfill (27)

   The market will be at the boundary of range 1 and range 2 described in proposition 3. And the expected profit of the gatekeeper is:

   $$E\pi_G = rS \frac{2(1 - L_2)(L_1 - L_2)}{(1 + L_1 - 2L_2)^2}.$$  \hfill (28)

**Proof.** See Appendix. □

Figure 3 demonstrates the optimal strategy of the gatekeepers in the loyal size space $L_1 - L_2$. The shaded area represents all the possible combinations that $L_1$ and $L_2$ can take, given the restriction that $L_1 \geq L_2$ and $L_1 + L_2 < 1$. The closer the pair $(L_1, L_2)$ is to the 45-degree line, the less asymmetric is the market. When the pair is in the vertical shaded area, the optimal fee for the gatekeeper is $\frac{rS}{4}$. And when the pair is in the horizontal shaded area, the optimal fee for the gatekeeper is $\frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2}$. When $3L_1 > 1 + 2L_2$, we have $\frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2} > \frac{rS}{4}$, which implies that as the market gets more asymmetric, the gatekeeper will charge higher advertising fee. The primary reason for this is that as the market gets more asymmetric, firm 2’s willingness to advertise increases and the gatekeeper increases the advertising fee to extract more profit from firm 2.
Figure 3: Illustration of Gatekeeper’s Optimal Fee Decision

5 Conclusions

The paper studies the equilibrium behavior of the firms and the information gatekeeper in a duopoly market with asymmetric sizes of loyal consumers. Our findings show that the asymmetry in the size of loyal consumer base does affect firms’ advertising and pricing strategy. In equilibrium, the firm with more loyal consumers tends to advertise less frequently but charge a lower average price whenever advertising. The reason it advertises less frequently is due to the higher opportunity cost by advertising and charging price that is lower than the reservation price. When it decides to advertise, however, it can afford to charge a lower price since it have a larger consumer base.

The gatekeeper’s behavior will also be affected by the asymmetric structure of the market. The optimal advertising fee charged by the information gatekeeper will depend on the degree of asymmetry of the market. As the market becomes more asymmetric, the gatekeeper will charge a higher advertising fee. The reason is that as the market becomes more asymmetric, the firm with less loyal customers is willing to pay a higher amount to compete for the shoppers.
References


6 Appendix

6.1 Proof of Proposition 3

Case 1

For case 1, we have \( \alpha_2 = 1 \) and \( 0 < \alpha_1 < 1 \). Therefore, firm 1 must be indifferent between advertising and not advertising, i.e. \( E\pi_1^A = E\pi_1^N(p) \). If we plug in the expression for the profits, we have

\[
p(L_1 + (1 - F_2(p))S) - \phi = L_1 r. \tag{29}
\]

Solve this gives the expression for \( F_2(p) \) as in Proposition 3. Plug in the limits value for the domain of \( F_2(p) \), we have \( F_2(p_1) = 0 \) and \( F_2(r) = 1 - \frac{\phi}{rS} < 1 \). This implies that firm 2’s price distribution has mass point at the reservation price \( r \). Therefore it must hold for firm 2 that: (1) \( E\pi_2^A(p_1) = E\pi_2^A(r) \), (2) \( E\pi_2^A(p) = E\pi_2^A(p_1) \).

Plugging in the profit expression gives us:

\[
\begin{align*}
p_1(L_2 + s) - \phi &= r(L_2 + (1 - \alpha_1)s) - \phi, \tag{30} \\
p(L_2 + (1 - \alpha_1 F_1(p))s) - \phi &= p(L_2 + s) - \phi. \tag{31}
\end{align*}
\]

Solving these two equations yields the solution for firm 1’s strategy in Proposition 3. It is easy to verify that \( F_1(p_1) = 0 \) and \( F_1(r) = 1 - \frac{\phi}{rS} \), which simplifies to \( \phi < rS \frac{L_1 - L_2}{L_1 + L_2} \). Since \( \frac{L_1 - L_2}{L_1 + L_2} < \frac{1}{2} \), the equilibrium that we characterized will only exist under a limited range of listing cost.

The equilibrium profit can be calculated as: \( E\pi_1 = E\pi_1^N \), \( E\pi_2 = E\pi_2^A(p) \).

The above proof for Case 1 is also valid when \( \phi = 0 \).

Case 3

For case 3, we have \( 0 < \alpha_i < 1 \). This can happen only when \( p_1 = p_2 \), which is equivalent to:

\[
\begin{align*}
\frac{rL_1 + \frac{1}{2} rS(1 - \alpha_2) + \phi}{L_1 + S} &= \frac{rL_2 + \frac{1}{2} rS(1 - \alpha_1) + \phi}{L_2 + S}. \tag{33}
\end{align*}
\]

Since both firm have mixed strategies in pricing, both firms will have positive probability density at the lowest price boundary, we have \( E\pi_1^A(p) = E\pi_1^A(p_1) \) and \( E\pi_2^A(p) = E\pi_2^A(p_2) \), which give us:

\[
\begin{align*}
pL_1 + pS(1 - \alpha_2 F_2(p)) - \phi &= p_1(L_1 + S) - \phi \tag{34} \\
pL_2 + pS(1 - \alpha_1 F_1(p)) - \phi &= p_2(L_2 + S) - \phi \tag{35}
\end{align*}
\]
Until now in our model, we are not sure which firm will have a positive density at r. There are three possibilities: (1) firm 1 has a mass point (2) firm 2 has a mass point (3) neither firm has a mass point so at equilibrium both firm will have a positive density at r.

If firm $i$ has positive probability density at r, then the condition $E\pi^A_i(r) = E\pi^A_i(p_i)$ has to hold. So we have:

$$rL_i + rS(1 - \alpha_j) - \phi = p_i(L_i + S) - \phi,$$

which yields

$$\alpha_j = 1 - \frac{2\phi}{rS}$$

First we argue that the third possibility can not happen. If both firm have positive density at r, then (37) will hold for both firms. Therefore we have $\alpha_1 = \alpha_2$. If we plug this into the expression for $p_1$, $p_2$, we would find $p_1 \neq p_2$ as long as $L_1 \neq L_2$, which contradicts with (33).

Now we consider possibility 1, if firm 1 has a mass point at r, then (37) will be valid for firm 1. Basically, the system of equilibrium strategies is fully described by equations (33),(34),(35), and (37) for firm 1.

We can solve for $F_1(p)$:

$$F_1(p) = \frac{1}{\alpha_1}(1 - rL_2 - pL_2 + \frac{1}{2}rS(1 - \alpha_1) + \phi) \frac{pS}{pS - 2\phi(2L_2 + S - L_1)}$$

$F_1(p_1) = 0$. However, $F_1(r) = \frac{S + L_2}{S + 2L_2 - L_1} > 1$. This contradicts with our assumption that firm 1 will have mass point at r. So possibility 1 is not a valid solution for equilibrium.

Now lets consider possibility 2, this time the system of equilibrium strategies is fully described by equations (33),(34),(35), and (37) for firm 2. Solving the systems of equations gives the following equilibrium solution as described in Proposition 3.

Notice that the solution satisfies required properties. It is obvious that $0 \leq \alpha_1 \leq 1$. When $\phi > \frac{rS}{S + 2L_2 - L_1}$, we have $0 \leq \alpha_2 < 1$. And it can be verified that $F_1(p_1) = 0$, $F_1(r) = 1$, $F_2(p_2) = 0$ and $F_2(r) = \frac{L_1 + S}{S + 2L_2 - L_1} < 1$. Therefore, at equilibrium, firm 2 will have mass point at r and firm 1 will not have positive density at r.

And the expected profits can be calculated as: $E\pi_1 = E\pi_1^{NA}$, $E\pi_2 = E\pi_2^{NA}$ since both firms are mixing in advertising in equilibrium.

### 6.2 Proof of Proposition 4

Since there are two ranges in our equilibrium. The approach of this proof is to find the optimal fee within each range, then to choose the larger one of the two local maxima.

The gatekeeper’s expected profit:

$$E\pi_G = (\alpha_1 + \alpha_2)\phi.$$  

In range 1 of the equilibrium, we have:

$$E\pi_G = \frac{L_2 + S}{L_1 + S}(1 - \frac{\phi}{rS}) + 1\phi.$$
This function is concave in $\phi$. Solving the first order condition w.r.t. $\phi$ gives the optimal fee, $\phi^* = \frac{rS}{2} (1 + \frac{L_1 + S}{L_2 + S}) > \frac{rS}{2}$. This optimal fee is out of the domain where range 1 equilibrium is sustained. So we have a corner solution for range 1:

$$\phi^1 = \frac{rS(L_1 - L_2)}{(1 + L_1 - 2L_2)},$$

with the corresponding profit:

$$E\pi^1_G = rS \frac{2(1-L_2)(L_1-L_2)}{(1+L_1-2L_2)^2}.$$  

In range 2 of the equilibrium, we have

$$E\pi_G = \frac{rS - 2\phi}{rS} \frac{2L_1 + S - L_2}{L_2 + S} \phi.$$  

The first order condition w.r.t. $\phi$ yields the local maximum:

$$\phi^* = \frac{rS}{4}$$  

Therefore, if $\frac{rS}{4} \geq rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$, i.e., $3L_1 \leq 1 + 2L_2$, the optimal fee in range 2 is: $\phi^2 = \frac{rS}{4}$. And the corresponding profit $E\pi^2_G = \frac{1}{2} rS \frac{L_1 + S}{L_2 + S} > E\pi^1_G$. Therefore, $\frac{rS}{4}$ is the advertising fee that global maximum is achieved. On the other hand, if $\frac{rS}{4} < rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$, i.e., $3L_1 > 1 + 2L_2$, we have $\phi^2 = \frac{rS(L_1 - L_2)}{(1 + L_1 - 2L_2)}$, which is the same solution as in range 1. As a result, it will also be the global maximum.

Summarizing the above arguments leads to the results in Proposition 4.
Estimation of Price Adjustment Cost Using Pricing Data

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Abstract

This paper examines an n-firm model of oligopoly pricing in online markets with heterogeneous marginal costs, costly price adjustments, and temporal cost shocks. The equilibrium of the model exhibits three characteristics: (1) spatial price dispersion; (2) temporal price dispersion; and (3) price rigidity. I propose a method to structurally estimate the model via a simulated maximum likelihood approach that requires only pricing data. This approach yields estimates of parameters, including price adjustment costs and the price sensitivity of consumers. Monte-Carlo results reveal that the proposed approach yields accurate and efficient estimates when the price adjustment frequencies generated by the true parameters are not too low. Additionally, the simulations suggest that when the initial values are randomly chosen, the numerical method must be iterated multiple times in order to recover reliable estimates of the parameters.

1 Introduction:

Price rigidity is well documented in the micro-economics literatures. For example, Carlton (1986) presented evidence of existence of price rigidity in individual transaction prices using Stigler-Kindah Data. Borenstein, Cameron and Gilbert (1997) showed that retail gasoline prices respond more quickly to increases than to decreases in crude oil prices. Hannan and Berger (1991) used data in the banking industry and found that price rigidity exists and is greater in market with higher level of concentration. Even in the online market which is considered to have much less friction than the traditional market, price rigidity is commonly observed. Baye and Arbatskaya (2004) studied the mortgage rates listed on website and found out that quoted price online are surprisingly rigid in the sense that many firms do not respond to cost shocks by adjusting their prices promptly. Bergen, Kauffman and Lee (2005) documented price rigidity of the online book market and also discovered different price adjusting behavior between Amazon and Barnes & Noble.

There are different theoretical explanations for price rigidity. Such as unobservable product quality (Allen, 1988), inventory policies (Amihud and Mendelson, 1983), and long-term contracts. A more dominant explanation, however, attributes price rigidity to the existence of price adjustment cost, or menu cost. The literatures include Barro (1972), Sheshinski and Yoram (1977, 1985), Rotemberg (1983) and Mankiw(1985). In terms of empirical evidence of price adjustment cost, Lach and Tsiddon (1996) presented research on the pricing behavior of multi-
product firms and discovered that the timing of the price changes of different products sold within the same store is highly synchronized, as the result of firms’ effort trying to reduce price adjustment cost.

In the literature, there are several attempts to measure the magnitude of adjustment cost. Levy et. al. (1997) measures the menu cost of supermarkets by directly calculating different component of the cost, such as labor cost of changing shelf prices, the cost of printing price tags, etc. Slade (1999) structurally estimates the price threshold for price adjustment in a dynamic oligopoly framework. Willis (2000) proposed a structure model that directly estimate the price adjustment cost in a dynamic monopolistic competition framework, using magazine prices.

Until now, there are very limited effort in measuring adjustment cost in an online market, where the price adjustment cost is deemed to be low compared to the traditional market. Therefore, the purpose of this paper is to propose an methodology to structurally estimate the price adjustment cost of the sellers in an online environment.

The paper first examines an n-firm oligopoly pricing model with logistic demand and heterogeneous cost. Logistic demand is based on the assumption that the firms provide horizontally different services and the consumers make their choices based on price and different preferences on services. With the logistic demand assumption, a pure strategy equilibrium exists in which the spatial price dispersion is generated by cost heterogeneity.

Then, the paper proposes an method to use maximum likelihood method to empirically estimate the price adjustment cost using only price data. The likelihood function is constructed based on the assumption that (1) Firms make pricing decisions to maximize the profit with the presence of price adjustment cost. (2) There is a cost shock to the firms each period to induce incentive for price change.

The results from Monte-Carlo experiments using different combination of parameter values shows that the method is able to recover the true value if (1) the simulated price data generated by the parameter value have adequate number of price adjustment. (2) The initial value of the numerical optimization is close to the true value of the parameters. When the initial value is not in the neighborhood of the true parameters, the numerical method is likely to stuck in local maximum and it would take multiple iterations for the estimates to converge to the true optimum.

In the rest of the paper, section 2 describes and solves the theoretical model. Section 3 adds stochastic structure to the model and describe the estimation strategy. Section 4 discusses the results using Monte-Carlo simulation. Section 5 concludes.

2 Static Model

In a market with homogeneous product, there are $N$ firms competing for a pool of consumers with size normalized to 1. I assume that the consumers have access to price information of all the firms in the market. In the real world, firms competing for shoppers visiting a price comparison website would be a good example. After obtaining all price information, a consumer makes choice on the firm to purchase from by maximizing its utility, which is formulated as:

$$u_{li} = v - \delta p_i + \epsilon_{li},$$  \hspace{1cm} (1)

where $l$ is the index for consumer and $i$ is the index for firm. $v$ is the utility of consuming the product by the consumer, which is assumed to be the same across consumers and across firms since this is a homogeneous
product. $\delta$ can be interpreted as the price sensitivity of the consumer, the larger $\delta$ is, the more utility consumer gains by switching from a higher priced firm to a lower priced firm. $\epsilon_{li}$ represents the utility the consumer derives from the features from the store that are unobservable for econometricians. In this paper, I assume that $\epsilon_{li}$ are i.i.d distributed with extreme type I distribution. Then it is straight forward to derive the probability consumer $l$ choose to buy from firm $i$, which is also the demand function for firm $i$ since the size of consumer base is normalized to 1:

$$Q_i(p) = \frac{\exp(-\delta p_i)}{\sum_{j=1}^{n} \exp(-\delta p_j)}$$  \hspace{1cm} (2)

The above demand function is first proposed by Anderson and Palma (1992). The the i.i.d random term $\epsilon_{li}$ represents the horizontally different features and services provided by different firms in the market. Although the firms are selling a homogeneous product, to avoid pure price competition that drives price to marginal cost, they can differentiate themselves by providing different features and services, such as different website interface, store channel, shipping method, etc. Also, consumers who have previous purchasing experience with certain store may prefer (or not) continuing to purchase from the same store if the past experience is good (bad). This demand structure has two advantages over the Bertrand assumption. First, it provides an explanation for the fact that firms charging higher than the lowest market price are able to sell product to consumers. Second, this demand structure makes it possible that a pure strategy equilibrium exists in which firms price above marginal cost. While in Bertrand demand structure, the equilibrium in which firms make positive profit only exists in mixed strategy.

The price elasticities of demand can be derived directly from the demand function.

$$\epsilon_i = \frac{\partial Q_i}{\partial p_i} \frac{p_i}{Q_i} = -\delta p_i \frac{\sum_{j\neq i} \exp(-\delta p_j)}{\sum_{j} \exp(-\delta p_j)}$$  \hspace{1cm} (3)

$$\epsilon_{ij} = \frac{\partial Q_i}{\partial p_j} \frac{p_j}{Q_i} = \delta p_j \frac{\exp(-\delta p_j)}{\sum_{j} \exp(-\delta p_j)}$$  \hspace{1cm} (4)

For simplicity, I assume that there are no vertical differences among the firms in the sense that if two firms are charging the same price, they will have equal demand in the market. As a matter of fact, vertical difference can be added to the demand function by incorporating observable store specific characteristics into the consumer’s utility function. The incorporating of vertical difference will be explored in future extension of this paper.

The marginal cost of supplying the product, $c_i$, is assumed to be different among firms. Although the firms might incur the same cost purchasing from the manufacturer, the differences in firm’s size, management efficiency and retail channel will result in differences in final cost of the firm.

At each point of time, firm $i$ makes price decisions to maximize its profit with absence of the price adjustment cost:

$$\pi_{i,t}(p_{i,t}) = (p_{i,t} - c_i) \frac{\exp(-\delta p_{i,t})}{\sum_{j=1}^{n} \exp(-\delta p_{j,t})}, \quad i = 1, ..., N$$  \hspace{1cm} (5)

and the first order conditions are:

$$p_{i,t} - c_i = \frac{1}{(1 - Q_{i,t}(p))\delta}, \quad i = 1, ..., N$$  \hspace{1cm} (6)
where

\[ Q_{i,t} = \frac{\exp(-\delta p_{i,t})}{\sum_{j=1}^{n} \exp(-\delta p_{j,t})} \]  

(7)

The system of equations (6) characterizes the equilibrium strategy of all the firms in the market. Anderson (1992) has provided the proof that there exist a pure strategy equilibrium. Basically, the proof showed that the profit functions are locally concave at the extreme location and therefore quasi-concave. Therefore, by Debreu (1952) [9], pure strategy equilibrium exists.

The left hand side of the first order condition (6) is the profit margin for firm \( i \). So this equation implies that, in equilibrium, the firm that charges lower price (thus has higher demand) also has higher profit margin, therefore it must has lower cost. Therefore we have:

**Proposition 1** In the oligopoly model in which firms competing in price and the profit function is characterized by Equation (5), there exists a pure strategy equilibrium in which firm with lower marginal cost will charge lower price and vice versa.

The equilibrium result of this model provides another theoretical basis for ubiquitously observed price dispersion in which consumers have complete price information of the market. Similar to Spulber (1995) [14], the price dispersion in the market is caused by cost heterogeneity of the firms and firms make positive profit in the equilibrium. The difference is that the marginal costs information is public information in this paper while it is private information in Spulber (1995). Assuming that the cost information is public is not unreasonable since firms can infer the costs of the other firm from historical prices charged in the market. The reason that the firms can make positive profit is horizontal differentiation in services, which allows firm to sell product to consumers even it is not charging the lowest price in the market.

### 3 Dynamic Model

The static model presented in the previous section provides theoretical foundation for spatial price dispersion in a homogeneous product market. In this section, I enrich the model to capture other characteristics in observed pricing data: temporal price dispersion and price rigidity. The enrichment is achieved by introducing price adjustment cost and shocks in marginal cost, and adding dynamics such that firm’s pricing decision is based on observed pricing information and realized cost structure.

First, instead of assuming that all firms making price decision simultaneously by solving a N-player pricing game, I consider an alternative assumption, which assumes that when each firm makes its price decision at each period, its expected profit is conditional on its own cost and the prices on the market it observed during the last period, i.e.,

\[ \pi_{i,t}(p_{i,t}) = (p_{i,t} - c_{i,t}) \frac{\exp(-\delta p_{i,t})}{\sum_{j\neq i} \exp(-\delta p_{j,t-1}) + \exp(-\delta p_{i,t})} \]  

(8)

With this assumption, each firm is solving its own optimization problem at each time period \( t \), without considering the possibility of price adjustment of other firms. There are two major reasons why this simplification assumption is not unreasonable. First, as mentioned above, the price adjustment frequency is relatively low.
Therefore, the price decision based on price information at $t-1$ should be very close to optimal since the realized price structure at $t$ is very similar to the price structure at $t-1$. Second, for popular product, there usually are dozens of firms competing for the shoppers, the decision of a single firm should have very small impact on the market and the other firms profit. What’s more, we can prove that the repeated optimizing behavior of single firm will lead the market to the equilibrium defined by the system equations of the static game described by Equation (6).

**Proposition 2** if the each firm keep adopting the the price strategy optimizing the expected profit as characterized by equation (8), the market will finally converge to the equilibrium of the simultanous move game as characterized by the Equation (6)

**Proof.** See Appendix ■

The first order condition for the new model should be re-written as

$$p_{i,t} - c_i = \frac{1}{(1 - Q_{A,i,t}(p))\delta}, \quad (9)$$

where

$$Q_{A,i,t}(p) = \frac{\exp(-\delta p_{i,t})}{\sum_{j \neq i} \exp(-\delta p_{j,t-1}) + \exp(-\delta p_{i,t})} \quad (10)$$

or

$$p_{i,t} - c_{i,t} = \frac{1}{\delta} + \frac{\exp(-\delta p_{i,t})}{\sum_{j \neq i} \exp(-\delta p_{j,t-1})\delta}, \quad i = 1, ..., N \quad (11)$$

Solving the equations in (11) is much less computational intensive than solving the system equations in (6) since each equation in (11) can be numerically solved individually.

**Price Adjustment Cost**

The result of the equilibrium characterized by equation (11) above suggests that a firm will adjust its price if there is change in market condition such as marginal cost, number of firms and prices charged by other firms. However, in real life, firms response to the changes in a much slower rate. As documented in many studies, such as Carlton (1986), Baye and Abatskaya (2004) and Bergen, Kauffman and Lee (2005), Price rigidity exists broadly in different types of markets for different products. Also, in the price data I collected from a price comparison site during a 189-day period, the overall price adjustment frequency is only 7%. In an online environment where price information is readily available each day, the frequency is relatively low.

To capture the dynamics of the price adjustment behavior of the firms, I assume that there is a fixed cost of price adjustment, $\theta$, that a firm need to incur whenever it changes its price relative to the previous period. In reality, $\theta$ could include the cost of managerial effort to collect information and making price decisions, the cost of the labor to change the price listings, and the potential cost when mistakes are made while adjusting prices.

Therefore, when a firm makes price decision each period, it must weigh the potential benefit and cost of adjusting prices. The price adjustment is profit improving only when the benefit of price adjustment (increased profit from sales) is higher than the cost of price adjustment.
Then the firm’s optimal pricing at time $t$ is:

\[
\begin{cases}
    p_{i,t}^*, & \text{if } \pi_{i,t}(p_{i,t}^*) - \pi_{i,t}(p_{i,t-1}) > \theta \\
    p_{i,t-1}, & \text{if } \pi_{i,t}(p_{i,t}^*) - \pi_{i,t}(p_{i,t-1}) \leq \theta
\end{cases}
\]

where $p_{i,t}^*$ is the optimal solution to the first order condition (11).

**Temporal Shock of Marginal Cost**

As shown in previous section, if the pricing game is played repeatedly, the market will finally converge to a steady-state equilibrium. With price adjustment cost, the steady state is such that each firm charges a price close enough to the equilibrium price and has no incentive to adjust further. To better suit the model to the real price observations, and also for empirical estimation purposes, stochastic structure needs to be added into the model.

To add stochastic structure into the model, I assume that, at each period, the firm faces i.i.d random cost shock that are normally distributed with mean 0 and variance $\sigma^2$, i.e.,

\[
c_{i,t} = c_i + \epsilon_{i,t}, \quad (12)
\]

where $\epsilon_{i,t} \sim N(0, \sigma^2)$. The source of this cost shock could be change of wholesale price, opportunity cost of inventory, change of operational cost, etc\(^1\).

The existence of cost shock to the firms is able to explain the observed price adjustment/rigidity in many oligopoly product market. In this setup, there are two causes of price adjustment. First, when the cost shock is large enough, a firm might find that the optimal price under the new cost yields significant higher profit than the current price, and thus adjusts its price. Second, the price adjustment of the other firm will change the market price and thus induce the firm to change its own price.

The assumption of heterogeneous average marginal cost and temporal cost shock is also related to the concept of spatial price dispersion and temporal price dispersion. Spatial price dispersion refers to the fact that different firms charge different prices at the same point of time, but the relative price ranking does not change over time. While temporal price dispersion refer to the fact that the same firm could charge different prices from the price distribution and its rank among different firms could change. With heterogeneous cost, spatial price dispersion arises in equilibrium where firm with lower marginal cost charges lower price. The cost shock, on the other hand, causes the firms to adjust price over time. If the size of cost shock is larger than the difference in average cost among firms, temporal price dispersion will arise.

### 4 Model Estimation Methodology

Next, I will construct the likelihood function for the parameters in the model $\{\delta, \sigma, \theta, C\}$, based on the assumption in the distribution of cost shocks.

Denote the observed data as $\{p_{i,t}, a_{i,t}\}_{i=1,\ldots,N, t=1,\ldots,T}$, where $p_{i,t}$ is the observed price and $a_{i,t}$ is the price adjustment indicator, i.e.,

\(^1\) Another way to introduce shock is to assume there is a shock to the demand function.
\[
\begin{cases}
    a_{i,t} = 1 \text{ if } p_{i,t} \neq p_{i,t-1} \\
    a_{i,t} = 0 \text{ otherwise.}
\end{cases}
\]

Real observed data are unbalanced panel since firms enter and exit the market during the sampling period. For notational simplification, I will assume balanced panel during the derivation. All the results can be applied to unbalanced panel data.

The pricing data observed online are similar to censored data with unknown threshold. When the potential gain of the price adjustment is larger than the cost of price adjustment cost, the firm makes the price adjustment and the adjusted price is observed. The adjustment price is the optimal price of the firm under the market condition and therefore can be used to infer the parameters of the model. On the other hand, if the potential gain of the price adjustment is smaller than the adjustment cost, then the firm will keep charging the old price and the optimal price is not observed.

The density function of \( \{a_{i,t}, p_{i,t}\} \) conditional on the parameters are:

\[
f(a_{i,t}, p_{i,t}|p_{t-1}, \delta, \theta, \sigma, C) = f(p_{i,t}|p_{t-1}, \delta, \theta, \sigma, C) f(a_{i,t}|p_{i,t}, p_{t-1}, \delta, \theta, \sigma, C)
\]

\[
= \begin{cases}
    f(p_{i,t}|p_{t-1}, \delta, \theta, \sigma, C) P[a_{i,t} = 1|p_{i,t}, p_{t-1}, \delta, \theta, \sigma, C], \text{ if } a_{i,t} = 1 \\
    P[a_{i,t} = 0|p_{t-1}, \delta, \theta, \sigma, C], \text{ if } a_{i,t} = 0
\end{cases}
\]

This function is similar to the likelihood function give in Carson and Sun (2007)[8] for Tobit model with unknown threshold. The added complexity of this paper is that the density is based on the structural model in which firms optimize their profit, rather than the linear reduced form in standard Tobit model.

When there is a price adjustment, we know the adjusted price is the optimal price to the market conditions. The distribution of the optimal price can be derived using the assumed distribution of the cost shock \( \epsilon_{i,t} \) and the relationship between \( p_{i,t} \) and \( \epsilon_{i,t} \) as characterized by Eq. (11):

\[
f(p_{i,t}|p_{t-1}, \delta, \theta, \sigma, C) = f_{\epsilon}(p_{i,t} - c_{i} - \frac{\exp(-\delta p_{i,t})}{B_{i,t} \delta} - \frac{1}{\delta})(1 + \frac{\exp(-\delta p_{i,t})}{B_{i,t}}),
\]

where \( f_{\epsilon} \) is the pdf of \( \epsilon_{i,t} \), which is normally distributed with mean 0 and variance \( \sigma^2 \), and \( B_{i,t} = \sum_{j \neq i} \exp(-\delta p_{j,t-1}) \).

The details of the derivation is included in the Appendix.

With the optimal price being observed, the cost shock can be calculated using Eq. (11) given particular set of parameter values. Then the exact profit increase by adjusting price can be calculated and directly compared with the price adjustment cost \( \theta \). Therefore,

\[
P[a_{i,t} = 1|p_{i,t}, p_{t-1}, \delta, \theta, \sigma, C] = 1\{\pi_{i,t}(p_{i,t}) - \pi_{i,t}(p_{i,t-1}) > \theta\}
\]

When there is no price adjustment, the density function equals to the probability that the difference between the optimal profit and profit of the current price is smaller than the adjustment cost, i.e.,

\[
P[a_{i,t} = 0|p_{t-1}, \delta, \theta, \sigma, C] = P[\pi_{i,t}(p_{i,t}^*) - \pi_{i,t}(p_{i,t-1}) \leq \theta]
\]
Since there is no closed form solution for the function \(\pi_{i,t}\), we can not derive the distribution of \(\pi_{i,t}\) from the distribution of \(\epsilon_{i,t}\). Therefore, a simulation process is needed to approximate the probability.

\[
P^s[a_{i,t} = 0|p_{t-1}, \delta, \theta, \sigma, C] = \frac{1}{S} \sum_{s=1}^{S} \mathbb{1}\{\pi_{i,t}(p_{i,t}^s|\epsilon_{i,t}^s) - \pi_{i,t}(p_{i,t-1}|\epsilon_{i,t}^s) \leq \theta\}
\]  

(17)

and \(\pi_{i,t}(p_{i,t}^s|\epsilon_{i,t}^s)\) needs to be solved numerically since there is no closed form solution.

Plug (14), (15) and (17) into (13) will give the full density function for firm \(i\) at time \(t\). If we assume that the cost shocks are independently distributed across firms and across time, then the conditional distributions of \(p_{i,t}, a_{i,t}\) are also independent. And the likelihood function is:

\[
L(\delta, \theta, \sigma, C) = \prod_{i,t} \left( f(p_{i,t}|p_{t-1}, \delta, \theta, \sigma, C) P[a_{i,t} = 1|p_{i,t}, p_{t-1}, \delta, \theta, \sigma, C] \right)^{a_{i,t}} \cdot \left( P^s[a_{i,t} = 0|p_{t-1}, \delta, \theta, \sigma, C] \right)^{1-a_{i,t}}
\]  

(18)

The estimation process will search for the parameters \((\delta, \theta, \sigma, C)\) to maximize the likelihood function. Notice that the likelihood function is not differentiable in the parameters since Eq. (13) and Eq. (15) are not differentiable in \(\theta\). Therefore, instead of using the gradient based method such as BHHH, I used the Nelder-Mead method, which is a simplex algorithm that search for maximum/minimum value iteratively.

In applications, the log of likelihood instead of the likelihood is maximized. For some parameter values (e.g. when \(\theta\) is large), Eq (15) could take the value of 0. Since the log of zero is infinity, to make sure the algorithm does not terminate abnormally, I assign a large negative number \(\eta\) to the log-likelihood when the value of Eq (15) equals to 0.

In reality, the dimension of the parameter \(C\) can be pretty large due to the fact that (1) for a popular product, there are many firms competing in the market at the same time, (2) firms enter and exit the market during the sample collection period so that the actual number firms is even larger. To increase the efficiency of the calculation, I made assumption to simplify the estimation of \(C\). I assume that when the firm first enters the market, the price is optimal and the cost shock \(\epsilon = 0\). With this assumption, the cost of each firm can be recovered by using the price information when the firm enters the market and do not need to be estimated as independent parameters.

5 Estimation Result with Simulated Data

In this section, I test the viability of the proposed model and estimation method by using Monte Carlo simulated data with different sets of parameters.

For simplicity, I assume that in the simulated data, there are a constant number of firms \(N\) in the market and competing for \(T\) period. And there is no firm entering or exiting the market. The simulation process is as follows:

1. Choose a set of true values for the parameters \(\{\delta, \theta, \sigma, C\}\).

2. At \(t = 1\), assume that the market is at the equilibrium status with no cost shock. Solve the pricing game numerically as characterized by Eq (6).
3. At each time point $t \geq 2$, generate a cost shock $\epsilon_{i,t}$ for each firm $i$, and solve for the optimal price strategy $p^*_{i,t}$ using Eq (11).

4. Compare the increase of profit by changing $p^*_{i,t}$ instead of the old price $p_{i,t-1}$. If it is larger than $\theta$, then set $p_{i,t} = p^*_{i,t}$, otherwise set $p_{i,t} = p_{i,t-1}$.

In the results presented below, I choose $N = 10$, $T = 100$. To test the robustness of the model under different market conditions, I picked a range of true values for the parameters.

For the marginal costs of the firm, I choose $C = \{184, 188, 192, 196, 200, 204, 208, 212, 216, 220\}$, with the marginal cost of each firm placed at equal distance.

For $\delta$, I choose four different values: $\delta_1 = 0.002$, $\delta_2 = 0.005$, $\delta_3 = 0.01$, $\delta_4 = 0.05$. According to Eq.(3), the self price elasticity is close to $-\delta p_i$ since $\frac{\sum_{j\neq i} \exp(-\delta p_j)}{\sum_j \exp(-\delta p_j)}$ is close to one for most firms when the number of firms is large. Therefore, the four values for $\delta$ approximately correspond to the price elasticity of 0.4, 1, 2 and 10, respectively.

Three different values are chosen for $\sigma$: $\sigma_1 = 2$, $\sigma_2 = 5$ and $\sigma_3 = 10$ and three different values are chosen for $\theta$: $\theta_1 = 0.005$, $\theta_2 = 0.02$ and $\theta_3 = 0.05$.

For each set of the parameters, I created 100 iterations of simulations. The parameters are estimated by maximizing the log-likelihood function. The parameters for each iteration. In the estimations, I set $S = 30$ and $\eta = -10000$.

Table 1-4 summarizes the result of the estimation under different true values of the parameters. For each set of true parameter values, several statistics are reported for the estimated values of main parameters $\delta, \sigma, \theta$ based on the estimation for the 100 iterations. The statistics include mean, minimum, maximum and standard deviation of the 100 estimates. Bias is defined as the difference between the mean value of the estimates and the true parameter value. Pseudo t is defined as the ratio between the mean and the standard deviation. For cost estimates, the mean and standard deviation are provided.

The accuracy and stability of the estimates are dependent on the combination of the true values. In general, the estimates tend to be less accurate when (1) $\delta$ gets smaller, (2) $\sigma$ gets smaller (3) $\theta$ gets larger. For example, in Table 2, we observe that in the case of $\delta = 0.01$, $\sigma = 2$ and $\theta = 0.02$, the estimates have large biases and larger standard deviation. The estimates become much more accurate and stable when the size of cost shock $\sigma$ gets larger or the adjustment cost $\theta$ gets smaller, as shown in other columns of Table 1. Similarly, when $\delta$ gets smaller as shown in Table 2, 3, 4, there are more cases in which the estimates become inaccurate and unstable.

To see why this happens, notice that the common effect of the trends in parameter value change mentioned above is that they all reduce the price adjustment frequency of the firms. When $\delta$ gets smaller, the consumers are less sensitive to the price difference between firms so the potential benefit of price adjustment gets smaller. When $\sigma$ gets smaller, there are less cost shocks that are larger enough to bring enough incentive for the firms to adjust prices. Finally, when the price adjustment cost $\theta$ gets larger, the firms will reduce the price adjustment frequency provided other things being equal. The estimation method becomes less accurate and unstable when price adjustment frequency is extremely low. As an extreme case, when there is no price adjustment observed, the method breaks down since the likelihood function becomes flat. Therefore, in Table 1-4, when there is no price adjustment in the simulated data for a certain parameter combination, the column is left empty.
<table>
<thead>
<tr>
<th>Estimates</th>
<th>Summary</th>
<th>( \delta = 0.05 )</th>
<th>( \sigma = 2 )</th>
<th>( \delta = 0.05 )</th>
<th>( \sigma = 5 )</th>
<th>( \delta = 0.05 )</th>
<th>( \sigma = 10 )</th>
<th>( \delta = 0.05 )</th>
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<td>0.0003</td>
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Table 1: Estimation Results Using Simulated Data (\( \delta = 0.05 \))
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<td>( \sigma = 2 )</td>
<td>( \sigma = 2 )</td>
<td>( \sigma = 2 )</td>
<td>( \sigma = 2 )</td>
<td>( \sigma = 2 )</td>
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<tr>
<td>Std Dev</td>
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Table 2: Estimation Results Using Simulated Data (\( \delta = 0.01 \))
## True Parameters

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<th>( \delta = 0.005 )</th>
<th>( \delta = 0.005 )</th>
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<td>0.0051</td>
<td>0.0051</td>
<td>0.0051</td>
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</tr>
<tr>
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<td>0.0006</td>
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<td>0.1867</td>
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<tr>
<td>( \theta ) Pseudo t</td>
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<td>54.2550</td>
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<td>3.5953</td>
<td>53.5280</td>
<td>48.6860</td>
<td>37.1070</td>
</tr>
</tbody>
</table>

| \( \hat{C}_1 \) Mean | 186.40 | 187.28 | 183.52 | 183.29 | 183.21 | 184.57 | 0.0143 |
| \( \hat{C}_2 \) Mean | 190.40 | 191.28 | 187.52 | 187.30 | 187.21 | 188.57 | 0.0143 |
| \( \hat{C}_3 \) Mean | 194.40 | 195.28 | 191.52 | 191.30 | 191.21 | 192.57 | 0.0143 |
| \( \hat{C}_4 \) Mean | 198.40 | 199.29 | 195.52 | 195.30 | 195.21 | 196.57 | 0.0143 |
| \( \hat{C}_5 \) Mean | 202.40 | 203.29 | 199.52 | 199.30 | 199.21 | 200.57 | 0.0143 |
| \( \hat{C}_6 \) Mean | 206.40 | 207.29 | 203.52 | 203.30 | 203.21 | 204.57 | 0.0143 |
| \( \hat{C}_7 \) Mean | 210.40 | 211.28 | 207.52 | 207.30 | 207.21 | 208.57 | 0.0143 |
| \( \hat{C}_8 \) Mean | 214.40 | 215.28 | 211.52 | 211.30 | 211.21 | 212.57 | 0.0143 |
| \( \hat{C}_9 \) Mean | 218.40 | 219.29 | 215.52 | 215.30 | 215.21 | 216.57 | 0.0143 |
| \( \hat{C}_{10} \) Mean | 222.40 | 223.28 | 219.52 | 219.30 | 219.21 | 220.57 | 0.0143 |

| Adj. Freq. | 4.80% | 0.00% | 0.00% | 47.43% | 13.74% | 0.57% | 71.69% | 47.81% | 25.70% |

Table 3: Estimation Results Using Simulated Data (\( \delta = 0.005 \))
<table>
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<th>Estimates Summary</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 5$</th>
<th>$\sigma = 10$</th>
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<td>0.0021</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\sigma$ Mean</td>
<td>1.6734</td>
<td>4.9747</td>
<td>9.9840</td>
</tr>
<tr>
<td>$\theta$ Mean</td>
<td>0.1187</td>
<td>0.0054</td>
<td>0.0052</td>
</tr>
<tr>
<td>$C_1$ Mean</td>
<td>595.19</td>
<td>196.23</td>
<td>159.84</td>
</tr>
<tr>
<td>$C_2$ Mean</td>
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<td>200.23</td>
<td>163.84</td>
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<tr>
<td>$C_3$ Mean</td>
<td>604.01</td>
<td>204.23</td>
<td>167.84</td>
</tr>
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<td>$C_4$ Mean</td>
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<td>208.23</td>
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<td>212.23</td>
<td>175.84</td>
</tr>
<tr>
<td>$C_6$ Mean</td>
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<td>216.23</td>
<td>179.84</td>
</tr>
<tr>
<td>$C_7$ Mean</td>
<td>619.87</td>
<td>220.23</td>
<td>183.84</td>
</tr>
<tr>
<td>$C_8$ Mean</td>
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<td>224.23</td>
<td>187.84</td>
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<tr>
<td>$C_9$ Mean</td>
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<td>191.84</td>
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<tr>
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<td>631.39</td>
<td>232.23</td>
<td>195.84</td>
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Adj. Freq. 0.01% 0.00% 0.00% 25.50% 0.46% 0.00% 56.87% 25.27% 4.62%

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<tr>
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<th>$\sigma = 0.02$</th>
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<td>0.0021</td>
<td>0.0021</td>
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<td>232.23</td>
<td>195.84</td>
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</tr>
</tbody>
</table>

Adj. Freq. 0.01% 0.00% 0.00% 25.50% 0.46% 0.00% 56.87% 25.27% 4.62%
When there are adequate price adjustment observed, as in 26 out of 36 combinations shown in the tables with price frequency larger than 4%, the estimates for $\delta, \sigma, \theta$ are very close to the true value and very stable.

One of the caveats of the Nelder-Mead method is that it occasionally gets stuck into a local maximum. Therefore, for the analysis above, I choose the initial value to be close to the true value of the parameters. In practice where real observed data is used, the Nelder-Mead method need to be iterated multiple times to make sure the estimates is not at local maximum. To be more specific, for each iteration, a new simplex is created with the best value of last iteration being one of the vertexes. To test the effectiveness of the iteration method for Nelder-Mead, I picked two sets of true parameter values and (1) use random number as the initial values (2) run 20 iterations for each of the 100 simulations. To determine if an estimates is the global maximum or stuck into a local maximum, I use the confidence interval based on the standard deviation calculated from Table 1 - 4. Specifically, if any of the estimates for $\delta, \sigma, \theta$ is three standard deviation away from the true value, the estimates is regarded as a local maximum instead of recovering the true value.

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<th>Iterations</th>
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<td>0.0005</td>
<td>0.0006</td>
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<td>9.9687</td>
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<td>Std Dev</td>
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<td>0.3127</td>
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<td>Std Dev</td>
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Table 5: Estimation Results Using Simulated Data when Initial Value is Randomly Picked

The first section of Table 5 shows the percentage of the 100 simulations having estimates within 3 standard deviation from the true value after $n$ iterations ($n = 1,...,20$). The percentage starts pretty low for both scenarios (29% for Scenario 1, 24% for Scenario 2 and 10% for Scenario 3) and increases significantly for the first five iterations. Within 20 iterations, the percentages raise above 70% for all three scenarios.

The second section of Table 5 lists the mean and the standard deviation at iteration 20, calculated based on the estimates that are within 3 standard deviation from the true value. With the estimates that stuck in local maximum taken out, the mean of estimates are very close to the true value and the estimates are also stable. The standard deviations are close to those given in Table 1 - 4, which indicates that as long as the estimates are
not stuck in local maximum, the performance of the estimation does not depend upon the choice of the initial value.

In practice, the fact that not all estimates converge to the global maximum after 20 iterations poses a challenge if real data are used. One method that can be used would be to use Bootstrap. For each Bootstrap sample, apply the Nelder-Mead method for 20 iterations. Then the estimates that fall into local maximum can be identified by plotting a histogram of the estimates for all the bootstrap samples.

6 Conclusion

The paper proposes an oligopoly pricing model with price adjustment cost and temporal marginal cost shock, which provides an explanation for (1) spatial price dispersion, (2) temporal price dispersion and (3) price rigidity. Then an empirical method is proposed to structurally estimate price adjustment cost which requires only pricing data.

The results of the Monte-Carlo study shows that the empirical method is able to recover the true values of the parameters. When the real value is unknown and the initial value of the numerical optimization has to be chosen randomly, the numerical method need to be iterated multiple times to give reliable results.

Proposed future work:

1. For the Monte-Carlo study, try larger number of repetitions.
2. Explore the possibility of allowing for heterogeneity in price adjustment cost among firms.
3. Adjust the model to fit the data with dynamic price trend.
References


Empirical Investigation of Price Adjustment Behavior in Online Digital Camera Market

Chenguang Li
Department of Economics
Indiana University
April 24, 2009

Abstract
The paper investigates the price adjustment behavior in online digital camera market using price data collected from PriceGrabber.com. The data provide evidences that support the existence of price adjustment cost in the online market: (1) relatively low price adjustment frequency (2) products that less popular have lower price adjustment frequency (3) products that are more expensive have lower price adjustment frequency. The paper then structurally estimates the price adjustment costs for different products.

1 Introduction

2 Data
The pricing data were collected from Pricegrabber.com. I picked the top 50 most popular digital cameras ranked by Pricegrabber.com as of Sept. 25, 2007, and collected the daily price listings for each camera from Sept. 25, 2007 to Mar. 31, 2008. The final dataset consists of the 189 days of prices for all the 50 products, for a total of 209,277 prices.

Table 1 summarizes the price listings by products. The price data exhibit ubiquitous price dispersion in homogeneous product market that has been documented widely in the literature. For most of the products, the differences between the highest price and lowest price are very large. 38 out of the 50 products has coefficients of variation that are larger than 10%. The statistics calculated here, although might overestimating the price dispersion since the prices are pooled across different days, provides an evidence that price dispersion exist in the online market of homogeneous products.

There are a total of 303 different retailers among the price listings. These retailers are different in many aspects, especially their participation of price listings and price adjustment. At one end, the retailer that having the most price listings is TriStateCamera.com, with 6,418 price listings. At the other end, there are 81(26.7%) firms having less than 20 listings in the 189-days span. Table 2 reports the top 10 retailers in terms of number of listings in the collected data. Among the 10 firms, only Circuit City has a significant brick-and-mortar presence.
<table>
<thead>
<tr>
<th>Product ID</th>
<th>Product Name</th>
<th>N</th>
<th>Min</th>
<th>Range</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>CV</th>
<th>Average Price</th>
<th>Price Adj.</th>
<th>Market Size</th>
<th>Freq.</th>
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<td>5,628</td>
<td>607.99</td>
<td>746.96</td>
<td>814.66</td>
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<td>17.21%</td>
<td>30.45</td>
<td>3.74%</td>
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<td>500</td>
<td>746.96</td>
<td>746.96</td>
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<td>500</td>
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<td>711.69</td>
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<td>16.67%</td>
<td>45.40</td>
<td>14.96%</td>
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<td>6.03%</td>
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<td>25.78</td>
<td>5.46%</td>
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<td>43.32</td>
<td>18.75%</td>
<td>39.08</td>
<td>19.70%</td>
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<td>686.38</td>
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</tr>
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</table>

Table 1: Summary of Prices by Product
The other nine firms are all relying the online market for most of their sales although some of them do have a physical store.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Retailer Name</th>
<th>Number of Listings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TriStateCamera.com</td>
<td>6,418</td>
</tr>
<tr>
<td>2</td>
<td>Abe’s of Maine</td>
<td>5,539</td>
</tr>
<tr>
<td>3</td>
<td>B&amp;H Photo-Video</td>
<td>5,158</td>
</tr>
<tr>
<td>4</td>
<td>Buydig.com</td>
<td>5,118</td>
</tr>
<tr>
<td>5</td>
<td>PC Connection</td>
<td>5,034</td>
</tr>
<tr>
<td>6</td>
<td>PCNation.com</td>
<td>5,001</td>
</tr>
<tr>
<td>7</td>
<td>Butterfly Photo</td>
<td>4,986</td>
</tr>
<tr>
<td>8</td>
<td>Circuit City</td>
<td>4,870</td>
</tr>
<tr>
<td>9</td>
<td>TigerDirect</td>
<td>4,754</td>
</tr>
<tr>
<td>10</td>
<td>42nd Street Photo</td>
<td>4,689</td>
</tr>
</tbody>
</table>

Table 2: Top 10 Retailers in Number of Price Listings

Since price adjustment is the main topic of this study, the rest of the section will be focusing on price adjustment for the rest of this section. First, I look at the frequencies at which the retailers are changing their prices. The last column of Table 1 lists the price adjustment frequency for each product. Price adjustment is defined as the event when the price of the same retailer for the same product is different from the previous day. Therefore, if a price listing does not have the same firm for the same product for the previous day, it is excluded from the adjustment frequency calculation. Overall, I found that on average, retailers change their prices only 6.96% of the time. This relatively infrequent price adjustment behavior suggests the existence of price adjustment cost in the online market environment. There are two interesting patterns of the price adjustment frequencies among different retailers: (1) cameras that are more expensive (mostly digital SLRs) have lower adjustment frequencies. The 11 models that have adjustment frequency larger than 10% are all low-end point-and-shoot cameras (8 of them are Canon Powershot Series). The reason for this is that the targeted consumers of high-end cameras are much less price sensitive so that the retailers are less likely to use price strategy to increase sales/profit. (2) Price adjustment frequencies are lower for cameras that are less popular. For example, Canon Powershot A640 and Sony Cyber-shot DSC-T100 are low-end cameras. But they both have pretty low price-adjustment frequency. The reason for this is that both the cameras are less popular models with only 574 and 619 price listings, respectively. Similarly, among the higher end cameras, the more popular models such as Nikon 40D and Canon 350D have higher adjustment frequencies. The reason for this fact might be that while the price adjustment cost is about the same for different product, the potential benefit of adjusting price is smaller for products that are less popular.

Several papers in the literature have studied the relationship between market structure and price change frequencies. Rotemberg and Saloner (1987) created a theoretical model to explain why monopolists change the nominal price less often than oligopolists. While Fisher and Konieczny (1995) found out that oligopolists change prices less often than monopolists using Canadian Daily Newspaper data. I studied this relationship using the collected price data. Figure 1 is the scatter plot of the adjust frequency vs. average number of firms for each product. The plot shows that there is a slight upper trend between adjustment frequency and average number of firms, but the relationship is not strong.
After exploring the frequency of price adjustment, I switch the focus to the magnitude of price adjustment. I found out that the price adjustment magnitude increases with the average price of the product. As show in figure 2, this relationship holds pretty well for the 50 cameras. Therefore, to provide a better price adjustment measure across different products, I introduce relative price adjustment (RPA), which is defined as the price change divided by the price before price change.

Figure 3 shows the histogram of RPA with the price adjustments for all the products and all the firms pooled together. Relative price change is defined as the price change divided by the price before price change.

First, the histogram shows that there are more price decrease than price increases and price decreases are more concentrated in ranges with small magnitude. Second, the distribution of the relative price change is roughly symmetrically distributed with mean at -0.012. This suggests that the price adjustment could be caused by cost shocks that are symmetrically distributed. Third, the distribution has a high density in the relatively small adjustment range. A more detailed look at the price change shows that there are more than 789 price adjustments...
having magnitude less than 1 dollar. Given the existence of price adjustment cost, a natural question to ask is that why firms make price adjustments with such a small magnitude since the potential benefit of such price adjustment might be too small to exceed the adjustment cost. The possible reasons for this might be: (1) the small change will move the firm to the lowest pricing firm. Although firm with the lowest firm will not capture all the shoppers due to horizontal differences among the firms, the firm with lowest price will be listed first and have more advantages. (2) Firm change prices for the purpose of experimenting instead of profit maximizing. (3) the price adjustment cost is very small.

After studying the magnitude of the price adjustments, I turn to the timing of the price adjustment. First, I look at the durations of price adjustments. The duration for a price adjustment is defined as the number of days the adjustment is from the last price adjustment. For price adjustment that is the first after a price re-list, I use the number of days from the first price listing. Figure 4 plots the histogram of the price adjustment duration. The histogram peaks at 1 day and decreases gradually. The mean duration is about 1 week and more than 61% of the durations are less than 5 days. The fact the firms make their pricing decisions in relatively short intervals implies that the marginal price adjustment cost might be pretty small. The histogram does have a small peak at 7 days, which implies that some firm might set their price-adjustment interval as one week. However, this pattern is not very strong in our data.

3 Estimating Price Adjustment Cost

In this section, I structurally estimates the price adjustment cost using the method proposed in Chapter 2. Compared to simulated data, fitting the model with real data is much more challenging. First, the real data is an unbalanced panel in the sense that firms do not list price on the price comparison site at exact same time frame. Also, the same firm might list its price on the price comparison site for separate intervals. To address this issue, I only uses observations that has the price listing for the same firm on the same product for the previous day since these are the valid observations that firm can make decision on whether to make price adjustment. Also, the firm’s base marginal cost is re-estimated at the beginning day of each continuous listing interval.

Second, digital camera is a type of product that has relatively short product cycle. Therefore, the overall price of the product could change significantly in relative short period of time. During the period I collect the pricing data, it is observed that over half of the 50 models had experienced considerable amount of price decreases. Figure 5 plotted the average price over time for a sample of the 50 models. The price trend of the products can be classified into three major categories. The first type of products experience significant amount of price decrease over relative short amount of time, such as product 1, 4, 6 and 9. The reason for the price decrease might be a common cost decrease, such as the price cut by the manufacturer. The second type of products have the price gradually decrease over the time periods of price collection, such as product 5 and 10. The reason for the gradual price decrease is more likely to be changes of the market condition. For example, as the model gets older, competition from newer models will force the price down to the level at which the consumer are willing to buy. Also, as time evolves, there will be less consumers with higher valuation of the product (thus less sensitive to price) and therefore the firms need to compete harder in price for consumers that are more price sensitive. The third type of products, including product 2, 3, 7 and 8, all have relatively constant price over the sampling period.
Figure 3: Relative Price Change Histogram
Figure 4: Histogram of Price Adjustment Duration
Figure 5: Average Price over Time
At current stage, the proposed model assumes that the average marginal cost for the same firm is constant over time. The overall price sensitivity of the market is also assumed to be constant over time. These two assumptions are overly-restrictive for product of the first two types and using the model on these two products is subject to mis-specification problems. Therefore, only the third type of products is used in estimation.

The estimation results of eight products of type three are shown in Table 3. From Eq. (3) of Chapter 2, the price elasticity is approximately $-\delta p$. For the products that included in the estimation, the approximate price elasticities are between 1 and 2 except for product 2 and 3. The relative sizes for the $\delta$ estimation for products 7-20 are consistent with intuition by having the sensitivity higher for lower priced items and lower for higher priced items.

<table>
<thead>
<tr>
<th>Product ID</th>
<th>Product</th>
<th>Ave Price</th>
<th>N</th>
<th>$\delta$</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>$-\log L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Pentax K100D SLR w/18-55mm Lens Kit</td>
<td>521.04</td>
<td>1,502</td>
<td>0.0197</td>
<td>13.43</td>
<td>0.1522</td>
<td>1.12E+05</td>
</tr>
<tr>
<td>3</td>
<td>Canon EOS 40D SLR Body Only</td>
<td>1272.82</td>
<td>4,759</td>
<td>0.0149</td>
<td>63.40</td>
<td>0.0029</td>
<td>9.10E+05</td>
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<tr>
<td>7</td>
<td>Nikon D40 SLR Kit</td>
<td>530.84</td>
<td>5,928</td>
<td>0.0024</td>
<td>10.25</td>
<td>0.0004</td>
<td>1.03E+06</td>
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<td>Canon PowerShot G9</td>
<td>489.66</td>
<td>5,629</td>
<td>0.0026</td>
<td>18.95</td>
<td>0.0021</td>
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<td>16</td>
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<td>348.06</td>
<td>3,387</td>
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<td>9.73</td>
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<td>0.0058</td>
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<td>0.0008</td>
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Table 3: Estimation Results Using Real Data

As for the magnitude of cost shock, the estimates are more sensible. Generally, the cost shock is higher for cameras with higher prices. For example, product 3 (Canon 40D), which has an average price of $1272, has a much higher cost shock than the other products included in the estimation.

The estimates for adjustment cost $\theta$ vary significantly for different products. At the first glance, the estimates seems to be low for most of the products. However, it should be kept in mind that $\theta$ is the adjustment cost per consumer in the market.

4 Conclusion and Future Work

Future work:

1. Use bootstrap method to calculate the standard error of the estimates.

2. Use multiple iterations for Nelder-Mead method together with the bootstrap method to identify the true global maximum from the local maximum.

3. Calculate (1) price elasticity (2) percentage of price adjustment in firm’s profit, based on the results of structural estimation.

4. Model specification testing. (1) homogeneous marginal cost vs heterogeneous marginal cost. (2) homogeneous price adjustment cost vs heterogeneous price adjustment cost.