International Antitrust Enforcement
and
Multi-Market Contact*

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Abstract
This paper analyzes international antitrust enforcement when multinational firms operate in several markets with antitrust authorities in each market. We are concerned with how the sustainability of collusion in one local market is affected by the existence of collusion in other markets when they are linked by demand relationships. The interdependence of collusion sustainability across markets leads to potential externalities in antitrust enforcement across jurisdictions. As a result, cartel prosecution can have a domino effect with the desistance of one cartel triggering the internal break-up of the cartel in the adjacent market. We further find that the equilibrium in antitrust authorities’ enforcement decisions may exhibit non-linearity due to a free-rider problem as the global economy is more integrated. We also analyze the equilibrium antitrust enforcement and compare it with the globally optimal antitrust enforcement policy.

JEL Classification: D41, F1, L13, L41

Key Words: Collusion, Antitrust Enforcement, Multi-Market Contact

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1 Introduction

This paper analyzes international antitrust enforcement when multinational firms operate in several markets with antitrust authorities in each market. We are concerned with how the sustainability of collusion in one local market is affected by the existence of collusion in other markets when they are linked by demand relationships. In the context of international trade, it would be natural to assume that the products in each local market are substitutes due to the possibility of arbitrage across markets.¹ The interdependence of collusion sustainability across markets leads to potential externalities in antitrust enforcement across jurisdictions. We analyze the equilibrium antitrust enforcement and compare it with the globally optimal antitrust enforcement policy.

To motivate our study, consider the "vitamin cartel" case of Empagran S.A. v. F. Hoffman-LaRoche. In this case, Empagran S.A. in Ecuador and other foreign companies filed a suit against F. Hoffman-LaRoche of Switzerland and numerous other foreign companies for an alleged international price-fixing conspiracy. An interesting aspect of the Empagran case is that it concerned a price-fixing conspiracy that allegedly took place overseas even though the case itself was filed in a US federal district court. Foreign plaintiffs, suing under the U.S. Foreign Trade Antitrust Improvement Act (FTAIA), claimed that "the cartel raised prices around the world in order to keep prices in equilibrium with United States prices in order to avoid a system of arbitrage" and therefore that "the foreign plaintiffs were injured as a direct result of the increases in United States prices even though they bought vitamins abroad."² In response, F. Hoffman-LaRoche moved to dismiss the suit, arguing that the court lacked subject matter jurisdiction under the federal antitrust laws, because the injuries plaintiffs sought to redress were sustained in transactions that lacked any direct connection to United States commerce. The Department of Justice (DOJ) also rejected the claim as there was no direct linkage between collusion in the US market and collusion in the plaintiff's home country. Even if these claims were correct, the DOJ argued, "they do not furnish a basis for jurisdiction under the FTAIA. To allow these claims would conflict directly with the rationales of the Supreme Court's decision, creating many

¹See Choi and Heiko (2009) for a general analysis of collusion in demand related markets that encompasses both the substitute and complement cases.
of the very harm to international antitrust enforcement that the Court sought to avoid."³

This case illustrates interdependency of collusive behavior across markets and raises the question about the sustainability of price fixing in demand-related (geographical or product) markets. In particular, the underlying argument by Empagran and foreign plaintiffs seems to be that collusion in one market is easier to sustain if there is also collusion in a related market. How does this argument actually work in theory? And does it depend, for example, on the degree of substitutability (here transportation cost) or complementarity of the products?

Furthermore, while looking at this argument is interesting in itself, it is obvious that based on the results important policy questions could be addressed. If, for example, collusion in one market is impossible without collusion in the other, then a targeted antitrust enforcement in one of the two markets is sufficient to desist both cartels. However, this possibility also raises the potential for a free rider problem in antitrust enforcement and may call for coordination between antitrust authorities in different jurisdictions if the enforcement is costly and the enforcement decision is made independently of each other.⁴ In contrast, if collusion is easier to sustain if the other market is more competitive, then a global policy effort in all markets might be more effective compared to targeted enforcement.

To address these issues, we construct a model of multi-market contact with antitrust authorities in each market and analyze the interplay of collusion incentives and antitrust enforcement incentives. We characterize equilibrium in firm behavior and antitrust enforcement policy. In particular, we show that the equilibrium may exhibit a non-linearity in antitrust authorities’ enforcement effort decisions as the global economy is more integrated due to a free-rider problem. We also compare the equilibrium enforcement efforts with the globally optimum and show that the equilibrium enforcement efforts are less than the globally optimal ones. This result suggests for the need to coordinate enforcement efforts across jurisdictions.

Our paper is related to two strands of the literature on multi-market contact and antitrust enforcement. The multi-market contact literature is concerned with how contact


⁴For an empirical analysis on the spillover effect of antitrust enforcement, see Block and Feinstein (1986).
across markets can affect the degree of collusion that firms can sustain in settings of repeated competition. The idea was first proposed by Edwards (1955). Bernheim and Whinston (1990) formalize Edwards’ idea that the multiplicity of contacts among conglomerate firms may induce "mutual forbearance" and "blunt the edge of their competition." They show that multi-market contacts can be used as a mechanism to pool the incentive constraints across markets. When there is slack in the incentive constraint in one market, the pooling of the incentive constraints allows the slack to be transferred to the other market where the constraint is binding, thereby aiding collusion in the market with the binding constraint. A recent paper by Bond and Syropoulos (forthcoming) extends Bernheim and Whinston’s model to an international context and explores the implication of reciprocal trade liberalization for collusion and welfare. They consider symmetric international markets segmented by trade costs in which firms compete in Cournot fashion in their domestic and export markets. However, both models are devoid of AA. As a result, the sustainability of collusion in each market is only constrained by the internal incentives to cheat against collusive outcomes. In contrast, we are interested in the interplay of AA’s external enforcement and internal incentives to cheat and the sustainability of collusion requires overcoming both internal and external hurdles.

Our paper also relates to the literature on cartel antitrust enforcement. In particular, there is a small, but growing, literature on corporate leniency programs that analyzes the effects of the programs on cartel stability. Motta and Polo (2003), for instance, conduct a positive analysis of leniency programs in which firms that reveal information about collusion to antitrust authorities receive reduced fines. They show that leniency programs make ex post enforcement more effective but may have an adverse ex ante incentive effect that encourages cartel formation by decreasing the expected cost of penalty associated with collusion.

Harrington (2008a), in contrast, investigates the optimal design of corporate leniency programs. However, none of these papers deals with multi-market contact.

Notable exceptions are Roux and von Ungern-Sternberg (2007) and Lefouili and Roux (2008). Roux and von Ungern-Sternberg analyze the impact of the so-called Amnesty Plus and Penalty Plus on the incentives for companies to reveal their collusive conduct when they are engaged in cartel activities in multiple markets simultaneously. At this point, Amnesty Plus and Penalty Plus are features unique to the US corporate leniency program.

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5See also Aubert, Rey, and Kovacic (2006) and Buccirossi and Spagnolo (2006).
They are proactive strategies aimed at soliciting information on cartel activities in markets other than currently inspected ones. Roux and von Ungern-Sternberg (2007) show that these programs strengthen firms’ reporting incentives in the other market once the firms are prosecuted in one of the markets. However, their effects on the initial incentives to apply for amnesty are ambiguous. The reason is that firms may be reluctant to denounce a cartel that would have been reported without these programs, if the initial revelation leads to discovery of a second cartel, especially when the benefit from collusion in the second market is very high. Their model, however, is static and does not investigate incentive constraints to sustaining collusion in the first place, focusing only on the issue of ex post desistance. Lefouili and Roux (2008) extend the analysis to investigate the effect of Amnesty Plus on cartel formation. They identify conditions under which the program has the desirable effect of weakening cartel stability. Otherwise, they show that the program is either neutral or even stabilizes a cartel. However, neither paper analyzes demand linkages across markets as we do in this paper. In addition, cartel enforcement is treated as exogenous in these papers, whereas we endogenize enforcement decisions and address the issue of enforcement externalities across jurisdictions in international antitrust enforcement.

The rest of this paper is organized as follows. In section 2, we set up the basic model and first analyze the benchmark case in which competition authorities always pursue any lead on collusive behavior in their own markets. Section 3 considers a simple model of antitrust enforcement by introducing the cost of enforcement. We analyze the interplay of collusion incentives and antitrust enforcement incentives. In section 4, we derive the globally optimal enforcement decisions and demonstrate the potential for free-riding in the enforcement efforts when the decisions are made independently. Section 5 concludes and suggests directions for future research.

2 The Model

In this section, we describe the basic model of multi-market contact when markets are linked through demand. We derive conditions under which collusion is sustainable in each market, under the assumption that competition authorities always pursue any lead on collusive behavior. The question of optimal antitrust enforcement policy is analyzed in the next section.
2.1 Benchmark model: collusion with active competition authorities

Consider the benchmark case with two local markets \( j = A, B \) and two firms, \( i = 1, 2 \) selling a homogenous product in both markets. Firms are engaged in a repeated game in which they decide in each period whether to collude on the monopoly price in the respective local market or whether to compete. Firms have a common discount factor of \( \delta \) and use eternal grim trigger strategies that punish deviations in one or both markets with competition in both markets. The linkage between the (otherwise symmetric) markets arises through demand. For simplicity, let us use the following "reduced form" set-up. Let \( \Pi_{11} \) and \( \Pi_{10}(\tau) \) denote the monopoly profits in market \( j \) if firms in the adjacent market collude and compete, respectively. The subscripts 1 and 0 indicate collusion and competition in each market, respectively. The parameter \( \tau \in [0, \bar{\tau}] \) measures the integration of the two markets and/or the transportation costs of the good. If \( \tau = \bar{\tau} \), the markets are completely independent, i.e. \( \Pi_{10}(\bar{\tau}) = \Pi_{11} \). For lower values of \( \tau \), competition in the adjacent market serves as a competitive fringe for the cartel. More specifically, the more integrated the markets are, the more restricted is the pricing of the cartel, \( \Pi_{10}'(\tau) > 0 \). For \( \tau = 0 \), the markets are completely integrated, \( \Pi_{10}(0) = 0 \). If firms compete in a market, local profits are zero independent of whether the other market is collusive or competitive, that is, \( \Pi_{01} = \Pi_{00} = \Pi_0 = 0 \).

There are three stages in the enforcement against cartel. First, price-fixing conspiracies need to be discovered. Second, discovered conspiracy schemes need to be prosecuted. Finally, successfully prosecuted cases need to be penalized to break up the existing cartels and deter any formation of future cartels. The final stage of punishment is captured by a fine imposed on successfully prosecuted cartel members. We assume that the fine for collusion is fixed by antitrust laws and we treat it as a fixed parameter value of the model. As pointed out by Harrington (2006), the role of the antitrust authorities in the discovery stage has been minimal in that they are typically a passive agent that responds to complaints by disgruntled employees and suspicious customers who typically provide initial leads on price-fixing schemes. To reflect this reality, we assume that if a cartel exists, AAs receive information about the cartel with a probability of \( \rho \) in each period. The information

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6An analysis of partial collusion would require introducing demand functions but shouldn’t change much of the qualitative results.

7In the US, punishment can entail both fines and incarceration of executives that are responsible for price-fixing.
provided to AAs is hard evidence and cannot be fabricated. In this section, we assume that AAs always pursue any leads on cartel and engage in prosecution of cartel. In the next section, we introduce a cost of prosecution and endogenize the prosecution decision of AAs.\(^8\)

With the assumption of active AAs, we can summarize antitrust enforcement mechanism as follows. In each market and period an antitrust authority (AA) discovers and prosecutions collusion with a probability of \(\rho\). Assume that this prosecution probability is independent across markets and over time. A convicted firm pays a fine of \(F(>0)\). Once convicted firms do not engage in collusion any more and earn competitive profits in all subsequent periods.

2.2 Sustainability of collusion in one market only

Suppose firms compete in one market, say market B. Then, the profits from collusion in market A are recursively defined as

\[
V_{10}^C = \frac{\Pi_{10}(\tau)}{2} + (1 - \rho)\delta V_{10} - \rho F
\]

which gives

\[
V_{10}^C = \frac{\Pi_{10}(\tau)/2 - \rho F}{1 - \delta(1 - \rho)}.
\]

Collusion is incentive compatible for both firms if these profits exceed the profits from a one-off deviation, i.e. if

\[
V_{10}^C = \frac{\Pi_{10}(\tau)/2 - \rho F}{1 - \delta(1 - \rho)} \geq \Pi_{10}(\tau)
\]

or

\[
F \leq F^*(\tau) = \frac{(\delta(1 - \rho) - 1/2)\Pi_{10}(\tau)}{\rho}.
\]

Note that this condition is never satisfied for any positive fine if \(\delta(1 - \rho) < 1/2\). In addition, the threshold increases in the industry profit \(\Pi_{10}(\tau)\), i.e. the more industry profits, the easier it is to sustain collusion. This implies that there is an externality across markets in the sustainability of collusion when the two markets are linked by demand relationships. In particular, as the two markets are more integrated (i.e., a lower \(\tau\)), collusion in one market

\(^8\text{As an alternative strategy of modeling the first two stages of enforcement, we can introduce a cost of monitoring and endogenize the monitoring decision of AAs. This alternative modeling strategy yields qualitatively identical results.}\)
is more difficult to sustain when the other market is competitive. It will be useful later to refer to condition (1) as a function of $\tau$. Condition (1) holds if $\tau \geq \tau^*$ where $\tau^*$ is implicitly defined by the equality of (1).

Let $V_{10}$ be the value function of a representative firm in one market when the other market is competitive. Then, we have

$$V_{10} = \begin{cases} 
V_{10}^C = \frac{\Pi_{10}(\tau)/2-\rho F}{1-\delta(1-\rho)}, & \text{if } F \leq F^*(\tau) \\
0, & \text{if } F > F^*(\tau)
\end{cases}$$

### 2.3 Sustainability of collusion in both markets

The present discounted value of a firm if collusion takes place in both markets is given by

$$V_{11}^C = \Pi_{11} + (1-\rho)^2\delta V_{11}^C + 2\rho(1-\rho)(\delta V_{10} - F) - 2\rho^2F$$

or

$$V_{11}^C = \frac{\Pi_{11} + 2\rho(1-\rho)\delta V_{10} - 2\rho F}{1 - \delta(1-\rho)^2}.$$  

If no cartel is detected, collusion persists in both markets. If one cartel is detected, firms receive the present discounted value from collusion in the other market, $V_{10}$, minus the fine. If both cartels are detected, firms are fined and stop colluding. Collusion in both markets is sustainable if

$$V_{11}^C \geq 2\Pi_{11}$$

or

$$F \leq \frac{2\delta(1-\rho)^2-1}{2\rho}\Pi_{11} + \delta(1-\rho)V_{10}. \quad (2)$$

Two cases have to be distinguished.

**Case 1** $F \leq F^*(\tau)$

In this case, collusion in one market would still be viable even if the cartel in the other market is discovered, that is, $V_{10} = V_{10}^C = \frac{\Pi_{10}(\tau)/2-\rho F}{1-\delta(1-\rho)}$. Therefore, collusion in both markets is sustainable if

$$F \leq F_{C}^{**}(\tau) \equiv \frac{[2\delta(1-\rho)^2-1][1-\delta(1-\rho)]}{\rho[1-\delta(1-\rho)^2]}\Pi_{11} + \frac{\delta(1-\rho)}{1-\delta(1-\rho)^2} \frac{\Pi_{10}(\tau)}{2}. \quad (3)$$
The subscript \( C \) in the critical value \( F^*_C(\tau) \) signifies that it is derived conditionally on that collusion in one market is viable, that is, \( V_{10} = V_{10}^C \).

**Case 2** \( F > F^*(\tau) \)

In this case, collusion in one market cannot be sustained without collusion in the adjacent market, i.e. \( V_{10} = 0 \). The incentive constraint for collusion in both markets is simply

\[
F \leq F^{**}_{NC} = \frac{2\delta(1-\rho)^2 - 1}{\rho} \Pi_{11} \tag{4}
\]

Once again, the subscript \( NC \) in the critical value \( F^{**}_{NC} \) signifies that it is derived conditionally on that collusion is not viable in a single market, that is, \( V_{10} = 0 \). Note that when collusion in the remaining market is not sustainable as cartel in the other market is discovered, the threshold value for collusion in both markets \( (F^{**}_{NC}) \) is independent of \( \tau \).

An examination of the three threshold values identified so far reveals the following relationships.

**Lemma 1** The relationships among \( F^*(\tau) \), \( F^*_C(\tau) \), and \( F^{**}_{NC} \) are as follows.

(i) For \( 1/2 < \delta(1-\rho)^2 \), we have \( F^*_C(\tau) \geq F^*(\tau) \) for all \( \tau \in [0, \tilde{\tau}] \), with the equality at \( \tau = \tilde{\tau} \).

(ii) For \( \delta(1-\rho)^2 < 1/2 < \delta(1-\rho) \), we have \( F^*_C(\tau) \leq F^*(\tau) \) for all \( \tau \in [0, \tilde{\tau}] \), with the equality at \( \tau = \tilde{\tau} \).

(iii) For \( \delta(1-\rho)^2 < 1/2 < \delta(1-\rho) \), there is a critical value of \( \tau^{**} \in (0, \tilde{\tau}) \) such that \( F^*(\tau) \geq F^{**}_{NC} \) if and only if \( \tau \geq \tau^{**} \).

**Proof.** First, note that at \( \tau = \tilde{\tau} \), we have \( \Pi_{10}(\tilde{\tau}) = \Pi_{11} \) by the definition of \( \tilde{\tau} \). Thus,

\[
F^*_C(\tilde{\tau}) = \frac{[2\delta(1-\rho)^2 - 1][1 - \delta(1-\rho)]}{\rho[1 - \delta(1-\rho)^2]} + \frac{\delta(1-\rho)}{1 - \delta(1-\rho)^2} \Pi_{11} = F^*(\tilde{\tau})
\]

In addition, \( F^{**}(\tau) \equiv \frac{(\delta(1-\rho) - 1/2)\Pi_{10}(\tau)}{\rho} > \frac{(1-\rho)}{1 - \delta(1-\rho)^2} \Pi_{10}(\tau) = F^*_C(\tau) \) if \( 1/2 < \delta(1-\rho)^2 \), which establishes (i). In contrast, for \( \delta(1-\rho)^2 < 1/2 < \delta(1-\rho) \), we have \( F^{**}(\tau) \equiv \frac{(\delta(1-\rho) - 1/2)\Pi_{10}(\tau)}{\rho} < \frac{\delta(1-\rho)}{1 - \delta(1-\rho)^2} \Pi_{10}(\tau) \), which proves (ii). Statement (iii) comes
from the fact that $F^*(\tau = 0) = 0 < F^*_{NC}$, with $\Pi_{10}(0) = 0$, $F^*(\tau = \tilde{\tau}) > F^*_{NC}$, and $F^*(\tau)$ is monotonically increasing in $\tau$ while $F^*_{NC}$ is a constant as a function of $\tau$. □

With the relationships among $F^*(\tau)$, $F^*_{C}(\tau)$, and $F^*_{NC}$, we can summarize the analysis so far as follows.

**Proposition 1**  
**[Collusion Equilibrium if AAs Always Prosecute]** Consider two markets whose degree of integration is represented by $\tau$.

- **A.** For $1/2 \leq \delta(1 - \rho)^2$ (See Figure 1)
  1. If $F \leq F^*(\tau)$, firms always collude regardless of the status of collusion in the other market (Region AC),
  2. if $F > F^*(\tau)$ and $F \leq F^*_{NC}$, there is collusion only if the other market is collusive (Region $C|C$),
  3. otherwise there is no collusion (Region NC).

- **B.** For $\delta(1 - \rho)^2 \leq 1/2 \leq \delta(1 - \rho)$ (See Figure 2)
  1. If $F \leq F^*_{C}(\tau)$, firms always collude regardless of the status of collusion in the other market (Region AC),
  2. if $F^*_{C}(\tau) < F \leq F^*(\tau)$, there is collusion only if the other market is competitive (Region $C|NC$),
  3. otherwise there is no collusion (Region NC).

- **C.** If $\delta(1 - \rho) \leq 1/2$, there is no collusion.

Proposition 1 reflects the interaction between internal cartel instability and antitrust enforcement. It identifies four possible cartel regimes which are illustrated in the $(\tau, F)$-diagrams in Figures 1 and 2. In region AC, the expected antitrust penalty is sufficiently low to allow firms to sustain cartels in all markets in which there has not been a successful cartel prosecution. In region NC, the expected antitrust penalty is sufficiently high to deter firms from colluding in any market.

Region $C|C$ represents an area of conditional collusion where firms can collude in a market only when the other market is also collusive. In this region, firms start colluding in both markets simultaneously. If one of the two cartels is discovered, firms stop their cartel activity not only in the prosecuted market but also in the adjacent market. In other words, cartel prosecution has a *domino effect* due to the negative demand externality from the prosecuted to the non-prosecuted market. After the break-up of the cartel in one market,
prices in the other market also drop to the competitive level. The reason for this result is that the competitive outcome in the prosecuted market presents an arbitrage opportunity and negatively affects cartel stability in the non-prosecuted market.

In region C|NC, antitrust intervention has the exact opposite effect. In this region, firms can collude in a market only when the other market is competitive. Cartel prosecution thus entails a waterbed effect; successful cartel prosecution in one market triggers cartel formation in the adjacent market. In this region, firms can sustain collusion in only one market. Firms start colluding in only one of the two markets, maintaining competition in the adjacent market. If the cartel is discovered in the collusive market and the price drops to the competitive level, firms start colluding in the adjacent market. After the break-up of the cartel in one market, prices in the other market increase to the collusive level.

Figure 1: Firms’ collusion incentives if both AAs always prosecute and \( \delta(1 - \rho)^2 \geq 1/2 \). C|C represents the area in which collusion is possible if the other market is also collusive.
Figure 2: Firms’ collusion incentives if both AAs always prosecute and
\[\delta(1 - \rho)^2 \leq 1/2 \leq \delta(1 - \rho).\] C|NC represents the area in which collusion is possible if the other market is noncollusive.

The results in Proposition 1 can be explained in the following way. There are two reasons for the break-down of collusion in this model: internal cartel instability and antitrust enforcement. To sustain collusion in both markets, the incentive constraint for cartel stability, condition (2), has to be satisfied. Rewriting (2) gives

\[
\frac{1}{1 - \delta(1 - \rho)^2} - 2\Pi_{11} + \frac{2\delta \rho (1 - \rho) V_{10} - 2\rho F}{1 - \delta(1 - \rho)^2} \geq 0.
\]

A further simplification yields the incentive constraint of sustaining collusion per market,

\[
[\delta(1 - \rho)^2 - 1/2]\Pi_{11} - \rho F + \delta \rho (1 - \rho) V_{10} \geq 0. \tag{5}
\]

The first term is the expected collusive surplus from one market if none of the two cartels has been successfully prosecuted. This surplus is positive if the expected, discounted value of not being caught in both markets is larger than the market share gain from deviation. The second term is the expected penalty from collusion in this market. The third term is the expected continuation value if collusion in the other market breaks down. This value is
higher, the higher the transportation cost and the higher $\Pi_{10}(\tau)$.

To see the intuition for the existence of conditional collusion region $C|\text{NC}$, set $F = 0$. In this case, collusion in one market is sustainable if $\delta(1 - \rho) \geq 1/2$. For lower values of $\delta$, the continuation value $V_{10}$ is zero and the first term in (5) is negative. Thus, no collusion is sustainable. If $\delta(1 - \rho)^2 \geq 1/2$, then the first term and the second term (since $\delta(1 - \rho) > \delta(1 - \rho)^2$) are positive. Thus, collusion is always possible. Finally, for intermediate values of $\delta$, $\delta(1 - \rho)^2 \leq 1/2 \leq \delta(1 - \rho)$, the first term is negative and the second positive, i.e. collusion in both markets cannot be sustained without a positive continuation value $V_{10}$. This implies for $\delta(1 - \rho)$ towards $1/2$, the second term approaches zero and the condition is never satisfied. Thus, full collusion in both markets is not sustainable whereas collusion in one market only is.\(^9\) The intuition for this result is that the incentive to deviate when colluding in both markets is higher than the incentive to deviate when colluding in one market only. However, the gains from collusion in both markets, i.e. $\Pi_{11}$, are curtailed by the possibility of being caught in two (instead of one) markets. If the per period profit of collusion in one market, $\Pi_{10}(\tau)$, is sufficiently high, then the continuation value $V_{10}$ is high and this could make up for the reduced gains from collusion in both markets. However, if $\Pi_{10}(\tau)$ is small and the discount factor intermediate, then region $C|\text{NC}$ exists.

### 3 Model of Antitrust Enforcement Decisions

#### 3.1 A simple model of antitrust intervention

To fix ideas for the international trade model suppose that in each market there is an antitrust authority (AA). Assume that each AA receives evidence of a cartel in their market with an exogenous probability $\rho$, $0 \leq \rho \leq 1$. This probability is independent across markets and over time. To convict the cartel with this evidence, the AA has to invest $C > 0$. In other words, a successful prosecution of a cartel requires incriminating evidence and a prosecution effort from the agency.\(^{10}\) Once a cartel has been prosecuted it will never be

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\(^9\) Also note that for $F = 0$, region $C|C$ never exist. Either there are no externalities between AAs of different countries (regions NC and AC) or they are negative (region C|NC).

\(^{10}\) The results should not change qualitatively with effort as a continuous choice variable, and/or if the probability of a successful prosecution would be less than one. We can also derive qualitatively similar results when we endogenize the AAs’ monitoring efforts rather than assuming an exogenous probability of detection.
Define $S_{11}$, $S_{10}(\tau)$ and $S_0$ as the per period consumer surplus in a local market if there is collusion in both markets, collusion in this market only, and no collusion, respectively. It holds that

$$S_{11} \leq S_{10}(\tau) \leq S_0$$

with $S_{11} \geq 0$. It is natural to assume that $S_{10}(\tau)$ is decreasing in $\tau$ with $S_{10}(0) = S_0$ and $S_{10}(\bar{\tau}) = S_{11}$.\footnote{If firms collude on the monopoly price $p^m$ and marginal costs are constant, the prohibitive level of transportation cost is defined by $\bar{\tau} = p^m - c$.} In words, local consumer surplus in a market in which firms collude is higher the lower the transportation cost, or the more integrated the markets are.\footnote{Consider the simplest case of inelastic demand functions in both markets. Consumers value each product at $v$; there is a marginal cost of $c$ and transportation cost $\tau$. In this case the link between the profit and surplus functions is straightforward. We would have $S_{11} = 0$, $S_0 = v - c = \Pi_{11}$, $S_{10}(\tau) = v - c - \tau$ and $\Pi_{10}(\tau) = \tau$.} Each agency is assumed to maximize the discounted, expected domestic consumer surplus net of its enforcement cost.

Let us start by assuming that the parameter constellation in the firms’ problem is such that firms would always collude independent of whether there is collusion in the adjacent market or not (i.e. region AC in Figures 1 and 2). After deriving the AAs’ optimal enforcement strategies we check under which conditions this is indeed optimal firm behavior given the AAs’ strategies.

### 3.2 Enforcement Decision with Collusion in Only One Market

First consider a situation where there is collusion in only one market and cartel in the other market is already broken up. Suppose the local competition authority has received evidence of the cartel and has to decide whether to prosecute or not. To investigate the AA’s optimal prosecution decision, let $W^P$ and $W^{NP}_{10}(\tau)$ denote the discounted expected domestic consumer surplus (net of enforcement cost, if any) when the AA actively prosecutes

\textsuperscript{11}This assumption is made for analytical convenience. For instance, AAs may closely monitor the previous collusive industry. In addition, AAs may uncover inside information on how the cartel operates in the process of previous investigation, which allows the formation of a new cartel more difficult.

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by pursuing any evidence it receives and does not engage in prosecution, respectively.\footnote{Note that prosecution leads to the competitive market outcome and thus the expected welfare from prosecution does not depend on the status of collusion in the other market, which explains the absence of subscripts and $\tau$ in the expression for $W^P$.}

\[ W^P = \frac{\delta}{1 - \delta} S_0 - C, \quad W_{10}^{NP}(\tau) = \frac{\delta}{1 - \delta} S_{10}(\tau) \]

Prosecution is beneficial if $W^P \geq W_{10}^{NP}(\tau)$, i.e., the stream of future competitive consumer surplus net of the enforcement cost outweighs the stream of future collusive consumer surplus:

\[ \frac{\delta}{1 - \delta} S_0 - C \geq \frac{\delta}{1 - \delta} S_{10}(\tau), \]

or

\[ C \leq \frac{\delta}{1 - \delta} (S_0 - S_{10}(\tau)) = C^*(\tau). \quad (6) \]

Note that $C^*(\tau)$ is increasing in $\tau$ with $C^*(0) = 0$. Thus, condition (6) is more stringent to satisfy as $\tau$ becomes smaller. If the markets are sufficiently integrated, the gains from stopping the cartel are low and the local competition authority prefers not to prosecute the cartel. Define $\rho \in \{0, \rho\}$ as the probability of successful antitrust enforcement by the local AA if the adjacent market is competitive. Then the \textit{ex ante} expected consumer surplus in this market is recursively given by

\[ W_{10} = S_{10}(\tau) + \delta(1 - \rho')W_{10} + \rho'\left(\frac{\delta}{1 - \delta} S_0 - C\right) \]

or

\[
\begin{align*}
W_{10} &= \frac{S_{10}(\tau)}{1 - \delta} + \frac{\rho'}{1 - \delta(1 - \rho')} \left[ \frac{\delta}{1 - \delta} (S_0 - S_{10}(\tau)) - C \right] \\
&= \frac{S_{10}(\tau)}{1 - \delta} + I_{\{C \leq C^*(\tau)\}}(C) \frac{\rho}{1 - \delta(1 - \rho)} \left[ \frac{\delta}{1 - \delta} (S_0 - S_{10}(\tau)) - C \right] \\
&= \begin{cases} \\
\frac{S_{10}(\tau)}{1 - \delta} + \frac{\rho}{1 - \delta(1 - \rho)} \left[ \frac{\delta}{1 - \delta} (S_0 - S_{10}(\tau)) - C \right], & \text{if } C \leq C^*(\tau) \\
\frac{S_{10}(\tau)}{1 - \delta}, & \text{if } C > C^*(\tau) \\
\end{cases}
\end{align*}
\]
3.3 Enforcement Decision with Collusion in Both Markets

Now suppose that both markets are collusive. First, consider the incentives for an equilibrium in which both agencies prosecute their domestic cartel if they find evidence. The expected welfare from prosecution is $W^P = \frac{\delta}{1-\delta} S_0 - C$ as before, which is independent of the other AA’s prosecution decision.

In contrast, the expected welfare from non-prosecution depends on the other AA’s prosecution decision due to demand externalities across markets. We consider a representative market $i$ and denote the other market as $-i$. Let $W^N_{11}(\tau, AA_{-i})$ denote the expected welfare from non-prosecution in market $i$ when the antitrust agency in the other market $-i$ takes the action of $AA_{-i}$, where $AA_{-i} = P, NP$.

Assume the agency in the other market $-i$ prosecutes whenever it finds hard evidence. Then the expected welfare from non-prosecution in market $i$ can be recursively defined as follows.

$$W^N_{11}(\tau, P) = \rho \delta W_{10} + (1 - \rho) \delta [S_{11} + W^N_{11}(\tau, P)]$$

or

$$W^N_{11}(\tau, P) = \frac{(1 - \rho) \delta S_{11} + \rho \delta W_{10}}{1 - \delta (1 - \rho)}.$$

When both markets collude and the AA in the other market actively prosecutes ($AA_{-i} = P$), the condition for active prosecution in market $i$ is given by $W^P \geq W^N_{11}(\tau, P)$. Note that $W^N_{11}(\tau, P)$ depends on $W_{10}$, which in turn depends on the value of prosecution cost $C$. Thus, we need to consider two cases.

**Case 1** $C \leq C^*(\tau)$

In this case, the local AA prosecutes when the adjacent market is competitive. We thus have $W_{10} = \frac{S_{10}(\tau)}{1-\delta} + \frac{\rho}{1-\delta (1-\rho)} \left[\frac{\delta}{1-\delta} (S_0 - S_{10}(\tau)) - C\right]$ and it is straightforward to show that $W^P \geq W^N_{11}(\tau, P)$. This implies that if an AA would make effort after the other has prosecuted its cartel, then it will surely prosecute when both markets are collusive.

**Case 2** $C > C^*(\tau)$

In this case, an AA is not willing to incur prosecution cost when the other has prosecuted
its cartel, and we have \( W_{10} = \frac{S_{10}(\tau)}{1-\delta} \). Then \( W^P \geq W_{11}^{NP}(\tau, P) \) holds if and only if

\[
C \leq \frac{\delta}{1-\delta} (S_0 - S_{10}(\tau)) + \frac{\delta(1-\rho)}{1-\delta(1-\rho)} (S_{10}(\tau) - S_{11}) \tag{7}
\]

\[
= C^*(\tau) + \frac{\delta(1-\rho)}{1-\delta(1-\rho)} (S_{10}(\tau) - S_{11}) \equiv C^{**}(\tau, P) \tag{8}
\]

Since \( C^{**}(\tau, P) > C^*(\tau) \), we have the following result. If \( C \leq C^{**}(\tau, P) \), then there exists an equilibrium in which both agencies make a prosecution effort after detecting the cartel. Two subcases can arise. If additionally \( C \leq C^*(\tau) \) holds, then an agency continues prosecution after the other agency was successful. If \( C \leq C^*(\tau) \) does not hold, an agency stops the prosecution once the other agency is successful. Note that the \( C^{**}(\tau, P) \) is increasing in \( \tau \). Thus, the more integrated the markets (or the lower transportation cost), the harder it is to sustain an equilibrium in which both agencies prosecute their domestic cartel.

Next consider the conditions under which there is an equilibrium in which no AA makes a prosecution effort. Suppose both markets are collusive and the AA in market B makes no prosecution effort after detection. In such a case, collusion in both markets will perpetuate without any prosecution in market A. Thus, the expected welfare from non-prosecution (NP) is given by

\[
W_{11}^{NP}(\tau, NP) = \frac{\delta}{1-\delta} S_{11}
\]

Non-prosecution is optimal if and only if \( W_{11}^{NP}(\tau, NP) \geq W^P \).

\[
\frac{\delta}{1-\delta} S_{11} \geq \frac{\delta}{1-\delta} S_0 - C \tag{9}
\]

or

\[
C \geq \frac{\delta}{1-\delta} (S_0 - S_{11}) \equiv C^{**}(NP) \tag{10}
\]

Note that the critical value \( C^{**}(NP) \) does not depend on \( \tau \). It follows straight from (7) and (10) that \( C^{**}(NP) \geq C^{**}(\tau, P) \) with the equality holding at \( \tau = \overline{\tau} \). It is immediate that for intermediate values of \( C \), \( C^{**}(\tau, P) \leq C \leq C^{**}(NP) \), two asymmetric equilibria (and a mixed-strategy equilibrium) exist in which exactly one AA exerts prosecution while the other one does not. Let us summarize this analysis in the following proposition.
Proposition 2 [Locally Optimal Prosecution Decisions if Firms Always Collude]:

(i) If $C \leq C^*(\tau)$, there is prosecution by both AAs whenever there is hard evidence.
(ii) If $C^*(\tau) < C \leq C^{**}(\tau, P)$, there is prosecution by both AA when the other market is also collusive, but not if the other market is competitive.
(iii) If $C^{**}(\tau, P) < C \leq C^{**}(NP)$, there is prosecution by one AA when the other market is also collusive and the other AA does not prosecute.
(iv) If $C > C^{**}(NP)$, there is no prosecution by either AA.

To derive this result we have assumed that firms collude independent of whether the adjacent market is collusive or competitive. Let us now check whether this is indeed optimal firm behavior. To narrow down all the cases to consider and illustrate the interplay between collusive behavior and antitrust enforcement decisions, we focus on the parameter space defined by

$$0 \leq F \leq F^*_{NC} = \frac{[2\delta(1-\rho)^2 - 1] \Pi_{11}}{\rho}$$  \hspace{1cm} (11)$$

which is satisfied for a low penalty $F$ and a low detection rate $\rho$ and a high discount factor $\delta$. This constraint is a sufficient condition for collusion in both markets not being deterred even if both AAs prosecute whenever there is evidence.

Two different cases arise with this assumption. First, if $C \geq C^{**}(\tau)$, then prosecution stops in one market as soon as the other market is competitive. This in turn implies that firms receive a higher continuation value, $V_{10} = (\Pi_{10}/2)/(1-\delta)$, than in region C|C of Figure 1, where collusion is conditional. In region C|C collusion in one market breaks down when the adjacent market becomes competitive. However, if (11) holds, collusion is sustainable in region C|C if the adjacent market is collusive, and hence collusion in both markets is always sustainable if prosecution stops when one market becomes competitive. This argument holds for the case where both AAs prosecute (i.e. $C^*(\tau) < C \leq C^{**}(\tau, P)$) and, a fortiori, for higher values of $C$ with less prosecution of both cartels. Thus, the prosecution equilibrium described in Proposition 2 is consistent with a firms’ equilibrium in which collusion persists in a local market independent of whether the adjacent market is collusive or competitive. Figure 3 depicts the antitrust enforcement equilibrium in the $(\tau, C)$ space. Regions D, E, F correspond to this first case.

In the second case, $0 \leq C < C^*(\tau)$, both AAs prosecute whenever there is evidence. This
is exactly the assumption behind our analysis of the firms’ behavior above (i.e. Proposition 1). Collusion is detected and prosecuted with a probability $\rho$ at all times. And, as shown above, if $\tau \geq \tau^*$, then firms collude independent of whether the adjacent market is collusive or competitive. Thus, in region B of Figure 3 firms always collude and AAs always prosecute.

In contrast, if $\tau < \tau^*$, then after successfully prosecuting one market, collusion in the adjacent market is deterred and breaks down. In this case, our initial assumption to derive the AAs’ prosecution equilibrium, i.e. firms colluding no matter what, is no longer valid. Let $\tilde{W}_{11}^{NP}(P)$ be the expected welfare from non-prosecution in market $i$ that reflects the domino effect when the antitrust agency in the other market $-i$ takes the action of AA$_{-i} = P$ given any lead.

What is the AAs’ prosecution equilibrium in this case? First, note that the condition for an equilibrium with no prosecution from either AA does not change, i.e. $C \geq C^{**}(NP)$. Then consider the condition for an equilibrium in which both AAs prosecute. The breakdown of collusion increases the continuation value with a competitive adjacent market to $S_0/(1-\delta)$. Thus, given the other AA prosecutes, not prosecuting hard evidence now yields

$$\tilde{W}_{11}^{NP}(P) = \delta(1-\rho)(1-\delta)S_{11} + \rho\delta S_0 + \rho\delta S_0 \left( S_{11} + \tilde{W}_{11}^{NP}(P) \right) \left( \frac{S_0}{1-\delta} \right)$$

or

$$\tilde{W}_{11}^{NP}(P) = \frac{\delta(1-\delta)(1-\rho)S_{11} + \rho\delta S_0}{(1-\delta)(1-\rho(1-\delta))} \left( \geq W_{11}^{NP}(\tau, P) \right).$$

Therefore an equilibrium in which both AA exert the prosecution effort exists if and only if

$$WP = \frac{\delta}{1-\delta} S_0 - C \geq \tilde{W}_{11}^{NP}(P) \tag{12}$$

or

$$C \leq \frac{\delta(1-\rho)}{1-\delta(1-\rho)} (S_0 - S_{11}) \equiv \tilde{C}^{**}(P).$$

Note that $\tilde{C}^{**}(P)$ is independent of $\tau$ and $\tilde{C}^{**}(P) \leq C^{**}(\tau, P)$, with the equality holding at $\tau = 0$. 
<table>
<thead>
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<th>Firm Behavior</th>
<th>AA Behavior</th>
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<tr>
<td>A</td>
<td>C</td>
<td>C (Conditional Collusion)</td>
</tr>
<tr>
<td>B</td>
<td>AC (Always Collude)</td>
<td>Always Enforce</td>
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<tr>
<td>C</td>
<td>C</td>
<td>C (Conditional Collusion)</td>
</tr>
<tr>
<td>D</td>
<td>AC (Always Collude)</td>
<td>Both AAs Enforce Only When Both Markets Collusive</td>
</tr>
<tr>
<td>E</td>
<td>AC (Always Collude)</td>
<td>Only One Prosecutes When Both Markets Collusive</td>
</tr>
<tr>
<td>F</td>
<td>AC (Always Collude)</td>
<td>No Enforcement</td>
</tr>
</tbody>
</table>

Figure 3: Antitrust Enforcement Equilibrium

Thus, in region A of Figure 3, both AAs prosecute if the adjacent market is collusive. When one cartel has been successfully prosecuted, the local AA in the collusive market
would prosecute the cartel if detected. This is anticipated by the firms and the cartel breaks down. In region B both AAs always prosecute collusion and firms collude until they are caught. Finally, as indicated in Figure 3, there might also exist a region C. Define the intersection value of $C^*(\tau)$ and $\tilde{C}^{**}(P)$ as $\tilde{\tau}$, i.e.

$$S_{10}(\tilde{\tau}) = S_{11} + \frac{\rho}{1 - \delta(1 - \rho)} (S_0 - S_{11}).$$

A necessary and sufficient condition for region C to exist is $\tilde{\tau} \leq \tau^*$ which holds for $\delta$ sufficiently high. In region C, only one AA prosecutes if there is collusion in both markets. If successful, collusion in the other market breaks down because firms anticipate that the local AA would prosecute any hard evidence. What is interesting about this region is that it introduces a non-linearity in the effort equilibrium of the AA. Consider a situation in which both markets are collusive. For $\tilde{\tau} \leq \tau \leq \tau^*$, both AAs prosecute if the cost of prosecution is low or medium-to-high (more precisely, if C lies in regions A or D, that is, $C < \tilde{C}^{**}(P)$ or $C^*(\tau) < C < C^{**}(\tau, P)$). However, One AA prosecuting is an equilibrium for low-to-medium $C$ (more precisely, region C where $\tilde{C}^{**}(P) < C < C^*(\tau)$). The reason is that for low to medium $C$, the cost of prosecution is low enough that a local AA can credibly commit to prosecute even if the other market is competitive. Therefore, collusion breaks down in a cascade: if one market is caught, the other one becomes competitive, too. This in turn, increases the free-rider problem for low $C$. In the Appendix, we show that such region C exists if $\delta$ is sufficiently large. The graph and table in Figure 3 summarize the conditions for the AAs’ prosecution effort equilibrium.

Alternatively, we can also derive a non-linearity result in enforcement equilibrium as we change $\tau$. Consider the effect of globalization on antitrust efforts. We model globalization as a decrease in $\tau$. Suppose that the cost of prosecution is just above $C^{**}(\tau, P)$ and initially $\tau$ is close to $\tilde{\tau}$. Then, as the two markets become more integrated and $\tau$ decreases, we move from area B to C to D. Once again, the antitrust equilibrium changes from both AAs enforcing to only one AA enforcing to both AAs enforcing when both markets are collusive.

4 Globally Optimal Antitrust Decisions

In section 3, we analyzed the equilibrium antitrust enforcement decisions when both AAs act independently to maximize their respective domestic consumer surplus net of enforce-
ment costs. However, the demand linkage across markets suggests that the enforcement equilibrium is not necessarily optimal from the global welfare viewpoint. In particular, cartel enforcement in one country can exert positive externalities by putting downward pressure on the highest collusive price sustainable in the other market and make collusion less desirable. As a result, there will be less enforcement in the equilibrium compared to the globally optimal level of enforcement.

To compare the equilibrium enforcement levels to the globally optimal one, we assume that there is no information sharing between the two AAs in deriving the globally optimal one. In other words, we consider the informationally constrained globally optimum policy in which each AA makes a decision without knowing whether the other agent possesses information that can lead to prosecution in the other market. The next proposition summarizes the globally optimal enforcement policy assuming that firms always collude.

**Proposition 3 [Globally Optimal Prosecution if firms always collude]:** The following characterizes the globally optimal antitrust policy where $C_G^*\tau, P > C_G^*\tau, P$ and $C_G^*\tau, NP > C_G^*NP$, which implies that there is too little enforcement efforts by AAs in equilibrium compared to the global optimum.

(i) If $C \leq C^*\tau$, there should be prosecution by both AA whenever there is hard evidence.
(ii) If $C^*\tau < C \leq C_G^*\tau, P$, there should be prosecution by both AA when the other market is also collusive, but not if the other market is competitive.
(iii) If $C_G^*\tau, P < C \leq C_G^*\tau, NP$, there should be asymmetric prosecution when both markets are collusive, with one AA actively prosecuting and the other not prosecuting.
(iv) If $C > C_G^*\tau, NP$, no prosecution by either AA is globally optimal.

**Proof.** See the Appendix

5 Concluding Remarks

In this paper, we have analyzed collusion incentives of multinational firms interacting in several local markets that are linked by substitute products in demand due to the arbitrage constraint across markets. Our focus was on how collusion incentives in one market can be influenced by competitive conditions in the other market. We have shown that this has implications for optimal antitrust enforcement decisions for each AA in charge of different local markets.
There are many unresolved issues. For instance, we have analyzed only the substitute product case in this paper since it is a natural setting in the context of international trade. However, our preliminary investigation [Choi and Gerlach (2009)] suggests that a very different picture emerge in terms of collusion and antitrust enforcement incentives in the case of complementary products. In addition, we have assumed that there is no information sharing between local antitrust authorities in the enforcement. In reality, however, investigation of cartel in one market often leads to additional pieces of evidence on cartel in other markets. In fact, one of the global initiatives in the fight against international cartel is to promote information sharing among antitrust agencies across different jurisdictions. Information sharing confers many benefits. For instance, timely sharing of critical information allows much more effective enforcement through coordinated searches and simultaneous dawn raids. However, many business groups have opposed information sharing in cartel investigations on the ground that it will undermine the efficacy of leniency programs, which have been one of the most effective instruments in the fight against cartels. Their argument against information sharing is that confidentiality is a necessary inducement to encourage leniency applications and restricting information sharing is necessary to protect the integrity of leniency programs. How information sharing among antitrust agencies plays out for the enforcement of international cartel can be an important issue in conjunction with the leniency programs in place. We plan to pursue these issues in our companion paper [Choi and Gerlach (in preparation)].
Appendix

Proof of Proposition 3. When there is collusion in only one market and cartel in the other market is already broken up, there is no externality from cartel enforcement in the remaining market. Therefore, the optimal policy in the remaining market is also globally optimal, which means that there should be prosecution if and only if $W^P \geq W^N_{10}(\tau)$, i.e., $C \leq C^* (\tau)$.

Now we analyze globally optimal enforcement decisions when both markets are collusive. Once again, it is straightforward to show that if $C \leq C^* (\tau)$ and an AA would make effort after the other has prosecuted its cartel, then it is not only domestically optimal but also globally optimal to prosecute when both markets are collusive regardless of the action of the other AA. From now on, I focus on the parameter region in which $C > C^* (\tau)$ and the enforcement is not carried out when the other market is competitive.

Let $GW^P (AA_{-i})$ denote the expected global welfare from prosecution in the representative market $i$ when the antitrust agency in the other market $-i$ takes the action of $AA_{-i}$ without any information sharing, where $AA_{-i} = P, NP$. In other words, $GW^P (AA_{-i}) = W^P + W^P_i (AA_{-i})$, where $W^P_i (AA_{-i})$ denotes the other country’s welfare when country $i$ prosecutes while the other country $-i$ takes the action of $AA_{-i}$.

To derive the condition under which it is globally optimal for both firms prosecute, consider the incentive for an AA to prosecute assuming that the other AA also actively pursue prosecution without any information sharing. The expected global welfare ($GW$) from prosecution is given by

$$GW^P (P) = W^P + W^P_{-i} (P),$$

where $W^P_{-i} (P)$ denotes the other country’s welfare when the other country also actively pursues prosecution ($AA_{-i} = P$). More specifically,

$$W^P_{-i} (AA_{-i} = P) = \rho \left[ \frac{\delta}{1 - \delta} S_0 - C \right] + (1 - \rho) \frac{\delta}{1 - \delta} S_{10}(\tau)$$

The expression above utilizes the fact that $AA_{-i}$ will not engage in prosecution once market $i$ is competitive. In contrast, if the country does not prosecute while the other one
does, the expected global welfare from non-prosecution is given by

\[ GW_{NP}(P) = W_{11}^{NP}(\tau, P) + W_{-i}^{NP}(P), \]

where \( W_{-i}^{NP}(P) \) denotes the other country’s welfare when the other one is the only one that actively pursues prosecution (i.e., \( AA_i = NP \) and \( AA_{-i} = P \)). The value for \( W_{-i}^{NP}(P) \) is recursively defined by

\[ W_{-i}^{NP}(P) = \rho[\frac{\delta}{1-\delta} S_0 - C] + (1-\rho)\delta[S_{11} + W_{-i}^{NP}(P)] \]

or

\[ W_{-i}^{NP}(P) = \rho[\frac{\delta}{1-\delta} S_0 - C] + \delta(1-\rho)S_{11} = \rho[\frac{\delta}{1-\delta} S_0 - C] + (1-\rho)\delta[\frac{\delta}{1-\delta} S_0 - C] + S_{11} \]

It is globally optimal for both countries to actively engage in prosecution if \( GW^P(P) \geq GW^{NP}(P) \). To analyze this condition, it is useful to define an externality term from one’s prosecution as:

\[ E^P(P) = W_{-i}^P(P) - W_{-i}^{NP}(P) \]

I first show that the externality term is always positive for \( C > C^*(\tau) \). To see this, note that when \( AA_{-i} = P \), the externality arises only in the event that the other country’s AA does not receive any evidence. Otherwise, \( AA_{-i} \) will engage in prosecution and the market will be competitive regardless of the action by \( AA_i \). Thus,

\[ E^P(P) = (1-\rho)\frac{\delta}{1-\delta}S_{10}(\tau) - (1-\rho)\delta\frac{\rho[\frac{\delta}{1-\delta} S_0 - C] + S_{11}}{1-\delta(1-\rho)} \]

For \( C > C^*(\tau) = \frac{\delta}{1-\delta} (S_0 - S_{10}(\tau)) \), we have

\[ E^P(P) \geq (1-\rho)\frac{\delta}{1-\delta}S_{10}(\tau) - (1-\rho)\delta\frac{\rho[\frac{\delta}{1-\delta} S_0 - C] + S_{11}}{1-\delta(1-\rho)} = \frac{(1-\rho)\delta}{1-\delta(1-\rho)}[S_{10}(\tau) - S_{11}] \geq 0 \]

Using the definition of \( E^P(P) \), the condition for \( GW^P(P) \geq GW^{NP}(P) \) can be rewritten as

\[ C \leq C^{**}(\tau, P) + E^P(P) \]
Notice that $C^*(\tau, P)$ is a constant and $E^P(P)$ is an increasing function of $C$. Let us define $\Phi(C) = C^*(\tau, P) + E^P(P)$. Then, $\Phi(C)$ is an increasing function of $C$ with the slope of $\frac{\delta}{1-\delta(1-\rho)} < 1$. Therefore, there is a unique $C^*_G(C, P)$ such that $C \leq \Phi(C)$ if and only if $C \leq C^*_G(\tau, P)$. We know that $C^*(\tau, P) \leq \Phi(C^*(\tau, P))$ because $E^P(P)$ evaluated at $C^*(\tau, P) (> C^*(\tau))$ is positive. Therefore, $C^*_G(\tau, P) > C^*(\tau, P)$.

By proceeding in a similar way, we can also derive the condition under which it is globally optimal for neither firms prosecute. Consider the incentive for an AA to prosecute assuming that the other AA does not actively pursue prosecution. The expected global welfare ($GW$) from prosecution can be written as

$$GW^P(NP) = W^P + W^P_{-i}(NP),$$

where $W^P_{-i}(NP)$ denotes the other country’s welfare when the other country does not actively pursues prosecution ($AA_{-i} = NP$). More specifically,

$$W^P_{-i}(NP) = \frac{\delta}{1-\delta}S_{10}(\tau)$$

Once again, the expression above utilizes the fact that $AA_{-i}$ will not engage in prosecution when market $i$ is competitive. In contrast, if both AAs do not pursue active prosecution, the expected global welfare is given by

$$GW^{NP}(NP) = W^{NP}_{11}(\tau, NP) + W^{NP}_{-i}(NP),$$

where $W^{NP}_{-i}(NP)$ denotes the other country’s welfare when neither of them pursues prosecution (i.e., $AA_{i} = NP$ and $AA_{-i} = NP$). Since cartel is never broken up in both countries, we have

$$W^{NP}_{11}(\tau, NP) = W^{NP}_{-i}(NP) = \frac{\delta}{1-\delta}S_{11}$$

It is globally optimal for neither county to actively engage in prosecution if $GW^{NP}(NP) \geq GW^P(NP)$. To analyze this condition, it is useful to define an externality term from one’s prosecution as:

$$E^P(NP) = W^P_{-i}(NP) - W^P_{-i}(NP) = \frac{\delta}{1-\delta}[S_{10}(\tau) - S_{11}]$$

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It is immediate to see that $GW^{NP}(NP) \geq GW^{P}(NP)$ if and only if $C \geq C^\ast\ast_G(\tau, NP) \equiv C^\ast\ast(NP) + E^P(NP) = \frac{\delta}{1-\rho}[S_0 - S_{11}] + \frac{\delta}{1-\rho}[S_{10}(\tau) - S_{11}]$. Thus, it is globally optimal for neither firm to engage in prosecution if $C \geq C^\ast\ast_G(\tau, NP)$, where $C^\ast\ast_G(\tau, NP) > C^\ast\ast(\tau, NP)$.

If $C$ is in the intermediate range of $[C^\ast\ast_G(\tau, P), C^\ast\ast_G(\tau, NP)]$, asymmetric prosecution in which only one country prosecutes is globally optimal.

**Existence of region C:** Suppose $\delta(1 - \rho)^2 \geq 1/2$. If $\delta$ is sufficiently large, then there always exist $(F, \rho)$ such that $\tau^* < \tau^*$.

Proof:

Assume

$$\delta(1 - \rho)^2 \geq 1/2. \tag{A}$$

$\tau^*$ is defined by

$$\Pi_{10}(\tau^*) = \frac{\rho F}{\delta(1 - \rho) - 1/2}.$$

Since $\tau^*$ increases in $F$, the maximum $\tau^*$, $\overline{\tau}^*$, compatible with (11) is given by

$$\Pi_{10}(\overline{\tau}^*) = \frac{\rho F^{**} |_{\nu_{10} = 0}}{\delta(1 - \rho) - 1/2} = \frac{2\delta(1 - \rho)^2 - 1}{2\delta(1 - \rho) - 1}\Pi_{11} \equiv \psi_1 \Pi_{11}.$$

Check that $\psi_1$ increases in $\delta$ with $\psi_1(\delta = 1/[2(1 - \rho)^2]) = 0$ and $0 \leq \psi_1(\delta = 1) \leq 1$ as long as (A) holds. Since $\Pi_{10}(\tau)$ increases in $\tau$ with $\Pi_{10}(0) = 0$ and $\Pi_{10}(\overline{\tau}) = \Pi_{11}$, it follows that $\overline{\tau}^*$ increases in $\delta$ and takes value 0 at $\delta = 1/[2(1 - \rho)^2]$ and value $\tau^{**}$ at $\delta = 1$ where $0 \leq \tau^{**} \leq \overline{\tau}$.

$\overline{\tau}$ is defined by

$$S_{10}(\overline{\tau}) = S_{11} + \frac{\rho}{1 - \delta(1 - \rho)} (S_0 - S_{11}) \equiv \psi_2.$$

Check that $\psi_2$ increases in $\delta$ with $\psi_2(\delta = 1/[2(1 - \rho)^2]) > S_{11}$ and $\psi_2(\delta = 1) = S_0$. Since $S_{10}$ decreases in $\tau$ with $S_{10}(0) = S_0$ and $S_{10}(\overline{\tau}) = S_{11}$, it follows that $\overline{\tau}$ decreases in $\delta$ and takes a strictly positive value at $\delta = 1/[2(1 - \rho)^2]$ and value 0 at $\delta = 1$.

Suppose $\Pi_{10}(\tau)$ and $S_{10}(\tau)$ are continuous in $\tau$. Then there exists a unique value $\tilde{\delta}$ with $1/[2(1 - \rho)^2] < \tilde{\delta} < 1$ such that if $\delta \geq \tilde{\delta}$, then $\overline{\tau} \geq \overline{\tau}$. It follows that if $\delta \geq \tilde{\delta}$, then there exists $F \leq F^{**} |_{\nu_{10} = 0}$ such that $\tau^* \geq \overline{\tau}$. □

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References


