News, Noise, and Fluctuations: An Empirical Exploration

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Abstract

We explore empirically a model of aggregate fluctuations with two basic ingredients: agents form anticipations about the future based on noisy sources of information; these anticipation affect spending decisions which determine output in the short run. Using post-war U.S. data, we attempt to separate fluctuations due to actual changes in fundamentals (news) from those due to temporary errors in the private sector’s estimate of these fundamentals (noise). We find that, as long as the econometrician has no informational advantage over the agents in the model, structural VARs cannot be used for this purpose. We show, however, that a structural Maximum Likelihood approach does work for identifying the model’s parameters. This approach suggests that noise shocks play a major role in short-run fluctuations.

Keywords: Aggregate shocks, business cycles, vector autoregression, invertibility.

JEL Codes: E32, C32, D83

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1 Introduction

In this paper, we explore empirically a model of fluctuations in which anticipations of the future affect output in the short run.

More specifically, we think of spending decisions as depending primarily on signals about the future. These signals may be news or they may be just noise. Based on these signals, agents solve a signal extraction problem and choose spending. Because of nominal rigidities, spending affects output in the short run. If ex post the signals turn out to have been news, agents adjust their expectations gradually to the new underlying fundamentals. If ex post the signals turn out to have been noise, then the economy returns to normal over time.

We explore this model for two reasons. The first is that it appears to capture many of the aspects often ascribed to fluctuations: the role of animal spirits in affecting demand—“spirits” coming here from a rational reaction to signals about the future—, the role of demand in affecting output in the short run, together with the notion that output eventually returns to its natural level.

The second is that it appears to fit the data in a more formal way. More specifically, it offers an interpretation of structural VARs based on the assumption of two major types of shocks: shocks with permanent effects and shocks with transitory effects on activity. As characterized by Blanchard and Quah (1989), Galí (1999), Beaudry and Portier (2006), among others, “permanent shocks” appear to lead to an increase in activity in the short run, building up to a larger effect in the long run, while—by construction—“transitory shocks” lead to a transitory effect on activity in the short run. It is tempting to associate shocks with permanent effects to news and shocks with transitory effects to noise.

Our model provides a useful laboratory to address two issues: a methodological one and a substantive one. First, can structural VARs indeed be used to recover news and noise shocks? Second, what is the role of news and noise shocks in short-run fluctuations?

On the first question, we reach a strong negative conclusion—one which came as an unhappy surprise for one of the coauthors. In models of expectation-driven fluctuations in which consumers solve a signal extraction problem, structural VARs can typically recover neither the news nor noise shocks, nor their propagation mechanisms. The reason is straightforward: If agents face a signal extraction problem, and are unable to separate news from noise, then the econometrician, faced with either the same data as the agents or a subset of these data, cannot do it either.

To address the second question, we then turn to structural estimation, first using a simple method of moments and then maximum likelihood. We find that our model fits well and gives
a clear description of fluctuations as a result of three types of shocks: shocks with permanent effects on productivity, which build up slowly over time; shocks with temporary effects on productivity, which decay slowly; and shocks to consumers’ signals about future productivity. All three shocks affect agents’ expectations, and thus affect demand and output in the short run. Noise shocks are a major source of short-run volatility. Namely, more than 70% of consumption volatility at a quarterly horizon is due to noise shocks and more than 50% at a yearly horizon is explained by noise shocks. This result is surprising when compared with variance decompositions from structural VAR exercises, such as Shapiro and Watson (1989) and Gali (1992), where transitory “demand shocks” typically account for a smaller fraction of aggregate volatility at the same horizons and permanent technology shock typically account for more than half of it. Our methodological analysis helps to explain the difference, showing that structural VARs tend to overstate the contribution of permanent productivity shocks to short-run volatility.

Recent efforts to empirically estimate models of news-driven business cycles include Christiano, Iliut, Motto and Rostagno (2007) and Schmitt-Grohé and Uribe (2008). These papers follow the approach of Jaimovich and Rebelo (2006), modeling news as advanced, perfect information about shocks affecting future productivity. We share with those papers the emphasis on structural estimation. The main difference is that we model the private sector information as a signal-extraction problem and focus our attention on disentangling the separate effects of news and noise.

The problem with structural VARs emphasized in this paper is essentially an invertibility problem, also known as a problem of non-fundamentalness. There is a resurgence of interest in the methodological and practical implications of invertibility problems, see, e.g., Sims and Zha (2006) and Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007). Our paper shows that non-invertibility problems are endemic to models where the agents’ uncertainty is represented as a signal-extraction problem. This idea has also recently surfaced in models that try to identify the effects of fiscal policy when the private sector receives information on future policy changes, see Leeper, Walker and Yang (2009).

The paper is organized as follows. Sections 2 and 3 present and solve the model. Section 4 looks at the use of structural VARs. Section 5 contains the results of our structural estimation. Section 6 explores a number of extensions and Section 7 concludes.
2 The model

For most of the paper, we focus on the following model, which is both analytically convenient, and, as we shall see, provides a good starting point for looking at post-war U.S. data.

We want to capture the notion that, behind productivity movements, there are two types of shocks. Shocks with permanent effects and shocks with only transitory effects. In particular, we assume that the effects of the first type of shock gradually build up over time, while the effects of the second gradually decay over time. One can think of the transitory component as either true or reflecting measurement error. This does not matter for our purposes.

We also want to capture the notion that spending decisions are based on agents’ expectations of the future, here future productivity. We assume that agents observe productivity, but not its individual components. To capture the idea that they have more information than just current and past productivity, we allow them to observe an additional signal about the permanent component of productivity. We will consider first a setup where this additional signal is not observed directly by the econometrician. Later we allow the econometrician to also observe this signal. Having solved the signal extraction problem, and based on their expectations, agents then choose spending. Because of nominal rigidities, spending determines output in the short run.

Thus, the dynamics of output are determined by three types of shocks, the two shocks to productivity, and the noise in the additional signal. For short, we shall refer to them as the “permanent shock”, the “transitory shock”, and the “noise shock”. Permanent shock is a slight (and common) misnomer, as it refers to a shock whose effects build up gradually.

Now to the specific assumptions.

2.1 Productivity

Productivity (in logs) is given by the sum of two components:

\[ a_t = x_t + z_t. \]  

(1)

The permanent component, \( x_t \), follows a unit root process given by

\[ \Delta x_t = \rho_x \Delta x_{t-1} + \epsilon_t. \]  

(2)
The transitory component, $z_t$, follows a stationary process given by

$$z_t = \rho z_{t-1} + \eta_t. \quad (3)$$

The coefficients $\rho_x$ and $\rho_z$ are in $[0, 1)$, and $\epsilon_t$ and $\eta_t$ are i.i.d. normal with variances $\sigma^2_\epsilon$ and $\sigma^2_\eta$. Agents observe productivity, but not the two components separately.

For most of the paper, we assume that the univariate representation of $a_t$ is a random walk

$$a_t = a_{t-1} + u_t, \quad (4)$$

with the variance of $u_t$ equal to $\sigma^2_u$, and restrict attention to the family of processes (1)-(3) that are consistent with this assumption. We do this for two reasons. The first is analytical convenience, as it makes our arguments more transparent. The second is that, as we shall see, this assumption provides a surprisingly good starting point when looking at post-war U.S. data. As will be clear however, our central results do not depend on this assumption.

In general, a given univariate process is consistent with an infinity of decompositions between a permanent and a transitory component with orthogonal innovations, as shown in Quah (1990, 1991). In our setup, there is a one-parameter family of processes for $x_t$ and $z_t$ which deliver a univariate random walk for $a_t$. These are all the processes with $\rho_x$ and $\rho_z$ equal to the same value $\rho \in [0, 1)$ and variance parameters equal to $\sigma^2_\epsilon = (1 - \rho)^2 \sigma^2_u$ and $\sigma^2_\eta = \rho \sigma^2_u$.\(^1\)

Productivity may be the sum of a permanent process with small shocks that build up slowly and a transitory process with large shocks that decay slowly (high $\rho$, small $\sigma^2_\epsilon$ and large $\sigma^2_\eta$), or it may be the sum of a permanent process which is itself close to a random walk and a transitory process close to white noise with small variance (low $\rho$, large $\sigma^2_\epsilon$ and small $\sigma^2_\eta$). An econometrician who can only observe $a_t$ cannot distinguish these cases. The sample variance of $\Delta a_t$ gives an estimate of $\sigma^2_u$. But the parameter $\rho$, and thus $\rho_x$, $\rho_z$, $\sigma^2_\epsilon$ and $\sigma^2_\eta$, are not identified. As we shall see, when consumers have access to some information on the permanent component $x_t$ and the econometrician has access to consumption data, it will be possible to identify $\rho$ and the remaining parameters.

\(^1\)To prove this result, notice that the spectral density of $\Delta a_t$, is equal to the sum of the spectral densities of $\Delta x_t$ and $\Delta z_t$, which are, respectively, $(1 - \rho e^{i\omega})^{-1}(1 - \rho e^{-i\omega})^{-1}\sigma^2_x$ and $(1 - e^{i\omega})(1 - e^{-i\omega})(1 - \rho e^{i\omega})^{-1}(1 - \rho e^{-i\omega})^{-1}\sigma^2_\eta$. Under the assumed parameter restrictions the sum yields a flat spectral density.
2.2 Consumption

We assume that consumption smoothing leads to the Euler equation

\[ c_t = E[c_{t+1} | \mathcal{I}_t], \tag{5} \]

where \( \mathcal{I}_t \) is the consumers’ information at date \( t \), to be specified below. For a generic variable \( X_t \), we use, when convenient, \( E_t [X_t] \) or \( X_t | I_t \) as alternative notation for \( E [X_t | I_t] \).

We drastically simplify the supply side, by considering an economy with no capital, in which consumption is the only component of demand and output is fully determined by the demand side. Output is given by \( y_t = c_t \) and the labor input adjusts to produce \( y_t \), given the current level of productivity. We impose the restriction that output returns to its natural level in the long run, namely that

\[ \lim_{j \to \infty} E_t [c_{t+j} - a_{t+j}] = 0. \]

In Appendix A, we show that this model can be derived as the limit case of a standard New Keynesian model with Calvo pricing when the frequency of price adjustment goes to zero.

Combining the last two equations gives

\[ c_t = \lim_{j \to \infty} E_t [a_{t+j}]. \tag{6} \]

Consumption, and by implication, output, depend on the consumers’ expectations of productivity in the long run.

To close the model we only need to specify the consumers’ information set. Consumers observe current and past productivity, \( a_t \). In addition, they receive a signal regarding the permanent component of the productivity process

\[ s_t = x_t + \nu_t, \tag{7} \]

where \( \nu_t \) is i.i.d. normal with variance \( \sigma_\nu^2 \). We assume that \( s_t \) is not observed by the econometrician. As we will see, in our benchmark model the econometrician is able to recover \( s_t \) exactly from the data, so this assumption is not essential for our results. Finally, we assume that consumers know the structure of the model, i.e., know \( \rho \) and the variances of the three shocks.
3 Solving the model

The solution to the model gives consumption and productivity as a function of current and lagged values of the three shocks, $\epsilon$, $u$, and $\nu$. It will be convenient for later to derive it in two steps. First, we solve for consumption as a function of productivity expectations. Second, we derive the dynamics of these expectations using the Kalman filter.

3.1 Step 1

From equations (2), (3), and (6) above,

$$c_t = x_{t,t} + \frac{\rho}{1-\rho}(x_{t|t} - x_{t-1|t})$$

or, equivalently,

$$(1-\rho)c_t = x_{t|t} - \rho x_{t-1|t}$$

Writing the corresponding expression for $c_{t-1}$, taking expectations of equation (2) at time $t-1$, and replacing, we can write consumption as

$$c_t = c_{t-1} + u_t^c,$$

with $u_t^c$ given by

$$u_t^c = \frac{1}{1-\rho}(x_{t|t} - x_{t|t-1}) - \frac{\rho}{1-\rho}(x_{t-1|t} - x_{t-1|t-1}).$$

Turning to productivity, equations (1) and (3) imply

$$a_t - \rho a_{t-1} = x_t + z_t - \rho(x_{t-1} + z_{t-1}) = x_t - \rho x_{t-1} + \eta_t.$$

Adding and subtracting $x_{t|t} - \rho x_{t-1|t}$ on the right-hand side, and substituting (8) and (9), gives

$$a_t = \rho a_{t-1} + (1-\rho)c_{t-1} + u_t^a,$$

with $u_t^a$ given by

$$u_t^a = x_t - x_{t|t-1} - \rho(x_{t-1} - x_{t-1|t-1}) + \eta_t.$$

The law of iterated expectations implies $E_{t-1}[u_t^j] = 0$ for $j = c, a$. Moreover, given that
past consumption is measurable with respect to past information, this implies

$$E\left[u_t^j | a_{t-1}, c_{t-1}, a_{t-2}, c_{t-2}, \ldots \right] = 0$$

for \( j = c, a \). Therefore, (9) and (10) give us the bivariate VAR representation of the joint process of consumption and productivity.

Note that, under our assumptions, the univariate representations of both productivity and consumption are random walks. For productivity, the result follows from our assumptions on the productivity process. For consumption, it follows from the behavioral assumption (5), independently of the productivity process. When we move to the bivariate representation, past productivity does not help predict consumption, but, if \( \rho \) is positive and consumption and productivity are not collinear, past consumption helps predict productivity as it captures the consumers’ information on the permanent component \( x_t \).

### 3.2 Step 2

The second step requires us to solve for the \( u \)'s as a function of the underlying shocks. Agents enter the period with beliefs \( x_{t|t-1} \) and \( x_{t-1|t-1} \) about the current and lagged values of the permanent component of productivity. They observe current productivity \( a_t = x_t + z_t \), and the signal \( s_t = x_t + \nu_t \) and update their beliefs applying the Kalman filter:

$$\begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} = A \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + B \begin{bmatrix} a_t \\ s_t \end{bmatrix}$$

where the matrices \( A \) and \( B \) depend on the underlying parameters (see Appendix B).

### 3.3 The dynamic effects of shocks

Equations (9)-(11), together with equations (1)-(3), fully characterize the dynamic responses of productivity and consumption to the different shocks. Except for two special cases to which we shall come back below (the case of a fully informative or a fully uninformative signal), these must be solved numerically.

Figure 1 gives the computed impulse responses of consumption and productivity to the three shocks. The parameters are chosen in line with the estimates obtained later, in Section 4. The time unit is the quarter. The parameter \( \rho \) is set to 0.89: this implies slowly building
Figure 1: Impulse Responses to the Three Shocks
permanent shocks and slowly decaying transitory shocks. The standard deviation of productivity growth, $\sigma_u$, is set to 0.67%, and this, together with $\rho$, implies standard deviations of the two technology shocks, $\sigma_\epsilon$ and $\sigma_\eta$, equal to 0.07% and 0.63%, respectively. The standard deviation of the noise shock, $\sigma_\nu$, is set to 0.89%, implying a fairly noisy signal.

In response to a one standard deviation increase in $\epsilon$, a permanent technology shock, productivity builds up slowly over time—the implication of a high value for $\rho$. Consumption also increases slowly. This reflects the fact that the standard deviations of the transitory shock, $\eta$, and the noise shock, $\nu$, are both large relative to the standard deviation of $\epsilon$. Thus, it takes a long time for consumers to be able to assess that this is really a permanent shock and to fully adjust consumption.

For our parameter values, consumption (equivalently, output) initially increases more than productivity, generating a transitory increase in employment. Smaller transitory shocks, or a more informative signal would lead to a larger initial increase in consumption, and thus a larger initial increase in employment. Larger transitory shocks, or a less informative signal, might lead instead to an initial decrease in employment.

In response to a one standard deviation increase in $\eta$, the transitory shock, productivity initially increases, and then slowly declines over time. As agents put some weight on it being a permanent shock, they initially increase consumption. As they learn that this was a transitory shock, consumption returns back to normal over time. For our parameter values, consumption increases less than productivity, leading to an initial decrease in employment. Again, for different parameters, the outcome may be an increase or a decrease in employment.

Finally, in response to a one standard deviation increase in $\nu$, the noise shock, consumption increases, and then returns to normal over time. The response of consumption need not be monotonic; in the simulation presented here, the response turns briefly negative, before returning to normal. By assumption, productivity does not change, so employment initially increases, to return to normal over time.

4 A structural VAR approach

The question we take up in this section is whether a structural VAR approach can recover the underlying shocks and their impulse responses.

The answer to this question is, generally, no. The basic intuition is the following: if the agents solve for the best estimate of the long run level of productivity to choose consumption, then an econometrician with access to the same data, or less, will not be able to separate news from noise based on the reduced form VAR innovations at time $t$. If the econometrician
could separate news from noise, so would the agents. But then it would be optimal for them not to respond to noise, and noise-driven fluctuations would disappear.

In the rest of this section we flesh out this intuition and show how it leads to a non-invertibility problem. As a benchmark, we use a simple long run identification restriction à la Blanchard and Quah (1990), but our argument extends to any identification scheme that attempts to map linear combinations of time $t$ innovations to the underlying shocks in the model.

We work out in more detail the case where the econometrician information set only includes $c_t$ and $a_t$, because this will be all the information we use in our empirical exercise below. In this case, the reason for the non-invertibility problem is simple: the model features three shocks and the econometrician only observes two variables. However, we will see that even when the econometrician can directly observe $s_t$ and runs a trivariate VAR in $(c_t, a_t, s_t)$, the problem remains. The non-invertibility problem does not depend on the number of observables, but it is inherent in the model representation of consumers’ uncertainty.

It is best to start with two special cases and then generalize.

### 4.1 A fully uninformative signal

Consider first the case of a fully uninformative signal, $\sigma_\nu = \infty$, so the consumers’ only information is given by current and past values of $a_t$.

Then, trivially, our random walk assumption for $a_t$ leads to $c_t = a_t$. In this case, the two innovations $u^c_t$ and $u^a_t$ coincide and are identical to the innovation $u_t$ in the univariate representation of $a_t$. The bivariate dynamics of consumption and productivity are given by

$$c_t = a_{t-1} + u_t, \quad a_t = a_{t-1} + u_t.$$

This characterization holds for any value of $\rho$. Thus, whatever the value of $\rho$ and the relative persistence and importance of the permanent and transitory components of productivity, a structural VAR with long-run restrictions will attribute all movements in productivity and consumption to permanent shocks, and none to transitory shocks. The impulse responses of productivity and consumption to $\epsilon$ will show a one-time permanent increase; the impulse responses of productivity and consumption to $\eta$ will be identically equal to zero.

In this case the decomposition between temporary and permanent shocks is essentially irrelevant, given that no information is available to ever separate the two. We could just
take the random walk representation as the fundamental starting point and just interpret \( u_t \) as the single, permanent shock. Unfortunately, this simplification will no longer be possible when consumers have access to some, imperfect information on \( x_t \).

### 4.2 A fully informative signal

Consider next the case of a fully informative signal, \( \sigma_\nu = 0 \), so consumers no longer face a signal extraction problem. They know exactly the value of the permanent component of productivity, \( x_t \)—and by implication, the value of the transitory component, \( z_t = a_t - x_t \). In this case, equations (9) and (10) simplify to:

\[

c_t = c_{t-1} + \frac{1}{1-\rho} \epsilon_t, \\
a_t = \rho a_{t-1} + (1-\rho) c_{t-1} + \epsilon_t + \eta_t.
\]

Consumption responds only to the permanent shock, productivity to both. In this case, a structural VAR approach will indeed work. Imposing the long-run restriction that only one of the shocks has a permanent effect on consumption and productivity will recover \( \epsilon_t \) and \( \eta_t \), and their dynamic effects.

### 4.3 The general case

Which of these two cases is exceptional? The answer is, unfortunately, the second. As soon as the signal is not fully informative a structural VAR approach fails.

Figure 2 shows the estimated impulse responses to the shocks with permanent and transitory effects obtained from structural VAR estimation, together with the true impulse responses to the three underlying shocks. The underlying parameters are the same as for Figure 1. The estimated impulse responses are obtained by generating a 100,000-period time series for consumption and productivity using the true model, and then running a structural VAR on them. The structural VAR is identified by imposing a long run restriction which distinguishes two orthogonal shocks: one with permanent effects on output and one with only transitory effects.

Look first at the true and estimated responses of productivity to a shock with permanent effects. The solid line in the top left quadrant plots the true response to a permanent technology shock, which replicates that in Figure 1, namely a small initial effect, followed by a steady buildup over time. The dashed line gives the estimated response from the structural VAR estimation: The initial effect is much larger, the later buildup much smaller. Indeed,
Figure 2: True and SVAR-based estimated impulse responses
simulations show that the less informative the signal, the larger the estimated initial effect, the smaller the later build up. (Remember that, when the signal is fully uninformative, the estimated response shows a one-time increase, with no further build up over time).

Turn to the true and estimated responses of consumption to a permanent shock in the bottom left quadrant. The solid line again replicates the corresponding response in Figure 1, showing a slow build-up of consumption over time. The dashed line shows the estimated response, namely a one-time response of consumption with no further build up over time.

The right quadrants show the true and estimated responses to shocks with transitory effects on output. The solid lines show the true responses to a transitory technology shock (thick line) and to a noise shock (thin line). The dashed lines give the estimated response to the single transitory shock from the structural VAR. They show that the estimated response of productivity to a transitory shock is close to the true response to a transitory technology shock, but the estimated response of consumption is equal to zero.

In short, the responses from the structural VAR overstate the initial response of productivity and consumption to permanent shocks, and thus give too much weight to these shocks in accounting for fluctuations. For productivity, the less informative the signal, the larger the overstatement. For consumption, the overstatement is independent of the informativeness of the signal.

The estimated responses of consumption to estimated permanent shocks (full initial response) and to estimated transitory shocks (no response) are particularly striking. In fact, it is possible to show that these features are the outcome of the random walk assumption for consumption and are independent of the model parameters. This is shown in the following proposition.

For generality, the proposition is proved allowing for any integrated process for productivity and for a general information structure for the consumers and the econometrician. The only restriction is that the econometrician has at most the same information as the consumers. We also allow for any form of structural identifying restriction. That is, we consider all possible identified shocks which are linear combinations of current and past innovations in the reduced form VAR. The proof is in Appendix C.

**Proposition 1** Suppose $a_t$ follows an I(1) stochastic process and consumption is given by $c_t = \lim_{j \to \infty} E[a_{t+j}|I_t]$. Suppose the econometrician observes a vector of variables $Y_t$, which are in the consumers’ date-t information set. Let $w_t$ be any identified shock in a structural VAR representation of $Y_t$. The shock $w_t$ either leads to a permanent change in consumption or has no effect on consumption at any horizon.
Why do structural VARs fail? Suppose there was an identified structural shock that could be mapped into the noise shock in the model. That means that there would be a linear combination of reduced form innovations at time $t$ that can be used to forecast the transitory increase in consumption depicted in panel (c) of Figure 1. Since the consumers have access to all the data used by the econometrician, they would be able to forecast this transitory fluctuation in consumption. But that would violate consumption smoothing, i.e., the random walk hypothesis for consumption. The same logic explains why the response to an identified permanent shock is flat.

One could enrich the model, e.g., adding preference shocks and allowing for changes in the real interest rate, so as to relax the random walk hypothesis for consumption. However, the essence of the argument remains: noise shocks that lead to transient “mistakes” by consumers cannot be detected using information available to consumers at date $t$. Any structural VAR identification scheme can only make use of that information and so is bound to fail.

Notice that Proposition 1 applies equally well to the case where $Y_t \equiv (c_t, a_t, s_t)$, so the problem is present also when the number of shocks and innovations is the same. In Section 5.5 below we will argue that in our model the observability of $s_t$ has a small effect on the econometrician’s ability to make inference on the underlying shocks. The crucial problem is not the number of observable variables, but whether the econometrician has access to some information not in the consumers’ information sets.

4.4 What if the econometrician has more information than the agents?

The argument above suggests two potential ways out, both based on the possibility that the econometrician may have access to more information about time $t$ than the agents had at the time.

The first is that, if we think of the transitory component as reflecting in part measurement error, and if the series for productivity is revised over time, the econometrician, who has access to the revised series, may be better able than the consumers to separate the permanent and the transitory components. To take an extreme case, if the transitory component reflects only measurement error, and if the revised series remove the measurement error, then the econometrician has access to the time series for the permanent component directly, and can therefore separate the two components. While this is extreme, this suggests that the bias from SVAR estimation may be reduced when using revised series rather than originally...
The dispersed information model in Lorenzoni (2009) goes in this direction, by assuming that consumers do not have access to real time information on aggregate output, but only to noisy local information. Under that assumption it is possible to map the noise shock in the model to the transitory shock from an identified VAR. However, also in models with dispersed information, once we enrich the consumers’ information sets, the problem raised here is bound to reappear.

The need for superior information on the econometrician’s side, suggests a second way out. In the end, the econometrician always has access to some superior information, as he can observe future realizations of variables that the consumer did not observe at time $t$. Then one may hope that a combination of past and future data may be used to identify the current shocks. More formally, the traditional invertibility problem is that the map from the economic shocks to the shocks in the VAR may not have an inverse that is one-sided in nonnegative powers of the lag operator. Maybe adding a sufficient number of lead terms an inverse can be found? Unfortunately, the answer is no. As we will show numerically in Section 5.5, even having access to an infinite sequence of past and future data the econometrician is never able to exactly recover the values of the shocks.

### 4.5 What does the structural VAR deliver?

A different way of looking at the problem is to understand what is the correct interpretation of the identified shocks that the structural VAR delivers. It turns out that the structural VAR allows us to recover the productivity process in its innovation representation. Namely, the process for $a_t$ can be represented by the alternative state-space system:

\begin{align}
\hat{x}_t &= \hat{x}_{t-1} + v^1_t \\
\rho \hat{a}_{t-1} + (1 - \rho) \hat{x}_{t-1} + v^2_t.
\end{align}

To prove this equivalence it is sufficient to define $\hat{x}_t \equiv a_t$, and use the results in Section 3, substituting $v^1_t = u^c_t$ and $v^2_t = u^a_t$.

But then why not start directly from (12)-(13) as our model for productivity dynamics? The reason why this is not particularly appealing as a primitive model is that the disturbances

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2A related article here is Rodriguez Mora and Schulstad (2007). They show that growth in period $t$ is correlated with preliminary estimates of past growth available in period $t$, not with final estimates, available later. One potential interpretation of these results is that agents choose spending in response to these preliminary estimates, and their spending in turn determines current output.

3See Anderson, Hansen, McGrattan, and Sargent (1996) for general conditions under which such a representation exists.
\(v_1^t\) and \(v_2^t\) in the innovation representation above are not mutually independent. In particular, our signal extraction model implies that \(v_1^t\) and \(v_2^t\) are positively correlated and that the correlation is higher the higher the value of \(\sigma_{\nu}\). As we shall see in the next section, this positive correlation is indeed present in the data. Our informational assumptions provide a rationale for it.

Going back to structural VARs, a long run identifying restriction will lead us to identify \(v_1^t\) as the permanent technology shock and will give a linear combination of \(v_1^t\) and \(v_2^t\) as the temporary shock. For some purposes, this representation may be all we are interested in. Clearly, that is not the case if we are trying to analyze the role of noise shocks in fluctuations.

5 Structural estimation

We now turn to structural estimation, proceeding in two steps. For our benchmark model structural estimation is particularly easy, and all parameters (save one) can be obtained using OLS; thus we start with it. For more general processes however, one must use maximum likelihood. We show how it can be done, and show estimation results.

5.1 A simple OLS approach

At this point, it is clear that we (the econometrician) cannot recover the three shocks from the two variables we observe, productivity and consumption. However, estimating equation (10), we can recover \(\rho\), and estimating equation (4) we can recover \(\sigma_u\). Given \(\rho\) and \(\sigma_u\), we immediately get \(\sigma_\epsilon\) and \(\sigma_\eta\). Recovering the variance of the noise shock is less straightforward, but it can be done by matching other moments. In particular, numerical results show that, given the remaining parameters, the coefficient of correlation between the reduced form innovations \(u_c^t\) and \(u_a^t\) is an increasing function of \(\sigma_\nu\). Therefore, we can recover this parameter by matching the correlation in the data. Having identified all the model parameters, we can fully characterize the dynamic effects of the various shocks.

How well does our simple benchmark model fit the time series facts for productivity and consumption? The answer is: fairly well. Although it clearly misses some of the dynamics in the data, it seems worth starting with it.

The basic characteristics of the two time series are shown in Table 1. We construct the productivity variable as the logarithm of the ratio of GDP to employment and the consumption variable as the logarithm of the ratio of NIPA consumption to population. We use quarterly data, from 1970:1 to 2008:1. An issue we have to confront is that, in
contradiction to our model, and indeed to any balanced growth model, the productivity and consumption variables have different growth rates over the sample (0.34% per quarter for productivity, versus 0.46% for consumption). This difference reflects factors we have left out of the model, from changes in participation, to changes in the saving rate, to changes in the capital-output ratio. For this reason, we allow for different linear trends in productivity and consumption and use the residuals from these trends in what follows.\footnote{We are aware that, in the context of our approach, where we are trying to isolate potentially low frequency movements in productivity, this is a rough and dangerous approximation. But, given our purposes, it seems to be a reasonable first pass assumption. The reason why we concentrate on the sample 1970:1 to 2008:1 is precisely because with longer samples we are less confident that the simple linear detrending adopted here does a satisfactory job. When we turn to variance decomposition, we will discuss the robustness of our results to extensions of the sample.}

Lines 1 and 2 of Table 1 show the results of estimated AR(1) for the first differences of the two variables. Recall that our model implies that both productivity and consumption should follow random walks, so the AR(1) term should be equal to zero. In both cases, the AR(1) term is indeed small, insignificant in the case of productivity, significant in the case of consumption.

Our model further implies a simple dynamic relation between productivity and consumption. Rewriting equation (10) as a cointegrating regression gives:

$$\Delta a_t = (1 - \rho)(c_{t-1} - a_{t-1}) + u^a_t$$

Line 3 shows the results of estimating this equation. Line 4 allows for lagged rates of change of consumption and productivity, and shows the presence of richer dynamics than implied by our specification, with small but significant coefficients on lagged rates of change consumption and productivity.

Our model’s dynamic implications on the relation between consumption and productivity can be extended to longer horizons. Specifically, (10) can be extended to obtain the following cointegrating regression, which holds for all $j \geq 0$:\footnote{This is obtained by induction. Suppose it is true for $j$, that is, $E_t[a_{t+j}] = (1 - \rho^j) c_t + \rho^j a_t$. Taking expectations at time $t - 1$ on both sides yields

$$E_{t-1}[a_{t+j}] = (1 - \rho^j) E_{t-1}[c_t] + \rho^j E_{t-1}[a_t]$$

$$= (1 - \rho^j) c_{t-1} + \rho^j ((1 - \rho) c_{t-1} + \rho a_{t-1})$$

$$= (1 - \rho^{j+1}) c_{t-1} + \rho^{j+1} a_{t-1},$$

the second equality follows from (5) and (10), the third from rearranging.}

$$a_{t+j} - a_t = (1 - \rho^j)(c_{t-1} - a_{t-1}) + u^a_{t+j},$$
where $u_{t,j}^a$ is a disturbance uncorrelated to the econometrician information at date $t$. Thus, according to the model, a larger consumption-productivity ratio should forecast higher future productivity growth at all horizons and the coefficient in this regression should increase with the horizon. Lines 5 to 7 explore this implication. We correct for the presence of autocorrelation due to overlapping intervals by using Newey-West standard errors. The results are roughly consistent with the model predictions. However, the increasing pattern in the regression coefficients only appears at long enough horizons, suggesting potentially richer productivity dynamics.

The regression in line 3 implies a value of $\rho$ of 0.95. Together with an estimated standard deviation $\sigma_\nu$ equal to 0.67%, this implies $\sigma_\epsilon = 0.03\%$ and $\sigma_\eta = 0.65\%$. In words, these results imply a very smooth permanent component, in which small shocks steadily build up over time, and a large transitory component, which decays slowly over time.

We can then compute the coefficient of correlation between $\Delta c$ and the residual of the regression on line 3, corresponding, respectively to $u_c^t$ and $u_a^t$. This coefficient of correlation is equal to 0.52. Notice that if the signal was perfectly informative this correlation would be equal to 0.05, while if the signal had infinite variance it would be 1. Therefore, the observed correlation is consistent with the model and allows us to identify a fairly large standard deviation of the noise shock, namely $\sigma_\nu = 2.1\%$.

The fact that we are able in our benchmark model to recover the model parameters by matching a few moments from the data, is clearly a special case. For more general

---

**Table 1: Consumption and Productivity Regressions.**

<table>
<thead>
<tr>
<th>Line</th>
<th>Dependent variable:</th>
<th>$\Delta a(-1)$</th>
<th>$\Delta c(-1)$</th>
<th>$(c - a)(-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Delta a$</td>
<td>-0.06 (0.09)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\Delta c$</td>
<td>0.24 (0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\Delta a$</td>
<td></td>
<td>0.05 (0.03)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\Delta a$</td>
<td>-0.21 (0.10)</td>
<td>0.32 (0.12)</td>
<td>0.03 (0.02)</td>
</tr>
<tr>
<td>5</td>
<td>$\Delta (8)a$</td>
<td></td>
<td>0.03 (0.15)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\Delta (20)a$</td>
<td></td>
<td>0.31 (0.30)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\Delta (40)a$</td>
<td></td>
<td>0.98 (0.43)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Sample: 1970:1 to 2008:1. $\Delta(j)a \equiv a(+j - 1) - a(-1)$. In parenthesis: robust standard errors computed using Newey-West window and 10 lags.

---

6These bounds can be derived from the analysis in Sections 4.1 and 4.2. To obtain the first, some algebra shows that under full information $Cov[u_c^t, u_a^t]/\sqrt{Var[u_c^t]Var[u_a^t]} = (1-\rho)/\sqrt{(1-\rho)^2 + \rho}$. The second bound is immediate.
specifications of productivity or consumption behavior, one must adopt a different approach. We now discuss this general approach, and then return to the data.

5.2 Maximum Likelihood

To estimate a model where consumers face a non trivial signal extraction problem, one can, generally, proceed in two steps.

• Take the point of view of the consumers. Write down the dynamics of the unobserved states in state space representation and solve the consumers’ filtering problem. In our case, the relevant state for the consumer is given by \( \xi_t \equiv (x_t, x_{t-1}, z_t) \), its dynamics are given by (2) and (3), the observation equations are (1) and (7), and Kalman filtering gives us the updating equation (11).

• Next, take the point of view of the econometrician, write down the model dynamics in state space representation and write the appropriate observation equations (which depend on the data available). In our case, the relevant state for the econometrician is given by \( \xi_t^E \equiv (x_t, x_{t-1}, z_t, x_{t|t}, x_{t-1|t}, z_{t|t}) \). Notice that the consumers’ expectations become part of the unobservable state and the consumers’ updating equation (11) becomes part of the description of the state’s dynamics. The observation equations for the econometrician are now (1) and (8), where the second links consumption (observed by the econometrician), to consumers’ expectations. The econometrician’s Kalman filter is then used to construct the likelihood function and estimate the model’s parameters.

Table 3 shows the results of estimation of the benchmark model presented as a grid over values of \( \rho \) from 0.0 to 0.99. For each value of \( \rho \), we find the values of the remaining parameters that maximize the likelihood function and in the last column we report the corresponding likelihood value. The table shows that the likelihood function has a well-behaved maximum at \( \rho = 0.89 \). The corresponding values of \( \sigma_e \) and \( \sigma_\eta \) are 0.08% and 0.6%, respectively. The standard deviation of the noise shock \( \sigma_\nu \) is 0.89%.

The results are roughly in line with those obtained in Section 5.1. Relative to those estimates, the maximum likelihood approach favors smaller values of \( \rho \) and \( \sigma_\nu \). However, comparing lines 6 and 8 in Table 2 shows that the likelihood gain from one set of parameters to the other is not large. In other words, the data are consistent with a range of different combinations of \( \rho \) and \( \sigma_\nu \). When we look at the model’s implications in terms of variance decomposition, we will consider different values in this range.
Table 2: Maximum Likelihood Estimation: Benchmark Model

A simple exercise, using this approach, is to relax the random walk assumption for productivity, allowing $\rho_x$ to differ from $\rho_z$, and allowing the variances of the shocks to be freely estimated. The estimation results are reported in Table 3 and are quite close to those obtained under the random walk assumption.

<table>
<thead>
<tr>
<th>Line</th>
<th>$\rho$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_\epsilon$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_\nu$</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.0067</td>
<td>0.0067</td>
<td>0.0000</td>
<td>0.0089</td>
<td>$-3 \times 10^{12}$</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.0183</td>
<td>0.0137</td>
<td>0.0092</td>
<td>0.0000</td>
<td>859.2</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>0.0102</td>
<td>0.0051</td>
<td>0.0072</td>
<td>0.0000</td>
<td>980.5</td>
</tr>
<tr>
<td>4</td>
<td>0.70</td>
<td>0.0077</td>
<td>0.0023</td>
<td>0.0065</td>
<td>0.0026</td>
<td>1042.6</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>0.0071</td>
<td>0.0014</td>
<td>0.0064</td>
<td>0.0056</td>
<td>1064.5</td>
</tr>
<tr>
<td>6</td>
<td>0.89</td>
<td>0.0067</td>
<td>0.0007</td>
<td>0.0063</td>
<td>0.0089</td>
<td>1073.2</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>0.0067</td>
<td>0.0007</td>
<td>0.0064</td>
<td>0.0099</td>
<td>1073.1</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
<td>0.0068</td>
<td>0.0003</td>
<td>0.0066</td>
<td>0.0234</td>
<td>1072.2</td>
</tr>
<tr>
<td>9</td>
<td>0.99</td>
<td>0.0063</td>
<td>0.0001</td>
<td>0.0063</td>
<td>0.0753</td>
<td>1068.5</td>
</tr>
</tbody>
</table>

Table 3: Maximum Likelihood Estimation: Unconstrained Model

5.3 Variance decomposition

What do our results imply in terms of dynamic effects of the shocks and variance decompositions? If we use the estimated parameters from the benchmark model (line 6 in Table 2), the dynamic effects of each shock are given in Figure 1 and were discussed in Section 3.3: A slow and steady build up of permanent shocks on productivity and consumption; a slowly decreasing effect of transitory shocks on productivity and consumption; and a slowly decreasing effect of noise shocks on consumption.

Figure 3 illustrates the implications in terms of variance decomposition. The main result is that noise shocks are a major source of short run volatility, accounting for more than 70% of output volatility at one quarter horizon and more than 50% at a four quarter horizon.
This is a surprising result when compared with traditional SVAR exercises, such as Shapiro and Watson (1989) and Gali (1992), where demand shocks typically explain a much smaller fraction of aggregate volatility. The analysis in Section 4 helps explain why SVAR approaches tend to understate the contribution of noise shocks to aggregate volatility.

In Table 4, we report the results of some robustness checks. On each line, we report the fraction of consumption variance due to noise shocks at 1, 4 and 8 quarters horizon, for different parameter values. Line 1 corresponds to our benchmark estimation. Line 2 reports the results obtained by setting $\rho$ at a higher level and choosing the remaining parameters by maximum likelihood (see line 8 of Table 2). The variance decomposition at short horizons is not very different, but noise shocks turn out to be more persistent under this parametrization and explain a much bigger fraction of variance at 8 quarter horizon. On line 3 we report the parameters obtained when estimating our model on a longer sample, 1948:1 to 2008:1. With this dataset the estimate of $\rho$ is larger and we obtain results analogous to the ones on line 2. Finally, in lines 4 and 5 we experiment with changing only the volatility of noise shocks.
shocks, keeping the other parameters fixed. In particular, relative to the benchmark, we first
decrease and then increase $\sigma_nu$ by one standard deviation (which is 0.0034 in our maximum
likelihood estimate). Interestingly, it is the lowest value of $\sigma_n$ that leads to the largest amount
of noise-driven volatility. A lower value of $\sigma_n$ makes the signal $s_t$ more precise, so consumers
rely on it more. In our parameters’ range this leads to greater short run volatility.

<table>
<thead>
<tr>
<th>Line</th>
<th>Parameters</th>
<th>Noise-driven consumption variance (fraction)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$</td>
<td>$\sigma_u$</td>
</tr>
<tr>
<td>1</td>
<td>benchmark</td>
<td>0.89</td>
</tr>
<tr>
<td>2</td>
<td>high $\rho$</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>sample 1948:1-2008:1</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>low $\sigma_\nu$</td>
<td>0.89</td>
</tr>
<tr>
<td>5</td>
<td>high $\sigma_\nu$</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 4: Variance Decomposition: Robustness Checks

5.4 Recovering the states: retrospective history

So far we have focused on using structural estimation to estimate the model’s parameters.
Now we turn to the question: what information about the unobservable states $x_t$ and $z_t$ and
the shocks $\epsilon_t$, $\eta_t$, and $\nu_t$ can be recovered from structural estimation? We begin from the
states.

Using the Kalman smoother it is possible to form Bayesian estimates of $x_t$ and $z_t$ using
the full time series available and obtain a retrospective history of the U.S. business cycle.
Figure 4 shows a plot of these estimates obtained using our benchmark model. The solid line
correspond to the estimated permanent component of productivity $x_t$, the dashed line to the
consumers’ real time estimate of the same variable $x_{t|t}$ (as estimated by the econometrician
using the full sample, recall that $x_{t|t}$ is also an unobservable state for the econometrician).

Looking first at medium run movements, the model identifies a gradual adjustment of
consumers’ expectations to the productivity slowdown in the 70s and a symmetric gradual
adjustment in the opposite direction during the faster productivity growth in the second half
of the 90s. Around these medium run trends, temporary fluctuations in consumers’ beliefs
produce short run volatility. However different episodes appear driven by different dynamics.
For example, the boom preceding the 90-91 recession seems driven by animal spirits and the
recession seems due to a unavoidable real adjustment of expectations to underlying productivity
dynamics, while the boom of the late 90s seems characterized by a belated catching up of
Figure 4: Smoothed estimates of the permanent component of productivity \( x_t \) and of the consumers’ real time estimate \( x_{t|t} \)

expectations to the underlying productivity acceleration. Obviously, the model is too stylized to give a credible account of all cyclical episodes. For example, given the absence of monetary policy shocks the recession of 81-82 is fully attributed to animal spirits.

The Kalman smoother also tells us how much information on the unobservable states is contained in past and future data. In particular, in Figure 5 we plot the root mean squared errors (RMSE) of the smoothed estimates of \( x_t \) and \( z_t \), when data up to \( t+j \) are available, for \( j = 0, 1, 2, \ldots \). Formally, these RMSE correspond to the square root of \( E_{t+j}[(x_t - E_{t+j}[x_t])^2] \), and can be computed using two different information sets: the econometrician’s, which only includes observations of \( c_t \) and \( a_t \), and the consumer’s, which also includes \( s_t \). For simplicity, we compute RMSE at the steady state of the Kalman filter, that is, assuming the forecaster has access to an infinite series of data up to time \( t+j \). In this case, the econometrician’s information set coincides with the consumer’s, that is, the econometrician can perfectly back up the current value of \( s_t \) from current and past observations of \( c_t \) and \( a_t \). This is a numerical result: computing the RMSE of \( s_t \) from the econometrician’s Kalman smoother, we find that it is equal to zero at \( j = 0 \). This implies that, in our model, with a sufficiently long data set, the direct observation of \( s_t \) does not add much to the econometrician’s ability to recover the unobservable states (or the shocks).

Figure 5 shows that the consumer’s and the econometrician’s contemporaneous estimate of the current state \( x_t \) has a standard deviation of 0.44%. By using future data, this standard deviation almost halves, to 0.28%. However, most of the relevant information arrives in the first six quarters, after that there are minimal gains in the precision of the estimate.
Turning to the shocks, we know from our discussion of structural VARs that the information in current and past values of \( c \) and \( a \) is not sufficient to derive the values of the current shocks. However, this does not mean that the data contain no information on the shocks. In particular, using the Kalman smoother the econometrician can form Bayesian estimates on \( \epsilon \), \( \eta \), and \( \nu \) using the entire time series available. Figure 6 plots these estimates for our benchmark model. As for the states, in Figure 7 we report the RMSE of the estimated shocks as a function of the number of leads available. To help the interpretation, each RMSE is normalized dividing it by the ex ante volatility of the respective shock, that is, \( \sigma_\epsilon \), \( \sigma_\eta \), and \( \sigma_\nu^2 \).

Notice that if the model was invertible, the RMSE would be zero at \( j = 0 \). The fact that all RMSE remain bounded from zero at all horizons shows that even an infinite data set does not allow us to recover the shocks exactly.

The transitory shock \( \eta \) is estimated with considerable precision already on impact and the precision of their estimates almost doubles in the long run. The noise shock \( \nu \) is less precisely estimated, but the data still tell us quite a lot about it, giving us an RMSE which is about 1/3 of the prior uncertainty in the long run. The shock that is least precisely estimated is the permanent shock \( \epsilon \). Even when infinite future data are available, the residual variance is about 94% of the prior uncertainty on the shock.

How do we reconcile the imprecision of the estimate of \( \epsilon \) with the fact that we have
Figure 6: Smoothed estimates of the shocks
Figure 7: Normalized RMSE of the estimated shocks at time $t$ using data up to $t + j$
relatively precise estimates of the state \( x \), as seen in Figure 5? The explanation is that the econometrician can estimate the cumulated effect of permanent productivity changes by looking at productivity growth over longer horizons, but cannot pinpoint the precise quarter in which the change occurred. Therefore, it is possible to have imprecise estimates of the past \( \epsilon \), while having relatively precise estimates of their cumulated effect on \( x \). This also helps to explain the high degree of autocorrelation of the estimated permanent shocks in Figure 6. The smoothed estimates of \( \epsilon \) in consecutive quarters tend to be highly correlated, as the econometrician does not know to which quarter to attribute an observed permanent change in productivity. This means that the autocorrelation of the estimated shocks is not a rejection of the assumption of i.i.d. shock, but purely a reflection of the econometrician’s information. In fact, performing the same estimation exercise on simulated data delivers a similar degree of autocorrelation of the estimated \( \epsilon \).

6 Extensions

We have shown how models where agents face signal-extraction problems cannot be estimated through SVARs, but can be estimated through structural estimation. Structural estimation however requires a full specification of the model, including the processes for the permanent and transitory components of productivity, the information structure, the behavior of consumers. And, unfortunately, the estimated parameters are likely to be sensitive to the specific assumptions.

There are at least two dimensions in which we think our benchmark model needs to be extended.

The first is motivated by the data. As we saw from Table 1, the dynamics of consumption, and the dynamic relation between productivity and consumption, are richer than those implied by the benchmark. These require at least a modification of our assumptions about consumption behavior. Our assumption about consumption implies that consumption follow a random walk for any productivity process and any standard deviation of the noise in the signal. As we have seen however, the univariate process for consumption, shown in line 2 of Table 2, shows evidence of richer dynamics.

The second is motivated by the discussion of labor hoarding, and pro-cyclical productivity in the research on the relation between output and employment. Our benchmark model has assumed that labor productivity is exogenous; there is however substantial evidence is however that, perhaps due to labor hoarding, some of the movements in productivity are in fact endogenous. Thus, in contrast to our assumption, a positive realization of the noise
shock may lead consumers to spend more, and lead in turn to an increase in productivity.

We consider these two extensions in turn.

6.1 Slow adjustment of consumption

To capture slow consumption adjustment, we adopt the simple specification, which replaces (6),

\[ c_t = \beta c_{t-1} + (1 - \beta) \lim_{j \to \infty} E_t[a_{t+j}] \]

In Table 4 we report the results from estimating this variant of the model, presented as a grid search over the value of the adjustment parameter \( \beta \). The data seem to prefer a small but positive value of \( \beta \).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \rho )</th>
<th>( \sigma_u )</th>
<th>( \sigma_e )</th>
<th>( \sigma_\zeta )</th>
<th>( \sigma_\nu )</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.8785</td>
<td>0.0068</td>
<td>0.0008</td>
<td>0.0063</td>
<td>0.0086</td>
<td>-1073.3</td>
</tr>
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<td>0.1</td>
<td>0.87</td>
<td>0.0071</td>
<td>0.0009</td>
<td>0.0066</td>
<td>0.008</td>
<td>-1075.9</td>
</tr>
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<td>0.2</td>
<td>0.8591</td>
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<td>0.0011</td>
<td>0.007</td>
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<td>-1074.8</td>
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<td>0.3</td>
<td>0.8412</td>
<td>0.0082</td>
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<td>0.0075</td>
<td>0.0062</td>
<td>-1068.8</td>
</tr>
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<td>0.4</td>
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<td>0.0002</td>
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<td>0.7126</td>
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<td>0.0037</td>
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<td>0.0003</td>
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<td>0.0061</td>
<td>0.0143</td>
<td>0.0006</td>
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</tr>
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<tr>
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<td>0.0567</td>
<td>0.02</td>
<td>0.0456</td>
<td>0.0033</td>
<td>-796</td>
</tr>
</tbody>
</table>

Table 5: Maximum Likelihood Estimation: Slow Consumption Adjustment

7 Conclusions

On the methodological side, this paper has emphasized the limits of SVARs in exploring news/noise models of the business cycle. This implies that to identify the role of news and noise in fluctuations one must rely more heavily on the model’s structure. In this paper, a central role for identification was played by the consumer’s Euler equation, that is, by the assumption that current movements in consumption are primarily driven by changes in the consumers’ expectations on the economy’s lung run potential.

On the empirical side, the data appear quite consistent with a view of fluctuations where the pattern of technological change is smooth, subject to random shocks which only build
up slowly, while most of the short term action in consumption and output comes from noisy information on these long run trends. Clearly, we need to extend the model in many dimensions before having confidence in these conclusions. In particular, adding investment seems an essential step to better distinguish the dynamics of employment, output, and consumption.
Appendix A. Relation of the model with the standard New Keynesian model

Consider a standard New Keynesian model, as laid out, e.g., in Gali (2008). Preferences are given by

\[ E \sum_{t=0}^{\infty} \beta^t U (C_t, N_t), \]

with

\[ U (C_t, N_t) = \log C_t - \frac{1}{1 + \zeta} N_t^{1+\zeta}, \]

where \( N_t \) are hours worked and \( C_t \) is a composite consumption good given by

\[ C_t = \left( \int_0^1 C_{j,t}^{\frac{1}{\gamma-1}} dj \right)^{\frac{\gamma-1}{\gamma}}, \]

\( C_{j,t} \) is the consumption of good \( j \) in period \( t \), and \( \gamma > 1 \) is the elasticity of substitution among goods. Each good \( j \in [0, 1] \) is produced by a single monopolistic firm with access to the linear production function

\[ Y_{j,t} = A_t N_{j,t}. \]

Productivity is given by \( A_t = \exp a_t \) and \( a_t \) follows the process (1)-(3). Firms are allowed to reset prices only at random time intervals. Each period, a firm is allowed to reset its price with probability \( 1 - \theta \) and must keep the price unchanged with probability \( \theta \). Firms hire labor on a competitive labor market at the wage \( W_t \), which is fully flexible.

Consumers have access to a nominal one-period bond which trades at the price \( Q_t \). The consumer’s budget constraint is

\[ Q_t B_{t+1} + \int_0^1 P_{j,t} C_{j,t} dj = B_t + W_t N_t + \int_0^1 \Pi_{j,t} dj, \]

where \( B_t \) are nominal bonds’ holdings, \( P_{j,t} \) is the price of good \( j \), \( W_t \) is the nominal wage rate, and \( \Pi_{j,t} \) are the profits of firm \( j \). In equilibrium consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (15) and a standard no-Ponzi-game condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires \( B_t = 0 \). The central bank sets the short-term nominal interest rate, that is, the price of the one-period nominal bond, \( Q_t \). Letting \( i_t = -\log Q_t \), monetary policy follows the simple rule

\[ i_t = i^* + \phi \pi_t, \]
where \( i^* = -\log \beta \) and \( \phi \) is a constant coefficient greater than 1.

Following standard steps, consumers’ and firms’ optimality conditions and market clearing can be log-linearized and transformed so as to obtain two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium. After substituting the policy rule we obtain:

\[
\begin{align*}
y_t &= E_t[y_{t+1}] - \phi \pi_t + E_t[\pi_{t+1}], \\
\pi_t &= \kappa (y_t - a_t) + \beta E_t[\pi_{t+1}],
\end{align*}
\]

where \( \kappa \equiv (1 + \zeta)(1 - \theta)(1 - \beta \theta) / \theta \) and where constant terms are omitted. As long as \( \phi > 1 \) this system has a unique locally stable solution where \( y_t \) and \( \pi_t \) are linear functions of the four exogenous state variables \( a_t, x_t|_t, x_{t-1}|_t, z_t|_t \),

\[
\begin{pmatrix}
y_t \\
\pi_t
\end{pmatrix} = D_\kappa
\begin{pmatrix}
a_t \\
x_t|_t \\
x_{t-1}|_t \\
z_t|_t
\end{pmatrix}.
\]

The matrix \( D_\kappa \) can be found using the method of undetermined coefficient as the solution to

\[
\begin{bmatrix}
1 & \phi \\
-\kappa & 1
\end{bmatrix} D_\kappa = \begin{bmatrix}
0 & 0 & 0 & 0 \\
-\kappa & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
1 & 1 \\
0 & \beta
\end{bmatrix} D_\kappa \begin{bmatrix}
0 & 1 + \rho & -\rho & \rho \\
0 & 1 + \rho & -\rho & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & \rho
\end{bmatrix}.
\]

The elements of \( D_\kappa \) are a continuous non-linear function of \( \kappa \) and some lengthy algebra (available on request) shows that

\[
\lim_{\kappa \to 0} D_\kappa = \frac{1}{1 - \rho} \begin{bmatrix}
0 & 1 & -\rho & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]

Since \( \kappa \to 0 \) when \( \theta \to 1 \), this completes the argument.
Appendix B. Kalman filter

Let

\[
C \equiv \begin{bmatrix}
1 + \rho_x & -\rho_x & 0 \\
1 & 0 & 0 \\
0 & 0 & \rho_z
\end{bmatrix},
\]

\[
D \equiv \begin{bmatrix}
1 & 0 & 1 \\
1 & 0 & 0
\end{bmatrix},
\]

and

\[
\Sigma_1 \equiv \begin{bmatrix}
\sigma^2_\epsilon & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \sigma^2_{\eta}
\end{bmatrix},
\]

\[
\Sigma_2 \equiv \begin{bmatrix}
0 & 0 \\
0 & \sigma^2_{\nu}
\end{bmatrix}.
\]

Then the process for \( \xi_t \equiv (x_t, x_{t-1}, z_t) \) is described compactly as

\[
\xi_t = C\xi_{t-1} + (\epsilon_t, 0, \eta_t)',
\]

and the observation equation for the consumers is

\[
(a_t, s_t) = D\xi_t + (0, \nu_t)'.
\]

Let \( P \equiv Var_{t-1} [\xi_t] \). The value of \( P \) is found solving the equation

\[
P = C \left[ P - PD' (DPD' + \Sigma_2)^{-1} DP \right] C' + \Sigma_1.
\]

The matrixes \( A \) and \( B \) in the text are then given by:

\[
A = (I - BD) C,
\]

\[
B = PD' (DPD' + \Sigma_2)^{-1}.
\]
Appendix C. Proof of Proposition 1

Given that \( w_t \) is a linear combination of current and past observables \( Y^t \equiv \{Y_t, Y_{t-1}, Y_{t-2}, \ldots\} \) and these variables are in \( \mathcal{I}_{t+k} \), for all \( k \geq 0 \), we can apply the law of iterated expectations to get

\[
E \left[ c_{t+k} | w_t, Y^{t-1} \right] = E \left[ \lim_{j \to \infty} E \left[ a_{t+k+j} | \mathcal{I}_{t+k} \right] | w_t, Y^{t-1} \right] = \lim_{j \to \infty} E \left[ a_{t+j} | w_t, Y^{t-1} \right],
\]

for all \( k \geq 0 \) and

\[
E \left[ c_{t+k} | Y^{t-1} \right] = E \left[ \lim_{j \to \infty} E \left[ a_{t+k+j} | \mathcal{I}_{t+k} \right] | Y^{t-1} \right] = \lim_{j \to \infty} E \left[ a_{t+j} | Y^{t-1} \right].
\]

It follows that the response of consumption to the shock \( w_t \) is constant and equal to

\[
E \left[ c_{t+k} | w_t, Y^{t-1} \right] - E \left[ c_t | Y^{t-1} \right] = \lim_{j \to \infty} E \left[ a_{t+j} | w_t, Y^{t-1} \right] - \lim_{j \to \infty} E \left[ a_{t+j} | Y^{t-1} \right],
\]

for all \( k \geq 0 \).

In particular, suppose \( w_t \) is the identified transitory shock under a Blanchard and Quah (1989) identification assumption. Then \( w_t \) is a linear combination of the VAR innovations such that

\[
\lim_{j \to \infty} E \left[ a_{t+j} | w_t, Y^{t-1} \right] - \lim_{j \to \infty} E \left[ a_{t+j} | Y^{t-1} \right] = 0,
\]

and the effect of the shock on consumption is zero.
References


