Policy Rule Parameters Driven by Latent Factors
Monetary and Fiscal Policy Interactions

Manuel Gonzalez-Astudillo
Department of Economics
Indiana University

March 11, 2010
Presentation

- Context.
- (Brief) Literature Review.
- Model Setup.
- Solution Method.
  - A simple Fisherian model (a bit on uniqueness).
  - A simple New Keynesian model (diagnosis).
- Bayesian Estimation.
- Future Work.
The core of the research

- Are there (nonlinear long-run) relationships driving monetary and fiscal policy interactions?
  - Stackelberg leadership? Fiscal leadership more desirable than Monetary leadership → Kirnanova, Stehn and Vines (2005)

- Commitment to a time varying policy rule?

- I aim to empirically determine if there are such relationships by estimating a New Keynesian DSGE model.
What is different here?

- I specify time-varying policy rule coefficients. Coefficients depend upon a persistent latent factor through a function that allows smooth transitions between states (logistic function).
- I introduce correlation between the latent factors driving monetary and fiscal policy rule coefficients, so that co-movements between policies are allowed.
- I propose a way to estimate parameters of the reduced-form model and then recover parameters of the structural model, in particular coefficients of the policy rules (avoiding endogeneity).
Time-varying policy rule coefficients?

Justification

- **Exogenous variations**
  - Dupor (2002) → Optimal policy may contain a purely random component (under monopolistic competition, high consumer prudence and low consumer aversion to risk).
  - Svensson and Williams (2007) → Allow central bank’s loss function to have time varying weights (model uncertainty).

- **Endogenous variations**

...monetary policy, just like other fields of human activity, always involves (in part) a mismatch between accumulated learning and the need to take specific policy actions.

Issing, 2001
Time-varying policy rule coefficients?

Modeling

Time-varying policy rule coefficients?

Estimation


- Davig and Doh (2008) → Bayesian estimation of a Markov-Switching New Keynesian model to investigate the mechanisms that lead to a decline in the persistence of inflation.
Time-varying policy rule coefficients?

Incorporating Fiscal Policy

Overview

I work with a conventional model with monopolistic competition and staggered pricing a la Calvo. Lump-sum taxes and nominal government debt are included to extend the analysis for fiscal policy. Agents in this model are:

- A representative household.
- Firms
  - Aggregation sector.
  - Intermediate goods sector.
- Fiscal authority.
- Monetary authority.
Households

A representative household solves

\[
\max_{\{C_{t+k}, L_{t+k}, B_{t+k}, M_{t+k}\}} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left( \log C_{t+k} - \chi L_{t+k} \frac{L_{t+k}^{1+\varphi}}{1+\varphi} + \chi M \frac{(M_{t+k}/P_{t+k})^{1-\sigma_M}}{1-\sigma_M} \right)
\]

subject to

\[
C_{t+k} + \frac{M_{t+k}}{P_{t+k}} + \frac{B_{t+k}}{P_{t+k}} + T_{t+k} \leq \frac{M_{t+k-1}}{P_{t+k}} + R_{t+k-1} \frac{B_{t+k-1}}{P_{t+k}} + \frac{W_{t+k}}{P_{t+k}} L_{t+k} + \frac{\Pi_{t+k}}{P_{t+k}}
\]

\[
\lim_{k \to \infty} MRS_{t,t+k} \frac{B_{t+k} + M_{t+k}}{P_{t+k}} = 0
\]

\[
M_{t-1} > 0, B_{t-1} \text{ given,}
\]

with \(0 < \beta < 1, \chi_N > 0, \varphi > 0, \chi_M > 0, \text{ and } \sigma_M > 0\)
Firms
Aggregation Sector

Given the composite good price, $P_t$, and individual good prices, $P_t(j)$, for $j \in [0, 1]$, producers ensemble intermediate goods, $Y_t(j)$, to obtain a composite final good, $Y_t$, according to a CES technology

$$Y_t = \left( \int_0^1 Y_t(j) \frac{\theta-1}{\theta} \, dj \right)^{\frac{\theta}{\theta-1}},$$

where $\theta \in [0, 1]$ is the price elasticity of demand.
Firms
Intermediate goods sector (1)

Intermediate goods firms produce type $j$ good according to the technology

$$Y_t(j) = A_t L_t(j),$$

where $L_t(j)$ are units of the work used by the producer of intermediate good $j$, and $A_t$ is an exogenous technology shock identical across producers following the stochastic process

$$A_t = \delta A_{t-1} \exp(\nu_t),$$

where $\delta$ is a trend and $\nu_t$ is a stochastic component following the process

$$\nu_t = \rho_{\nu} \nu_{t-1} + \varepsilon_t^\nu,$$

with $\rho_{\nu} \in (0,1)$ and $\varepsilon_{\nu t} \sim \text{iid}(0, \sigma_{\nu}^2)$.
Firms
Intermediate goods sector (2)

Producers in the intermediate good sector take wages as given and behave as monopolistic competitors in their good markets choosing the price for their product, taking the demand
\[ Y_t^d(j) = Y_t \left( P_t(j)/P_t \right)^{-\theta} \]
as given. I assume staggered pricing a la Calvo (1983) across firms, where the probability that a firm cannot adjust its price in a given period is given by \( \xi \). Those firms that cannot adjust their price, set it to be \( P_t(j) = \bar{\pi} P_{t-1}(j) \), where \( \bar{\pi} \) is the gross inflation rate in steady state. Under this setup, a firm that changes its price at time \( t \) chooses \( P_t(j) \) to maximize

\[
\mathbb{E}_t \sum_{k=0}^{\infty} \text{MRS}_{t,t+k} \xi^k \left[ \left( \frac{\bar{\pi}^k P_t(j)}{P_{t+k}} \right)^{1-\theta} - \Psi_{t+k} \left( \frac{\bar{\pi}^k P_t(j)}{P_{t+k}} \right)^{-\theta} \right] Y_{t+k}.
\]
Government Sector

The government demands goods as a proportion of aggregate output, $G_t = \zeta_t Y_t$, where $\zeta_t \in (0, 1)$ is a exogenous process given by the transformation $g_t = 1/(1 - \zeta_t)$ with

$$
\ln g_{t+1} = (1 - \rho_g) \ln g + \rho_g \ln g_t + \varepsilon_{t+1}^g
$$

where $\rho_g \in (0, 1)$ and $\varepsilon_{t+1}^g \sim \text{iid}\mathcal{N}(0, \sigma_g^2)$. \{G_t, T_t, M_t, B_t\}_{t=0}^\infty must satisfy the government budget constraint given by

$$
G_t = T_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} - \frac{R_{t-1}B_{t-1}}{P_t},
$$

given $M_{-1} > 0$ and $R_{-1}B_{-1}$.
Policy Rules

General Specification

Let $\varrho_t$ be a time varying coefficient of a policy rule.

$$
\varrho_t \equiv \varrho(z_t) = \varrho_0 + \frac{\varrho_1}{1 + \exp \left( -\varrho_2 (z_t - \varrho_3) \right)},
$$

where $z_t = \rho_z z_{t-1} + u_t$, $\rho_z \in (0, 1]$ and $u_t \sim \text{iid} N(0, 1)$. 

![Graph](attachment:image.png)
Policy Rules

- **Monetary policy rule**

\[ R_t = R_{t-1}^\phi \tilde{R}_t^{1-\phi} \exp(\varepsilon_t^R), \]

where \( \phi \in (0, 1) \), \( \varepsilon_t^R \sim \text{iid } \mathcal{N}(0, \sigma_R^2) \) and

\[ \tilde{R}_t = R \left( \frac{\bar{\pi}_t}{\bar{\pi}} \right)^{\alpha_t^\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\alpha_t^y}. \]

- **Fiscal Policy**

\[ \tau_t = \tau \left( \frac{b_{t-1}}{\tilde{b}} \right)^{\gamma_t^b} \left( \frac{Y_t}{Y_t^*} \right)^{\gamma_t^y} \left( \frac{\zeta_t}{\bar{\zeta}} \right)^{\gamma_t^g} \exp(\varepsilon_t^\tau), \]

where \( \varepsilon_t^\tau \sim \text{iid } \mathcal{N}(0, \sigma_\tau^2) \).
To introduce co-movements between policies, we specify the latent factors driving the policy parameters as follows:

\[
\begin{align*}
\hat{z}_t^R &= \rho_z \hat{z}_{t-1}^R + u_t^R, \\
\hat{z}_t^{\tau} &= \rho_z \hat{z}_{t-1}^{\tau} + u_t^{\tau},
\end{align*}
\]

where \( u_t^R \) and \( u_t^{\tau} \) are normally distributed with mean zero, unit variance and \( \text{cov}_t(u_t^R, u_t^{\tau}) = \kappa \).
Equilibrium Dynamics

\[
\begin{align*}
\hat{x}_t &= \mathbb{E}_t \hat{x}_{t+1} - (\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1}) + (1 - \rho_g) \frac{\varphi}{1 + \varphi} \hat{\nu}_t + \rho \nu_t \\
\hat{\pi}_t &= \frac{(1 - \beta \xi)(1 - \xi)(1 + \varphi)}{\xi} \hat{x}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \\
\hat{\nu}_t &= \left[ 1 + \frac{1}{\sigma_M} \left( \frac{\varphi}{1 + \varphi} \right) \right] \hat{\nu}_t + \frac{1}{\sigma_M} \frac{1}{R - 1} \hat{R}_t + \left( 1 - \frac{1}{\sigma_M} \right) \hat{x}_t \\
b_{\theta t} &= \frac{1}{\hat{\nu}_t} \hat{\nu}_t - \tau \hat{\pi}_t + \frac{1}{\hat{v}_t} \hat{v}_t - \frac{1}{\hat{v}_t \pi} (\hat{v}_{t-1} + \hat{\pi}_{t-1}) + \frac{R b}{\pi} (\hat{R}_{t-1} + b_{t-1} - \hat{\pi}_{t-1}) \\
\hat{R}_t &= \phi \hat{R}_{t-1} + (1 - \phi) \left( \alpha_\pi \hat{\pi}_t + \alpha_\nu \hat{x}_t \right) + \epsilon^R_t \\
\hat{\tau}_t &= \gamma_\theta \hat{b}_{t-1} + \gamma_y \hat{x}_t + \gamma_g \hat{\nu}_t + \epsilon^\tau_t \\
\nu_t &= \rho \nu \nu_{t-1} + \epsilon^\nu_t \\
\hat{\nu}_t &= \rho \nu \hat{\nu}_{t-1} + \epsilon^\nu_t \\
\alpha_i^t &= \alpha_\theta^t + \frac{\alpha_i^1}{1 + \exp(-\alpha_2^i \hat{z}_t^R)} \quad i = \pi, y \\
\gamma_i^t &= \gamma_y^t + \frac{\gamma_1^i}{1 + \exp(-\gamma_2^i \hat{z}_t^R)} \quad i = b, y, g \\
\hat{z}_t^R &= \rho_z \hat{z}_{t-1}^R + u_t^R \\
\hat{z}_t^\tau &= \rho_z \hat{z}_{t-1}^\tau + u_t^\tau.
\end{align*}
\]
Setup

\[ i_t = \mathbb{E}_t \pi_{t+1} + r_t \]  \hspace{1cm} (1)

\[ r_t = \rho_r r_{t-1} + \varepsilon_t \]

\[ i_t = \alpha(z_t) \pi_t \]  \hspace{1cm} (2)

\[ \alpha(z_t) = \alpha_0 + \frac{\alpha_1}{1 + \exp(-\alpha_2 z_t)} \]

\[ z_t = \rho_z z_{t-1} + u_t \]

\[ u_t, \varepsilon_t \text{ Gaussian, independent of each other.} \]

“Equilibrium” is given by equating the Taylor rule equation (3) with the Euler equation (1) to get

\[ \alpha(z_t) \pi_t = \mathbb{E}_t \pi_{t+1} + r_t. \]
Solution (1)

Posit the following MSV solution:

\[ \pi_t = a(z_t) r_t, \]

where

\[ a(z_t) = a_0 + \frac{a_1}{1 + \exp(-a_2 z_t)}, \]

for \( a_0, a_1, a_2 \) being parameters of the MSV solution to be found.

Notice that

\[ \mathbb{E}_t \pi_{t+1} = \mathbb{E}_t a(z_{t+1}) r_{t+1} \]
\[ = \rho_r r_t \mathbb{E}_t a(z_{t+1}) \text{ by independence} \]
\[ = \rho_r r_t \int_{-\infty}^{\infty} a(z_{t+1}) p(z_{t+1}|z_t) \, dz_{t+1} \]
Solution (2)

Recall the “equilibrium” condition

$$\alpha(z_t)\pi_t = \mathbb{E}_t\pi_{t+1} + r_t.$$  

Replacing the MSV solution for $\pi_t$, and $\mathbb{E}_t\pi_{t+1}$ yields

$$\alpha(z_t)a(z_t)r_t = r_t \left[ \rho_r \int_{-\infty}^{\infty} a(z_{t+1})p(z_{t+1}|z_t) \, dz_{t+1} + 1 \right].$$

Since the solution has to hold for all $r_t$, we write the above equation as

$$\frac{\alpha(z_t)a(z_t) - 1}{\rho_r} = \int_{-\infty}^{\infty} a(z_{t+1})p(z_{t+1}|z_t) \, dz_{t+1},$$

and find the values of $a_0, a_1, a_2$ that satisfy this equation.
Parameterized Solution

$\alpha_0 = 0.9, \alpha_1 = 1.3, \alpha_2 = 1, \rho_r = 0.9, \rho_z = 0.98.$
$a_0 = 3.1, a_1 = -2.3, a_2 = 1.4.$
Uniqueness

\[ \alpha_2 = 1, \rho_z = 0.8 \]
Setup

\[ x_t = \mathbb{E}_t x_{t+1} - (R_t - \mathbb{E}_t \pi_{t+1}) + u_t \]

\[ \pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + v_t \]

\[ R_t = \alpha^\pi (z_t) \pi_t + \alpha^x (z_t) x_t \]

\[ \alpha^j (z_t) = \alpha^j_0 + \frac{\alpha^j_1}{1 + \exp(-\alpha^j_2 z_t)} \quad j = x, \pi \]

\[ z_t = \rho z_{t-1} + \varepsilon^z_t, \quad \rho_z \in (0, 1] \]

\[ u_t = \rho u u_{t-1} + \varepsilon^u_t, \quad \rho_u \in (0, 1) \]

\[ v_t = \rho v v_{t-1} + \varepsilon^v_t, \quad \rho_v \in (0, 1) \]

\[ \varepsilon^z_t, \varepsilon^u_t, \varepsilon^v_t \text{ Gaussian, independent of each other.} \]
Policy Rule Parameters Driven by Latent Factors
---
Solution Method
---
Solving a simple New Keynesian model

**Solution (1)**

We posit the following MSV solution:

\[
\begin{align*}
  x_t &= a^x(z_t)(d^x_u u_t + d^x_v v_t), \\
  \pi_t &= a^\pi(z_t)(d^\pi_u u_t + d^\pi_v v_t),
\end{align*}
\]

where

\[
a^j(z_t) = a^j_0 + \frac{a^j_1}{1 + \exp(-a^j_2 z_t)},
\]

for \( j = x, \pi, \) and \( d^x_u, d^x_v, d^\pi_u, d^\pi_v, a^j_0, a^j_1, \) and \( a^j_2 \) are parameters of the MSV solution to be found.
Solution (2)

Using the method of undetermined coefficients yields the following system:

\[
\begin{align*}
    a^\pi(z_t) d^\pi_v &= \frac{1 + \alpha^x(z_t) + \tilde{a}^\pi(z_t) \rho_v d^\pi_v (\beta (1 + \alpha^x(z_t)) + \kappa)}{1 + \alpha^x(z_t) + \kappa \alpha^\pi(z_t)} + \tilde{a}^x(z_t) \rho_v d^x_v \kappa \\
    a^\pi(z_t) d^\pi_u &= \frac{\kappa + \tilde{a}^\pi(z_t) \rho_u d^\pi_u (\beta (1 + \alpha^x(z_t)) + \kappa) + \tilde{a}^x(z_t) \rho_u d^x_u \kappa}{1 + \alpha^x(z_t) + \kappa \alpha^\pi(z_t)} \\
    a^x(z_t) d^x_v &= \frac{\tilde{a}^\pi(z_t) \rho_v d^\pi_v (1 - \beta \alpha^\pi(z_t)) + \tilde{a}^x(z_t) \rho_v d^x_v - \alpha^\pi(z_t)}{1 + \alpha^x(z_t) + \kappa \alpha^\pi(z_t)} \\
    a^x(z_t) d^x_u &= \frac{\tilde{a}^\pi(z_t) \rho_u d^\pi_u (1 - \beta \alpha^\pi(z_t)) + \tilde{a}^x(z_t) \rho_u d^x_u + 1}{1 + \alpha^x(z_t) + \kappa \alpha^\pi(z_t)},
\end{align*}
\]

where \( \tilde{a}^j(z_t) \equiv \int_{-\infty}^{\infty} a^j(z_{t+1}) p(z_{t+1}|z_t) dz_{t+1}, \ j = x, \pi. \)
Parameterized Solution

\[ \rho_u = 0.9, \rho_v = 0.9, \rho_z = 0.95, \alpha_0^\pi = 0.9, \alpha_1^\pi = 1.3, \alpha_2^\pi = 1.5, \]
\[ \alpha_0^x = 0.15, \alpha_1^x = 0.15, \alpha_2^x = 1, \beta = 0.99, \kappa = 0.17, \sigma_u = 0.23, \sigma_v = 0.5. \]
\[ \Rightarrow d_\pi^u = 0.55, d_\pi^u = 0.08, d_x^v = -0.56, d_x^u = 0.14 \]
Diagnosis

Given that the solution above corresponds to only one realization of \( \{z_t\} \), I performed 100 simulations to obtain pseudo R-squares to measure the deviations of the solutions with respect to the “true” equalities, as follows: Let

\[
\hat{y}_t^L = [\hat{a}^\pi(z_t) \hat{d}_u^\pi, \hat{a}^\pi(z_t) \hat{d}_u^\pi, \hat{a}^x(z_t) \hat{d}_u^x, \hat{a}^x(z_t) \hat{d}_u^x]',
\]

and define \( \hat{y}_t^R \) accordingly for the right-hand side of the system above. The pseudo R-squared is defined as

\[
1 - \frac{(\hat{y}_t^L - \hat{y}_t^R)'(\hat{y}_t^L - \hat{y}_t^R)}{\hat{y}_t^L \hat{y}_t^L}.
\]

Values for the pseudo R-squared range from 98.6% to 99.8%, indicating that the solution is accurate.
Obtaining densities (1)

Fisherian model

Recall the MSV solution

\[ \pi_t = a(z_t) r_t \]

Given the dynamics for \( r_t \), we can write the MSV solution as

\[
\begin{align*}
\pi_t &= a(z_t)(\rho r_{t-1} + \varepsilon_t) \\
&= a(z_t) \left[ \rho r \frac{\pi_{t-1}}{a(z_{t-1})} + \varepsilon_t \right],
\end{align*}
\]

so that

\[
\pi_t | z_t, \pi_{t-1}, z_{t-1} \sim \mathcal{N} \left( \rho r \frac{a(z_t)}{a(z_{t-1})}, a(z_t)^2 \sigma^2_{\varepsilon} \right).
\]
Obtaining densities (2)

Let \( \Theta = \{a_0, a_1, a_2, \rho_r, \sigma^2_\varepsilon\} \), and denote

\[
p(\Pi, Z|\Theta) = p(\pi_n, \ldots \pi_1, \pi_0, z_n, \ldots, z_1, 0|\Theta).
\]

Then

\[
p(\Pi, Z|\Theta) = p(\pi_1|z_1 \pi_0, 0, \Theta) \times p(z_t|0, \Theta) \times p(\pi_0|\Theta)
\]

\[
\times \prod_{t=2}^{n} p(\pi_t|z_t, \pi_{t-1}, z_{t-1}, \Theta) \times p(z_t|z_{t-1}, \Theta).
\]
The posterior density of $Z$ and $\Theta$ becomes

$$p(Z, \Theta | \Pi) \propto p(\Pi, Z | \Theta)p(\Theta),$$

where $p(\Theta)$ is the prior density of the reduced form parameters. To obtain the structural parameters, we do reverse engineering from

$$\frac{\alpha(z_t)a(z_t) - 1}{\rho_r} = \int_{-\infty}^{\infty} a(z_{t+1})p(z_{t+1}|z_t) \, dz_{t+1},$$

given the estimated values for $a(z_t)$ and $z_t$ at their posterior means.
Obtaining densities \((1)\)

**New Keynesian model**

Let

\[
A(z_t) \equiv \begin{bmatrix} a^x(z_t) & 0 \\ 0 & a^\pi(z_t) \end{bmatrix} \begin{bmatrix} d^x_u & d^x_v \\ d^\pi_u & d^\pi_v \end{bmatrix}.
\]

From the MSV solution to the New Keynesian model

\[
x_t = a^x(z_t)(d^x_u u_t + d^x_v v_t),
\]

\[
\pi_t = a^\pi(z_t)(d^\pi_u u_t + d^\pi_v v_t),
\]

for \(y_t \equiv [x_t, \pi_t]'\) and \(w_t \equiv [u_t, v_t]',\) we have

\[
y_t = A(z_t)w_t.
\]
Obtaining densities (2)

New Keynesian model

Since

\[ w_t = P w_{t-1} + \varepsilon_t, \]

where

\[ P \equiv \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_v \end{bmatrix}, \]

and \( \varepsilon_t \equiv [\varepsilon^u_t, \varepsilon^v_t]' \), we have

\[ y_t = A(z_t)P A^{-1}(z_{t-1}) y_{t-1} + A(z_t) \varepsilon_t \]

so that

\[ y_t | y_{t-1}, z_t, z_{t-1} \sim \mathcal{N} \left( A(z_t)P A^{-1}(z_{t-1}) y_{t-1}, A(z_t) \Sigma A'(z_t) \right). \]
To Do

- Obtain the MSV solution when lagged states are present.
- Obtain the densities in the case of lagged state variables.
- Perform the Bayesian estimation.