Durable Goods and Persistent Recession

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Abstract

In this paper, we develop a two-sector DSGE model with infrequent purchases of durable goods and use it to address the following three issues. Does this model help explain the recent weak recovery? Can this model help resolve the absence of co-movement between durable and non-durable consumption? Is the dynamic behavior of this model similar with that of the (durable goods) stock adjustment-cost model? First, we allow for the possibility of (temporary) future durable-goods price shocks with ambiguous signals in expectations of households. We then illustrate that when anticipation of these future durable-goods price shocks takes place, the duration of recession can increase. Second, we demonstrate that the presence of idiosyncratic preference shocks to leisure in the absence of complete state-contingent financial market helps explain co-movement between durable and non-durable consumption. The main reason for this result is that, with heterogenous households and financial market incompleteness, we derive the aggregate consumption function by aggregating individual consumption functions that connect each household’s consumption directly to its wealth including housing wealth. Third, we demonstrate that since infrequent purchases of durable-goods generate inter-temporal links of durable-goods holdings, the dynamic responses of our model with respect to interest rate shocks can be similar to those of (durable goods) stock-accumulation models with convex adjustment costs widely used in recent literature.

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1 Introduction

The important role of durable consumption goods in the aggregate business cycles has been emphasized and remarked by academic researchers and policy makers. For example, it is not easy to deny the importance of the housing sector in the recent recession. On the other hand, the recent Great Recession has led us to witness the effective zero bound on the short-term nominal interest rates in many countries around the world. Thus, the desirable time-duration of the effective zero bound has become an important issue in academia as well as policy makers, while it is closely associated with the overall performance of the aggregate economy.

In this paper, we develop a two-sector DSGE model with infrequent purchases of durable goods and use it to address the following three issues. Does this model help explain the recent weak recovery? Can this model help resolve the absence of co-movement between durable and non-durable consumption? Is the dynamic behavior of this model similar with that of the (durable goods) stock adjustment-cost model?

We allow for the possibility of (temporary) future durable-goods price shocks with ambiguous signals in expectations of households. More specifically, with recursive multiple-priors utility, actions of each household are evaluated under the worst conditional probability that minimizes conditional mean (real) prices of used durable goods (multiplied by marginal utilities of consumption) following Epstein and Zin (2008). In this framework, we illustrate that when anticipation of these future durable-goods price shocks takes place, the duration of recession can increase.

We also demonstrate that the presence of idiosyncratic preference shocks to leisure in the absence of complete state-contingent financial market helps explain co-movement between durable and non-durable consumption. The main reason for this result is that, with heterogenous households and financial market incompleteness, we derive the aggregate consumption function by aggregating individual consumption functions that connect each household’s consumption directly to its wealth including housing wealth. In this sense, our model’s spill-over effect of durable-goods sector’s price drops into the aggregate economy takes a different route from those used in Iacoviello and Neri (2010).

Our model builds on the durable-goods model of Caplin and Leahy (1999, 2005) in which households determine both timings and amounts of durable-goods purchases endogenously. We incorporate random decision costs for durable-goods purchases into our model, thereby leading to state-dependent probabilities of durable-goods purchases in the similar way as is done for state-contingent pricing decisions in Dotsey, King, and Wolman (1999). We also allow for the possibility that each agent faces exogenously-determined random timings of durable-goods transactions each period as is done in the Calvo-pricing models for firms. The most apparent distinction made by the
introduction of infrequent durable-goods purchases (vs. every-period purchases) are two-fold. First, it helps incorporate the present-value of future resale values of durable-goods into consumption functions of individual households. Second, it helps incorporate the impact of the anticipation of future price shocks with ambiguous signals on housing prices or durable-goods prices. The reason for this argument is that when durable-goods purchases are made infrequently, a present-value of somewhat far future resale values of durable goods is included in the optimization condition of durable-goods purchases, whereas it is absent in the model of every-period purchases of durable goods. In turn, this feature facilitates to analyze the impacts of the anticipation of future price shocks on durable-goods prices.

It is also possible to revisit the non-comovement puzzle between durable and non-durable consumption in sticky price models. Explicitly, Barsky, House and Kimball (2007) demonstrated that non-durable and durable consumptions move in the opposite direction when prices are sticky in the non-durable goods sector but fully flexible in the non-durable goods sector and labor services move freely across the two sectors. In their model, the real price of durable goods (deflated by the price of non-durable goods) is affected only by the expected discounted value of current and future marginal utilities of durable goods. In particular, this feature slows down the movements of real durable-goods prices and thus generates the substitutability between durable and non-durable labor inputs in the aggregate labor-supply equation, leading to the non-comovement puzzle between durable and non-durable consumption. In this paper, however, the presence of heterogenous households and financial market incompleteness eliminates the substitutability between durable and non-durable labor inputs in the aggregate labor-supply equation because the aggregate consumption directly depends on aggregate value of durable-goods (housing wealth) and labor income.

Furthermore, the aggregate demand curve in this model is affected by the current price of durable goods and the expected capital gain of holding durable goods as well as the expected future aggregate demand and the anticipated real interest rate. Given this specification of the aggregate demand, we demonstrate that our model is remarkably similar with (durable goods) stock-accumulation models with convex adjustment costs such as Erceg and Levin (2006) in terms of their responses to exogenous interest rates shocks.

The rest of our paper is organized as follows. In section 2, we elaborate related literature survey. In sections 3 and 4, we spell out a durable-goods model outlined above. In section 5, we analyze the resale value of durable goods that can arise when ambiguous-averse households anticipate future shocks to prices of used goods and price shocks generate only ambiguous noisy signals. In section 6, we derive the aggregate consumption function the presence of heterogenous households and financial market incompleteness. Section 7 contains the discussion of calibration. In section 8, we extend
our analysis to the case where endogenous determination on timings of durable-goods transactions are endogenously determined. In section 9, we discuss our future research directions.

2 Literature Survey

An important question in the literature on macro models with the housing sector is to ask if fluctuations in the housing market propagate to other forms of expenditure. For example, Iacoviello and Neri (2010) argue that the spill overs from the housing market to the broader economy are non-negligible, concentrated on consumption rather than business investment, and have become more important over time, to the extent that financial innovation has increased the marginal availability of funds for credit-constrained agents. In addition, they report the following regression result on the relation between aggregate consumption and aggregate housing wealth:

$$\Delta \log C_t = 0.0039 + 0.122 \Delta \log HW_{t-1}$$

where $C_t$ is the consumption level at period $t$, $HW_t$ is the housing wealth at period $t$ and numbers in parenthesis are standard errors.

Although our primary aim is more theoretic than taking models to actual data, it is also important to have a positive response of aggregate consumption with respect to an increase in housing wealth. In this vein, a novel element in our approach lies in how to attain a positive response of aggregate consumption with respect to an increase in aggregate housing wealth. Most of related papers have tried to create co-movement between non-durable and durable-goods sectors. In our approach, we use budget constraints of each household to directly express the household’s consumption in terms of housing wealth and labor incomes in the presence of incompleteness in financial markets.

Another important question might be what are driving forces behind fluctuations in the housing market. Davis and Heathcote (2005) use a multi-sector model (construction, manufactures, and services) to illustrate that their model can replicate observed facts: Residential investment is more than twice as volatile as business investment and consumption, residential investment, and non-residential investment comove positively. In their model, sectoral technology shocks are important driving forces, while there is no demand shock in the housing market. On the other hand, Iacoviello and Neri (2010) estimate their multi-sector model to show that preference shocks to housing service play an important role in housing market fluctuations, though technology and monetary shocks are still important. Specifically, the inclusion of demand shocks in the housing market helps to replicate the observed positive correlation between housing price and housing investment. In addition, Lambertini, Caterina and Punzi (2010) analyze the possibility of expectations-driven cycles in the
housing market, stressing that peaks of real housing price tend to be followed by macroeconomic recession in the U.S. data. In their model, expectations regarding sector-specific shocks can generate booms in the housing market but only expectations of future expansionary monetary policy that are not fulfilled can generate macroeconomic recessions. In this regard, our interest is not to identify new driving forces behind housing-market fluctuations. Rather, our analysis illustrates that whatever shocks drive a recession, there is a possibility of having a deeper and persistent recession when households feel lack of confidence in their information regarding future shocks that do not necessarily need to be fulfilled.

Turning to models of infrequent purchases of durable goods, Ghent (2009) analyzes a DSGE model with infrequent adjustment of housing where its slow adjustment is imposed through a staggering structure similar to the mechanism Taylor (1980) uses to model wages. In addition, Thomas (2002) incorporates lumpy investment decisions (for production capital) into a DSGE model in a similar way as is done for state-contingent pricing decisions in Dotsey, King, and Wolman (1999).

3 The Durable-Goods Market Model with Infrequent Purchases of Durable Goods

In this section, we describe a simple DSGE model in which time intervals of durable-goods purchases are exogenously determined. Each infinitely-lived household consists of a lot of agents who determine purchases of durable goods, while their measure is 1. In addition, these agents also share non-durable goods consumptions and leisures within the household.

Each period, a faction of agents, $1 - \theta$, sell their durable goods, while other agents, $\theta$, keep their durable goods. The value of $\theta$ is an exogenously-determined constant. We also assume that each agent’s utility function is additively separable between durable and non-durable consumptions and between durable goods and leisure. Specifically, if an agent purchases durable goods at period $t$, its period utility at period $t$ is $U(C_t, 1 - N_t) + \hat{U}(D_t^*)$, where $C_t$ is non-durable consumption, $N_t$ is the aggregate hours worked, and $D_t^*$ is the amount of durable goods at period $t$. If the agent has not purchased durable goods after period $t - i$, its period utility at period $t$ is $U(C_t, 1 - N_t) + \hat{U}((1 - \delta_d)^iD_{t-i}^*)$, where $\delta_d$ is the depreciation rate of durable goods. The functions $U(C, H)$ and $\hat{U}(D)$ are concave and continuously twice differentiable in their arguments respectively.

On the other hand, used durable-goods are the same as new durable-goods if depreciations from their vintages are taken into account. In addition, dealers alone can intermediate between buying and selling of durable goods. In other words, there are no durable-goods transactions between households so that household should sell their durable goods to dealers and then dealers
sell them to households. The reason for this restriction is to allow for the possibility of resale market imperfection, leading to the difference between selling and purchasing prices of durable goods. Specifically, the (real) selling price (denoted by $\tilde{P}_{u,t}$) is $\kappa$ times the (real) purchasing price (denoted by $\tilde{P}_{d,t}$) so that $\tilde{P}_{u,t} = \kappa \tilde{P}_{d,t}$ with $0 \leq \kappa \leq 1$ in each period $t = 0, 1, \cdots, \infty$. The real prices of durable goods are deflated by the nominal price of non-durable goods (denoted by $P_{n,t}$).

In this framework, agents who purchase durable goods at period $t$ maximize the following equation with respect to $D^*_t$: 

$$
\sum_{i=0}^{\infty} (\beta \theta)^k \tilde{U}'((1 - \delta d)^i D^*_t) + (1 - \theta) \sum_{i=1}^{\infty} ((1 - \delta d)\beta)^i \theta^{i-1} E_t[\Lambda_{t+i} \tilde{P}_{u,t+i}]D^*_t - \Lambda_t \tilde{P}_{d,t} D^*_t
$$

where $\Lambda_t$ is the marginal utility of non-durable goods consumption at period $t$. We note that agents take the marginal utility of non-durable goods as given when they make decisions on durable-goods purchases, while this optimization problem is expressed in terms of utility level at period $t$. In addition, the first term of this equation corresponds to the discounted sum of current and future utility benefits of holding durable goods and the second term is the resale value of new durable goods. The optimization condition for $D^*_t$ turns out to be

$$
\sum_{k=0}^{\infty} (\theta \beta)^k \tilde{U}'((1 - \delta d)^k D^*_t) + Z_t = \Lambda_t \tilde{P}_{d,t}
$$

(3.1)

where $Z_t$ has the following recursive representation:

$$
Z_t = (1 - \theta) \beta (1 - \delta d) E_t[\Lambda_{t+1} \tilde{P}_{u,t+1}] + \theta \beta (1 - \delta d) E_t[Z_{t+1}].
$$

(3.2)

In this equation, $Z_t$ summarizes expectations of households regarding future resale prices of durable goods. Given current-period’s (real) market price of durable goods, the demand of durable goods rises as the resale value increases. Hence, it acts like a demand shock in durable-goods or housing market.

Having described the demand side of durable goods market, we now characterize the supply side of durable-goods market. We first characterize the law of motion for the market supply of used goods. In doing so, we first note that households would sell $(1 - \theta)(1 - \delta d)\mathcal{F} D^*_{t-1}$ in the market at period $t$ if they all purchased their goods at period $t - 1$ (when $\mathcal{F}$ denotes the fraction of agents who purchase durable goods in each period). In the same way as is done before, $(1 - \theta)\theta^{i-1}(1 - \delta d)^i\mathcal{F} D^*_{t-i}$ is the amount of used durable goods that households would sell at period $t$ if they purchased them at period $t - i$ for $i = 1, 2, \cdots, \infty$. In turn, the market supply at period $t$ of used goods can be written as $M_{d,t} = (1 - \theta) \sum_{i=1}^{\infty} \theta^{i-1}(1 - \delta d)^i\mathcal{F} D^*_{t-i}$. Hence, our discussion implies that there is a recursive representation for the market supply at period $t$ of used goods:

$$
M_{d,t} = (1 - \delta d)(\theta M_{d,t-1} + (1 - \theta)\mathcal{F} D^*_{t-1}).
$$

(3.3)
We also allow for inventory holdings of durable goods producers. The introduction of inventory holdings can be rationalized as follows. To the extent which the market supply of used goods exists in our model, we cannot exclude the possibility that the market supply of used goods can exceed the aggregate demand of durable goods. Especially, if we frequently run into the case that the market supply of used goods is greater than the aggregate demand of durable goods, our equilibrium analysis may not be interesting. Allowing for each firm’s inventory holdings then helps avoid this situation.

Furthermore, durable-goods producers not only produce their own products as monopolistically competitive firms but also purchase used goods from households. The real cash flow of each firm (denoted by $\Upsilon_t(f)$) can be written as

$$\Upsilon_{d,t}(f) = S_{d,t}(f)\tilde{P}_{d,t}(f) - mc_t Y_{d,t}(f) - \tilde{P}_{u,t}(f) M_{d,t}(f)$$

where $mc_t$ is the real marginal cost (measured in units of non-durable goods) that is independent of output flows of individual firms, $S_{d,t}(f)$ is firm $f$’s sales, $Y_{d,t}(f)$ is the firm’s output at period $t$, and $M_{d,t}(f)$ is the amount of used goods that the firm purchases from households. The inventory equation of firm $f$ can be written as

$$I_{d,t}(f) = (1 - \delta_m) I_{d,t-1}(f) + Y_{d,t} + M_{d,t}(f) - S_{d,t}(f)$$

where $I_{d,t}(f)$ is firm $f$’s inventory holdings at the end of period $t$ and $\delta_m$ is the deprecations rate for inventories of firms. The demand curve facing each firm $f$ is given by

$$S_{d,t}(f) = \left(\frac{P_{d,t}(f)}{\mu_d}\right)^{-\epsilon} S_{d,t}; \quad P_{d,t}^{1-\epsilon} = \int_0^1 (P_{d,t}(f))^{1-\epsilon} df; \quad S_{d,t} = (\int_0^1 (P_{d,t}(f))^{\epsilon-1} df)^{\frac{1}{\epsilon-1}}$$

where $\epsilon$ is the elasticity of demand. Since prices are fully flexible in the durable-goods market, the profit-maximization condition for sales turns out to be $\tilde{P}_{d,t}(f) = \mu_d mc_t$, where $\mu_d (= \epsilon/(\epsilon - 1))$ is the markup of durable-goods producers. Putting this equation into the cash flow equation for firm $f$ leads to the following equation: $\Upsilon_{d,t}(f) = mc_t (\mu_d(S_{d,t}(f) - \kappa M_{d,t}(f)) - Y_{d,t}(f))$. We also impose the zero-profit condition for each retail firm in the durable-goods market. The zero-profit condition implies that supply curves for new durable goods should take the following form: $Y_{d,t}(f) = \mu_d(S_{d,t}(f) - \kappa M_{d,t}(f))$. Substituting this supply curve equation into the inventory equation specified above, the law of motion for durable-goods inventories can be rewritten as follows:

$$I_{d,t}(f) = (1 - \delta_m) I_{d,t-1}(f) + (\mu_d - 1) S_{d,t}(f) + (1 - \kappa_t \mu_d) M_{d,t}(f).$$

The profit maximization condition implies that each firm sells the same amount in each period so that $S_{dt}(f) = S_{dt}$, while we also impose $M_{dt}(f) = M_{dt}$ for all $f \in [0, 1]$. In this symmetric equilibrium
where each firm produces the same amount of durable goods in each period, the aggregate supply of durable goods turns out to be

$$Y_{dt} = \mu_d(S_{dt} - \kappa M_{dt}).$$  \hspace{1cm} (3.4)

We thus find that the aggregate output of new durable goods increases with the aggregate demand of durable goods (both of new and used goods), whereas it is crowded out by increases in the market supply of used durable goods. Moreover, our analysis is focused on the case of $S_{dt} > \kappa M_{dt}$ in order to have a non-negative equilibrium output of durable goods. The aggregate inventory of durable goods (held by dealers and firms) can be written as

$$I_{dt} = (1 - \delta_m)I_{dt-1} + (\mu_d - 1)S_{dt} + (1 - \kappa \mu_d)M_{dt}.$$

(3.5)

### 4 Production of Durable and Non-Durable Goods

We will describe the production side of the model. We assume that labor and capital inputs can move freely between non-durable and durable goods sectors without having any costs. In addition, an individual wholesale firm hires labor and capital to produce wholesale goods $Y_{n,t}$ and new durable goods (or new houses) $Y_{d,t}$ at the same time, following Davis and Heathcote (2005) and Iacoviello and Neri (2010). In addition, prices of wholesale firms can be obtained by deflating retailers’ prices by markups of retailers. Hence, the nominal cash flow at period $t$ of each wholesale firm is

$$(P_{n,t}/\mu_{n,t})Y_{c,t} + (P_{d,t}/\mu_{d,t})Y_{d,t} - P_{n,t}(R_t(K_{n,t} + K_{d,t}) + W_t(N_{n,t} + N_{d,t}))$$

where $\mu_{n,t}$ and $\mu_{d,t}$ are markups of retailers for wholesale goods and new houses respectively. The production technologies of non-durable and durable goods are both represented by a Cobb-Douglas production function with respect to capital and labor, $Y_{n,t} = N_{n,t}^{1-\nu}K_{n,t}^{\nu}$ and $Y_{d,t} = N_{d,t}^{1-\nu}K_{d,t}^{\nu}$ where parameter $\nu$ is positive and less than one. The optimization conditions for hiring production inputs are

$$(\nu/\mu_{n,t})N_{n,t}^{1-\nu}K_{n,t}^{\nu-1} = R_t; \quad (\nu/\mu_{d,t})N_{d,t}^{1-\nu}K_{d,t}^{\nu-1} = R_t; \quad (1-\nu)/\mu_{n,t})N_{n,t}^{1-\nu}K_{n,t}^{\nu} = W_t; \quad ((1-\nu)\tilde{P}_{d,t}/\mu_{d,t})N_{d,t}^{1-\nu}K_{d,t}^{\nu-1} = W_t.$$  

We find from these optimization conditions that $\mu_{n,t}^{-1} = P_{d,t}/\mu_{d,t}$.

Hence, when we define $mc_t = \mu_{n,t}^{-1}$, the real price of new durable goods turns out to be $\tilde{P}_{d,t} = \mu_d mc_t$.

In addition, the cost-minimization conditions can be rewritten in terms of aggregate production inputs as follows:

$$\nu mc_t N_{t}^{1-\nu}K_{t}^{\nu-1} = R_t; \quad (1-\nu)mc_t N_{t}^{1-\nu}K_{t}^{\nu} = W_t$$

where $N_{t} = N_{n,t} + N_{d,t}$ and $K_{t} = K_{n,t} + K_{d,t}$. The aggregate production function is also given by

$$Y_{n,t} + Y_{d,t} = N_{t}^{1-\nu}K_{t}^{\nu}.$$  

The market-clearing condition for non-durable goods sector is

$$C_t + K_{t+1} - (1 - \delta_k)K_t = Y_{n,t}.$$
We thus add the market clearing condition of durable goods sector to this equation to yield
\[ C_t + \mu_d(FD^*_t - \kappa M_{d,t}) + K_{t+1} - (1 - \delta_k)K_t = N^{1-\nu}_t K^\nu_t. \]

In what follows, we describe the pricing behavior of non-durable goods producers. In the market for non-durable goods, there are a lot of monopolistically competitive firms that follow the Calvo-type staggered price-setting. The introduction of nominal price rigidity into the non-durable goods sector makes the real marginal cost \( mc_t \) vary over time. Each period, a fraction of firms \( 1 - \alpha_n \) set a new price, while the remaining firms do not change their prices. Specifically, firms that are allowed to set a new price in period \( t \) solve the following problem:
\[
\max_{P^*_n,t} \sum_{i=0}^{\infty} (\alpha_n \beta)^i \frac{\Lambda_{t+i}}{\Lambda_t} Y_{n,t+i} \left( \left( \frac{P^*_n,t}{P_{n,t+i}} \right)^{-\epsilon} P^*_n,t - mc_{t+i} \left( \frac{P^*_n,t}{P_{n,t+i}} \right)^{-\epsilon} \right)
\]

The optimization condition can be written as
\[
\hat{P}_{n,t} = \left( \frac{\epsilon}{\epsilon - 1} \right) \left( \frac{Z_{2,t}}{Z_{1,t}} \right)
\]
where \( \hat{P}_{n,t} \) is the real price of the optimal price at period \( t \). The recursive representations of \( Z_{1,t} \) and \( Z_{2,t} \) are
\[
Z_{1,t} = \Lambda_t Y_{n,t} + \alpha_n \beta E_t[\Pi_{n,t+1}^\epsilon Z_{1,t+1}]
\]
\[
Z_{2,t} = \Lambda_t Y_{n,t} mc_t + \alpha_n \beta E_t[\Pi_{n,t+1}^{\epsilon-1} Z_{2,t+1}]
\]
where \( \Pi_{n,t} \) is the inflation between periods \( t \) and \( t - 1 \). Moreover, the nominal price index of non-durable goods is \( P_{n,t}^{1-\epsilon} = (1 - \alpha_n)(P^*_n,t)^{1-\epsilon} + \alpha_n P_{n,t-1}^{1-\epsilon} \). Hence, under the Calvo pricing, the price inflation of non-durable goods sector also becomes
\[
1 = (1 - \alpha_n)(\hat{P}_{n,t})^{1-\epsilon} + \alpha_n \Pi_{n,t}^{\epsilon-1}
\]

5 **Durable-Goods Price Shocks Ambiguous Signals**

We will allow for the possibility that agents are temporarily uncertain about future relations between selling and purchasing prices. In particular, we demonstrate consequences of this perceived uncertainty regarding future prices of used goods on current resale value of durable goods.

**Definition 5.1 (Recursive multiple-priors utility, Epstein and Schneider (2008))** The state space at period \( t \), \( S_t = S \), is identical for all times. Each household’s information at period \( t \) consists of history \( s^t = (s_1, \ldots, s_t) \), while a state vector \( s_t \) is observed in period \( t \). At any period \( t = 0,1,\ldots \), given a history of states \( s^t \), preferences over future consumption and leisure are represented by a conditional utility function \( \mathcal{V}_t \) defined recursively by
\[
\mathcal{V}_t(C, s^t) = \min_{P_t \in \mathcal{P}_t(s^t)} \mathcal{E}^P[U(C_t, N_t) + \hat{U}(D^*_t) + \beta\mathcal{V}_{t+1}(C, s^t, s_{t+1})]
\]
The set $\mathcal{P}(s^t)$ of probability measures on $S$ captures conditional beliefs about the next observation $s_{t+1}$. Thus, beliefs are determined by the whole process of conditional one-step-ahead belief sets $\mathcal{P}(s^t)$.

In this framework, each household begins with a prior on the distribution of the noisy signal at period 0, while the household revises its perceived distribution on the noisy signal as time goes. In this sense, the beliefs of households are constructed recursively over time.

**Definition 5.2 (Ambiguous-averse household)** With recursive multiple-priors utility, actions of each household are evaluated under the worst conditional probability. The worst conditional probability minimizes conditional mean (real) prices of used durable goods (multiplied by marginal utilities of consumption).

Specifically, at the beginning of each period, each household chooses the distribution of current-period’s signal for next-period’s price shock by minimizing conditional mean prices of used goods, while such a choice is repeated over time.

**Definition 5.3 (Real price shock with ambiguous signal)** In each period, a shock can hit the relation between selling and purchasing prices of durable goods, while the shock is evaluated in terms of utils. Specifically, when the shock takes a value $u_{t+1}$ in period $t+1$, the real price of used goods at period $t+1$ can be written as $\tilde{P}_{u,t+1} = \kappa \tilde{P}_{d,t+1} + u_{t+1} \Lambda^{-1}_{t+1}$. At period $t$, the signal for $u_{t+1}$ is realized with an additive noise: $s_{u,t} = u_{t+1} + \epsilon_{s,t}$, while the variance of this noise $\epsilon_s$ is known only to lie in some range, $\sigma^2_s \in [\sigma^2_s, \sigma^2_s]$.

More specifically, we assume that $u_{t+1}$ is an independently and identically distributed random variable whose distribution is $\mathcal{N} \sim (0, \sigma^2_u)$, while the one-period ahead signal for this shock (which is realized at period $t+1$) is observed at period $t$. The signal (denoted by $s_{u,t}$) is a noisy estimate of $u_{t+1}$ because it is affected by an independently and identically distributed random variable whose distribution is $\mathcal{N} \sim (0, \sigma^2_s)$. In addition, since the variance of the shock $\epsilon_s$ is known only to lie in some range, $\sigma^2_s \in [\sigma^2_s, \sigma^2_s]$, this structure of ambiguous signals captures the agent’s lack of confidence in the signal’s precision, following Epstein and Schneider (2008).

We now discuss the impact of this ambiguous signal on expectations of an ambiguous-averse household. Under the assumption of normal distribution, the conditional expectation of $u$ based on a realization of $s$ is the same as the projection of next-period’s shock on current-period’s signal (projection of $u$ on $s$). We thus have $E[u|s] = \gamma(\sigma^2_s)s$ where $\gamma(\sigma^2_s) = \sigma^2_u/(\sigma^2_u + \sigma^2_s)$. The minimization of this expectation in terms of $\sigma^2_s$ depends on the realized value of signal $s_u$, leading to
the following equation:

\[
\begin{align*}
\min_{\sigma^2_u \in [\sigma^2, \pi^2]} E[u|s_u] &= \gamma_{\text{max}} s_u \quad \text{if} \quad s_u < 0 \\
&= \gamma_{\text{min}} s_u \quad \text{if} \quad s_u \geq 0
\end{align*}
\]

where \(\gamma_{\text{min}}\) and \(\gamma_{\text{max}}\) are defined as \(\gamma_{\text{min}} = \sigma_u^2/(\sigma_u^2 + \sigma_s^2)\) and \(\gamma_{\text{max}} = \sigma_u^2/(\sigma_u^2 + \sigma_s^2)\).

On the other hand, the minimized expected-value of the projection specified above can be determined by calculating the mean of a truncated normal variable first and then choosing a value of the variance of the noise (to minimize the resulting mean). First, the expected value of the projection is given by

\[
E[\gamma s_u] = \frac{\gamma_{\text{max}}}{2} E[s_u|s_u < 0] + \frac{\gamma_{\text{min}}}{2} E[s_u|s_u > 0] = - (\gamma_{\text{max}} - \gamma_{\text{min}}) \frac{\sigma_{su}}{\sqrt{2\pi}}
\]

where \(\gamma\) takes either \(\gamma_{\text{min}}\) or \(\gamma_{\text{max}}\) depending on the sign of a realized value of \(s_u\). In this equation, the second equality is based on the expected value of a truncated normal random variable:

\[
E[x|x < 0] = \frac{2}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{0} x \exp(-\frac{x^2}{2\sigma_x^2})dx = -\sigma_x \sqrt{\frac{2}{\pi}}
\]

where \(x\) is a normal random variable whose distribution is \(N(0, \sigma_x^2)\), and for the second equality we use a change-of-variable method for integration: \(\int_{-\infty}^{0} x \exp(-x^2/(2\sigma_x^2))dx = 2\sigma_x^2 \int_{-\infty}^{0} y \exp(-y^2)dy = -\sigma_x^2\) with \(y = x/\sigma_x \sqrt{2})\). Second, the minimization of this expected value can be done as follows:

\[
\min_{\sigma^2_u \in [\sigma^2, \pi^2]} \{-(\gamma_{\text{max}} - \gamma_{\text{min}}) \frac{\sqrt{\sigma_u^2 + \sigma_s^2}}{\sqrt{2\pi}}\} = -(\gamma_{\text{max}} - \gamma_{\text{min}}) \frac{\sqrt{\sigma_u^2 + \sigma_s^2}}{\sqrt{2\pi}} = -(\gamma_{\text{max}} - \gamma_{\text{min}}) \frac{\sigma_{su}}{\sqrt{2\pi} \gamma_{\text{min}}}
\]

where we use the definition equation of \(\sigma_{su} = \sqrt{\sigma_u^2 + \sigma_s^2}\) for the left side of this equation and the second equality reflects the definition of \(\gamma_{\text{min}}\).

**Assumption 5.1 (Temporary durable-goods price shocks with ambiguous signals)** The household at period 0 expects that temporary ambiguous signals on resale values of durable goods will take place at periods 1 and 2. The following equations describe the expectation at period 0 of the household regarding resale-values at periods 0, 1, and 2:

\[
\begin{align*}
Z_0^\dagger &= (1 - \theta)\beta(1 - \delta_d)E_0[k\Lambda_1 \tilde{P}_{d1} + u_1] + \theta \beta(1 - \delta_d)E_0[Z_1] \\
Z_1^\dagger &= (1 - \theta)\beta(1 - \delta_d)E_1[k\Lambda_2 \tilde{P}_{a2} + u_2] + \theta \beta(1 - \delta_d)E_1[Z_2] \\
Z_2 &= (1 - \theta)\beta(1 - \delta_d)\kappa E_2[A_3 \tilde{P}_{d3}] + \theta \beta(1 - \delta_d)E_2[Z_3].
\end{align*}
\]

In this equation, \(Z_0^\dagger\) and \(Z_1^\dagger\) are used to distinguish the presence of temporary shocks with ambiguous signal from the absence of such shocks in the households’ expectations formed at period 0.
We now show how to compute the resale value when there are temporary ambiguous signals about future prices of used durable goods. In doing so, we note that the resale-value equation can be written as follows:

$$Z_t = (1 - \theta) \beta (1 - \delta_d) E_t [\Lambda_{t+1} \tilde{P}_{u,t+1}] + \theta \beta (1 - \delta_d) E_t [Z_{t+1}]$$

Given the assumption for the temporary durable-goods price shocks with ambiguous signals, the household at period 0 expects that the ambiguous averse agents at period 1 computes the resale value of used durable goods as follows:

$$Z_{0}^{\dagger} = (1 - \theta) \beta (1 - \delta_d) \kappa E_1 [\Lambda_2 \tilde{P}_{d2}] + \theta \beta (1 - \delta_d) E_1 [Z_2] + \gamma_1 s_{u,1}$$

where $\gamma_1 s_{u,1} = \min_{\sigma^2 \in [\sigma^2, \sigma^2]} E_1 [u_2]$ and the derivation of this term is discussed above. We also note the following minimization problem: $\min_{\sigma^2 \in [\sigma^2, \sigma^2]} E_0 [\gamma_1 s_{u,1}] = - (\gamma - \gamma) \sigma_u / \sqrt{2 \pi \gamma}$, which was already discussed before. We thus summarize the resale value at period 0 as follows:

$$Z_{0}^{\dagger} = Z_0 + (1 - \theta) \beta (1 - \delta_d) \gamma_0 s_{u,0} - \theta \beta (1 - \delta_d) \frac{(\gamma - \gamma) \sigma_u}{\sqrt{2 \pi \gamma}}$$

where $Z_1 = (1 - \theta) \beta (1 - \delta_d) \kappa E_1 [\Lambda_2 \tilde{P}_{d2}] + \theta \beta (1 - \delta_d) E_1 [Z_2]$ and $Z_0 = (1 - \theta) \beta (1 - \delta_d) \kappa E_0 [\Lambda_2 \tilde{P}_{d1}] + \theta \beta (1 - \delta_d) E_0 [Z_1]$.

Furthermore, one might want to continue our analysis even when price shocks of used durable goods with ambiguous signals are expected to take place beyond period 2. For example, agents at period 0 might expect that ambiguous signals continue through period $t + 1$ for $t = 1, 2, \ldots, T$ and $T$ is a finite number. In this case, the resale value factor at period $t$ is given by

$$Z_{0}^{\dagger} = Z_0 + (1 - \theta) \beta (1 - \delta_d) \gamma_0 s_{u,0} - \theta \beta (1 - \delta_d) \frac{1 - (\theta \beta (1 - \delta_d))^t (\gamma_{\text{max}} - \gamma_{\text{min}}) \sigma_u}{1 - \theta \beta (1 - \delta_d)} \frac{(\gamma_{\text{max}} - \gamma_{\text{min}}) \sigma_u}{\sqrt{2 \pi \gamma_{\text{min}}}}.$$  

On the other hand, the impact of ambiguous signals still remains in a log-linear approximation of this equation around the deterministic steady state because we assume that price shocks exist only for a finite number of time periods. Specifically, the log-linearization of this equation leads to

$$\hat{Z}_{0}^{\dagger} = \hat{Z}_0 + (1 - \theta) \beta (1 - \delta_d) Z^{-1} \gamma_0 s_{u,0} - \theta \beta (1 - \delta_d) \frac{1 - (\theta \beta (1 - \delta_d))^t (\gamma_{\text{max}} - \gamma_{\text{min}}) \sigma_u}{1 - \theta \beta (1 - \delta_d)} \frac{(\gamma_{\text{max}} - \gamma_{\text{min}}) \sigma_u}{\sqrt{2 \pi \gamma_{\text{min}}}}$$

where $\hat{Z}_0$ represents the log-deviation of $\hat{Z}_0$ from its deterministic steady-state value, $\hat{Z}_0$ represents the log-deviation of $Z_0$ from its deterministic steady-state value, and the deterministic steady-state value of $Z$ is $Z = (1 - \theta) \beta (1 - \delta_d) \Lambda \hat{P}_u / (1 - \theta \beta (1 - \delta_d))$.

**Result 5.1 (Perceived resale value with ambiguous signals)** The household at period 0 expects that future price shocks with ambiguous signals take place from period 1 through period $t + 1$.
for $t = 1, 2, \cdots, T$ and $T$ is a finite natural number. Given this assumption, the perceived resale value at period 0 of durable goods (by the ambiguous-averse household) is given by

$$Z^\dagger_0 = Z_0 + (1 - \theta)\beta(1 - \delta_d)\gamma_0 \sigma_u \theta(1 - \delta_d) \left[ \frac{\left( \frac{\gamma_{\max}}{\gamma_{\min}} \right) \theta \beta(1 - \delta_d) \gamma_0 \sigma_u}{\sqrt{2\pi \gamma_{\min}}} \right]$$

where $Z_0$ can be computed by using the recursive formula for the resale value of durable goods that holds in the absence of ambiguous signals.

### 6 Determination of Aggregate Non-Durable Consumption

In this section, we demonstrate that the presence of idiosyncratic preference shocks to leisure in the absence of complete state-contingent financial market helps explain co-movement between durable and non-durable consumption. The main reason for this result is that, with heterogenous households and financial market incompleteness, the aggregate consumption function by aggregating individual consumption functions that connect each household’s consumption directly to its wealth including housing wealth.

We now derive the aggregate non-durable consumption function by solving individual households’ utility maximization problems subject to their budget constraints. We also assume that each household’s preferences are represented by log utility functions for non-durable consumption and leisure respectively and affected by idiosyncratic preference shocks to leisure. Specifically, the period utility function for each household $h$ at period $t$ is given by

$$U(C_t(h), 1 - N_t(h)) = \log C_t(h) + (b_n + \epsilon_t(h)) \log(1 - N_t(h))$$

where $\epsilon_t(h)$ denotes an idiosyncratic preference shock to leisure. We choose this logarithmic utility function to facilitate the aggregation of individual consumption functions, while it has been widely used in the literature. Since household $h$ takes $\epsilon_t(h)$ as given, the utility-maximization condition at period $t$ for household $h$ leads to the following labor supply equation:

$$W_t(1 - N_t(h)) = (b_n + \epsilon_t(h))C_t(h).$$

We also assume the absence of complete state-contingent market for both aggregate and idiosyncratic risks, so that we derive the aggregate consumption function by summing up all present-value budget constraints of individual households. The optimization condition for investment for physical production capital is

$$1 = E_t[\beta_{t+1}(h)(R_{t+1} + 1 - \delta_k)]$$

where $R_{t+1}$ is the real rental at period $t + 1$ for capital and $\beta_{t+1}(h)$ is the inter-temporal marginal rate of substitution between non-durable consumptions period $t$ and $t + 1$ of household $h$. 

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We now derive present-value budget constraint of individual households and aggregate them to express the aggregate consumption in terms of financial and human wealths. Each household needs to accumulate their wealths in order to reduce variations in its consumption expenditures over time and across different states flows in the absence of the complete state-contingent financial markets. In period $t$, the flow budget constraint of each household $h$ can be written as follows:

$$
(1 + b_h + \epsilon_t(h))C_t(h) + \mathcal{F}\tilde{P}_{dt}D_t^*(h) + E_t[\beta_{t,t+1}(h)\Omega_{t+1}(h)] \leq (1 - \theta)\tilde{P}_{u,t}D_{u,t}(h) + \Omega_t(h),
$$

$$
\Omega_t(h) = K_t(h)(R_t + 1 - \delta_k) + \tilde{P}_{s,t}(h) + \mathcal{H}_t(h) + \frac{B_t(h)}{\theta} + T_t(h)
$$

where $D_{u,t}(h)$ denotes the total amount of durable goods held by household $h$ at the end of period $t-1$, $\tilde{P}_{s,t}(h)$ is the real price of each firm’s share, $\mathcal{H}_t(h)$ is the present value of full income flows, and $T_t(h)$ is the present value of real lump-sum transfers. Specifically, $\tilde{P}_{s,t}(h)$, $\mathcal{H}_t(h)$, and $T_t(h)$ are defined as follows:

$$
\tilde{P}_{s,t}(h) = Y_t + E_t[\beta_{t,t+1}(h)\tilde{P}_{s,t+1}(h)]
$$

$$
\mathcal{H}_t(h) = W_t + E_t[\beta_{t,t+1}(h)\mathcal{H}_{t+1}(h)]
$$

$$
T_t(h) = T_t + E_t[\beta_{t,t+1}(h)T_{t+1}(h)]
$$

We also note that there is a recursive law of motion for $D_{u,t}(h)$ as follows:

$$
D_{u,t}(h) = \mathcal{F}(1 - \delta_d)D_{t-1}^*(h) + \theta(1 - \delta_d)D_{u,t-1}(h).
$$

Since this recursive law of motion for $D_{u,t}(h)$ implies that $D_{u,t+k}(h) = (\theta(1 - \delta_d))^kD_{u,t}(h) + (1 - \delta_d)\mathcal{F}\sum_{i=0}^{k-1}((1 - \delta_d)\theta)^{k-i-1}D_{t+i}^*(h)$, forward iterations of flow budget constraints lead to a present value budget constraint of the following form:

$$
E_t[\sum_{k=0}^{\infty} \beta_{t,t+k}(h)(1 + b_h + \epsilon_{t+k}(h))C_{t+k}(h) + \mathcal{F}D_{t+k}^*(h)\mathcal{U}_{t+k}(h))] \leq ((1 - \theta)\tilde{P}_{u,t} + \theta\mathcal{R}_t(h))D_{u,t}(h) + \Omega_t(h)
$$

where $\mathcal{U}_{t+k}(h)$ is the user cost of current durable-goods holdings and $\mathcal{R}_t(h)$ is the resale value of durable goods:

$$
\mathcal{U}_{t+k}(h) = \tilde{P}_{d,t+k} - (1 - \theta)(1 - \delta_d)E_t[\sum_{i=1}^{\infty} \beta_{t+k,t+k+i}(h)((1 - \delta_d)\theta)^{i-1}\tilde{P}_{u,t+k+i}]
$$

$$
\mathcal{R}_t(h) = (1 - \theta)(1 - \delta_d)E_t[\sum_{k=1}^{\infty} \beta_{t,t+k}(h)\theta(1 - \delta_d)^{k-1}\tilde{P}_{u,t+k}]
$$

In order to derive this present-value budget constraint, the following conditions should hold:

$$
\lim_{k \to \infty} E_t[\beta_{t,t+k}\Omega_{t+k}(h)] = 0, \quad \lim_{k \to \infty} E_t[\beta_{t,t+k}(h)(\theta(1 - \delta_d))^{k-1}\tilde{P}_{u,t+k}] = 0, \quad \lim_{k \to \infty} E_t[\beta_{t,t+k}] = 0.
$$

On the other hand, since the first-order condition for durable goods is $\nu_dC_t(h) = D_t^*(h)\mathcal{U}_t(h)$, substituting this equation into the present-value budget constraint of household $h$ leads a consumption function for household at period $t$ as follows:

$$
\frac{(1 + b_h + \nu_d\mathcal{F}}{1 - \beta} + \epsilon_t(h))C_t(h) = (1 - \theta)\tilde{P}_{u,t}D_{u,t}(h) + \theta\mathcal{R}_t(h)D_{u,t}(h) + \Omega_t(h)
$$

for $t = 0, 1, 2, \cdots, \infty$. 

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Before aggregating these flow-budget constraints across different households, we note that aggregating labor-supply curves of individual households leads to

\[ W_t(1 - N_t) = b_n C_t + \Delta_{c,t}, \quad \Delta_{c,t} = \int_0^1 C_t(h)\epsilon_t(h)dh \]

Hence, aggregating individual consumption functions and then substituting this equation into the resulting equation, we have

\[ C_t = \frac{1 - \beta}{1 + \beta b_h + \nu dF}((1 - \theta)\tilde{P}_{s,t}D_{u,t} + \theta\Delta_{b,t}R_{t}D_{u,t} + \tilde{\Omega}_t), \quad \Delta_{b,t} = \int_0^1 \frac{R_t(h)D_{u,t}(h)}{R_t D_{u,t}}dh. \]

where \( \tilde{\Omega}_t \) summarizes all non-housing (or durable-goods) wealths so that \( \tilde{\Omega}_t = \Omega_t - W_t + W_t N_t \). Specifically, this non-housing wealth variable (non-durable goods wealth) can be written as

\[ \tilde{\Omega}_t = mc_t N_t^{1-v}k^{\nu} + K_t(1 - \delta_k) + Y_{n,t}(1 - mc_t) + E_t[\beta_{t,t+1}\mathcal{H}_{t+1} + \tilde{P}_{s,t+1} + \tilde{T}_t] + (B_t/P_t) + \tilde{T}_t \]

where \( \tilde{P}_{s,t}, \mathcal{H}_t, \) and \( \tilde{T}_t \) are defined as follows:

\[ \tilde{P}_{s,t} = \mathcal{Y}_t + E_t[\beta_{t,t+1}\tilde{P}_{s,t+1}], \quad \mathcal{H}_t = W_t + E_t[\beta_{t,t+1}\mathcal{H}_{t+1}], \quad \tilde{T}_t = T_t + E_t[\beta_{t,t+1}T_{t+1}] \]

Given the assumption of the production structure, we now elaborate the expression of \( \tilde{\Omega}_t \) discussed above.

\[ \tilde{\Omega}_t = mc_t N_t^{1-v}k^{\nu} + K_t(1 - \delta_k) + Y_{n,t}(1 - mc_t) + E_t[\beta_{t,t+1}(\mathcal{H}_{t+1} + \tilde{P}_{s,t+1} + \tilde{T}_{t+1})] + (B_t/P_t) + \tilde{T}_t \]

7 Calibration

We log-linearize equilibrium conditions around the deterministic steady state of the model to compute numerical solutions. We include a set of linearized equilibrium conditions in the appendix, while Dynare is used to compute numerical solutions. In what follows, we describe how we assign numerical values to model parameters.

We note that we assign logarithmic functions to non-durable and durable consumption and leisure in order to facilitate exact aggregation. Since we take the unit decision horizon to be a quarter in our numerical solutions, we set the time discount factor to be \( \beta = 0.9925 \), which corresponds to a 3 percent of annul real interest rate. The depreciation rates of durable goods and production capital are the same: \( \delta_d = \delta_k = 0.025 \) (which means 10 percent annual depreciation rate). The output elasticity of labor input is set to be \( \nu = 0.52 \), following King, Plosser and Rebelo (1988). The demand elasticity of retailer goods is set to be \( \epsilon = 11 \) for both durable and non-durable goods sectors so that steady-state mark-ups in the two sectors are the same: \( \mu_d = \mu_n = 1.1 \).

In order to compute the relative size of durable and non-durable goods sectors, we set \( s_n = 0.87 \) for the output share of non-durable goods sector, following Erceg and Levin (2006). On the other
hand, the average ownership period of new cars is set to be 5 years in Caplin and Leahy (2005). We follow their calibration for the average time length of durable-goods ownership so that we set $\theta = 0.95$. The price of used durable goods is 80 percent of the price of new goods so that we set $\kappa = 0.8$. The coefficient for inflation rate is $\phi_\pi = 1.5$ and the coefficient of output gap is $\phi_y = 0.5$ in the interest-rate rule. The slope coefficient of the Phillips curve in the non-durable goods sector is $\lambda_n = 0.035$, while this value of slope coefficient belongs to the range of empirical estimates reported in the literature.

8 Allowing for Endogenous Transaction Probabilities of Durable-Goods Purchases

In this section, we extend our analysis to allow for the possibility that agents can determine timings of durable-goods purchases endogenously. More explicitly, each agent who is in charge of durable goods purchases in a household is randomly assigned to a fixed utility cost $a \xi_t$. The random variable $\xi_t$ is independently and identically distributed across agents and over time. All agents still share non-durable goods consumption and leisure within the household.

It may be worthwhile to describe how budget constraints and period utility functions change depending whether or not agents purchase durable goods. For agents who purchase durable goods at period $t$, their period utility function and flow budget constraints are

$$U(C_t, N_t) + \hat{U}(D_t^*) - a\xi_t$$

$$C_t + \hat{P}_{dt}D_t^* + K_{t+1} + E_t[q_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] \leq \hat{P}_{ut}(1 - \delta_d)^iD_{t-i}^* + W_tH_t + R_t(K_t + 1 - \delta_k) + \frac{B_t}{P_t} + T_{0,t}$$

where $q_{t,t+1}$ is the nominal stochastic discount factor used for computing the value at period $t$ of one unit of non-durable goods at period $t + 1$. In this section, we temporarily assume that there is a complete financial market in order to simplify our exposition, though our analysis can go with the absence of the centralized complete state-contingent market. The random variable $\xi_t$ is independently and identically distributed across agents and over time and the value of its cumulative probability distribution is $G(\xi)$ when $\xi_t = \xi$, while $a$ is a positive constant. For agents who do not purchase durable goods at period $t$, their period utility function and flow budget constraints are

$$U(C_t, N_t) + \hat{U}((1 - \delta_d)^iD_{t-i}^*)$$

$$C_t + K_{t+1} + E_t[q_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] \leq W_tH_t + R_t(K_t + 1 - \delta_k) + \frac{B_t}{P_t} + T_{i,t}$$

The consolidated period utility function and flow budget constraint of each household at period $t$ is

$$U(C_t, N_t) + \sum_{i=1}^{J} \omega_{i,t+1}U((1 - \delta_d)^{i-1}D_{t-i+1}^*) - \Gamma_t$$

$$C_t + \mathcal{F}_t\hat{P}_{dt}D_t^* + K_{t+1} + E_t[q_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] \leq \hat{P}_{ut}M_{dt} + W_tH_t + R_t(K_t + 1 - \delta_k) + \frac{B_t}{P_t} + \Upsilon_t$$

where $\omega_{i,t+1}$ represents the fraction (at the end of period $t$) of agents within the household that hold durable goods purchased at period $t - i$ so that $\sum_{i=1}^{J} \omega_{i,t+1} = 1$, and $J$ is the maximum number of
time periods for which agents can own durable goods without selling. In addition, \( \Gamma_t \) denotes the amount of utility costs that are related to the determination of holding and selling durable goods (this cost will be specified later).

In particular, these random costs play an important role in determining whether to purchase durable goods. For example, let’s suppose that an agent (who holds \((1 – \delta_d)^i D_{t-i}^*\) at the beginning of period \(t\)) is considering whether to sell its durable goods at period \(t\). In addition, let’s denote the utility benefit of purchasing \(D_t^*\) in period \(t\) (after paying the price of durable goods) by \(V_{0,t}\). The utility benefit of holding \((1 – \delta_d)^i D_{t-i}^*\) in period \(t\) is \(V_{i,t}\). In this case, the agent will sell \((1 – \delta_d)^i D_{t-i}^*\) and purchase \(D_t^*\) if \(V_{0,t} + \bar{P}_{u,t} \Lambda_t (1 – \delta_d)^i D_{t-i} – V_{i,t} > a \xi\), while that agent will hold \((1 – \delta_d)^i D_{t-i}^*\) if \(V_{0,t} + \bar{P}_{u,t} \Lambda_t (1 – \delta_d)^i D_{t-i} – V_{i,t} \leq a \xi\). Hence, the adjustment cost of the marginal household is \(a \xi(\eta_{i,t})\) if \(\eta_{i,t}\) is the fraction of households who purchase durable goods at period \(t - i\). Accordingly, the fraction of agents who keep \((1 – \delta_d)^i D_{t-i}^*\) in period \(t\) is \(\theta_{i,t} = 1 – \eta_{i,t}\). In sum, fractions of households who purchase durable goods are determined as follows:

\[
\begin{align*}
V_{0,t} + \bar{P}_{u,t} \Lambda_t (1 – \delta_d)^i D_{t-i}^* – V_{i,t} &= a \xi(\eta_{i,t}) \quad \text{for } i = 1, \ldots, J - 1 \\
V_{0,t} + \bar{P}_{u,t} \Lambda_t (1 – \delta_d)^i D_{t-J}^* – V_{J,t} &> a \xi(1)
\end{align*}
\]

The average utility cost for agents who sell \((1 – \delta_d)^i D_{t-i}^*\) in period \(t\) is \(\bar{\xi}_{i,t} = \int_0^{\xi_{i,t}} \xi g(\xi) d\xi / \eta_{i,t}\). while the aggregate utility cost in period \(t\) is \(\Gamma_t = a \sum_{i=1}^J \eta_{i,t} \bar{\xi}_{i,t}\). On the other hand, the distribution of agents (depending on vintages of purchases) evolves over time according to the following equation:

\[
\omega_{i+1,t+1} = \theta_{i,t} \omega_{i,t} \quad \text{for } i = 1, 2, \ldots, J - 1; \quad \omega_{1,t+1} = \sum_{i=1}^J \eta_{i,t} \omega_{i,t}
\] (8.10)

Having characterized holding and selling probabilities in each period, we find that agents who purchase \(D_t^*\) in period \(t\) solve the following optimization problem:

\[V_{0,t} = \max_{D_t^*} \{\hat{U}(D_t^*) – \bar{P}_{d,t} \Lambda_t D_t^* + \beta E_t[\theta_{1,t+1} V_{1,t+1} + \eta_{1,t+1} (V_{0,t+1} + \bar{P}_{u,t+1} \Lambda_{t+1} (1 – \delta_d) D_t^* – a \bar{\xi}_{1,t+1})]\}.
\]

The agents who do not purchase durable goods and hold \((1 – \delta_d)^i D_{t-i}^*\) in period \(t\) have the following value function:

\[V_{i,t} = \hat{U}((1 – \delta_d)^i D_{t-i}^*) + \beta E_t[\theta_{i+1,t+1} V_{i+1,t+1} + \eta_{i+1,t+1} (V_{0,t+1} + \bar{P}_{u,t+1} \Lambda_{t+1} (1 – \delta_d)^{i+1} D_{t-i}^* – a \bar{\xi}_{i+1,t+1})]\]

for \(i = 1, 2, \ldots, J - 1\). The optimization condition for \(D_t^*\) is

\[
\sum_{i=0}^{J-1} ((1 – \delta_d) \beta)^i \varphi_{i,t+i} \hat{U}'((1 – \delta_d)^i D_t^*) + E_t[\sum_{i=1}^{J-1} ((1 – \delta_d) \beta)^i \varphi_{i-1,t+i-1} \eta_{i,t+i} \Lambda_{t+i} \bar{P}_{u,t+i}] = \Lambda_t \bar{P}_{d,t} \] (8.11)

The first-term of the left-hand side of this equation is the discounted sum of utility benefits of holding durable goods and the second-term represents the present value of future resale values (measured in the current utility level). The right-hand side is the cost of purchasing durable goods in period \(t\).
9 Future Research Directions

We have assumed that housing and consumption goods are produced by different production inputs but the same technology in order to facilitate the exact aggregation of production functions. However, it is realistic to assume that housing and consumption goods are produced using different technologies. We abstract from land inputs in the production of durable goods. In addition, our analysis has not reflected the fact that fluctuations in house prices can affect the borrowing capacity of a fraction of households as emphasized in Iacoviello (2005). Hence, our ongoing extension is to allow for the possibility in this current framework that housing wealth can be used as a collateral for borrowing of households.

In particular, Iacoviello and Neri (2010) demonstrate that collateral effects are the key feature of their model that generates a positive and persistent response of consumption following an increase in housing demand. Absent from this effect, in fact, an increase in the demand for housing would generate an increase in housing investment and housing prices, but a fall in consumption. However, in our model, the presence of idiosyncratic preference shocks and financial market incompleteness help generate a positive response of consumption following an increase in housing price. Hence, an addition of collateral constraints for borrowing of an individual household to the current framework would strengthen the connection between the housing market and aggregate fluctuations.
Appendix

A Linearized Equilibrium Conditions

In this linearized equilibrium conditions, we present a minimal set of conditions that are relevant for our numerical exercises but self-sufficient. The followings correspond to the model of time-dependent purchases.

Sticky Price Equilibrium We use 19 linearized equilibrium conditions to compute linear decision rules of 19 endogenous variables such as \( Z_t, \Lambda_t, \beta_{t,t+1}, C_t, Y_{d,t}, Y_{n,t}, K_t, N_t, P_{s,t}, H_t, X_t, \hat{P}_{d,t}, S_{d,t}, D_t^a, M_{d,t}, mc_t, \pi_{n,t}, \pi_{d,t} \) and \( r_t \). The demand side of the durable-goods market consists of two linearized conditions: One is resale value equation and the other is demand curve for durable goods. The resale value equation is

\[
\hat{Z}_t = \theta \beta (1 - \delta) E_t[\hat{Z}_{t+1}] + (1 - \theta \beta (1 - \delta_d)) E_t[\hat{A}_{t+1} + \hat{P}_{d,t+1} - \hat{A}_t - \hat{P}_{d,t}].
\]

The demand curve is

\[
\hat{A}_t + \hat{P}_{d,t} = \gamma \hat{Z}_t - \sigma_d \hat{D}_t^a
\]

where \( \gamma \) is defined as

\[
\gamma = \frac{\theta \beta (1 - \delta)(1 - \theta) \kappa}{1 - \theta \beta (1 - \delta)(1 + \kappa(1 - \theta))}.
\]

The supply side of the durable-goods market can be described by the following three equations:

\[
\hat{S}_{d,t} = \hat{D}_t^a; \quad M_{d,t} = (1 - \delta) \theta \hat{M}_{d,t-1} + (1 - \theta (1 - \delta_d)) \hat{D}_{t-1}^a; \quad \hat{Y}_{d,t} = \gamma_s \hat{S}_{d,t} - \gamma_m \hat{M}_{d,t}
\]

where \( \gamma_s \) and \( \gamma_m \) are defined as

\[
\gamma_s = \frac{1 - \theta (1 - \delta)}{1 - (1 - \delta)(1 + \kappa(1 - \theta))}; \quad \gamma_m = \frac{\kappa(1 - \theta)(1 - \delta)}{1 - (1 - \delta)(1 + \kappa(1 - \theta))}.
\]

In the non-durable goods sector, its linearized Phillips curve is given by

\[
\pi_{n,t} = \beta E_t[\pi_{n,t+1}] + \lambda_n \hat{mc}_t.
\]

The aggregate consumption function can be written as follows:

\[
\hat{C}_t = \frac{m \lambda}{1 - \theta \beta (1 - \delta_d)} \hat{M}_{d,t} + m (1 - \theta) \lambda \hat{P}_{d,t} + \gamma_r (\hat{A}_t + \hat{Z}_t) + m \hat{X}_t
\]

where \( \gamma_r \) is defined as

\[
\gamma_r = \frac{\beta \theta \lambda m (1 - \theta)(1 - \delta_d)}{1 - \beta \theta (1 - \delta_d)}.
\]

In addition, \( \hat{X}_t \) is defined as

\[
\hat{X}_t = \frac{mc(1 - s_n)}{s_C} \hat{mc}_t + \frac{mc(1 - \nu)}{s_C} \hat{N}_t + \frac{s_n(1 - mc)}{s_C} \hat{Y}_{n,t} + a_K \hat{K}_t + a_{d} E_t[\hat{d}_{t+1}] + a_{H} E_t[\hat{H}_{t+1}] + a_{P_s} E_t[\hat{P}_{s,t+1}]
\]

where \( a_K, a_d, a_H, \) and \( a_{P_s} \) are defined as

\[
a_K = \frac{mc}{s_C} (\frac{\nu + \beta (1 - \delta_k)}{1 - \beta (1 - \delta_k)}), \quad a_d = \alpha_H + a_{P_s}, \quad a_H = \frac{\beta mc(1 - \nu)}{(1 - \beta)s_C H}, \quad a_{P_s} = \frac{\beta (1 - mc)}{(1 - \beta)s_C}.
\]
Furthermore, log-linearized equations of human wealth and real stock price of retail firms in the non-durable goods sector are

\[ \hat{H}_t = (1 - \beta)(\hat{mc}_t + \nu(\hat{K}_t - \hat{N}_t)) + \beta E_t[\hat{H}_{t+1}] \]

\[ \hat{P}_{s,t} = (1 - \beta)(\hat{Y}_{n,t} - \frac{mc}{1 - mc}\hat{mc}_t) + \beta E_t[\hat{P}_{s,t+1}] \]

The aggregate production function is

\[ s_n\hat{Y}_{n,t} + (1 - s_n)\hat{Y}_{d,t} = (1 - \nu)\hat{N}_t + \nu\hat{K}_t. \]

The aggregate resource constraint is

\[ s_C\hat{C}_t + (1 - s_n)(\gamma_s\hat{S}_{d,t} - \gamma_m\hat{M}_{d,t}) + r_k(\hat{K}_{t+1} - (1 - \delta_k)\hat{K}_t) = (1 - \nu)\hat{N}_t + \nu\hat{K}_t \]

where \( r_k = \frac{K/Y} \) is defined as

\[ r_k = \frac{mc\beta}{1 - \beta(1 - \delta_k)}. \]

The market clearing condition for the non-durable goods sector is

\[ \frac{s_C}{s_n}\hat{C}_t + \frac{r_k}{s_n}(\hat{K}_{t+1} - (1 - \delta_k)\hat{K}_t) = \hat{Y}_{n,t}. \]

The aggregate real stochastic discount factor is

\[ E_t[\beta_{t,t+1}] = \hat{C}_t - E_t[\hat{C}_{t+1}] \]

The Euler equation for bond holdings is

\[ E_t[\beta_{t,t+1}] = -(r_t - E_t[\pi_{n,t+1}]). \]

The marginal utility of consumption is

\[ \hat{\Lambda}_t = -\hat{C}_t. \]

The final one includes the interest rate rule and two definition equations:

\[ r_t = \max\{r_t^n + \phi_x\pi_t + \phi_y(y_t - y_t^f), 0\}; \quad \hat{mc}_t = \hat{P}_{d,t}; \quad \pi_{d,t} = \pi_{n,t} + \hat{P}_{d,t} - \hat{P}_{d,t-1}. \]
References


