Technological Learning and Labor Market Dynamics*

Martin Gervais  
*University of Iowa  
martin-gervais@uiowa.edu

Nir Jaimovich  
Duke University and NBER  
nj41@duke.edu

Henry E. Siu  
University of British Columbia and NBER  
hankman@mail.ubc.ca

Yaniv Yedid-Levi  
University of British Columbia  
yaniv@mail.ubc.ca

October 20, 2011

Abstract

The search-and-matching model of the labor market fails to match two important business cycle facts: (i) a high volatility of unemployment relative to that of labor productivity, and (ii) a mild correlation between these two variables. We address these shortcomings by focusing on the implementation process of technological innovation, that is, the empirical finding that new technologies are subject to a learning process. We consider a novel source of business cycles, namely, fluctuations in the ease or speed of technological learning and show that a search-and-matching model featuring such shocks can account for both facts. Moreover, our model provides a new interpretation of “news shocks” discussed in the recent business cycle literature.

*We thank Manuel Amador, David Andolfatto, Gadi Barlevy, Paul Beaudry, Aspen Gorry, Jean-Olivier Hairault, Andreas Hornstein, Patrick Kehoe, Guido Menzio, Richard Rogerson, Peter Rupert, Martin Schneider, Shouyong Shi, Robert Shimer, and workshop participants at Bocconi, CREI, LSE, Minneapolis Fed, Princeton, Richmond Fed, Tel Aviv, UC Santa Cruz, the 2011 SED Meeting, the 2011 Duke Macroeconomics Conference, the 2011 CEPR/ENPAP Labor Market Workshop, and the 2011 Vienna Macro Workshop for helpful comments. Siu and Yedid-Levi thank the Social Sciences and Humanities Research Council of Canada for support.
1 Introduction

A long-standing challenge in macroeconomics is accounting for labor market dynamics over the business cycle. This challenge is particularly acute in the seminal model of equilibrium unemployment due to Diamond (1982), Mortensen (1982) and Pissarides (1985)—hereafter, the DMP model. When applied to business cycles analysis through the introduction of stochastic shocks to technology, the model features two primary shortcomings.

First, as discussed in Andolfatto (1996), Shimer (2005a), Hall (2005), and Costain and Reiter (2008), the model fails to generate sufficient volatility of unemployment relative to that of labor productivity, in comparison to postwar U.S. data. This discrepancy indicates that the textbook DMP model embodies weak amplification of technology shocks into unemployment fluctuations, and has been referred to as the unemployment volatility puzzle in the literature. The second shortcoming relates to the correlation between unemployment and labor productivity. In U.S. data, these two variables are only mildly negatively correlated, whereas in the model, the correlation is near minus one. We refer to this as the unemployment-productivity correlation puzzle. Mortensen and Nagypál (2007) argue that this discrepancy points to the omission of an important driving force in the cyclical analysis of the DMP framework.

Within this context, we propose a DMP model that addresses both shortcomings. As in much of the literature, our model retains an important relationship between technological change and the business cycle. However, we depart by focusing on technological learning: the notion that it takes workers time using a technology before it reaches its productive potential. Indeed, the idea that technology is subject to a learning process is well documented.\footnote{The literature documenting technological learning has a long history in management and economics. See Wright (1936) for an early study of learning-by-doing in airplane manufacturing. For recent examples, see Argote and Epple (1990), Irwin and Klenow (1994), and the references therein.}

The novel aspect of our framework is to model the nature of technology arrival to be stochastic in terms of its “ease of learning.” That is, innovations are not uniform in the amount of learning time required to make them fully productive.\footnote{Again, the literature documenting variation in learning rates for new technologies is too vast to summarize completely; see, for instance, Argote and Epple (1990) and Balasubramanian and Lieberman (2010).} We embed this idea in a simple search-and-matching framework where workers have the ability to increase their labor productivity through the learning of technology. Periods in which technologies are easier to learn or more “user-friendly” imply an acceleration in the rate of technological learning, resulting in an increase in the rate of labor productivity growth. The arrival of
technologies that are harder to learn generate a period of falling productivity growth.  

In the DMP framework, firms create job vacancies in expectation of future profit flows when matched with a worker. In our model, potential profit gains that are due to the learning of a technology constitute an important component of this profit flow. Hence, shocks to the ease of learning generate pronounced fluctuations in expected future profit. This in turn generates fluctuations in job creation and unemployment. However, these shocks have only an indirect effect on aggregate productivity as workers gain technological proficiency. This enables the model to address the unemployment volatility puzzle. Furthermore, shocks to the learning rate of technology naturally break the tight correlation between unemployment and productivity, thereby addressing the correlation puzzle. To summarize, learning rate shocks generate an immediate response of job creation and unemployment; but given that the learning process takes time, the impact on labor productivity is persistent and cumulative.

In this respect, our analysis is related to the recent literature on “news shocks.” Specifically, Beaudry and Portier (2006) find that a substantial fraction of business cycle variation in U.S. data can be attributed to shocks to long-run TFP that: (1) have immediate effects on measures such as consumption, employment, and stock market value; but (2) have effectively no effect on TFP upon impact; instead the effect on productivity is persistent, observed over a horizon of at least 8 to 10 quarters. Beaudry and Portier refer to these as “news shocks” since they signal changes in future productivity, and find that such shocks account for at least half of the variation in hours worked at business cycle frequencies.

In our model, shocks to the learning rate generate dynamic responses in unemployment, output and stock market value that conform with the responses to empirically identified news shocks. But importantly, our shock is not a news shock as modeled in the recent literature. In these papers, news shocks are signals of conventional technology shocks that are to arrive in the future. In contrast, (positive) shocks to the learning rate represent the current arrival of innovations that are easier to implement; nonetheless, the process of learning-by-doing must still be undertaken. More importantly, we show that conventionally

3This contrasts with conventional technology shocks considered in the RBC framework, which represent the stochastic arrival of new technology that are immediately implemented, and at times, more or less productive. See Rebelo (2005) for a recent survey of that literature, and a discussion on the sources of cyclical shocks. See Merz (1995) and Andolfatto (1996) for early models of productivity driven cycles in search models of the labor market.

4See also Beaudry and Lucke (2010) and Schmitt-Grohe and Uribe (2010) who find similar results.

5See, for instance, Beaudry and Portier (2007), Jaimovich and Rebelo (2009), and den Haan and Kaltenbrunner (2009). See also Comin et al. (2009) who study a model in which economic activity devoted to technology adoption varies in response to the stochastic arrival rate of ‘frontier’ technologies.
modeled news shocks do not resolve either of the two shortcomings in the DMP model.

Our work is also related to a growing body of research studying the cyclical implications of the DMP framework. Most of this work addresses the unemployment volatility puzzle.\textsuperscript{6} By maintaining the assumption of a technology shock-driven cycle, these papers do not make progress on the unemployment-productivity correlation puzzle.\textsuperscript{7} By focusing on an alternative interpretation of technological change over the cycle—shocks to the ease of learning—we show that the DMP framework is able to generate substantial volatility of unemployment relative to productivity, and to deliver a correlation between the two variables that is much closer to the data.

The remainder of the paper is organized as follows. In Section 2, we present a very simple search-and-matching model of the labor market, in which technological learning takes time. In Section 3, we derive analytical results regarding the implications of shocks to the rate of learning, and their effects on job creation and unemployment. In Section 4, we show that a calibrated version of our model generates much greater unemployment volatility relative to the standard DMP model. This is due to the fact that the volatility of the job finding rate, relative to that of labor productivity, is very close to that observed in the U.S. data. Moreover, the model delivers a correlation of unemployment and productivity that is much smaller (in absolute value) than one, and indeed, is very close to that observed in the data. Finally, in Section 5, we provide both macro- and micro-level evidence for the relevance of learning rate shocks for business cycles, and consider an extension of our labor market model to a general equilibrium, RBC framework.

\section{Economic Environment}

We study a search-and-matching model of the labor market. The matching process between unemployed workers and vacancy posting firms is subject to a search friction. The ratio of vacancies to unemployed determines the economy’s match probabilities. Workers differ in

\textsuperscript{6}The literature is too vast to provide a complete summary; see, for instance, Shimer (2004), Hall (2005), Hall and Milgrom (2008), Pries (2008), Gertler and Trigari (2009), and Menzio and Shi (2011). Hagedorn and Manovskii (2008) find that, for specific calibrations, the DMP model does not suffer from a volatility puzzle. For discussion, see Hornstein et al. (2005), Mortensen and Nagypál (2007), Costain and Reiter (2008), van Rens et al. (2008), Pissarides (2009), and Brugemann and Moscarini (2010).

\textsuperscript{7}In the RBC literature, a similar puzzle exists regarding the correlation between hours worked and the real wage. Early papers addressed this by introducing shocks to labor supply (see, for example, Benhabib et al. (1991) and Christiano and Eichenbaum (1992)); unfortunately, empirical evidence for the relevance of these shocks in accounting for postwar business cycles is limited. The only paper in the search framework to address this puzzle is Hagedorn and Manovskii (2010), who follow Benhabib et al. (1991) by introducing home production/preference shocks to the DMP model.
their proficiency with production technology, and this proficiency is reflected in the output in a worker-firm match. The process of gaining proficiency for a worker takes time, as emphasized in the learning-by-doing literature. Within this environment, the novel aspect of our analysis is to consider how technological learning drives the business cycle.

To isolate the role of technological learning, we make the following assumptions on innovations to the production technology. First, innovations arrive deterministically over time. These innovations affect the productivity of all workers proportionately. The cost of adoption is sufficiently small that, upon arrival, all worker-firm matches find it advantageous to adopt the innovation. Finally, these innovations do not affect the proficiency of workers per se; that is, those that were proficient with the technology remain so, and those that were not, remain not. Thus, the only aspect along which innovations differ is the rate at which proficiency is learned; this is the stochastic element in our analysis.

These assumptions allow us to focus on a model where productivity is stationary in the long-run (has no growth trend). Becoming proficient with technology is represented as a ‘level shift’: a jump in the worker’s productivity from low to high. As a simple example, consider the case of an accountant or secretary, where the current mode of production requires the use of personal computing technology. Proficiency with the technology requires understanding how to use the PC’s operating system. Attaining this understanding requires time; this is represented by a hazard rate, $\lambda \in (0, 1]$, which we refer to as the “learning rate”. An employed worker currently without proficiency in the PC technology produces output $f_L$. With probability $\lambda$ she “figures out” the technology; in the next period, she produces $f_H > f_L$. With probability $(1 - \lambda)$ she remains at productivity $f_L$.

Our analysis focuses on shocks to the learning rate, $\lambda$—shocks to the ease at which innovations can be learned. Returning to our example, consider a PC operating system such as Microsoft DOS, that is relatively difficult to learn. In this case, it takes workers a long time to become proficient with the PC, and the learning rate is low. The introduction of Microsoft Windows would represent a positive shock to the learning rate. Given that it is substantially more user-friendly, it increases the probability or speed at which a worker successfully becomes proficient with the PC technology.

2.1 Market Tightness

A worker’s proficiency or ‘type’ is perfectly observable. Accordingly, a firm can maintain a vacancy for workers of either type, $i \in \{L, H\}$. The cost of maintaining a vacancy for either type is $\kappa$. There is free entry into vacancy posting on the part of firms. We define market
tightness in market \(i\), \(\theta_i\), as the ratio of the number of vacancies maintained by firms to the number of workers looking for jobs of productivity \(i\). While the tightness of each market is an equilibrium object, they are taken parametrically by firms and workers.

We denote the probability that a worker will meet a vacant job in market \(i\) by \(\mu(\theta_i)\), where \(\mu : \mathbb{R}_+ \rightarrow [0, 1]\) is a strictly increasing function with \(\mu(0) = 0\). Similarly, we let \(q(\theta_i)\) denote the probability that a firm with a vacancy meets a worker in market \(i\), where \(q : \mathbb{R}_+ \rightarrow [0, 1]\) is a strictly decreasing function with \(q(\theta) \rightarrow 1\) as \(\theta \rightarrow 0\), and \(q(\theta) = \mu(\theta)/\theta\).

Allowing for market segmentation across low and high type workers is useful for a number of reasons. As will become clear, it affords analytical and computational tractability, as equilibrium is block recursive in the sense that agents’ value functions and decision rules are independent of the distribution of workers across types and employment status (see Shi (2009)). In addition, it makes the economic mechanism transparent, highlighting the role of learning rate shocks on the incentive for job creation.

2.2 Contractual Arrangement and Timing

We specify the compensation in a match as being determined via Nash bargaining with fixed bargaining weights, as in Pissarides (1985). As such, our results do not rely on mechanisms that change the relative bargaining power of workers and firms over the cycle.

When an unemployed worker and a firm match, they begin producing output in the following period. In all periods that a worker and firm are matched, the compensation is bargained with complete knowledge of the worker’s productivity. We let \(\omega_i\) denote the compensation of a type \(i\) worker.

2.3 Technological Learning in Worker-Firm Matches

We define \(U_L\) as the value of being unemployed for a low productivity worker:

\[
U_L = z + \beta E \left[ \mu(\theta_L)W_L' + (1 - \mu(\theta_L))U_L' \right].
\]

Here, \(z\) is the flow value of unemployment, \(W_L\) is the worker’s value of being employed in a match, and primes (\('\)) denote variables one period in the future. An unemployed worker

---

8If we did not allow for segmented markets, the qualitative implications of technological learning would be preserved. However, the computation of equilibrium with aggregate uncertainty would be extraneously burdensome, because of the need to track the distribution of types in unemployment.

9See, for instance, Hall and Milgrom (2008) and Gertler and Trigari (2009).
transits to employment in the following period with probability \( \mu(\theta_L) \); we refer to this as the job finding probability.

The value of being employed for a low productivity worker is:

\[
W_L = \omega_L + \beta E \left[ \lambda \left( (1 - \delta)W_H' + \delta U_H' \right) + (1 - \lambda) \left( (1 - \delta)W_L' + \delta U_L' \right) \right].
\]

During the period, the employed worker becomes proficient with probability \( \lambda \), which is stochastic. At the end of the period, the match is separated with (exogenous and constant) probability \( \delta \in (0, 1] \). In the case where the worker figures out the technology but is separated from her match, she enters the next period with value \( U_H \). That is, the proficiency that the worker acquires on-the-job is retained when unemployed and can be applied to future matches. In this sense, the technology being learned is not firm- or match-specific. Note also that learning happens only when a worker is matched. That is, since technological proficiency is acquired though learning-by-doing, the worker cannot transit from type \( L \) to \( H \) while unemployed.

There is a large number of firms that can potentially maintain vacancies, as long as they pay the cost, \( \kappa \). The value of maintaining a vacancy for low productivity workers is:

\[
V_L = -\kappa + \beta E \left[ q(\theta_L)J_L' + (1 - q(\theta_L)) \max_j (V_j', 0) \right],
\]

where \( q(\theta_L) \) denotes the firm’s job filling probability. The maximization within the expectation term implies that firms who do not find a worker may choose to maintain a vacancy in either market, or be inactive in the following period. The firm’s value of being matched with a type \( L \) worker is:

\[
J_L = f_L - \omega_L + \beta E \left[ (1 - \delta) \left( (1 - \lambda)J_H' + \lambda J_L' \right) + \delta \max_j (V_j', 0) \right].
\]

This value is composed of the contemporaneous profit—output minus the worker’s compensation—plus the expected discounted value from next period on. This latter part (conditional on the match surviving) consists of the value of being in a match of type \( H \) which occurs with probability \( \lambda \), or being in a type \( L \) match with complementary probability.

### 2.4 High Productivity Workers

To close the model description, we present the value functions associated with high productivity workers:

\[
U_H = z + \beta E \left[ \mu(\theta_H)W_H' + (1 - \mu(\theta_H))U_H' \right],
\]
\[ W_H = \omega_H + \beta E \left[ (1 - \delta)W_H' + \delta U_H' \right]. \]  

(6)

A worker of type \( H \) transits from unemployment to employment with probability \( \mu(\theta_H) \), and transits from employment to unemployment with probability \( \delta \).

Again, a large number of inactive firms can potentially maintain vacancies for type \( H \) workers. The value of maintaining such a vacancy is:

\[ V_H = -\kappa + \beta E \left[ q(\theta_H)J_H' + (1 - q(\theta_H)) \max_j (V_j', 0) \right]. \]  

(7)

Finally, \( J_H \) is simply the expected discounted value of flow profits:

\[ J_H = f_H - \omega_H + \beta E \left[ (1 - \delta)J_H' + \delta \max_j (V_j', 0) \right]. \]  

(8)

Note that the type \( H \) market is identical to the standard DMP model.

2.5 Defining Equilibrium

An equilibrium with Nash bargaining is a collection of value functions, \( V_L, J_L, V_H, J_H, \)
\( U_L, W_L, U_H, W_H \), compensations, \( \omega_L, \omega_H \), and tightness ratios, \( \theta_L, \theta_H \), such that:

1. Workers are optimizing. That is, workers that are matched prefer to remain matched rather than be unemployed, \( W_L > U_L, W_H > U_H \), and workers prefer to be of high productivity as opposed to low productivity, \( W_H > W_L, U_H > U_L \).

2. Firms are optimizing. That is, the value of maintaining a vacancy is equalized across markets and is no less than the value of remaining idle, \( V_L = V_H \equiv V \geq 0 \), and firms that are matched must prefer to remain matched as opposed to maintaining a vacancy, \( J_L, J_H > V \).

3. Compensations solve the Nash bargaining problems:

\[ \omega_i = \arg \max (W_i - U_i)^\tau (J_i - V)^{1-\tau}, \]

for \( i \in \{L, H\} \), where \( \tau \) denotes the bargaining weight of workers.

4. The free entry condition is satisfied; that is, \( V = 0 \).

Nash bargaining prescribes a very simple relationship between the worker’s surplus, \( W_i - U_i \), and firm’s surplus, \( J_i \), in a match. Let \( TS \) denote the total surplus from a match:

\[ TS_i = W_i + J_i - U_i, \quad i \in \{L, H\}. \]

Under Nash bargaining, the worker and firm receive a constant, proportional share of the total surplus, \( W_i - U_i = \tau TS_i \) and \( J_i = (1 - \tau)TS_i \).
2.6 Discussion

The model has been kept simple for exposition. In particular, we have modeled only two types of workers. Type L workers represent labor force members who have the potential to upgrade their productivity via learning-by-doing. Type H workers are those who no longer have the ability to do so.

In our analysis, we focus attention on type L workers. This represents our presumption that, in reality, most workers have the ability to increase their productivity while on the job. In this sense, the presence of type H workers represents an analytical device, allowing us to specify a well-defined dynamic problem for type L workers, those who represent the majority of labor force members in the economy.

However, our model naturally introduces a source of heterogeneity relative to the standard DMP model. That is, while the standard model features heterogeneity in the employment status of workers (of the same ability), those in our model are also distinguished by the scope of their upgrade potential. Given this, it is interesting to consider the implications of this heterogeneity in a richer framework. We do this by extending the model to include N worker types, where N > 2. This is done in Section 4, where we explore the quantitative properties of our model.

In our exposition, we have specified compensation as being determined by Nash bargaining. However, our results do not rely on this assumption. For example, the model’s implications are identical to a version with wage posting on the part of firms, as in the competitive search framework; this is true when Hosios (1990)’s condition is met. Furthermore, when the Hosios condition is met, the model’s equilibrium is efficient. We refer the reader to the Appendix for details.

Note also that our model nests the standard DMP model in two cases. The first is when there is no difference in productivity across worker types, i.e., \( f_L = f_H \). In this case, upgrading is meaningless, and our model collapses to the standard one. Alternatively, when \( \lambda = 0 \) there is no scope for productivity upgrading for type L workers. In this case, the model features two unrelated labor markets, each of which behaves identically to the standard DMP model.

Finally, we note that while workers have the potential for productivity upgrading, they face no chance of downgrading. Hence, our model would feature a degenerate distribution of worker types in steady state, with all workers being of type H. To address this, we introduce an exogenous probability of death: in each period, all workers (regardless of
employment status or productivity) die with probability $\phi$. These workers are “re-born” as type $L$ workers. As such, the discount factor, $\beta$, represents a composite of a true subjective discount factor and a survival probability, $1 - \phi$.

3 Analytical Results

In this section, we provide analytical results characterizing some key properties of our model. We begin by characterizing the model’s steady state equilibrium. We then discuss the key differences between our model and the standard DMP model, and their implications for business cycle fluctuations.

3.1 Characterizing Steady State

Our analysis begins with a useful lemma.

**Lemma 1** In any steady state equilibrium, if $U_L < U_H$, then $\theta_L < \theta_H$.

**Proof.** The result follows directly from the expressions for the value of unemployment, (1) and (5). Taking the difference and evaluating in the steady state:

$$U_H - U_L = \frac{\tau \kappa (\theta_H - \theta_L)}{(1 - \tau)(1 - \beta)}.$$

A number of results follow immediately from this lemma, collected in the following corollary:

**Corollary 1** If $U_L < U_H$, then:

1. $\mu(\theta_H) > \mu(\theta_L)$;
2. $q(\theta_H) < q(\theta_L)$;
3. $J_H > J_L$;
4. $TS_H > TS_L$;
5. $W_H - U_H > W_L - U_L$;
The next proposition establishes that the steady state value of unemployment is increasing in type.

**Proposition 1** In any steady state equilibrium, \( U_L < U_H \).

**Proof.** The proof is by contradiction. Suppose \( U_L > U_H \); from Lemma 1, this implies that \( \theta_L > \theta_H \), so that \( q(\theta_L) < q(\theta_H) \). From the free entry condition, this implies \( J_H < J_L \). From Nash bargaining, this implies \( TS_H < TS_L \). Total surplus is given by:

\[
TS_H = f_H + \beta \left[ (1 - \delta)TS'_H + U'_H \right] - U_H,
\]

\[
TS_L = f_L + \beta \left[ (1 - \delta) \left[ \lambda TS'_H + (1 - \lambda)TS'_L \right] + \lambda U'_H + (1 - \lambda)U'_L \right] - U_L.
\]

Using the fact that \( TS'_i = TS_i \) and \( U'_i = U_i \) in steady state, and gathering terms:

\[
[1 - \beta(1 - \delta)(1 - \lambda)](TS_L - TS_H) = f_L - f_H + [\beta(1 - \lambda) - 1] (U_L - U_H).
\] (9)

Both terms on the LHS are positive (the first by construction, the second by assumption; \( f_L - f_H < 0; [\beta(1 - \lambda) - 1] < 0 \) and \( (U_L - U_H) > 0 \) by assumption; therefore, the RHS is negative. This is a contradiction. ■

This result is important for a number of reasons. It ensures that in steady state equilibrium, the assumptions implicit in the model exposition are verified. In particular, a high productivity unemployed worker would prefer to maintain her type, as opposed to reverting to low productivity (see Subsection 2.5). From Corollary 1, it also ensures that an employed worker would prefer to be of high productivity than low productivity.

More importantly, it allows us to understand the incentives for job creation in our model relative to the standard model. In steady state, the free entry condition of the standard DMP model can be expressed as:

\[
\kappa = q(\theta_{DMP})\beta(1 - \tau)TS_{DMP},
\]

where the subscript \( DMP \) refers to the standard model. Hence, the number of vacancies firms post per unemployed worker, \( \theta \), depends on the profit conditional on being matched, \( \beta(1 - \tau)TS \), which is proportional to total surplus. Total surplus in the standard model can be expressed as:

\[
TS_{DMP} = f - z - \bar{\tau}\kappa\theta_{DMP} + \beta(1 - \delta)TS_{DMP},
\]

11
where \( \bar{\tau} \equiv \tau/(1-\tau) \). In words, the total surplus from a match consists of a contemporaneous surplus plus its continuation value; the contemporaneous surplus is the output from the match \( (f) \), net of the foregone flow value \( (z) \) and option value \( (\bar{\tau}k\theta_{DMP}) \) of unemployment.

In our model, the analogous free entry condition must hold in each market:

\[
\kappa = q(\theta_i)\beta (1-\tau) TS_i, \quad i \in \{L,H\}.
\]  

(10)

Total surplus for type \( H \) matches is identical to the standard DMP model. This is not the case for type \( L \) matches. In the market for workers with the possibility of learning:

\[
TS_L = f_L - z - \bar{\tau}k\theta_L + \beta(1-\delta)TS_L + \lambda\beta[(1-\delta)(TS_H-TS_L) + (U_H-U_L)] \equiv \Delta.
\]  

(11)

Relative to the standard model, the total surplus of a type \( L \) match involves the additional term, \( \Delta \), which we refer to as the value of learning. This reflects a capital gain due to the fact that technological learning may occur when a worker and firm are matched.

Conditional on learning, there is an upgrade to a high productivity match in the next period. Hence, the total surplus includes the change in the worker’s and firm’s values, weighted by \( \beta \) and \( \lambda \). With probability \( (1-\delta) \) the match survives, so that learning reflects a change in the matched value of both the worker and the firm. With probability \( \delta \) the match is separated, and the learning is reflected only in a change in the unemployed worker’s value. Hence:

\[
\Delta = \lambda\beta[(1-\delta)(W_H-W_L + J_H-J_L) + \delta(U_H-U_L)]
= \lambda\beta[(1-\delta)(TS_H-TS_L) + (U_H-U_L)].
\]

Moreover, given Proposition 1 and Corollary 1, \( U_H - U_L > 0 \) and \( TS_H - TS_L > 0 \), so that the value of learning is positive, \( \Delta > 0 \).

### 3.2 Deviations from Steady State

In this subsection, we provide results from log-linearizing the model’s steady state conditions. These are useful in providing insight into the business cycle properties of our model.

The equations governing equilibrium job creation—more specifically, market tightness—are the free entry conditions. From (10), it is clear that the response of market tightness to a shock depends on the response of total surplus. Intuitively, a shock that causes total surplus to rise implies a rise in the flow of profits to a matched firm. Since there is free entry, firms respond by creating more vacancies per unemployed worker. Job creation occurs until the
point where the rise in profit is offset by the fall in the probability that any given vacancy is filled.

The next proposition relates to the model’s response to learning rate shocks. Consider total surplus in type $L$ matches with the possibility of learning. From equation (11), it is clear that the effect of a $\lambda$ shock on total surplus operates through its influence on the value of learning, $\Delta$. We first establish that $TS_L$ increases in response to a positive learning rate shock.

**Proposition 2** A positive (negative) shock to $\lambda$ causes $TS_L$ to rise (fall).

The proof is provided in the Appendix. The intuition is straightforward. An increase in the learning rate increases the value of learning, $\Delta > 0$, which is positive (see the previous subsection). As the probability that the match upgrades from low to high productivity increases, the expected profit rises as well; that is, the “upside risk” of the match has improved. Hence, a positive shock to $\lambda$ causes an increase in total surplus. Via free entry, this causes a rise in $\theta_L$: job creation of matches with the possibility of upgrading rises.

It is also straightforward to see that total surplus in high productivity matches is unaffected by shocks to the learning rate. As discussed in Section 2, the $H$ type market is simply a standard DMP model, and independent of the type $L$ market. Hence, job creation in this market is unresponsive to shocks to the learning rate, $\lambda$.

## 4 Numerical Results

In this section, we provide numerical results for our model. Subsection 4.1 discusses issues in calibration; the details are specific to the benchmark model presented in Section 2, but the calibration strategy extends to the quantitatively richer version of our model (with $N > 2$) studied in subsection 4.5. In subsections 4.2 and 4.3, we present business cycle statistics for the postwar U.S. economy and demonstrate that the standard DMP model does poorly in replicating them. Subsections 4.4 and 4.5 present results for the benchmark model and the extended model, respectively. Subsection 4.6 illustrates why our model improves upon the standard DMP model in terms of the volatility of unemployment and its correlation with labor productivity.
4.1 Calibration

Many of our model features are standard to the DMP literature, so our calibration strategy is to maintain comparability wherever possible. As in Hagedorn and Manovskii (2008), the model is calibrated to a weekly frequency. As such, the discount factor is set to $\beta = 0.999$ to accord with an annual risk free rate of 5%.

We assume that the matching function in each market is Cobb-Douglas, so that:

$$\mu(\theta) = \theta q(\theta) = \theta^\alpha.$$

Summarizing a large literature that directly estimates the matching function using aggregate data, Petrongolo and Pissarides (2001) establish a plausible range for $\alpha$ of $0.3 - 0.5$. Refining the inference approaches of Shimer (2005a) and Mortensen and Nagypál (2007), Brugemann (2008) obtains a range of $0.37 - 0.46$. In our benchmark calibration, we specify $\alpha = 0.4$ to be near the mid point of these ranges (see also Pissarides (2009)).

For comparability with previous work, we specify the parameter in the Nash bargaining problem as $\tau = 1 - \alpha$. As in the standard DMP model, this implies that the Hosios (1990) condition is met and the equilibrium is efficient (see the Appendix for details).

The vacancy cost, $\kappa$, pins down the aggregate job finding rate, $\mu(\theta)$. We target a weekly job finding rate of $\mu(\theta) = 0.139$, which corresponds with a monthly rate of 45%, as in Shimer (2005a). Given this aggregate job finding rate, we set $\delta = 0.0081$ to correspond with a steady state unemployment rate of 5.5%.

Following Hall and Milgrom (2008), Mortensen and Nagypál (2007), and Pissarides (2009), we specify $z$, the flow value of unemployment, to equal 73% of the average return to market work. The interpretation is that $z$ is composed of two components: a value of leisure or home production, and a value associated with unemployment benefits. As in their work, the return to leisure/home production is equated to 43% of the average return to market work. Given this target, the model’s Nash bargained compensation, and the steady state distribution of worker types, we set $z = 0.444$. This implies an unemployment benefit replacement rate of roughly 40% for type $L$ workers, and 20% for type $H$ workers; this accords with the range of replacement rates reported by Hall and Milgrom (2008).

Relative to the standard DMP model, our model adds a number of new parameters: $f_H$, $f_L$, $\lambda$, and $\phi$. We normalize $f_H = 1$. For the remaining parameters we follow the calibration

---

$^{10}$As discussed in Shimer (2005a), note that the exact value of $\kappa$ is irrelevant. That is, by introducing a multiplicative constant, $\xi$, to the matching function, $\kappa$ can be scaled by a factor of $x$ and $\xi$ by a factor of $x^\alpha$, leaving the job finding rate unchanged.
strategy of Ljungqvist and Sargent (1998, 2004) and den Haan et al. (2005). Specifically, we choose these three parameters to match three observations from the empirical life-cycle earnings profile estimated by Murphy and Welch (1990) and others. First, in the data, the maximal lifetime wage gain for a typical worker represents an approximate doubling of earnings. Given this, we set the difference between low and high productivity at $f_H/f_L = 2$. Second, in the data, this doubling occurs after the typical worker has accumulated 25-30 years of experience. In our model, the steady state learning rate, $\lambda$, governs the ‘speed’ at which technological learning takes place. We set $\lambda = 0.0008$ so that in steady state, it takes the average worker 25 years to realize the productivity upgrade from $f_L$ to $f_H$. Third, in the data, the average worker’s earnings cease to increase from the age of approximately 50 years old onward. In the model, the value of $\phi$ determines the proportion of the workforce that no longer has the potential for productivity upgrading. As such, we calibrate $\phi$ so that 25% of workers are type $H$ in steady state. This corresponds to the average fraction of the labor force over the age of 50 years in postwar U.S. data.

To investigate the quantitative predictions of our model, we log-linearize around the steady state, simulate to obtain 250,000 observations at the weekly frequency, then time-aggregate these observations to obtain quarterly data. Following Shimer (2005a), we HP filter the (logged) data with smoothing parameter $10^5$ to obtain second moment statistics.

4.2 U.S. Business Cycle Facts

Column 1 of Table 1 presents selected business cycle statistics for the U.S. economy, 1953:I–2009:IV. To isolate cyclical fluctuations, we HP filter the data in the same manner as the model simulated data. See the Appendix for detailed information on data sources used throughout the paper. Here, we highlight a number of well established observations and discuss their implications for the quantitative analysis of search-and-matching models.

The first is that the aggregate unemployment rate is very volatile over the business cycle.

---

11 Though the emphasis of their work is very different, these papers also study a DMP framework in which workers face a stochastic process of productivity upgrading (and downgrading). In particular, their papers focus on the implications of ‘turbulence’ in the form of a high depreciation rate on productivity or ‘human capital’ on steady state levels of unemployment. As such, they do not characterize the impact of learning and upgrading on incentives for job creation, nor the implications for cyclical fluctuations.

12 While these calibrated parameter values apply to our benchmark model (as presented in Section 2), the calibration strategy is identical for the extended version presented in subsection 4.5; we defer discussion of the extended model and the application of the strategy to its own subsection.

13 We have also solved the model by obtaining (numerically) exact solutions for the case when the exogenous shock is assumed to follow a finite state Markov process. The results are essentially identical using either the approximate, log-linear solution method or the exact, non-linear approach.
relative to labor productivity. The standard deviation of unemployment relative to that of labor productivity is 9.34; unemployment is nearly an order of magnitude more volatile than productivity. Hence, models that rely on shocks to productivity as the driving force require strong amplification.

Secondly, we report statistics relating to the cyclicality of job finding since this is the source of all unemployment volatility in the model. Since the job finding rate, $\mu(\theta)$, is a function of the vacancy-unemployment (or tightness) ratio, $\theta$, we present statistics for this variable as well. Column 1 of Table 1 indicates that both the job finding rate and tightness ratio are very volatile over the cycle. Relative to labor productivity, the standard deviation of these variables are 6.05 and 18.20, respectively.

Moreover, there exists a robust relationship between unemployment and vacancies over the business cycle—the “Beveridge Curve”. In postwar U.S. data, this correlation is highly negative at $-0.89$. This summarizes the fact that recessions are periods when firms stop hiring (vacancies fall) and unemployment soars; booms are periods when hiring is brisk and unemployment is low.

Finally, we highlight the correlation between labor productivity and unemployment. Labor productivity provides a measure of the return to work effort, while unemployment measures work effort itself. Over the business cycle, these two measures are only mildly (negatively) related, with a correlation of $-0.41$; periods when work effort rises, and unemployment falls, are only weakly associated with higher productivity. This weak relationship is mirrored in the correlations of labor productivity with both the job finding rate (0.44) and the vacancy-unemployment ratio (0.39). These weak correlations are informative regarding the relevant business cycle impulses that should be incorporated in our models.

4.3 Technology Shocks

We first review the properties of business cycle fluctuations in the standard DMP framework driven by technology shocks. Specifically, we consider AR(1) disturbances to the productivity of all worker-firm matches. This is done by setting $f_L = f_H = f$ in our model, and specifying:

$$f_t = f \exp(x_t), \quad x_t = \rho f x_{t-1} + \varepsilon_t.$$

---

14 Job separations in the model are constant at the exogenous rate $\delta$. This simplifying assumption accords with the findings of Shimer (2005a) and Hall (2005), namely that the primary determinant of unemployment fluctuations is variation in the job finding rate. See also Fujita and Ramey (2009) and Elsby et al. (2009). They arrive at similar conclusions, though with a slightly larger contribution to job separation rates.
Calibrating the technology shock process is difficult since productivity data is not available at the weekly frequency, and extrapolating from quarterly data is problematic. As such we choose the standard deviation of the shock to match the quarterly standard deviation of labor productivity in the U.S. data, and the persistence of the shock to match the Beveridge curve relation between vacancies and unemployment. This requires $\rho_f = 0.981$ and $\sigma_\epsilon = 0.005$.

The results for the standard DMP model are presented in Column 2 of Table 1. The model delivers a standard deviation of unemployment relative to productivity of 1.21, which is approximately 8 times smaller than in the data. This large discrepancy between theory and data has been discussed extensively in the literature. Since the model’s unemployment fluctuations are driven solely by the response of job creation, it is not surprising to see that the model performs poorly with respect to the job finding rate and the tightness ratio as well. On these dimensions, the model misses by a factor of approximately 5 and 6, respectively.

The standard DMP model also dramatically overpredicts the correlation of labor productivity to labor market measures. Consider first the correlation with the job finding rate
and the tightness ratio. These are ‘jump’ variables in the model and the correlation with productivity is perfect. In the data, these correlations are far from perfect. Unemployment is a state variable in the model. As such, its correlation with productivity is smaller than minus one, but still very close at −0.96. In the data, this correlation is only −0.41. For Mortensen and Nagypál (2007), this evidence points to the importance of other driving forces that are omitted from the standard analysis focusing solely on technology shocks. In the next subsection, we illustrate how shocks to the technological learning rate represents a potentially important omitted shock.

### 4.4 Technological Learning Rate Shocks

In this subsection, we document the cyclical properties of our benchmark model when fluctuations are driven by shocks to the technological learning rate, $\lambda$. We model the learning rate as following an AR(1) process:

$$\lambda_t = \lambda \exp(x_t), \quad x_t = \rho \lambda x_{t-1} + \epsilon_t.$$  

To maintain comparability with our analysis of technology shocks, we calibrate the volatility of the shock to match the standard deviation of labor productivity observed in the data, and the persistence to match the Beveridge curve. This implies a first-order persistence of $\rho = 0.979$ with a standard deviation of the shock innovation of $\sigma_\epsilon = 0.161$.

To be clear, we are interested in the quantitative evaluation of our benchmark model, in so far as it illustrates the importance of the technological learning mechanism. Because of the model’s simplicity in featuring only two types of workers, its predictions for cyclical fluctuations may be sensitive to a more detailed or ‘realistic’ setup. In order to match life-cycle earnings profiles, the productivity upgrade from type $L$ to $H$ is large. Moreover, given that the average duration to realizing this upgrade is long, shocks to the learning rate imply large swings in this duration over the business cycle. As such, we consider a richer version of the model in the following subsection to demonstrate robustness to having smaller upgrades and smaller fluctuations in the learning duration.

The results for the benchmark model are presented in Column 3 of Table 1. Learning rate shocks generate substantial amplification in labor market variables. The volatility of unemployment relative to productivity is more than 5 times that of the standard DMP model (displayed in Column 2); the same is true regarding the volatility of the job finding rate and the tightness ratio, relative to productivity.\(^{15}\)

\(^{15}\)These results are not due to the mechanisms stressed by Hagedorn and Manovskii (2008). We verify this
These results represent a marked improvement in terms of matching the U.S. data. Fluctuations due to learning rate shocks account for 72% of the observed volatility of unemployment. That the model does not capture all of its volatility is not surprising considering that there is no role for cyclical variation in job separation. In terms of job creation, the model’s amplification properties are clear. The model generates a volatility of the tightness ratio relative to productivity, 16.59, that is very close to its observed volatility, 18.20. With respect to the job finding rate, the model overpredicts its cyclical volatility relative to the data. Hence, our model makes substantial progress toward resolving the unemployment volatility puzzle, especially when viewed from the perspective of cyclicality in job creation.

Moreover, our model makes substantial progress toward resolving the unemployment-productivity correlation puzzle. The model generates a correlation between these two variables of −0.27. This is close to the value of −0.41 observed in the data, and far from the value near minus one generated by the standard DMP model driven by technology shocks.

This is mirrored in our model’s ability to generate realistic correlations of labor productivity with the job finding rate and the tightness ratio. In the data, these correlations are 0.44 and 0.39, respectively; in the model, they are 0.18 and 0.19. Learning rate shocks effectively decouple the dynamics of productivity from that of the labor market. This implies business cycle behavior that is closer to that observed in the U.S. data, relative to models where fluctuations are driven by shocks to technology.

4.5 Richer Heterogeneity

As discussed above, the calibration of our benchmark, two type model implies strong effects of fluctuations in the learning rate on job creation. Given this, we consider a richer version of the model with \( N > 2 \) worker types. Each worker type faces a higher steady state learning rate and a smaller productivity gain associated with any single upgrade, relative to our benchmark model (with \( N = 2 \)). In terms of notation, let \( f_i \) denote the level of output produced in a match between a firm and worker of type \( i \), where \( i = 1, \ldots, N \). For simplicity, we assume that a worker increases her productivity by one level at a time. Let \( \lambda_i \) denote the probability, while matched, that a worker of type \( i \) upgrades to type \( i + 1 \). All other aspects of the model remain unchanged.

The characterization of the highest productivity, type \( N \), market is identical to that of

---

by solving a version of our benchmark model (with \( N = 2 \)) when driven solely by conventional technology shocks. This version of the model features the same magnification result as in the standard DMP model. For brevity we do not present the results here, but they are available from the authors upon request.
the high productivity market summarized by the value functions, (5) through (8) (obviously, with the \(H\)-subscripts replaced by \(N\)’s). The value functions for all other workers and firms with the potential for technological learning (types \(i = 1, \ldots, N - 1\)) are given by:

\[
U_i = z + \beta E \left[ \mu(\theta_i) W'_i + (1 - \mu(\theta_i)) U'_i \right],
\]

(12)

\[
W_i = \omega_i + \beta E \left[ \lambda_i \left[ (1 - \delta) W'_{i+1} + \delta U'_{i+1} \right] + (1 - \lambda_i) \left[ (1 - \delta) W'_i + \delta U'_i \right] \right],
\]

(13)

\[
V_i = -k + \beta E \left[ q(\theta_i) J'_i \right],
\]

(14)

\[
J_i = f_i - \omega_i + \beta E \left[ (1 - \delta) \left[ \lambda_i J'_{i+1} + (1 - \lambda_i) J'_i \right] \right].
\]

(15)

To study the quantitative implications of this model extension, we pursue the same calibration strategy as before. As such, we do not discuss the specification of the more conventional model parameters, which is done in the identical manner to subsection 4.1. Here, we focus our discussion on the calibration of \(\{f_i, \lambda_i\}_{i=1}^N\) and the process of ‘death and rebirth’. For the sake of clarity and comparability, our strategy is to do this in as simple a manner as possible.

As before, we set these parameters to match empirical life-cycle earnings profiles. The average worker takes approximately 25 years to achieve the maximal earnings gains due to labor market experience; during this time, earnings double. Given this, we choose \(N = 25\) and set the ratio of the lowest-to-highest productivity to be \(f_N/f_1 = 2\). For simplicity, we follow Ljungqvist and Sargent (1998) and specify all productivity levels to be equally spaced, so that \(f_1 = 0.5, f_2 = 0.5208, \ldots, f_{N-1} = 0.9792,\) and \(f_N = 1\). We restrict the steady state learning rate between any two adjacent productivity levels to be symmetric; that is, \(\lambda_i = \lambda\) for all \(i < N\) (and, obviously, \(\lambda_N = 0\)). Finally, we set \(\lambda = 0.0185\), so that it takes the average worker 54 weeks (or approximately one year) to realize a productivity upgrade; in this way, it takes the average worker 25 years to progress from the lowest to highest productivity level.

We choose \(\phi\), the constant probability of death, such that in the model’s steady state, 25% of workers are of type \(N\) with no further potential for upgrade. Finally, we must specify how ‘dead’ workers are ‘reborn’ in order to maintain a constant unit mass of workers. For symmetry, we do this so that in the model’s steady state, there is an equal measure of workers of types 1 through \(N - 1\) (specifically, \(75\% / 24 = 3.13\%\) of each type).\(^{16}\)

\(^{16}\)As a point of reference, we note that this is a close approximation to the observed age distribution of the labor force. During the postwar period, the average fraction of labor force participants in 5-year age bins between the ages of 20-24 years and 45-49 years ranges from a low of 10.3% to a high of 12.5%; as discussed in Section 4, the average fraction over the age of 50 years is 25%.
Table 2: Business Cycle Statistics: U.S. Data and Technological Learning Model

<table>
<thead>
<tr>
<th></th>
<th>U.S. data</th>
<th>benchmark λ shocks</th>
<th>λ shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>labor productivity</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>relative to labor productivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment</td>
<td>9.34</td>
<td>6.69</td>
<td>5.54</td>
</tr>
<tr>
<td>job finding rate</td>
<td>6.05</td>
<td>7.45</td>
<td>6.18</td>
</tr>
<tr>
<td>tightness ratio</td>
<td>18.20</td>
<td>16.59</td>
<td>13.80</td>
</tr>
<tr>
<td><strong>correlations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beveridge curve</td>
<td>−0.89</td>
<td>−0.89</td>
<td>−0.89</td>
</tr>
<tr>
<td><strong>relative to labor productivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment</td>
<td>−0.41</td>
<td>−0.27</td>
<td>−0.31</td>
</tr>
<tr>
<td>job finding rate</td>
<td>0.44</td>
<td>0.18</td>
<td>0.22</td>
</tr>
<tr>
<td>tightness ratio</td>
<td>0.39</td>
<td>0.19</td>
<td>0.22</td>
</tr>
</tbody>
</table>

*Notes*: All data are logged and HP filtered. U.S. data: 1953:I–2009:IV, various sources, see text. Models: quarterly averages of simulated data, 250,000 observations at weekly frequency.

The results are presented in Column 3 of Table 2. In columns 1 and 2, we display the second moment statistics from Table 1 for the U.S. data and our benchmark model driven by learning rate shocks, respectively. Overall, our results are remarkably robust when the number of worker types is extended from \( N = 2 \) to \( N = 25 \). Again, the model displays strong cyclical volatility. The standard deviation of HP-filtered unemployment is 5.54 times that of labor productivity, which is not appreciably different from the value of 6.69 derived from the benchmark model.17 Hence, the extended model still accounts for approximately 60% of the amplification observed in the data. In terms of job creation, the model fares even better. The model generates a relative volatility of the job finding rate, 6.18, that is very close to the data, 6.05. Similar results are obtained for the tightness ratio. We conclude that our model’s performance in terms of cyclical volatility is robust, and importantly, makes progress in resolving the unemployment volatility puzzle.

Moreover, the results regarding cross correlations are robust. In the benchmark model, the correlation between unemployment and productivity is −0.27. For the extended model,

17In simulation experiments, we find that the model’s volatility of unemployment declines monotonically as \( N \) increases from 2 to 25, but ‘flattens’ substantially for \( N > 10 \). For instance, between \( N = 10 \) and \( N = 25 \), the standard deviation of unemployment relative to productivity changes only from 5.62 to 5.54.
this is largely unchanged, at −0.31. Again, our model’s predicted correlation is not far from the U.S. data. The same is true regarding the correlations of productivity with the job finding rate and the tightness ratio. The model generates realistic job creation dynamics that are robust.

How large must the $\lambda$ shocks be to account for the volatility of U.S. business cycles? Our calibration strategy for the exogenous shock process is identical to before. This implies a persistence of $\rho_\lambda = 0.979$ and a standard deviation of the innovation of $\sigma_\epsilon = 0.185$ for our extended model. To quantify this volatility, note that the steady state learning rate is calibrated so that the average employed worker realizes a productivity upgrade every 54 weeks. In the stochastic model, the 68% coverage region around the median learning duration ranges from approximately 6 months to just over 2 years.

4.6 Understanding the Mechanism

To better understand our model relative to the standard DMP model, we present impulse response functions for unemployment and labor productivity in Figure 1. The vertical scale of both panels is identical to facilitate comparison across models.

Panel A presents responses for the conventional business cycle impulse, specifically, the response to a positive one standard deviation technology shock. Technology shocks have a direct impact on matched workers’ productivity. Hence, labor productivity jumps upon impact of the shock, gradually declining to steady state thereafter. This shock implies an immediate impact on firm profit. From the free entry condition, vacancies respond immediately. Because of the high empirical job finding rate that the model is calibrated to, unemployment responds quickly, peaking 15 weeks (or about 1 quarter) after the impact period of the shock.

These responses clearly illustrate the shortcomings of technology shock-driven cycles in the DMP model. Because the peak response of both unemployment and labor productivity occur in the short-run, this implies a counterfactually strong correlation of the two variables over the business cycle. Moreover, the response of unemployment is of the same order of magnitude as that of productivity. Hence, as discussed extensively in the literature, the model displays much weaker amplification of unemployment, relative to that observed in the data.

Panel B displays the response to a positive one standard deviation learning rate shock in our extended ($N = 25$) model. The jump in the learning rate creates a jump in the
surplus from a match. From the free entry condition, vacancies respond immediately, and unemployment soon after. The response of unemployment peaks 16 weeks (about 1 quarter) after the impact period of the shock.

In contrast to a technology shock, a learning rate shock has only an indirect effect on labor productivity via the type composition of the workforce. After a positive shock to $\lambda$, the economy-wide upgrading rate rises. This causes productivity to rise as workers shuffle from lower to higher types at a faster rate. But because the technological learning process must still be undertaken, the dynamic response of productivity is persistent, and only peaks about 120 periods (or about 2 years) after the shock. As a result, our model naturally decouples the dynamics between unemployment and labor productivity. While the response of unemployment peaks in the short-run, productivity peaks in the long-run. Hence, learning rate shocks generate a low correlation between these two variables, as observed in the data.

Moreover, learning rate shocks generate a substantially stronger effect on unemployment than on productivity. To understand the response of unemployment, consider the benchmark ($N = 2$) version, and the return to job creation, namely total surplus in type
matches, displayed in equation (11). This surplus is determined largely by the value of learning, \( \Delta \), which represents the expected future profit gain from an upgrade. Since \( \lambda \) shocks have a direct impact on the learning value, they have strong effects on job creation and unemployment.

On the other hand, the effect of learning rate shocks on labor productivity is quantitatively weak. This can be seen analytically from the log-linearized response of productivity to a \( \lambda \) shock. Given the model’s timing, there is no impact in the period of the shock, since upgrading is reflected in output with a one period lag. The response of labor productivity in the following period, \( \hat{LP}_{t+1} \), is given by:

\[
\hat{LP}_{t+1} = X \left\{ \left( \frac{n_L + n_H}{n_H} \right) \left[ \lambda (1 - \delta) \right] \hat{\lambda} t - \left( \frac{\alpha \theta_L u_L}{n_L} \right) \hat{\theta}_{Lt} \right\}.
\]

Here, \( n_i (u_i) \) denotes the steady state measure of employed (unemployed) workers of type \( i \), \( \theta_L \) is the steady state tightness ratio in market \( L \), \( X \) is a constant (a function of parameters and steady state values), and the circumflex represents log-linearized deviations from steady state. The first term in the curly brackets indicates the effect of the \( \lambda \) shock. Its strength depends on the steady state distribution of worker types, \( (n_L + n_H)/n_H \) (the larger the fraction of \( L \) types with the potential to upgrade, the bigger the effect), and importantly, the level of the steady state learning rate, as captured by the term in square brackets. In order to account for life-cycle earnings dynamics, our calibration requires a small value for \( \lambda \). Hence, the response of labor productivity to a learning rate shock is quantitatively small.\(^{18}\)

Finally, we note that the quantitative success of our model does not depend on the details of the filtering procedure used. In particular, because the response of labor productivity is smooth and persistent, it is possible that its cyclical fluctuations are subsumed into the HP-filtered trend. In this case, our results may be overstating the volatility of unemployment relative to productivity. To investigate this possibility, we derive second moment statistics from our model and the U.S. data using a larger smoothing parameter in the HP filter.\(^{19}\) We increase it by a factor of 1000, from \( 10^5 \) (as in Shimer (2005a)) to \( 10^8 \). Increasing the smoothing parameter makes the trend smoother, magnifying deviations from the trend.

\(^{18}\)Deriving log-linearized expressions for labor productivity at longer time horizons is difficult, given the need to track the dynamic response of the distribution of workers across types and employment status. Note also that in the equation above, the second term in curly brackets is negative. This reflects the fact that a positive \( \lambda \) shock generates a response of job creation for \( L \) types (and no response in creation for \( H \) types). Hence, the distribution of employment shifts toward low productivity workers. This negative composition effect offsets the positive effect of faster upgrading on the response of labor productivity. However, in our numerical experiments, this offsetting effect is small. See subsection 5.3 for further discussion.

\(^{19}\)We thank Robert Shimer for suggesting this analysis to us.
impact of this magnification is quantitatively larger for productivity than unemployment in our model generated data. Using this smoothing parameter, the relative standard deviation of unemployment to labor productivity in our benchmark ($N = 2$) model is 4.25, and 3.59 in our extended ($N = 25$) model. When using this alternative filter, the corresponding value in the U.S. data is 5.53. Hence, even with this extremely smooth filter, our model accounts for $65\% - 77\%$ of the observed volatility of unemployment. In terms of the relationship between unemployment and productivity, the correlation in our $N = 2$ model is $-0.41$, and $-0.45$ in our $N = 25$ model. In the U.S. data, this is $-0.24$ using this filter. Hence, our model continues to deliver a correlation similar to that found in the U.S. data.$^{20}$

5 Further Analysis

In this section, we provide evidence for the relevance of learning rate shocks for business cycle analysis. In subsection 5.1, we relate our model’s results to the recent “news shock” literature. In subsection 5.2, we relate our model’s implications for the cyclical behaviour of wages to evidence from panel data. Finally, in subsection 5.3, we embed our labor market framework into a general equilibrium business cycle model, to further illustrate the robustness of our results.

5.1 Learning Rate Shocks and “News Shocks”

The decoupling of productivity and unemployment dynamics induced by shocks to the learning rate have important implications for our understanding of business cycle impulses. In a recent paper, Beaudry and Portier (2006) use a number of structural VAR techniques to identify shocks to productivity. They find that shocks to long-run productivity have essentially no effect on productivity upon impact.$^{21}$ Instead, productivity is found to respond in a smooth, persistent manner. On the other hand, measures such as the stock market index and employment are found to respond immediately (i.e., within the first quarter) to these long-run TFP shocks.

Technological learning rate shocks generate dynamic responses that share these features.$^{22}$

$^{20}$In comparison, applying this filter to the standard DMP model does not change its results: the relative standard deviation of unemployment to productivity remains near one, and the correlation between the variables remains near minus one.

$^{21}$In their benchmark bivariate system, shocks identified to have a permanent impact on productivity are found to have a small, negative effect on productivity upon impact (though the response is statistically indistinguishable from zero). And interestingly, shocks to stock market prices that are orthogonal to productivity upon impact generate a nearly identical dynamic response to TFP.
Hence, shocks to the learning rate provide a theoretical interpretation of empirically identified “news shocks.” This is illustrated in Figure 2, where we plot impulse response functions to a learning rate shock in our extended, $N = 25$ model of subsection 4.5. In Panel A, we display the response of the model’s stock price index. We construct this index as a weighted average of the present discounted value of firm profits in all match types, $\{J_i\}_{i=1}^N$, where the weights are the proportions of each type in the model’s steady state. A positive $\lambda$ shock causes the value of type $i = 1, \ldots, N - 1$ matches to jump immediately as the ‘upside risk’ of these matches increases.\footnote{Recall that the value of type $N$ matches is unaffected.} Hence, the stock price index jumps upon impact of the shock; the response gradually returns to zero as $\lambda$ returns to its steady state value.

Panel B displays the response of the aggregate job finding rate. The learning rate shock causes firm surplus for all type $i < N$ firms to jump upon impact. From the free entry condition, vacancies and job finding rates jump. Panel C displays the response of the aggregate unemployment rate. Unemployment is a state variable, and therefore does
Table 3: Business Cycle Statistics: U.S. Data and News Shock Model

<table>
<thead>
<tr>
<th></th>
<th>U.S. data</th>
<th>standard model</th>
<th>1 week ahead model</th>
<th>2 quarters ahead model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>relative standard deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment to productivity</td>
<td>9.34</td>
<td>1.21</td>
<td>1.22</td>
<td>1.32</td>
</tr>
<tr>
<td><strong>correlation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment and productivity</td>
<td>−0.41</td>
<td>−0.96</td>
<td>−0.97</td>
<td>−0.99</td>
</tr>
</tbody>
</table>

**Notes:** All data are logged and HP filtered. U.S. data: 1953:1–2009:IV, various sources, see text. Models: quarterly averages of simulated data, 250,000 observations at weekly frequency.

not respond in the period of the shock, but responds very quickly after impact. Hence, economic activity, as measured by stock prices, job creation, and unemployment respond within the quarter of the learning rate shock.

In contrast, labor productivity responds in a persistent, protracted manner. This is evidenced in Panel D. Shocks to the learning rate induce gradual changes in the productivity composition of the workforce. As a result, the productivity response is smooth, peaking approximately 120 periods—or over 2 years—after the initial shock. Hence, the productivity response to learning rate shocks are observed in the long run. These features are consistent with the responses identified by Beaudry and Portier (2006).

Finally, while our model conforms with the empirical evidence for news shocks, the economic mechanism embodied by our learning rate shocks are distinct from those in recent models. In those papers, news shocks are signals of technology shocks that are to arrive a number of quarters in the future. Upon arrival, these innovations immediately affect productivity. In contrast, shocks to the learning rate represent the arrival of innovations that vary in their ease of implementation; their effects on labor productivity are realized in a delayed manner, via the process of technological learning-by-doing.

This distinction is not just a matter of interpretation: while learning rate shocks make progress on rationalizing labor market dynamics in the DMP framework, conventionally modeled news shocks do not. This is illustrated in Table 3, where news shocks are introduced in the standard DMP model in the usual way—as the arrival of information at date 0 of a change in productivity in period $\hat{t}$. For brevity, we do not present details regarding this version of the model, and instead, make them available upon request.
the standard technology shock. Columns 3 and 4 present results for a one-week ahead ($\hat{t} = 1$) and two-quarter ahead ($\hat{t} = 26$) news shock, respectively. As is obvious, the amplification of unemployment volatility relative to productivity is essentially unchanged; the same is true for the variables’ correlation. Both statistics are vastly different from that observed in the U.S. data, as presented in Column 1. Hence, the manner in which “news shocks” are modeled is important for rationalizing labor market dynamics in the DMP framework.

5.2 Learning Rate Shocks and the Return to Experience

Our analysis emphasizes the importance of learning rate shocks in accounting for the cyclical behavior of aggregate unemployment and productivity. At its core, the idea is that the individual workers gain technological proficiency and productivity while on the job, and that the rate at which this happens varies over the business cycle. Shocks to the learning rate tilt the life-cycle earnings profile, making it steeper when the shock is positive, and flatter when the shock is negative. Hence, our model puts emphasis on the return to labor market experience and its cyclical properties.

Relatively little empirical work has been devoted to identifying the cyclicality of the return to experience. A notable exception is that of French et al. (2006). Their work focuses on the evolution of employment and earnings for a cohort of young workers (18 to 28 year olds) in the Census Bureau’s Survey of Income and Program Participation (SIPP). Using wage data for continuously employed workers who remain with the same employer, they are able to identify time variation in the return to experience. French et al. (2006) find this return to be strongly procyclical. In their baseline specification, a one percent rise in the unemployment rate generates a 1.1% fall in the return to experience, that is statistically significant at the 5% level.24

In light of this, we perform the same exercise in simulated data from the extended ($N = 25$) version of our model. Specifically, we track the wages of a cohort of young workers over a 25 year period; this is done by “interviewing” the workers at four month

---

24French et al. (2006) also consider a second specification which attempts to control for a time effect that is common to all workers. This common time effect might capture, for instance, technology shocks that affect all wages, independent of a worker’s experience. As French et al. (2006) point out, separately identifying such a time effect is not at all straightforward in their framework. In the end, they choose to proxy for this using the wages of new labor market entrants, with the idea that workers with no experience would not be affected by changes in the return to experience. However, this identifying assumption is clearly violated in our model. With Nash bargaining, wages are forward-looking and respond to changes in match surplus due to changes in the learning rate; this is true regardless of whether a worker has accumulated labor market experience or not. As such, we believe future empirical work attempting to disentangle common time effects from the return to experience would be of clear value.
intervals (the same frequency of interview waves in the SIPP).\textsuperscript{25} As in French et al. (2006), we estimate the return to experience at any point in time as the (cross-sectional) average log wage change from the previous interview, for those who were continuously employed. Repeating this period-after-period over the 25 years obtains a time series for the return to experience. We then determine its cyclicality by regressing it on a constant, time trend, and the aggregate unemployment rate, as in French et al. (2006).

We repeat this simulation exercise 100 times and report the median coefficient estimate. In our model, a one percent rise in unemployment generates a 1.6% fall in the return to experience. This is in line with the 1.1% value reported in French et al. (2006), though is slightly greater in magnitude. Note, however, that there are many factors that may be contributing to this discrepancy. For instance, wages in our model are determined by Nash bargaining on a period-by-period basis. As such, they are highly responsive to shocks to the learning rate. On the other hand, wage changes in the SIPP (and especially those at the four month frequency) are potentially shielded from cyclical fluctuations in the return to experience for reasons emphasized in the implicit contracting literature.\textsuperscript{26} The inclusion of such considerations into our model would likely dampen wage cyclicality. More importantly, our model analysis assumes that all fluctuations in unemployment are due to shocks to the learning rate. In reality, there are likely to be other shocks contributing to the cycle that do not affect the return to experience.\textsuperscript{27} Hence, the inclusion of such shocks in a more elaborate model would also dampen the estimated covariance of unemployment with the return to experience.

5.3 A Real Business Cycle Model

In this subsection, we embed our labor market framework into a general equilibrium, real business cycle (RBC) model. The RBC model includes a number of features that are

\textsuperscript{25}Our cohort of workers is initialized by presuming that all workers enter the labor force at age 18 as unemployed, type \( i = 1 \) workers. Applying the steady state job finding, job separation, and learning rates to these workers for a six month period allows us to generate the distribution of 18 year olds across employment statuses and types. The distribution of 19 year olds is obtained from a further 12 months of transitions for the initial cohort, and so on, until we have distributions for workers of all ages between 18 and 28 (the same age group studied in French et al. (2006)). These workers of different ages are then used to generate a representative SIPP cohort by weighting them according to the age distribution of 18 to 28 year olds in the U.S. labor force, as found in 1984 (the initial year of the SIPP) in the CPS.

\textsuperscript{26}See, for instance, Beaudry and DiNardo (1991).

\textsuperscript{27}For instance, while our model does well in replicating the volatility of job creation, it understates that of unemployment. Recent work attributes one quarter to one third of unemployment variability to cyclical job destruction. Hence, one could imagine shocks to job separation rates as generating fluctuations in unemployment that are unrelated to the return to experience.
absent from the DMP framework, notably, diminishing marginal returns in production and a consumption/investment choice. The primary purpose of this analysis is to show robustness of the cyclical properties of unemployment to the inclusion of these neoclassical features. A secondary purpose is to ensure that our model maintains the success of the standard RBC model regarding the cyclical behavior of consumption, investment, and output.

5.3.1 Description

Our model is a simplified version of Andolfatto (1996), modified to include stochastic technological learning as in Section 2. As such, we keep our exposition brief. The economy is populated by a representative household who provides consumption insurance to all of its members (a continuum of individuals with measure one). Each individual member derives utility from consumption, and incurs a utility cost of working denoted $\gamma$. Let $n_L$ and $n_H$ denote the measures of employed type $L$ and type $H$ individuals, and $u_L$ and $u_H$ denote the measures of unemployed individuals.

Let $c$ denote consumption, and $k$ denote capital. The household’s budget constraint is given by:

$$c + k' = (1 - d + r)k + \omega_L n_L + \omega_H n_H + b(u_L + u_H) + \pi. \quad (16)$$

Here $r$ is the rental rate on capital, $d$ is its depreciation rate, and $b$ is an unemployment benefit; $\pi$ represents profits earned by firms (and distributed to the household) minus lump-sum taxes paid to the government. The government maintains a balanced budget, and levies taxes to cover unemployment benefits. Let $V$ denote the value function of the household, specifically:

$$V(k, n_L, n_H, u_L, u_H) = \max_{c,k'} \left\{ \ln c - \gamma(n_L + n_H) + \beta E V(k', n'_L, n'_H, u'_L, u'_H) \right\},$$

subject to constraint (16) and the following laws of motion:

$$
\begin{align*}
n'_L & = (1 - \delta)(1 - \lambda)n_L + \mu(\theta_L)u_L, \\
n'_H & = (1 - \delta)n_H + (1 - \delta)\lambda n_L + \mu(\theta_H)u_H, \\
u'_L & = \delta(1 - \lambda)n_L + (1 - \mu(\theta_L))u_L, \\
u'_H & = \delta n_H + \delta \lambda n_L + (1 - \mu(\theta_H))u_H.
\end{align*}
$$

The representative firm has access to two ‘production lines’: one operated by type $L$ workers, the other by type $H$ workers. Both production lines use capital and labor as inputs in a Cobb-Douglas production function to produce a homogenous final good. This final good
serves either as consumption or investment. As will be discussed shortly, the assumption of two production lines facilitates comparison to our model presented in Section 2.

The firm faces a dynamic problem, with state variables consisting of the stock of current employment, \( n_L \) and \( n_H \). In each period, it chooses the number of vacancies to maintain, and the amount of capital to rent. The value of the firm, \( J \), is defined as:

\[
J(n_L, n_H) = \max_{v_L, v_H, k_L, k_H} \left\{ A_L k_L n_L^{1-a} + A_H k_H n_H^{1-a} - r(k_L + k_H) - \omega_L n_L - \omega_H n_H - \kappa(v_L + v_H) + E [QJ(n'_{L}, n'_{H})] \right\},
\]

subject to

\[
\begin{align*}
n'_L &= (1 - \delta)(1 - \lambda)n_L + q(\theta_L)v_L, \\
n'_H &= (1 - \delta)n_H + (1 - \delta)\lambda n_L + q(\theta_H)v_H.
\end{align*}
\]

Here, \( a \) is the capital share of income, and \( Q \) is the stochastic discount factor (which, in equilibrium, equals the household’s intertemporal marginal rate of substitution). In posting vacancies, the firm takes the job filling probabilities as given.

The first order conditions with respect to capital deliver the following no arbitrage condition:

\[
r = aA_L (n_L/k_L)^{1-a} = aA_H (n_H/k_H)^{1-a}. \tag{17}
\]

The first order conditions with respect to vacancies deliver the usual free entry entry conditions:

\[
\begin{align*}
\kappa &= q(\theta_L)E [QJ_{n'_L}], \\
\kappa &= q(\theta_H)E [QJ_{n'_H}],
\end{align*}
\]

where the values of the marginal workers are given by:

\[
\begin{align*}
J_{n_L} &= MP_{n_L} - \omega_L + (1 - \delta) \left\{ (1 - \lambda)E [QJ_{n'_L}] + \lambda E [QJ_{n'_H}] \right\}, \\
J_{n_H} &= MP_{n_H} - \omega_H + (1 - \delta)E [QJ_{n'_H}],
\end{align*}
\]

and \( MP_{n_L} = (1-a)A_L(k_L/n_L)^a, \) \( MP_{n_H} = (1-a)A_H(k_H/n_H)^a. \) As before, we assume that worker compensations are the result of generalized Nash bargaining between the firm and worker.

Finally, feasibility requires that

\[
c + k' + \kappa(v_L + v_H) = A_L k_L n_L^{1-a} + A_H k_H n_H^{1-a} + (1 - d)k,
\]

where \( k_L + k_H = k. \)
5.3.2 Analysis

We first discuss a number of the RBC model features, before proceeding to the numerical analysis. In the standard DMP model, match output is given parametrically. Hence, the output of an additional type $L$ worker is always $f_L$; the marginal product of a type $H$ worker is always $f_H$. In contrast, the Cobb-Douglas production function in this model features diminishing marginal product of labor. This implies that as employment rises, all else equal, labor productivity falls.

Despite diminishing marginal product of labor, our assumption of two production lines ensures that the productivity upgrade for an individual worker is constant. From the definitions of $MP_{nL}$ and $MP_{nH}$, and the no arbitrage condition, (17):

$$\frac{MP_{nH}}{MP_{nL}} = \left(\frac{A_H}{A_L}\right)^{\frac{1}{1-a}}.$$ 

This facilitates comparison with our benchmark model, where the productivity upgrade is given parametrically by $f_H/f_L$.

We now turn to quantifying the cyclical behavior of unemployment and productivity in this RBC framework. Our calibration strategy is identical to that described in subsection 4.1. For brevity, we discuss only the specification of the new parameters.\(^{28}\) The capital share of income is set to $a = 0.3$. Given this, we set $A_L$ and $A_H$ so that $MP_{nH}/MP_{nL} = 2$, as in our previous analysis. The capital depreciation rate is set to $d = 0.002$ (at the weekly frequency), to match a steady state investment to output ratio of 0.2.

In Table 4, we present the key business cycle statistics for our model driven by learning rate shocks; this is in Column 3. Columns 1 and 2 present the same statistics for the U.S. data and the RBC version of the standard DMP model driven by technology shocks, respectively. The RBC version of our technological learning model generates substantial unemployment volatility. The relative standard deviation of unemployment to productivity is 6.42. This is approximately 70% of the amplification observed in the U.S. data, and is more than 6 times greater than that of the standard model driven by technology shocks. Indeed, the degree of amplification in the RBC version of the technological learning model is very close to that from the partial equilibrium, DMP framework discussed in Section 4.

The RBC version of our model generates a correlation between unemployment and labor productivity that is near zero. Hence, the model understates the degree to which these variables are (negatively) related in the data. This is in contrast to the standard

---

\(^{28}\) Further details are available from the authors upon request.
Table 4: Business Cycle Statistics: U.S. Data and Various RBC Models

<table>
<thead>
<tr>
<th></th>
<th>U.S. data</th>
<th>standard DMP version</th>
<th>technological learning version</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative standard deviation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment to productivity</td>
<td>9.34</td>
<td>1.03</td>
<td>6.42</td>
</tr>
<tr>
<td>correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment and productivity</td>
<td>−0.41</td>
<td>−0.97</td>
<td>−0.03</td>
</tr>
<tr>
<td>std devs relative to output</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>employment</td>
<td>0.56</td>
<td>0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>consumption</td>
<td>0.85</td>
<td>0.43</td>
<td>0.60</td>
</tr>
<tr>
<td>investment</td>
<td>3.91</td>
<td>3.86</td>
<td>2.88</td>
</tr>
<tr>
<td>correlations with output</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>employment</td>
<td>0.78</td>
<td>0.97</td>
<td>0.38</td>
</tr>
<tr>
<td>consumption</td>
<td>0.88</td>
<td>0.71</td>
<td>0.92</td>
</tr>
<tr>
<td>investment</td>
<td>0.90</td>
<td>0.96</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Notes: All data are logged and HP filtered. U.S. data: 1953:I–2009:IV, various sources, see text. Models: quarterly averages of simulated data, 250,000 observations at weekly frequency.

model driven by technology shocks, where the correlation of −0.97 is much too strong.

To illustrate why our model generates such a mild correlation, Figure 3 plots the impulse response functions for unemployment and labor productivity to a positive one standard deviation learning rate shock. The response of unemployment is very similar to that displayed previously. On the other hand, the response of productivity differs, notably in the very short run. For the first 10 weeks after the shock, labor productivity actually falls relative to steady state, before rising in a protracted manner, peaking after 150 periods (approximately 3 years).

To understand these short-run dynamics, note that a positive shock to λ causes productivity to rise, as type L workers upgrade to type H at a faster rate. This is offset by two factors in the short-run. First, the shock generates job creation in the L market (where technological learning is relevant). This creates a negative composition effect, as the distribution of employment shifts toward type L workers. In addition, employment is subject to diminishing marginal returns: as more type L workers are hired, their average productivity is dragged down (an effect that is absent from the DMP framework). These two countervailing effects dominate the positive effect of the λ shock on productivity in
the short run, and are responsible for the very mild correlation between productivity and unemployment. Of course, one could easily introduce mechanisms that are common in RBC analysis, to mitigate these countervailing effects (for example, cyclical capital utilization). However, given the primary emphasis of this exercise, this is beyond the scope of this paper.

Finally, the technological learning model does a good job on the dimensions typically studied in RBC models, namely the cyclical behavior of output, consumption, investment, and employment. Our model’s predictions regarding the relative volatility of consumption and investment to output are on par with those from the standard RBC model driven by technology shocks. And not surprisingly, our model greatly outperforms the standard model with respect to the relative volatility of employment.

Finally, we note that learning rate shocks generate business cycle comovement: output, employment, consumption and investment all move together. Hence, our model generates realistic business cycle dynamics. Moreover, our model delivers comovement while simultaneously generating impulse responses resembling those of empirically identified news shocks. This last task has proven problematic in standard RBC models driven by anticipated technology shocks. A comprehensive analysis of the RBC version of our model with respect to the new shock literature would require extending the model in a number of directions (for instance, including an ‘intensive margin’ of labor effort). This is beyond the scope of this
paper, and we leave this to ongoing and future work.

6 Conclusion

In this paper, we have focused on two key labor market observations in postwar U.S. data. The first is that unemployment is very volatile over the business cycle relative to labor productivity. The second is that cyclical fluctuations in unemployment and productivity are only mildly negatively correlated. The canonical model of equilibrium unemployment, when driven by technology shocks, fails to account for either of these facts.

We propose a model of technological learning that makes progress on both shortcomings. Specifically, we construct a tractable search-and-matching model in which: (a) it takes time for workers to become fully productive with new technology, and (b) shocks to the speed or ease of learning are a source of business cycle fluctuations. Quantitative analysis indicates that our model accounts for 60% – 70% of the relative volatility of unemployment to productivity, and essentially all of the relative volatility of job finding rates, observed in the U.S. data. Moreover, our model delivers a correlation between unemployment and labor productivity that is very close to the data.
A  Data Sources

Our measure of unemployment is the quarterly average of the seasonally adjusted monthly series constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Our measure of labor productivity is (quarterly, seasonally adjusted) output divided by employment in the nonfarm business sector as constructed by the BLS Major Sector Productivity program. To construct the job finding rate, we follow the approach of Shimer (2005a,b, 2007), using monthly data on employment, unemployment, and short-term unemployment tabulated from the CPS. We use vacancy data provided by Barnichon (2010), divided by unemployment, to construct the tightness ratio. Specifically, we use his “composite Help-Wanted Index” which combines information on the number of newspaper and online job advertisements compiled by the Conference Board.

Quarterly data on aggregate output and its expenditure components are obtained from the National Income and Product Accounts. Specifically, real output, consumption, and investment refer to seasonally adjusted gross domestic product, personal consumption expenditures, and gross private domestic investment, respectively, expressed in chained 2005 dollars. Finally, employment refers to the quarterly average of the seasonally adjusted monthly series for the civilian employment-population ratio, constructed by the BLS from CPS data.

B  Efficiency

To show that our equilibrium is efficient, we derive equations that fully characterize the solution to a planner’s problem and show that the same equations characterize equilibrium under Hosios (1990)’s condition.

B.1  Compensations

To begin, we derive the compensations from generalized Nash bargaining. These are required for the characterizations below. The market for high productivity workers is identical to that of the standard DMP model. Therefore, the compensation in such matches is entirely standard:

$$\omega_H = \tau(f_H + \kappa\theta_H) + (1 - \tau)z. \quad (18)$$

To obtain the compensation in low productivity matches, begin with proportionality of surplus:

$$(1 - \tau) (W_L - U_L) = \tau J_L.$$ 

Worker surplus can be expressed as:

$$(W_L - U_L) = \omega_L - z + \beta E\left\{ (1 - \delta)[\lambda (W'_H - U'_H) + (1 - \lambda) (W'_L - U'_L)] \right\} - \mu(\theta_L) (W'_L - U'_L) + \lambda (U'_H - U'_L),$$

36
or:

\[(1 - \tau) (W_L - U_L) = (1 - \tau)(\omega_L - z) + \beta E \left\{ (1 - \delta) \tau [\lambda J'_L + (1 - \lambda) J'_H] - \tau \mu(\theta_L) J'_L + (1 - \tau) \lambda (U_H' - U_L') \right\}.\]

Equating this with \(\tau J_L\) and using free entry, we get:

\[
\omega_L = \tau (f_L + \kappa \theta_L) + (1 - \tau) z - (1 - \tau) \lambda \beta E [U_H' - U_L'].
\] (19)

The last term is \((1 - \tau)\) times the worker’s value of learning.

The worker’s value of learning can be simplified by noting that:

\[
U_H - U_L = \beta E \left[ \mu(\theta_H) (W_H' - U_H') - \mu(\theta_L) (W_L' - U_L') + [U_H' - U_L'] \right].
\]

Using the proportionality of surplus and the free entry condition again, we get:

\[
U_H - U_L = \bar{\tau} \kappa (\theta_H - \theta_L) + \beta E [U_H' - U_L']
= \bar{\tau} \kappa E \sum_{s=0}^{\infty} \beta^s (\theta_H^s - \theta_L^s)\] (20)

where \(\bar{\tau} = \tau / (1 - \tau)\), \(\theta_i^0 = \theta_i\), \(\theta_i^1 = \theta_i'\), \(\theta_i^2 = \theta_i''\), and so on.

**B.2 A Planner’s Problem**

Let \(V(u_L, u_H, n_L, n_H)\) denote the value function of the planner who inherits unemployment for type \(i \in \{L, H\}\) and employment for type \(i \in \{L, H\}\):

\[
V(u_L, u_H, n_L, n_H) = \max_{\{\theta_i, u_i', n_i'\}, i=(L,H)} \left\{ (u_L + u_H)z + n_L f_L + n_H f_H - \kappa (u_L \theta_L + u_H \theta_H) + \beta V(u_L', u_H', n_L', n_H') \right\},
\]

subject to the following laws of motion:

\[
u_L' \geq (1 - \mu(\theta_L)) u_L + (1 - \lambda) \delta n_L,
\]
\[
u_H' \geq (1 - \mu(\theta_H)) u_H + \lambda \delta n_L + \delta n_H,
\]
\[
n_L' \leq \mu(\theta_L) u_L + (1 - \lambda)(1 - \delta) n_L,
\]
\[
n_H' \leq (1 - \delta) n_H + \mu(\theta_H) u_H + \lambda (1 - \delta) n_L.
\]

Setting \(u_L + u_H + n_L + n_H = 1\) and letting \(\lambda_{u_i}\) be the multiplier on the law of motion for \(u_i\) and \(\lambda_{n_i}\) be the multiplier on the law of motion for \(n_i\), the first-order conditions with respect to \(\theta_i, i \in \{L, H\}\), are:

\[-\kappa + \mu'(\theta_L)(\lambda_{u_L} + \lambda_{n_L}) = 0,
\]
\[-\kappa + \mu'(\theta_H)(\lambda_{uH} + \lambda_{nH}) = 0.\]

The first order conditions with respect to \(u'_i\) and \(n'_i\), \(i \in \{L,H\}\), are:

\[
\beta V_{u_i}(u'_L, u'_H, n'_L, n'_H) + \lambda_{u_i} = 0,
\]

\[
\beta V_{n_i}(u'_L, u'_H, n'_L, n'_H) - \lambda_{n_i} = 0.
\]

Finally, the envelope conditions are:

\[
V_{u_L} = z - \kappa \theta_L - \lambda_{u_L} (1 - \mu(\theta_L)) + \lambda_{n_L} \mu(\theta_L),
\]

\[
V_{u_H} = z - \kappa \theta_H - \lambda_{u_H} (1 - \mu(\theta_H)) + \lambda_{n_H} \mu(\theta_H),
\]

\[
V_{n_L} = f_L - \lambda_{u_L} (1 - \lambda) \delta - \lambda_{n_L} \lambda \delta (1 - \lambda) + \lambda_{n_L} \lambda (1 - \delta),
\]

\[
V_{n_H} = f_H - \lambda_{u_H} \delta + \lambda_{n_H} (1 - \delta).
\]

Combining the first-order and envelope conditions, the following equations characterize a steady state:

\[
\lambda_{uL} + \lambda_{nL} = \frac{\kappa}{\mu'(\theta_L)}
\]

(21)

\[
\lambda_{uH} + \lambda_{nH} = \frac{\kappa}{\mu'(\theta_H)}
\]

(22)

\[
\beta \left( z - \kappa \theta_L + \mu(\theta_L) (\lambda_{uL} + \lambda_{nL}) - \lambda_{uL} \right) + \lambda_{uL} = 0
\]

(23)

\[
\beta \left( z - \kappa \theta_H + \mu(\theta_H) (\lambda_{uH} + \lambda_{nH}) - \lambda_{uH} \right) + \lambda_{uH} = 0
\]

(24)

\[
\beta \left( f_L - \delta [(1 - \lambda) (\lambda_{uL} + \lambda_{nL}) + \lambda (\lambda_{uH} + \lambda_{nH})] + (1 - \lambda) \lambda_{nL} + \lambda \lambda_{nH} \right) - \lambda_{nL} = 0
\]

(25)

\[
\beta \left( f_H - \delta (\lambda_{uH} + \lambda_{nH}) + \lambda_{nH} \right) - \lambda_{nH} = 0
\]

(26)

It will prove convenient to use these equations get expressions for some of the multipliers. From equation (23), we have:

\[
\lambda_{uL} = -\frac{\beta(\alpha z + (1 - \alpha) \theta_L \kappa)}{\alpha(1 - \beta)}.
\]

(27)

Similarly, from equation (24), we have:

\[
\lambda_{uH} = -\frac{\beta(\alpha z + (1 - \alpha) \theta_H \kappa)}{\alpha(1 - \beta)},
\]

(28)

and from equation (26) we have:

\[
\lambda_{nH} = \frac{\beta(\alpha f_H - \delta \theta_H \kappa / \mu(\theta_H))}{\alpha(1 - \beta)}.
\]

(29)
B.2.1 Market $H$

Notice that equations (22), (28) and (29) fully characterize $\theta_H$ in an unrelated way to $\theta_L$, or submarket $L$ in general. Using (28) and (29) in (22):

$$\frac{\beta(\alpha f_H - \delta \theta_H \kappa / \mu(\theta_H) - \alpha z - (1 - \alpha) \theta_H \kappa)}{\alpha(1 - \beta)} = \frac{\kappa}{\mu'(\theta_H)}.$$  

Using the fact that $\mu'(\theta) = \alpha \mu(\theta) / \theta$, we have:

$$\beta \left( \alpha(f_H - z) - (1 - \alpha) \theta_H \kappa - \frac{\delta \theta_H \kappa}{\mu(\theta_H)} \right) = \frac{(1 - \beta) \alpha \theta_H \kappa}{\alpha \mu(\theta_H)}.$$  

Rearranging, $\theta_H$ is characterized by:

$$\frac{\beta(\alpha(f_H - z) - (1 - \alpha) \theta_H \kappa)}{1 - \beta(1 - \delta)} = \frac{\theta_H \kappa}{\mu(\theta_H)}. \quad (30)$$

B.2.2 Market $L$

Rewrite equation (25) as follows:

$$\lambda_{n_L} = \bar{\beta}(f_L - \delta (1 - \lambda)(\lambda_{u_L} + \lambda_{n_L}) - \delta \lambda(\lambda_{u_H} + \lambda_{n_H}) - \lambda(\lambda_{n_L} - \lambda_{n_H})),$$

where $\bar{\beta} = \beta / (1 - \beta)$. We now need an expression for $\lambda_{n_L} - \lambda_{n_H}$. From equations (21) and (22), we have

$$\lambda_{n_L} - \lambda_{n_H} = \frac{\kappa}{\mu'(\theta_L)} - \frac{\kappa}{\mu'(\theta_H)} + \lambda_{u_H} - \lambda_{u_L}.$$  

Using the expressions for $\lambda_{u_H}$ and $\lambda_{u_L}$ from equations (27) and (28), this is

$$\lambda_{n_L} - \lambda_{n_H} = \frac{\kappa}{\mu'(\theta_L)} - \frac{\kappa}{\mu'(\theta_H)} + (\bar{\beta} / \alpha)(1 - \alpha)(\theta_L - \theta_H) \kappa.$$  

So $\lambda_{n_L}$ becomes

$$\lambda_{n_L} = \bar{\beta} \left[ f_L - \delta (1 - \lambda)(\lambda_{u_L} + \lambda_{n_L}) - \delta \lambda(\lambda_{u_H} + \lambda_{n_H}) ight. \quad \left. - \lambda \left( \frac{\kappa}{\mu'(\theta_L)} - \frac{\kappa}{\mu'(\theta_H)} + (\bar{\beta} / \alpha)(1 - \alpha)(\theta_L - \theta_H) \kappa \right) \right],$$

or, rearranging,

$$\lambda_{n_L} = \bar{\beta} \left[ f_L - [\delta (1 - \lambda) + \lambda] \frac{\kappa}{\mu'(\theta_L)} + (1 - \delta) \lambda \frac{\kappa}{\mu'(\theta_H)} - \lambda(\bar{\beta} / \alpha)(1 - \alpha)(\theta_L - \theta_H) \kappa \right].$$  

Using this last equation together with the expression for $\lambda_{u_L}$ from equation (27) in equation (21), we have

$$\bar{\beta} \left[ f_L - z - (1 - \alpha) \theta_L \kappa / \alpha - [\delta (1 - \lambda) + \lambda] \frac{\kappa}{\mu'(\theta_L)} + (1 - \delta) \lambda \frac{\kappa}{\mu'(\theta_H)} - \lambda(\bar{\beta} / \alpha)(1 - \alpha)(\theta_L - \theta_H) \kappa \right] = \frac{\kappa}{\mu'(\theta_L)}.$$  

39
Grouping terms and rearranging,
\[
\tilde{\beta}\left[f_L - z - (1-\alpha)\theta_L \kappa / \alpha + (1-\delta)\lambda \frac{\kappa}{\mu'(\theta_H)} + \lambda(\tilde{\beta}/\alpha)(\theta_H - \theta_L)\kappa\right] = \frac{\kappa}{\mu'(\theta_L)}, \tag{31}
\]
where \(\tilde{\beta} = \beta/(1-\beta(1-\delta)(1-\lambda))\). Given the value of \(\theta_H\) which solves equation (30), this last expression characterizes \(\theta_L\).

B.3 Relation to Equilibrium

B.3.1 Market \(H\)

The steady state value of the firm in market \(H\) is
\[J_H = \frac{f_H - \omega_H}{1 - \beta(1-\delta)}.\]

Using the wage equation (18) in the free-entry condition \(\beta\mu(\theta_H)J_H = \theta_H \kappa\), we have
\[
\frac{\beta((1-\tau)(f_H - z) - \tau\theta_H \kappa)}{1 - \beta(1-\delta)} = \frac{\theta_H \kappa}{\mu(\theta_H)}.
\]
Clearly, this is equivalent to its counterpart (30) from the Planner’s problem if \(\tau = 1 - \alpha\).

B.3.2 Market \(L\)

The free entry condition in that market reads
\[\beta\mu(\theta_L)J_L = \theta_L \kappa,\]
where
\[J_L = f_L - \omega_L + \beta(1-\delta)[\lambda J_H + (1-\lambda)J_L].\]

We can use the free entry condition \(J_H = \theta_H \kappa / \beta\mu(\theta_H)\) to get
\[
J_L = \frac{f_L - \omega_L + \beta(1-\delta)\lambda \theta_H \kappa / \beta\mu(\theta_H)}{1 - \beta(1-\delta)(1-\lambda)}.
\]
From equations (19) and (20), the wage in market \(L\) can be written as
\[\omega_L = \tau(f_L + \kappa \theta_L) + (1-\tau)z - \bar{\beta}\lambda \tau \kappa(\theta_H - \theta_L).\]

Using this wage in \(J_L\), the free entry condition becomes
\[
\frac{\beta\mu(\theta_L)}{1 - \beta(1-\delta)(1-\lambda)} \left[f_L - \left(\tau(f_L + \kappa \theta_L) + (1-\tau)z - \bar{\beta}\lambda \tau \kappa(\theta_H - \theta_L)\right) + (1-\delta)\lambda \theta_H \kappa / \mu(\theta_H)\right] = \theta_L \kappa.
\]
Rearranging, we have
\[ \tilde{\beta} \left[ (1 - \tau)(f_L - z) - \tau \kappa \theta_L + \tilde{\beta} \lambda \tau \kappa (\theta_H - \theta_L) + (1 - \delta) \lambda \alpha \kappa / \mu' (\theta_H) \right] = \alpha \kappa / \mu' (\theta_L). \] (32)

If \( \alpha = 1 - \tau \), this becomes
\[ \tilde{\beta} \left[ f_L - z - (1 - \alpha) \theta_L \kappa / \alpha + (1 - \delta) \lambda \kappa / \mu' (\theta_H) + \lambda (\tilde{\beta} / \alpha) (1 - \alpha) \kappa (\theta_H - \theta_L) \right] = \kappa / \mu' (\theta_L), \]
which is identical to its counterpart (31) from the Planner’s problem.

\section{Directed Search}

Given the previous result regarding efficiency, it is perhaps not surprising that the equilibrium of our model is robust to the decentralization and determination of wages. In particular, it is robust to the following “wage posting” setup.

To proceed, we maintain the assumption that the labor market is segmented by worker type; that is, a type \( L \) worker cannot search for a vacancy posted for a type \( H \) worker, and vice versa. The interpretation is as follows. In the market for type \( H \), firms post a wage that they commit to pay upon matching with a worker in each period that they are productive. A firm who meets with a type \( H \) worker is certain of the match output until exogenous separation. This allows a firm to post a single wage. As will become obvious, this implies that competitive search equilibrium in this market is identical to that in a standard DMP model.

In the type \( L \) market, firms post wages to attract unemployed workers where matches face a probability of upgrading to a type \( H \) match. Therefore, a firm posts a wage, \( \omega_L \), that it commits to while the match produces output \( f_L \), and must also post a wage commitment in the event that the match upgrades. For simplicity, assume that firms commit to pay the (unique) equilibrium wage that is being posted by firms in the type \( H \) market.

Workers observe all posted wages and choose which wage submarket to search in (within the type market). Both workers and firms observe the market tightness, and hence, the job finding and filling rates implied by their choice. Given this setup, it suffices to show that the tightness ratios in each market are unique, and that the wages posted in each market are identical to the Nash bargained wages from our model.

\subsection{H Market}

For simplicity, we focus on deterministic steady state in our model. Using equation (6), the value function for a type \( H \) unemployed worker can be rearranged as:
\[ (1 - \beta) U_H = z + \hat{\beta} \mu (\theta_H) [\omega_H - (1 - \beta) U_H], \] (33)
where \( \hat{\beta} = \beta / (1 - \beta (1 - \delta)) \). Given that the worker is directing her search, she chooses the wage submarket that offers her the highest \( U_H \). For a given level of \( U_H \), (33) gives the
indifference trade-off between wage and tightness: the worker can be compensated for a lower $\omega_H$ through an appropriately higher job finding rate, $\mu(\theta_H)$.

From equations (7) and (8), we obtain the value of maintaining a type $H$ vacancy in steady state:

$$V_H = -\kappa + \hat{\beta}q(\theta_H)(f_H - \omega_H).$$

(34)

The firm’s problem is to choose the $(\theta_H, \omega_H)$ pair that maximizes $V_H$. Let $U_H$ denote the highest utility a type $H$ worker can attain by applying to some firm’s vacancy. The profit maximizing firm knows that in order for any workers to search for its vacancy, its $(\theta_H, \omega_H)$ must provide at least as much utility as $U_H$. Hence, the firm maximizes $V_H$ subject to the constraint (33).

At this point it is useful to consider a change of variable for the sake of algebra. Let $v = \theta^{-1}$. Previewing our choice in Section 4, assume a Cobb-Douglas matching function, and define:

$$\bar{\mu}(v) = v^{-\alpha} = \theta^\alpha = \mu(\theta),$$

$$\bar{q}(v) = v^{1-\alpha} = \theta^{\alpha-1} = q(\theta).$$

Note that $\bar{q}(v) = v \bar{\mu}(v)$.

Subbing (33) into (34) and using the change of variable, we obtain the following as the firm’s problem:

$$\max_{v_H} \left\{ -\kappa + \hat{\beta}v_H \bar{\mu}(v_H)f_H - v_H \left[ (1 - \beta)U_H \left( 1 + \hat{\beta} \bar{\mu}(v_H) \right) - z \right] \right\}.$$  

(35)

The FONC can be written as:

$$\hat{\beta} \left[ \bar{\mu}(v_H) + v_H \bar{\mu}'(v_H) \right] \left[ f_H - (1 - \beta)U_H \right] = (1 - \beta)U_H - z.$$  

(36)

Given the Cobb-Douglas functional form, $\bar{\mu}(v_H) + v_H \bar{\mu}'(v_H) = (1 - \alpha)\bar{\mu}(v_H) = (1 - \alpha)\mu(\theta_H)$. Since the firm takes the level of $U_H$ as given, and since $\mu(\theta_H)$ is monotone increasing, there is a unique $\theta_H$ that satisfies (36); all firms post the same wage, implying the same market tightness.

To find the posted wage, rearrange (36) as:

$$(1 - \beta)U_H \left[ 1 + (1 - \alpha)\hat{\beta}\mu(\theta_H) \right] = (1 - \alpha)\hat{\beta}\mu(\theta_H)f_H + z.$$  

Subing this into (33) for $(1 - \beta)U_H$, and using the free entry condition, obtains:

$$\omega_H = (1 - \alpha)f_H + (1 - \alpha)\theta_H \kappa + \alpha z.$$  

Hence, the steady-state per period wage with wage posting and directed search is identical to the case with Nash bargaining, when $\tau = 1 - \alpha$.  

42
C.2 L Market

Proceeding as above, the value of a type L unemployed worker can be expressed as:

\[
(1 - \beta)U_L = z + \tilde{\beta}\mu(\theta_L)\left[\omega_L + \beta\lambda[(1 - \delta)(W_H - U_H) + (U_H - U_L)] - (1 - \beta)U_L\right],
\]

where \(\tilde{\beta} = \beta/(1 - \beta(1 - \delta)(1 - \lambda))\). Using the change of variable and rearranging we get:

\[
\tilde{\beta}\mu(v_L)\omega_L = [1 + \tilde{\beta}\mu(v_L)](1 - \beta)U_L - z - \tilde{\beta}\mu(v_L)\beta\lambda[(1 - \delta)(W_H - U_H) + (U_H - U_L)].
\]

The value function for a type L vacancy is:

\[
V_L = -\kappa + \tilde{\beta}q(\theta_L)[f_L - \omega_L + \beta\lambda(1 - \delta)J_H].
\]

Making the change of variable and subbing in from above we get:

\[
\max_{v_L} \left\{-\kappa + \tilde{\beta}v_L\mu(v_L)f_L + \beta\lambda(1 - \delta)J_H - v_L(1 - \beta)U_L(1 + \tilde{\beta}\mu(v_L)) - z - \tilde{\beta}\mu(v_L)\beta\lambda[(1 - \delta)(W_H - U_H) + (U_H - U_L)]\right\}. \quad (39)
\]

After rearranging, the FONC is:

\[
\left[1 + (1 - \alpha)\tilde{\beta}\mu(v_L)\right](1 - \beta)U_L = (1 - \alpha)\tilde{\beta}\mu(v_L)f_L + \beta\lambda(1 - \delta)(J_H + W_H - U_H) + \beta\lambda(U_H - U_L) + z.
\]

Given that the firm takes \(J_H, W_H, U_H\) and \(U_L\) as given, this establishes that \(\theta_L\) is unique.

Subbing this into (37) and rearranging obtains:

\[
\omega_L + \alpha\beta\lambda(1 - \delta)(W_H - U_H) + \alpha\beta\lambda(U_H - U_L) = \alpha z + (1 - \alpha)f_L + (1 - \alpha)\beta\lambda(1 - \delta)J_H + (1 - \alpha)\tilde{\beta}\mu(\theta_L)f_L - \omega_L + \beta\lambda(1 - \delta)J_H.
\]

Using the free entry condition this simplifies to:

\[
\omega_L = (1 - \alpha)f_L + (1 - \alpha)\theta_L\kappa + \alpha z - \alpha\beta\lambda(U_H - U_L) + \beta\lambda(1 - \delta)[(1 - \alpha)J_H - \alpha(W_H - U_H)].
\]

Given that \((1 - \alpha)J_H = \alpha(W_H - U_H)\), the last term cancels out. Hence, the expression for \(\omega_L\) is the exact expression obtained with Nash bargaining, when \(\tau = 1 - \alpha\).

D Proof of Proposition 2

We begin by establishing a lemma.

Lemma 2 With respect to log-linear deviations from steady state in response to a \(\lambda\) shock, \((\theta_L, U_L, TS_L, \Delta)\) comove positively.
Proof. Using the free entry condition, equation (10), and assuming a Cobb-Douglas matching function so that \( q(\theta) = \theta^{\alpha-1} \), we obtain the log-linearized relationship:

\[
\hat{T}S_L = (1 - \alpha) \hat{\theta}_L.
\]

From the expression for the value of unemployment, equation (1), in steady state:

\[
\hat{U}_L = \left[ \frac{\bar{\tau}_\kappa \theta_L}{(1 - \beta) U_L} \right] \hat{\theta}_L.
\]

From the definition of total surplus, equation (11):

\[
\hat{\Delta} = \left[ TS_L \left[ 1 - \beta(1 - \delta)(1 - \alpha) + \theta_L \bar{\tau}_\kappa \right] \right] \hat{\theta}_L.
\]

Given this lemma, proving Proposition 2 requires showing that \( \theta_L \) responds positively to a \( \lambda \) shock.

Proof. Since \( \Delta = \lambda \beta \left[ (1 - \delta)(TS_H - TS_L) + (U_H - U_L) \right] \), log-linearizing we obtain:

\[
\Delta \hat{\Delta} = \Delta \hat{\lambda} - \lambda \beta \left[ (1 - \delta)TS_L \hat{TS}_L + U_L \hat{U}_L \right].
\]

This uses the fact that \( \hat{TS}_H = \hat{U}_H = 0 \) in response to \( \hat{\lambda} \). Using the derivations from Lemma 2:

\[
[1 - \beta(1 - \delta)](1 - \alpha)TS_L \hat{\theta}_L + \bar{\tau}_\kappa \theta_L \hat{\theta}_L = \Delta \hat{\lambda} - \lambda \beta \left[ (1 - \delta)TS_L \hat{TS}_L + U_L \hat{U}_L \right],
\]

\[
[1 - \beta(1 - \delta)](1 - \alpha)TS_L \hat{\theta}_L + \bar{\tau}_\kappa \theta_L \hat{\theta}_L = \Delta \hat{\lambda} - \lambda \beta(1 - \delta)(1 - \alpha)TS_L \hat{\theta}_L - \frac{\lambda \beta}{1 - \beta} \bar{\tau}_\kappa \theta_L \hat{\theta}_L.
\]

Therefore:

\[
\Delta \hat{\lambda} = \left\{ [1 - \beta(1 - \delta)(1 - \lambda)](1 - \alpha)TS_L + \left[ 1 + \frac{\lambda \beta}{1 - \beta} \right] \bar{\tau}_\kappa \theta_L \right\} \hat{\theta}_L.
\]

Since both \( \Delta \) and the term in curly brackets are positive, this implies that \( \theta_L \) comoves positively with a \( \lambda \) shock.
References


Brugemann, B. (2008). What elasticity of the matching function is consistent with u.s. aggregate labor market data?


