Cournot and Bertrand Games

Steven R. Beckman

Abstract: The author describes a series of matrix choice games illustrating monopoly, shared monopoly, Cournot, Bertrand, and Stackelberg behavior given either perfect complements or perfect substitutes. The games are created by using a spreadsheet to fill out a profit table given the choices of two players. One player selects the column, the other the row, and the table gives the profit of the row chooser. Because each player has a table, each thinks of him- or herself as the row chooser and the other as the column chooser. The games may be applied to international trade through the traditional Boeing v. Airbus story or, more currently, through foreign sales corporations. Addition of Bertrand competition allows discussion of price wars, and addition of perfect complements allows discussion of the proposed Microsoft breakup.

Key words: Bertrand, Cournot, games, Microsoft
JEL codes: A22, F12

Cournot and Bertrand competition can be difficult to teach, but some simple games do a remarkably good job of introducing the basic concepts.[1] Business students with little patience for economic abstractions dominate my international trade class. They want to see the point right away. These games satisfy us both. I introduce them to shared monopolies, Stackelberg leadership, Stackelberg warfare, perfect complements, perfect substitutes, quantity competition, and price competition in a set of four games that may be played in two class periods. Rather than drawing complicated reaction functions, the students observe the strategies their classmates use and attempt to devise effective countermoves in these simple games. Once the games have been played, it is easy to connect the games to applications in international trade and industrial organization. Prominent among these applications are the traditional Boeing v. Airbus story and the proposed breakup of Microsoft. More convincing for my students is that the games help explain current business research on how to fight a price war.

THE FIRST CLASS

The game, in a form that may be reproduced and handed out, is presented in Figure 1.[2] The instructor should read through the game before class and think about how students might play the game. Experienced gamers may instantly find the monopoly, shared monopoly, and Cournot and Stackelberg solutions, but that is well outside my experience. It is more likely that a few students will play domi-

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FIGURE 1
Profit Games

In this game, you and your partner choose outputs and record your profits below. There are a number of ways to play the game. Cournot assumed firms play as if the other firms's output choice is constant. This makes the game simple. For example, if your playing partner chooses 3 then, looking down column 3, the "best response" is 5 because your profits would be 20, and any other choice reduces profit. (It will prove convenient to define the Cournot best response as the larger of any tied values.) Furthermore, because you aren't trying to change the other player's behavior, there is no other consideration. Alternatively, you can try to manipulate the choice of your playing partner. Obviously it is best for you if your partner picks 0 and you pick 6. But watch out, if you try to manipulate your partner, then he or she may try to manipulate you.

The table is constructed from a simple demand function, $P = 12 - (Q_1 + Q_2)$, where Q_1 and Q_2 are the output choices of you and your partner. For demand, only the sum of output matters. Apparently the consumers consider the two goods to be perfect substitutes because only the sum matters and not which firm produces more. The profits reported are just PQ_1, that is, there are no costs. For example, if your partner chooses 3 and you choose 5, then the output of the substitutes is 8, the price is $12 - 8 = 4$, and your profit is $4 \times 5 = 20$. A bit more formally, firm one's profit is $Q_1[12 - (Q_1 + Q_2)]$.

My Firm's Profit Table

		Playing partner's choice												
		0	1	2	3	4	5	6	7	8	9	10	11	12
My choice	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	11	10	9	8	7	6	5	4	3	2	1	0	–1
	2	20	18	16	14	12	10	8	6	4	2	0	–2	–4
	3	27	24	21	19	15	12	9	6	3	0	–3	–6	–9
	4	32	28	24	20	16	12	8	4	0	–4	–8	–12	–16
	5	35	30	25	20	15	10	5	0	–5	–10	–15	–20	–25
	6	36	30	24	18	12	6	0	–6	–12	–18	–24	–30	–36
	7	35	28	21	14	7	0	–7	–14	–21	–28	–35	–42	–49
	8	32	24	16	8	0	–8	–16	–24	–32	–40	–48	–56	–64

Form groups of two and play the game. Write down your choice and cover it with your hand. Once your partner has made a choice, simultaneously reveal your decisions and record your profit. You are welcome to talk.

Round	My choice	Partner's choice	My profit
1			
2			
3			
⋮			
10			

nated strategies, about one-third will converge on the shared monopoly solution of 3, and another third will randomly oscillate around the Cournot solution of 4. Very few, perhaps 1 in 20, will play the Stackelberg warfare game.[3] Some will simply fill out the form without actually playing the game.

Therefore it is important that the instructor go into the class with a set of strategies that will encourage active learning. Before students can be active gamers, they must understand the game. Instructors should take the time to discuss construction of the table, go through two or three examples showing how the profit levels are calculated, and discuss dominated strategies. A mock round should be conducted, and a form filled out for each player on an overhead. However, I leave discussions of Cournot, shared monopoly, and Stackelberg strategies for after the game. The introduction should take about 5 to 10 minutes. Once students begin, the instructor should walk about and observe, and, if students play dominated strategies, point that out. If the instructor sees people just filling out the cooperative solution on the form, they should be given a new form and told that actual play introduces interesting features. In this case, it is rational to defect from the shared monopoly in the last round because retaliation is impossible. If the student believes the other student will defect in the last round, then it is rational to defect in the second to last and so on. Most of all, talk to students about their strategies and draw them out in the discussion following the game. Talking to students and letting some play a second time may take 20 to 30 minutes. This will still leave time for discussion.

Analyzing the Game

The game may be analyzed in a series of steps. The first step eliminates dominated strategies. No matter what column is selected, rows 7 and 8 have lower payoffs. No matter what someone thinks the other player will choose, it is never rational to choose a number higher than 6. Next, if the other player chooses 0, the best response is 6. This is the monopoly solution with profits of 36. The obvious shared monopoly is 3 each with profits of 18 for each firm. The difficulty with the shared monopoly is that the best response to 3 is not 3 but 5. That is, because the profit from picking 5 is 20, there is an incentive to defect from the cooperative solution.[4] If both players play best response, the only equilibrium is at 4 each with profits of 16. This is, of course, the Cournot solution.[5]

If the other player may be relied on to play best response, then a cunning strategy is possible. If a player knows the other will play best response and begins to consider which choice maximizes profit, the player will soon see the optimal choice is 6. The best response is 3, and the Stackelberg leader's profit is 18 (column 3, row 6). The follower, who chooses best response, nets only 9 (column 6, row 3). It is easy to check that this is the best the leader can do. If instead the leader picked 5, the best response is 4, and the leader's profit is only 15 (column 4, row 5). Of course, any choice above 6 was eliminated as a dominated strategy. Should the other player reciprocate and attempt to be the leader, Stackelberg warfare breaks out with both picking 6 and both earning zero profit.[6]

Discussion and Application of Cournot Competition

Following the game, the instructor should get students to share their strategies and observations. Let them talk enough that the class gets a sense for which strategies are popular and then elicit comments on nuances. For example, someone generally notices that the shared monopoly and Cournot solutions form a prisoner's dilemma. Just as in a prisoner's dilemma, there is an incentive to mislead by claiming cooperation while planning defection. A number of questions may be asked to keep discussion going: Is simultaneous choice important? Would it matter if someone always chose first? (It does, Stackelberg leadership is often referred to as the first mover advantage.) If someone fills out 6 for all the rounds and shows it to their playing partner, how will such a precommitment change the game? Are there sanctions for being dishonest? Should there be? Would it matter if the game had three players rather than two? Are friendships and trust important? If the players could sign a contract before play began, what provisions should be included?

The quality of discussion seems to vary with the level of previous game experience. It is probably best to play a prisoner's dilemma game before trying this game. However, even with novice gamers, once the game has been played, students readily understand the concepts of Cournot, Stackelberg, shared monopoly, and cheating. With this background, applications abound. The most traditional application comes from Brander and Spencer (1985). Given two firms, one home and one foreign, serving the same market in a third country, then subsidizing the home firm so that it will play Stackelberg leader can force the foreign firm into the role of follower. The shift in profits to the home firm makes the strategy worthwhile. However, if the foreign firm retaliates, Stackelberg warfare ensues and both lose. The World Trade Organization restricts export subsidies partly in the recognition that subsidies are an attempt to shift profits that may well trigger retaliation and collective losses. The most prominent textbook case is Boeing v. Airbus, whereas the example garnering the most headlines now involves foreign sales corporations.[7]

I find that this is about all one can do in a single class.

THE SECOND CLASS

The second class period begins with a brief review and a new game. The game is presented in Figure 2 and models price competition given perfect substitutes.[8] Students rapidly find the shared monopoly solution in which each charge a price of 6 and earn profits of 18. However, the incentive to cheat is enormous. Dropping the price to 5 raises profits to 35. Most students rapidly converge on prices of 1, each with profits of 5. The classroom discussion often centers on why the small change from quantity to price competition creates such a huge behavioral change. Of course, the reason is that shading the price of a perfect substitute is a bid for the competitor's entire market whereas increasing quantity is far less threatening.

Applications of Bertrand Competition

The game clearly illustrates why price wars may be destructive. Rao, Bergen, and Davis (2000) discussed how best to survive a price war. Essentially, they argued that price wars with perfect substitutes should be avoided and that smart firms will find nonprice methods to compete or find ways to differentiate their

FIGURE 2
A Bertrand Game with Perfect Substitutes

If two firms produce the same good then if one has the lower price, it takes the whole market. If they charge the same price, they split the market. We use our familiar demand curve: $P = 12 – Q$ or $Q = 12 – P$. Revenue for the firm with the lower price is $P_1(12 – P_1)$, whereas revenue if the prices are tied is $.5P_1(12 – P_1)$. The firm with the higher price disposes of unsold inventory at a cost of 0.1, regardless of inventory volume. Form groups of two and play the game. Write down your choice and cover it with your hand. Once the other player has made his or her choice, simultaneously reveal your decisions and record your profit. You are welcome to talk. The form to record play is the same as in Figure 1.

My Firm's Profit Table

My price choice	Playing partner's price choice								
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	5.5	11	11	11	11	11	11	11
2	0	–0.1	10	20	20	20	20	20	20
3	0	–0.1	–0.1	13.5	27	27	27	27	27
4	0	–0.1	–0.1	–0.1	16	32	32	32	32
5	0	–0.1	–0.1	–0.1	–0.1	17.5	35	35	35
6	0	–0.1	–0.1	–0.1	–0.1	–0.1	18	36	36
7	0	–0.1	–0.1	–0.1	–0.1	–0.1	–0.1	17.5	35
8	0	–0.1	–0.1	–0.1	–0.1	–0.1	–0.1	–0.1	16

product. However, if a firm is caught in a price war, they advise that the firm reveal its strategic intentions. For example, when Food Lion cut prices in North Carolina, the Big Star market announced it would meet or beat Food Lion's prices. Winn-Dixie made a similar announcement, and Food Lion raised prices. If firms understand that retaliation is imminent, the incentive to cut prices is reduced (Rao, Bergen, and Davis 2000, 109).[9] As another example, given severe price cutting in Asia, the manager of the Ritz-Carlton in Malaysia chose to compete by meeting travelers at the plane, with "music, mimosas, discount coupons, and a model room" (Rao, Bergen, and Davis 2000, 110). The manager's preservation of quality in a difficult time helped preserve the long-term reputation of the hotel.

Price Competition, Perfect Complements, and Microsoft

The Ritz-Carlton example points out the advantages of product differentiation and the need to expand the game repertoire. Some insight may be gained into the effects of product differentiation by making the opposite assumption that goods are perfect complements. For most consumers, Windows 98 and Office 2000 are perfect complements. It does not matter how the purchase price of software is divided between the two software bundles because both are necessary. Therefore, the demand equation becomes $Q = 12 – (P_1 + P_2)$. The sum of the prices, not the individual prices, matters. Continuing to assume zero cost, the profit for firm one is $P_1[12 – (P_1 + P_2)]$. Recall that profit for the quantity-choice game in Figure 1 is $Q_1[12 – (Q_1 + Q_2)]$. The two functions are the same except that price and quantity are reversed. Therefore, the table for quantity competition and perfect substitutes is the table for price competition and perfect complements! The immediate result is that, as the firms are separated, each will have the incentive to play noncooperatively and raise prices. This example is particularly important given recent attempts by the U.S. Department of Justice to break Microsoft into two firms, one that produces the operating system and another the office software.

Although the math is simple, it is worthwhile providing some intuition. Assume the two firms operate as a shared monopoly with perfect substitutes and quantity competition. If one firm increases the quantity, the other firm will see its revenue fall as the price falls. A monopolist would take this effect into account, but the separate firms respond only to their own revenue and ignore effects on the other firm. Each has the incentive to expand beyond the monopoly output. Similarly, starting from a shared monopoly solution given price competition and perfect complements, if one firm raises its price, the revenue of the other firm falls as fewer bundles of output are sold. A monopoly takes the effect into account, but the separate firms do not and each raises price beyond the monopoly level.

Paul Krugman provides the following colorful presentation of this situation in a *New York Times* column (April 26, 2000):

> Baron Wilhelm von Gates was the lord of two castles, each commanding a strategic bottleneck along the Rhine. From these castles he was able to demand money from all the travelers who passed by. This made him wealthy, but also much disliked. Eventually the Holy Roman Emperor was persuaded to curb the robber baron's power; he split up the Gates domain, giving one of the castles to the baron's nephew.
>
> But the results of this breakup were not quite what the emperor's legal department had promised. In fact, travelers complained that things had gotten even worse. Not only did they now face the nuisance of dealing with two different robber barons, but they said they were paying more for each trip than they had before.

The Justice Department recently seems to have abandoned hopes of breaking Microsoft into two separate companies. Whether well-publicized criticisms that the remedy would not have the desired effect influenced their decision is unknown. If the Justice Department had succeeded in breaking up Microsoft into one company that produced operating systems and another that produced application software, then each would have the incentive to raise prices. The current monopoly resists further price increases because it will reduce sales. Given separate firms, each would worry only about their own firm's lost sales, not the other firm's, and each would have the incentive to raise prices a bit more. The data in Figure 2 make the arithmetic clear. At the shared monopoly price of \$3 for each bundle, 6 units are sold: $Q = \$12 – (\$3 + \$3) = 6$. Profits then are $\$3 \times 6 = \$18$, and the best response is to charge \$5. Then $Q = \$12 – (\$3 + \$5) = 4$, and profit is $4 \times \$5 = \$20$. It is even possible the two firms will vie for Stackelberg leadership, charge prices of $6 each, and drive profit to zero. Here, both the firms and customers lose. It is not clear why the Department of Justice was interested in creating two firms producing complements given the likely response.[10]

Quantity Competition, Perfect Complements, and Microsoft

The previous examples underscored the important distinction between price competition and quantity competition given substitute products. I should also consider what would happen if the two firms producing complements compete over quantities and not prices. The relevant data are in Figure 3. The key feature is that the firm with the lowest output determines the sales volume because no one will buy operating systems without software or software without operating systems. The small disposal cost of $0.10 introduced in Figure 2 and continued in Figure 3 ensures that no strategy is dominant.[11] I still assume zero production cost so that producing more than the other firm produces does not reduce profits.[12]

After eliminating dominated strategies, any symmetric solution is a Nash equilibrium. However, the shared monopoly is a reasonably safe bet. For example, assume a player knows that the other player will pick 5 or 6 but is uncertain about the relative frequency of 5 or 6. Let α represent the probability the other player will pick 5 and $(1 - \alpha)$ the probability he or she will pick 6. Then picking 6 produces the higher payoff if $17.4\alpha + 18(1 - \alpha) > 17.5\alpha + 17.5(1 - \alpha)$.[13] The inequality holds if $\alpha < .883$. As long as the other player is expected to select 5

FIGURE 3
A Quantity Choice Game with Perfect Complements

Consumers need one of each. Therefore, whichever firm produces the least determines sales. The price of the combined bundle is given by: $P = 12 - \min(Q_1,Q_2)$. For simplicity, we assume each piece sells for half the combined price, then profit is $P \times \min(Q_1,Q_2)$. The firm with the higher price disposes of unsold inventory at a cost of 0.1, regardless of inventory volume. The form to record play is the same as in Figure 1.

My quantity choice	Playing partner's quantity choice									
		0	1	2	3	4	5	6	7	8
	0	0	0	0	0	0	0	0	0	0
	1	0	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5
	2	0	5.4	10	10	10	10	10	10	10
	3	0	5.4	9.9	13.5	13.5	13.5	13.5	13.5	13.5
	4	0	5.4	9.9	13.4	16	16	16	16	16
	5	0	5.4	9.9	13.4	15.9	17.5	17.5	17.5	17.5
	6	0	5.4	9.9	13.4	15.9	17.4	18	18	18
	7	0	5.4	9.9	13.4	15.9	17.4	17.9	17.5	17.5
	8	0	5.4	9.9	13.4	15.9	17.4	17.9	17.4	16

less than 83 percent of the time, it is best to pick 6. Not surprisingly, the shared monopoly dominates play.

If Microsoft is able to avoid price competition and compete over quantities instead, the monopoly behavior may well survive the breakup. If the shared monopoly does not survive, the firms will reduce output. This would be more likely the higher the disposal cost. Once again, consumers lose. Under either quantity or price competition, breaking up Microsoft to create complements will not benefit consumers.

CONCLUSION

I have used a spreadsheet program to construct profit tables under different circumstances. Students then interact given the profits created. The games created are fun to play and introduce a wide variety of game concepts. The games are readily applied to international trade, antitrust, and price wars. It will take more than one class to introduce and discuss the games. However, in my experience, the time is well spent. For example, it is simple to use the tables to draw traditional reaction functions by graphing best responses. In the case of a tie, use the average of tied values. The set of three figures sumarizes what has been learned in a way that is easy to remember and now has deep meaning for students.

NOTES

1. Portraits and biographical sketches of Cournot and Bertrand may be found at http://cepa.newschool.edu/het/.
2. The games are inspired by experiments conducted by Charles Holt (1985).
3. A reviewer pointed out that designating one player as leader could increase the frequency of leadership.
4. The profit from selecting 4 or 5 is 20. The tie emerges because the true best response is 4.5. Picking the higher number produces examples more consistent with theory.
5. The subgame consisting of shared monopoly and Cournot is a prisoner's dilemma because each member of the shared monopoly has the incentive to play noncooperatively, but if both defect, both lose.
6. The math, for the example, is not too difficult; let $P = A - B(Q_1 + Q_2)$. For a monopoly, marginal revenue is $A - 2BQ$, and, given zero cost, profit maximizing output is $A/(2B)$. The marginal revenue of the Cournot firm is $P + P'Q_1 = A - B(Q_1 + Q_2) - BQ_1$. I can find the reaction function by setting this equal to zero and solving for Q_1. The function is: $Q_1 = A/(2B) - (Q_2)/2$. The profit of the Stackelberg leader is PQ_2 assuming firm one is the follower. Then the leader's profit is $\{A - B[A/(2B) - (Q_2)/2 + Q_2]Q_2\}$. Setting the derivative with respect to Q_2 equal to zero, I find $Q_2 = A/(2B)$, which is, of course, the monopoly solution. Therefore, for linear demand, the leader's best strategy is the monopoly output choice.
7. A sketch of the WTO's rules about subsidies may be found at http://www.wto.org/english/thewto_e/whatis_e/tif_e/agrm7_e.htm. The appellate body report on foreign sales corporations is at http://www.wto.org/english/tratop_e/dispu_e/621d.pdf. The official U.S. position on foreign sales corporations is available at http://www.ustr.gov/releases/2000/02/00-13.pdf.
8. For the first time, there may be unsold inventory, and the table introduces a small disposal cost. This cost will play a role in the next game.
9. This can be illustrated easily. Have players consider their strategy if they know their price will be matched. Then only the payments in the diagonal matter, and the largest is at the shared monopoly with prices of 6 and output of 3 each.
10. Expert witnesses focused on the tendency of Microsoft to use both the operating system and software monopolies to restrict entry. Presumably the hope is that a software company and an operating system company will not present as formidable a barrier to entry. The expert testimony is available at the Department of Justice's Web site: from the main index at http://www.usdoj.

gov/atr/cases/ms_index.htm, click on the remedies subsection and then on the name of the expert witness. The most likely objection to creating two Microsoft companies producing substitute software products is that the standardization Microsoft provides is valuable. However, standards are usually thought to be the proper role of government.

11. Without the disposal cost, all strategies other than picking 6, are weakly dominated. In a more complex and realistic example with positive production costs, the domination would not exist. The disposal cost is a device that maintains the simplicity of the exercise while remaining true to the underlying theory.
12. The assumption that Microsoft may expand output at no cost is not all that unrealistic. The added cost of software is essentially the cost of the box and the CD, whereas the purchase price is often over a hundred dollars. Similarly, disposal costs are low.
13. The equation simply multiplies probabilities times payoffs for different circumstances. The left side of the equation is the expected payoff from selecting 6. It is the probability that the other player picks 5 times the payoff in column 5, row 6, plus the probability the other player picks 6 and the payoff in column 6, row 6. The right side is the expected payoff from selecting 5.

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