Update on Determination of Fragmentation Functions

Shunzo Kumano
High Energy Accelerator Research Organization (KEK)
J-PARC Center (J-PARC)
Graduate University for Advanced Studies (GUAS)
http://research.kek.jp/people/kumanos/

Collaborators: M. Hirai, H. Kawamura, K. Saito

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**Fragmentation Functions**

Fragmentation function is defined by

\[ F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} \]

\[ \sigma_{tot} = \text{total hadronic cross section} \]

A fragmentation process occurs from quarks, antiquarks, and gluons, so that \( F^h \) is expressed by their individual contributions:

\[ F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i \left( \frac{z}{y}, Q^2 \right) D^h_i(y,Q^2) \]

**Variable \( z \)**
- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

\[ z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2 \]

**Non-perturbative**
(determined from experiments)

**Calculated in perturbative QCD**

\[ C_i(z,Q^2) = \text{coefficient function} \]

\[ D^h_i(z,Q^2) = \text{fragmentation function of hadron } h \text{ from a parton } i \]
Momentum (energy) sum rule

\[ D_i^h (z, Q^2) = \text{probability to find the hadron } h \text{ from a parton } i \]

with the energy fraction \( z \)

Energy conservation: \( \sum_h \int_0^1 d z \, D_i^h (z, Q^2) = 1 \)

\[ h = \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, p, \bar{p}, n, \bar{n}, \ldots \]

Favored and disfavored fragmentation functions

Simple quark model: \( \pi^+ (u\bar{d}), K^+ (u\bar{s}), p(uu\bar{d}), \ldots \)

**Favored** fragmentation: \( D_{u}^{\pi^+}, D_{\bar{d}}^{\pi^+}, \ldots \)

(from a quark which exists in a naive quark model)

**Disfavored** fragmentation: \( D_{d}^{\pi^+}, D_{\bar{u}}^{\pi^+}, D_{s}^{\pi^+}, \ldots \)

(from a quark which does not exist in a naive quark model)
Semi-inclusive reactions have been used for investigating

- **origin of proton spin** \( \bar{e} + \bar{p} \rightarrow e' + h + X, \ \bar{p} + \bar{p} \rightarrow h + X \) (RHIC-Spin)

  Quark, antiquark, and gluon contributions to proton spin
  (flavor separation, gluon polarization)

- **properties of quark-hadron matters** \( A + A' \rightarrow h + X \) (RHIC, LHC)

  Nuclear medium effects (energy loss, …)

- **exotic-hadron search**

\[
\sigma = \sum_{a,b,c} f_a(x_a,Q^2) \otimes f_b(x_b,Q^2) \otimes \hat{\sigma}(ab \rightarrow cX) \otimes D^\pi_c(z,Q^2)
\]
Comparison with various parametrizations in pion

Disfavored-quark and gluon fragmentation functions have large uncertainties.

(KKP) Kniehl, Kramer, Pötter

(AKK) Albino, Kniehl, Kramer

(HKNS) Hirai, Kumano, Nagai, Sudoh

(DSS) De Florian, Sassot, Stratmann
Exotic-hadron search by fragmentation functions

“Favored” and “disfavored” (unfavored) fragmentation functions Possibility of finding exotic hadrons in high-energy processes

Possibilities for $f_0(980)$: $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $s\bar{s}$, $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$, $K\bar{K}$, or $gg$

e.g. if $f_0(980) = s\bar{s}$: favored $s$, $\bar{s} \rightarrow f_0$; disfavored $u$, $d$, $\bar{u}$, $\bar{d} \rightarrow f_0$, ...

Pion case

Belle analysis is possible for $f_0$ in principle.

$M_{2nd} = \int_0^1 dz \, z \, D^\pi_i(z)$

2nd moments of

There are distinct differences between the favored and disfavored 2nd moments. → It could be used for exotic-hadron studies.

Global analysis of fragmentation functions

Refs. M. Hirai, SK, T.-H. Nagai, K. Sudoh
M. Hirai, H. Kawamura, S. Kumano, K. Saito,
research in progress.

Code for calculating the fragmentation functions is available at
**Determination of fragmentation functions**

- At the Z-pole (LEP/SLD)
  \[
  F^h(z, M^2_Z) \approx (c_v^{u2} + c_A^{u2}) \left[ D^{h}_{u^+}(z, M^2_Z) + D^{h}_{c^+}(z, M^2_Z) \right] \\
  + (c_v^{d2} + c_A^{d2}) \left[ D^{h}_{d^+}(z, M^2_Z) + D^{h}_{s^+}(z, M^2_Z) + D^{h}_{b^+}(z, M^2_Z) \right] \\
  \approx 0.33 \sum_q D^{h}_{q^+}(z, M^2_Z) \\
  D^{h}_{q^+} \equiv D^{h}_q + D^{h}_{ar{q}} \\
  \text{flavor singlet combination}
  \]

- Far from the Z-pole (Belle, TASSO/TPC/HRC/TOPAZ)
  \[
  F^h(z, Q^2) \approx \frac{4}{9} \left[ D^{h}_{u^+}(z, Q^2) + D^{h}_{c^+}(z, M^2_Z) \right] + \frac{1}{9} \left[ D^{h}_{d^+}(z, Q^2) + D^{h}_{s^+}(z, Q^2) + D^{h}_{b^+}(z, Q^2) \right]
  \]

1. c-quark, b-quark FFs are determined from the flavor tagged data.
2. If we have very precise data at and far from the Z-pole, we can determine 2 independent components of the quark FFs.
3. Remaining flavor decomposition & determination of the gluon FF come from the mixing through scale evolution.
New development for an update: precise Belle measurements

$D_i^h(z, Q^2)$

Scale evolution of $D_i^h$
→ gluon fragmentation function

Scale evolution of $F_2$
→ gluon distribution

Initial functions for pion

Note: constituent-quark composition $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$

$$D^{\pi^+}_u(z,Q_0^2) = N^\pi_u z^{\alpha^\pi_u} (1-z)^{\beta^\pi_u} = D^\pi_d (z,Q_0^2)$$

$$D^{\pi^+}_{\bar{u}}(z,Q_0^2) = N^{\pi}_{\bar{u}} z^{\alpha^\pi_{\bar{u}}} (1-z)^{\beta^\pi_{\bar{u}}} = D^\pi_{\bar{d}} (z,Q_0^2) = D^\pi_s (z,Q_0^2) = D^\pi_{\bar{s}} (z,Q_0^2)$$

$$D^{\pi^+}_c(z,m_c^2) = N^\pi_c z^{\alpha^\pi_c} (1-z)^{\beta^\pi_c} = D^\pi_{\bar{c}} (z,m_c^2)$$

$$D^{\pi^+}_b(z,m_b^2) = N^\pi_b z^{\alpha^\pi_b} (1-z)^{\beta^\pi_b} = D^\pi_{\bar{b}} (z,m_b^2)$$

$$D^{\pi^+}_g(z,Q_0^2) = N^\pi_g z^{\alpha^\pi_g} (1-z)^{\beta^\pi_g}$$

Constraint: 2nd moment should be finite and less than 1

$$N = \frac{M}{B(\alpha + 2, \beta + 1)}$$

$$M \equiv \int_0^1 zD(z)dz$$

(2nd moment), $B(\alpha + 2, \beta + 1) = \text{beta function}$

$$0 < M^h_i < 1 \quad \text{because of the sum rule } \sum_h M^h_i = 1$$
At the stage of 2012
(with only preliminary Belle data)

Note: The preliminary Belle data were obtained by personal communications, so that they should not be used after the Belle final data are published in PRL.
Comparison with pion data

\[ F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} \]

Orninate: \[ \frac{F^\pi(data) - F^\pi(theory)}{F^\pi(theory)} \]

Good agreement with the Belle data

as well as the others like SLD, LEP data

Note: Preliminary Belle data of 2012 (≠ PRL 111 (2013) 062002) are used in this presentation.
Significant improvement due to the Belle measurements

Impact of the Belle data for pion

without Belle data

with Belle data

Leading order (LO)

Next-to-leading order (NLO)
Impact of the Belle data

2nd moment: $M = \int_0^1 dz D_i^h(z, Q^2)$

- without Belle data
- with Belle data

Significant improvement due to the Belle measurements

Leading order (LO)

Next-to-leading order (NLO)
Impact of the Belle data for kaon

Significant improvement due to the Belle measurements

without Belle data

with Belle data

Leading order (LO)

Next-to-leading order (NLO)
So far so good, but …

At the stage of 2013

(with final Belle and BaBar data)
Comparison of various data with theory

\[ \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} \]

**Issue:** It seems that the Belle and BaBar data are not consistent with the HKNS07 (and possibly with other parametrizations) which are determined mainly by SLD and LEP measurements.
Comparison of Belle / BaBar data with theory

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} = M_z = 10.54 \text{ GeV}
\]

Belle and BaBar data are not consistent with each other and with the other data (essentially SLD and LEP), and their errors are tiny.

It seems that a real global analysis is not possible.

Question: Why BaBar > Belle ?!
   Belle: ISR
   BaBar: Lifetime cut (\(\tau < 3 \cdot 10^{-11} \text{ sec}\)
How to use the Belle 2013-final data tables in our global analysis

Total fragmentation function: \( F = \int \frac{d\sigma^h}{dz} \)

Belle data table: \( \left( \frac{d\sigma^h}{dz} \right)_{E_{ISR}<0.5\% \cdot \sqrt{s}/2} = \frac{d\sigma^h}{dz} \cdot c \)

\( c = \) normalization correction factor due to the kinematical cut \( E_{ISR} < 0.5\% \cdot \sqrt{s} / 2 \).

Total fragmentation function \( F \)

\[
F_{\text{final Belle data}} = \frac{1}{(\sigma_{\text{total}})_{E_{ISR}<0.5\% \cdot \sqrt{s}/2}} \left( \frac{d\sigma^h}{dz} \right)_{E_{ISR}<0.5\% \cdot \sqrt{s}/2}
\]

\[
= \frac{1}{c \cdot (\sigma_{\text{total}})_{\text{no cut for } E_{ISR}}} \cdot c \cdot \left( \frac{d\sigma^h}{dz} \right)_{\text{no cut for } E_{ISR}}
\]

\[
= \frac{1}{c \cdot (\sigma_{\text{total}})_{\text{no cut for } E_{ISR}}} \left( \frac{d\sigma^h}{dz} \right)_{\text{final Belle data}}
\]

\[
= \frac{1}{\sigma_{\text{total}}} \frac{d\sigma^h}{dz}, \quad \text{if } \frac{1}{(\sigma_{\text{total}})_{\text{no cut for } E_{ISR}}} \left( \frac{d\sigma^h}{dz} \right)_{\text{no cut for } E_{ISR}} = \frac{1}{\sigma_{\text{total}}} \frac{d\sigma^h}{dz}
\]

\[
\left( \frac{1}{\sigma_{\text{total}}} \frac{d\sigma^h}{dz} \right)_{\text{Theory}} \Leftrightarrow c \cdot (\sigma_{\text{total}})_{\text{no cut for } E_{ISR}} \left( \frac{d\sigma^h}{dz} \right)_{\text{final Belle data}} = \frac{1}{c \cdot \sigma_{\text{total}}} \left( \frac{d\sigma^h}{dz} \right)_{\text{final Belle data}}
\]

\( c = 0.65 \) (Monte Carlo simulation, Belle paper)
**Preliminary results on pion**

**Analysis with Belle data**

\[ \chi^2_{\text{tot}} = 782 \text{ for 342 data, } \chi^2 / \text{dof} = 2.4 \]
\[ \chi^2(\text{Belle}) = 117 \text{ for 78 data} \]

**Analysis with Belle and BaBar data**

\[ \chi^2_{\text{tot}} = 1076 \text{ for 377 data, } \chi^2 / \text{dof} = 3.0 \]
\[ \chi^2(\text{Belle}) = 74 \text{ for 78 data} \]
\[ \chi^2(\text{BaBar}) = 185 \text{ for 35 data} \]

**Issues**
- Belle and BaBar data are not very consistent with other data (e.g. SLD, LEP data) + DGLAP evolution.
- Belle and BaBar data are not consistent with each other.
Preliminary results on kaon

Analysis with Belle data

\[ \chi^2_{\text{tot}} = 324 \text{ for } 323 \text{ data}, \quad \chi^2 / \text{dof} = 1.1 \]
\[ \chi^2 (\text{Belle}) = 4.6 \text{ for } 78 \text{ data} \]

Issues
- Belle data are consistent with other data (e.g. SLD, LEP data) + DGLAP evolution.
- BaBar data are not very consistent with other data.

Analysis with Belle and BaBar data

\[ \chi^2_{\text{tot}} = 1357 \text{ for } 368 \text{ data}, \quad \chi^2 / \text{dof} = 3.9 \]
\[ \chi^2 (\text{Belle}) = 86 \text{ for } 78 \text{ data} \]
\[ \chi^2 (\text{BaBar}) = 717 \text{ for } 45 \text{ data} \]
Issues and Comments

- Kinematical cuts of Belle and BaBar data: ISR, lifetime of final hadrons
  ⇒ Why Belle < BaBar?
- The other data are not consistent with Belle-pion, but they are consistent with Belle-kaon.
  ⇒ May not be an ISR correction issue of Belle.
- $z$-dependent functions are similar in kaon between Belle and BaBar
  ⇒ The issue could be simply the normalization??
- Belle and BaBar data are not consistent with other data (+DGLAP evolution).
Summary

Precise new data by Belle and BaBar

Wide range of $z$ (0.05 < $z$ < 0.97), Far from $Z$ mass (E~10 GeV)

Global analyses with the Belle and BaBar data

Significant improvements are expected on the determination of fragmentation functions.

Particularly, uncertainties of gluon and light-quark fragmentation functions should be reduced significantly in principle.

Comments:

• Our studies are still preliminary.
• It seems that the Belle (pion) and BaBar data are not consistent with current parametrization (namely with other data + DGLAP evolution).
• Due to tiny errors of Belle and BaBar, a global analysis is difficult.

→ Solving these issues, we should be able to obtain accurate fragmentation functions, which becomes one of major achievements of the Belle and BaBar experiments.
The End