Structure and Geometry of Black Holes and the Universe

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Supported in part by NSF, ONR and Chinese NSF

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25th Midwest Relativity Meeting, 2015         Northwestern University
I. Blackhole Theorem

Blackhole Theorem (Ma-Wang, 2014). Assume the validity of the Einstein theory of general relativity, then the following assertions hold true:

1. black holes are closed: matters can neither enter nor leave their interiors;

2. black holes are innate: they are neither born to explosion of cosmic objects, nor born to gravitational collapsing; and

3. black holes are filled and incompressible, and if the matter field is non-homogeneously distributed in a black hole, then there must be sub-blackholes in the interior of the black hole.
Key observations:

- The singularity at the Schwarzschild radius \( R_s = \frac{2MG}{c^2} \) is **physical**:

\[
(1) \quad ds^2 = -\left[1 - \frac{R_s}{r}\right]c^2 dt^2 + \left[1 - \frac{R_s}{r}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2.
\]

At \( r = R_s \), time freezes, and there is no motion.

The **Eddington-Findkelstein** coordinate transformation

\[
r^* = r + \frac{2GM}{c^2} \ln \left| \frac{r}{R_s} - 1 \right|.
\]

is **not differentiable**, and is **not allowed** both physically and mathematically.

The **same problem occurs for all other coordinate transformations.**
The geometrical realization of a black hole is dictated by the Schwarzschild and the TOV metrics, and shows also that nothing will get in the black hole:

\[ ds^2 = \left[ 1 - \frac{R_s}{r} \right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad \text{for } r > R_s, \]

\[ ds^2 = \left( 1 - \frac{r^2}{R_s^2} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad \text{for } r < R_s. \]
• Take for example the supermassive black-hole at the center of our galaxy, the Milky Way. By the classical theory, this black-hole would continuously gobble matters nearby. Then the radius $R_s$ would increase in cubic rate. Then it would not hard to see the black hole will consume the entire Milky Way, and eventually the entire Universe.

• A rigorous proof is also given in terms of energy conservation law by coupling the gravitational field equations and the fluid dynamic equations through a symmetry-breaking mechanism.
Supernovae Explosion and AGN Jets. Consider e.g. an active galactic nucleus (AGN), which occupies a spherical annular shell region $R_s < r < R_1$, where $R_s$ is the Schwarzschild radius of the black hole core of the galaxy.

The relativistic effect is then reflected in the radial force, gives rise to a huge explosive force near $r = R_s$:

$$\frac{\nu}{1 - R_s/r} \frac{R_s^2}{r^4} P_r.$$ 

The basic mechanism of the formation of AGN jets is that the radial temperature gradient causes vertical Bénard convection cells. Each Bénard convection cell has a vertical exit, where the circulating gas is pushed by the radial force, and then erupts leading to a jet. Each Bénard convection cell is also an entrance, where the external gas is attracted into the nucleus, is cycloaccelerated by the radial force as well, goes down to the interior boundary $r = R_s$, and then is pushed toward to the exit. Thus the circulation cells form jets in their exits and accretions in their entrances.

This mechanism can also be applied to supernovae explosion.
IV. Structure of the Universe

Thm (Ma-Wang, 2014). Assume the Einstein theory of general relativity, and the cosmological principle, then the following assertions hold true:

1. our Universe is not originated from a Big-Bang, and is static;

2. the topological structure of our Universe is the 3D sphere $S^3$ such that to each observer, the corresponding equator with the observer at the center of the hemisphere can be viewed as the black hole horizon;

3. the total mass $M_{total} = \frac{3\pi M}{2}$ in the Universe includes both the cosmic observable mass $M$ and the non-observable mass, regarded as dark matter, due to the space curvature energy; and

4. a negative pressure is present in our Universe to balance the gravitational attracting force, and is due to the gravitational repelling force, also called dark energy.
**Redshift problem.** The natural and important question that one has to answer is the consistency with astronomical observations, including the cosmic edge, the flatness, the horizon, the redshift, and the cosmic microwave background (CMB) problems. These problems can now be easily understood based on the structure of the Universe and the blackhole theorem we derived.

There are three sources of redshifts: the *Doppler effect*, the *cosmological redshift*, and the *gravitational redshift*. Due to black hole properties of our Universe, the black hole and cosmological redshifts cannot be ignored. Due to the horizon of the sphere, for an arbitrary point in the spherical Universe, its opposite hemisphere relative to the point is regarded as a black hole. Hence, we derive the following redshift formula, which is consistent with the observed redshifts:

\[
1 + z = \frac{1}{\sqrt{\alpha(r)(1 - \frac{R_s}{r})}} = \frac{\sqrt{2R_s - r}}{\sqrt{\alpha(r)(R_s - r)}} \quad \text{for } 0 < r < R_s.
\]
**CMB problem.** Based on the unique scenario of our Universe we derived, it is the most natural thing that there exists a CMB, because the Universe has always been there as a black-body, and CMB is a result of blackbody equilibrium radiation.

**Some Observations:**

- If our Universe were born to the Big-Bang, assuming at the initial stage, all energy is concentrated in a ball with radius $R_0 < R_s$, by the theory of black holes, then the energy contained in $B_{R_0}$ must generate a black hole in $\mathbb{R}^3$ with fixed radius $R_s = 2MG/c^2$.

- If we assume that at certain stage, the Universe were contained in ball of a radius $R$ with $R_0 < R < R_s$, then we can prove that the Universe must contain a sub-black hole with radius $r$ given by

$$r = \sqrt{\frac{R}{R_s}}R.$$
Based on this property, the expansion of the Universe, with increasing $R$ to $R_s$, will give rise to an infinite sequence of black holes with one embedded to another. Apparently, this scenario is clearly against the observations of our Universe, and demonstrates that our Universe cannot be originated from a Big-Bang.

- By the cosmological principle, given the energy density $\rho_0 > 0$ of the Universe, based on the Schwarzschild radius, the Universe will always be bounded in black hole, which is an open ball of radius:

\[
R_s = \sqrt{\frac{3c^2}{8\pi G \rho_0}}.
\]
V. PID-Cosmological Model (Ma-Wang, 2015)

New Gravitational Field Equations (Ma-Wang, 2012):

The presence of dark matter and dark energy implies that the energy-momentum tensor of visible matter $T_{\mu\nu}$ may no longer be conserved: $\nabla^{\mu}T_{\mu\nu} \neq 0$.

Then the variation of $L_{EH}$ must be taken under div-free constraints representing energy-momentum conservation:

$$\frac{d}{d\lambda}\bigg|_{\lambda=0} L_{EH}(g_{\mu\nu} + \lambda X_{\mu\nu}) = 0 \quad \forall X = \{X_{\mu\nu}\} \text{ with } \nabla^{\mu}X_{\mu\nu} = 0,$$

which leads to a new of gravitational field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_{\mu}\Phi_{\nu},$$

$$\nabla^{\mu} \left[ \frac{8\pi G}{c^4} T_{\mu\nu} + \nabla_{\mu}\Phi_{\nu} \right] = 0.$$
Law of gravity (Ma-Wang, 2012)

1. (Einstein’s PE). The space-time is a 4D Riemannian manifold \( \{ \mathcal{M}, g_{\mu\nu} \} \), with the metric \( \{ g_{\mu\nu} \} \) being the gravitational potential;

2. The Einstein PGR dictates the Einstein-Hilbert action;

3. The gravitational field equations (4) are derived using PID, and determine gravitational potential \( \{ g_{\mu\nu} \} \) and its dual vector field \( \Phi_\mu = \nabla_\mu \phi \);

4. Gravity can display both attractive and repulsive effect, caused by the duality between the attracting gravitational field \( \{ g_{\mu\nu} \} \) and the repulsive dual vector field \( \{ \Phi_\mu \} \), together with their nonlinear interactions governed by the field equations (4).

5. It is the nonlinear interaction between \( \{ g_{\mu\nu} \} \) and the dual field \( \Phi_\mu \) that leads to a unified theory of dark matter and dark energy phenomena.
Metric of a homogeneous spherical universe:

\[ ds^2 = -c^2 dt^2 + R^2 \left[ \frac{dr^2}{1 - r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right], \]

where \( R = R(t) \) is the cosmic radius. By (4), \( T_{\mu\nu} = \text{diag}(c^2 \rho, g_{11} p, g_{22} p, g_{33} p) \), and with \( \varphi = \phi'' \), we have

\[ R'' = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} + \frac{\varphi}{8\pi G} \right) R, \]

\[ (R')^2 = \frac{1}{3} (8\pi G \rho + \varphi) R^2 - c^2, \]

\[ \varphi' + \frac{3R'}{R} \varphi = -\frac{24\pi G}{c^2} \frac{R'}{R} p. \]
The model describing the static Universe is in the form (Ma-Wang, 2015):

\[ \varphi = -8\pi G \left( \rho + \frac{3p}{c^2} \right), \]

(7)

\[ p = -\frac{c^4}{8\pi GR^2}, \]

\[ p = f(\rho, \varphi). \]

The negative pressure contains two parts:

\[ p = -\frac{1}{3} \rho c^2 - \frac{c^2}{24\pi G} \varphi = \text{observable energy} + \text{dark energy}. \]

(8)

The CMB and the WMAP measurements manifest that the cosmic radius \( R \) is greater than the blackhole radius of the normal energy. The deficient energy is compensated by the dual gravitational potential, i.e. by the second term of (8).
VI. Summary


- Inspired by the Albert Einstein’s vision, our general view of Nature is as follows:

  *Nature speaks the language of Mathematics: the laws of Nature are represented by mathematical equations, are dictated by a few fundamental principles, and take the simplest and aesthetic forms.*

- We have derived experimentally verifiable laws of Nature based only on a few fundamental first principles, *guided by experimental and observation evidences.*

- The results are established with solid mathematical foundation, are in agreement with the existing observations and experiments we are aware of, solve a number of longstanding challenges, and offer new predictions.
We have discovered three fundamental principles:

- the principle of interaction dynamics (PID), (MA-Wang, 2012),
- the principle of representation invariance (PRI) (Ma-Wang, 2012), and

PID takes the variation of the action under the energy-momentum conservation constraints, and is required by the dark matter and dark energy phenomena (for gravity), by the quark confinements (for strong interaction), and by the Higgs field (for the weak interaction).

For example, the following gravitational field equations (Ma-Wang, 2012) is due to PID:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu. \]
• PRI requires that the gauge theory be independent of the choices of the representation generators. These representation generators play the same role as coordinates, and in this sense, PRI is a coordinate-free invariance/covariance, reminiscent of the Einstein principle of general relativity.

PRI is purely a logic requirement for the gauge theory.

• PSB offers an entirely different route of unification from the Einstein unification route which uses large symmetry group. The three sets of symmetries — the general relativistic invariance, the Lorentz and gauge invariances, as well as the Galileo invariance — are mutually independent and dictate in part the physical laws in different levels of Nature.

For a system coupling different levels of physical laws, part of these symmetries must be broken.
These three new principles have profound physical consequences, and, in particular, provide a new route of unification for the four interactions:

1) the general relativity and the gauge symmetries dictate the Lagrangian;
2) the coupling of the four interactions is achieved through PID and PRI in the field equations, which obey the PGR and PRI, but break spontaneously the gauge symmetry;
3) the unified field model can be easily decoupled to study individual interaction, when the other interactions are negligible; and
4) the unified field model coupling the matter fields using PSB.

The new theory gives rise to solutions and explanations to a number of longstanding mysteries in modern theoretical physics, including e.g. 1) dark matter and dark energy phenomena, 2) the structure of black holes, 3) the structure and origin of our Universe, 4) the unified field theory, 5) quark confinement, and 6) mechanism of supernovae explosion and active galactic nucleus jets.