DUALITY THEORY OF STRONG INTERACTION

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Abstract. The main objective of this article is to explore the duality of strong interaction derived from an $SU(3)$ gauge theory based on two basic principles: the principle of interaction dynamics (PID) and principle of representation invariance (PRI). Intuitively, PID takes the variation of the action functional under energy-momentum conservation constraint, and PRI requires that physical laws be independent of representations of the gauge group. The new $SU(3)$ gauge field equations establish a natural duality between strong gauge fields $\{S^k_\mu\}$, representing the eight gluons, and eight bosonic scalar fields.

With the duality, we derive three levels of strong interaction potentials: the quark potential Φ_q, the nucleon/hadron potential Φ_n and the atom/molecule potential Φ_a. These potentials clearly demonstrates many features of strong interaction consistent with observations. In particular, they provide a clear explanation for both quark confinement and asymptotic freedom. Also, in the nuclear level, the new potential is an improvement of the Yukawa potential. As the distance between two nucleons is increasing, the nuclear force corresponding to the nucleon potential Φ_n behaves as repelling, then attracting, then repelling again and diminishes, consistent with experimental observations.

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1. Introduction

Strong interaction, electromagnetic interaction, weak interaction and gravity are four fundamental interactions of Nature. The main objective of this article is to establish a duality theory for strong interaction based on an $SU(3)$ gauge theory, the principle of interaction dynamics (PID) and the principle of representation invariance (PRI), postulated recently by the authors [14, 15].

Intuitively, PID takes the Lagrangian action with energy-momentum conservation constraint. The original motivation for PID was to derive gravitational field equations taking into account of the presence of dark energy and dark matter [16]. The key point was that due to the presence of dark energy and dark matter, the energy-momentum tensor $\{T_{\mu\nu}\}$ is no longer conserved, and the variation of the classical Einstein-Hilbert functional has to be taken under the divergence-free constraint. Namely, the variational element must be energy-momentum conserved.

With PID at our disposal, we derive in [14, 15] a unified field model. This model leads not only to consistent results with the standard model, but also to many new insights and predictions. One important feature of the unified field model is a natural duality between the interacting fields (g, A, W^a, S^k), corresponding to graviton, photon, intermediate vector bosons $W^{\pm}$ and Z and gluons, and the dual bosonic fields $(\Phi_\mu, \phi^0, \phi^a_w, \phi^k_s)$.

The second principle PRI requires that all $SU(N)$ gauge theories should be invariant under transformations of different representations of the gauge group $SU(N)$. In a sense, this principle has the same spirit as the principle of general relativity, where physical laws should be independent of the coordinate system. For gauge theories in particle physics, it is equally natural to require physical laws be independent of different representations of the underlying Lie group $SU(N)$. It is with PRI that we substantially reduced the large number of to-be-determined parameters in the unified model to two $SU(2)$ and $SU(3)$ constant vectors $\{\alpha^1_\mu\}$ and $\{\alpha^2_k\}$, containing 11 parameters, which represent the portions distributed to the gauge potentials by the weak and strong charges.

The field equations can be decoupled easily to study individual interactions. In this article, the new gauge field equations for strong interaction are derived by applying PID to the standard $SU(3)$ gauge action functional in QCD. The new model leads to consistent results as the classical QCD, and, more importantly, to a number of new results and predictions.

First, this model gives rise to a natural duality between the $SU(3)$ gauge fields S^k_μ $(k = 1, \cdots, 8)$, representing the gluons, and the dual scalar fields ϕ^k_s.

Second, for the first time, we derive several levels of strong interaction potentials, including the quark potential S_q, the nucleon potential S_n and the atom/molecule potential S_a. They are given as follows:

$$\Phi_q = g_s\left[\frac{1}{r} - \frac{A_q}{\rho}(1+k_0 r)e^{-k_0 r}\right], \tag{1.1}$$

$$\Phi_n = 3g_s\left(\frac{\rho_0}{\rho_n}\right)^3\left[\frac{1}{r} - \frac{A_n}{\rho_n}(1+k_1 r)e^{-k_1 r}\right], \tag{1.2}$$

$$\Phi_a = 3Ng_s\left(\frac{\rho_0}{\rho_a}\right)^3\left[\frac{1}{r} - \frac{A_a}{\rho_a}(1+k_1 r)e^{-k_1 r}\right], \tag{1.3}$$

where g_s is the strong charge[1], A_q, A_n and A_a are dimensionless constants, $r_0 = 1/k_0 = 10^{-16}$cm, $r_1 = 1/k_1 = 10^{-13}$cm, ρ_0 is the effective quark radius, ρ_n is the effective radius of a nucleon, ρ_a is the radius of an atom/molecule, and N is the number of nucleons in an atom/molecule. These potentials match very well with experimental data, and offer a number of physical conclusions. Hereafter we shall explore a few important implications of these potentials.

Third, with these potentials, the binding energy of quarks can be estimated as

$$E_q \sim \left(\frac{\rho_n}{\rho_0}\right)^6 E_n \sim 10^{24}E_n \sim 10^{22}\text{GeV}, \tag{1.4}$$

where $E_n \sim 10^{-2}$GeV is the binding energy of nucleons. Consequently, close to the Planck energy scale is required to break a quark free. Hence these potential formulas offer a clear mechanism for quark confinement.

Fourth, with the quark potential, there is a radius $\bar{r}$, as shown in Figure 6.1, such that two quarks closer than $\bar{r}$ are repelling, and for r near $\bar{r}$, the strong interaction diminishes. Hence this clearly explains asymptotic freedom.

Fifth, in the nucleon level, the new potential is an improvement of the Yukawa potential. The corresponding Yukawa force is always attractive. However, as the distance between two nucleons is increasing, the nucleon force corresponding to the nucleon potential S_n behaves as repelling, then attracting, then repelling again and diminishes. This is exactly the picture that the existing observations tell us.

Sixth, the factor $\left(\frac{\rho_0}{\rho_2}\right)^3$ in (1.3) indicates the short-range nature of the strong interaction, in agreement with observations. In particular, beyond molecular level, strong interaction diminishes.

Before the end of this Introduction, we would like to give a brief historical account of the related problems studied in this article.

1. The use of gauge symmetry and the non-abelian gauge theory for particle physics go back to [12, 29], motivated also by the work of [28]. An important revolution regarding was [25], where the Yang-Mills theory was shown to be renormalizable, even after symmetry breaking.

With the great achievements in field theory and particle physics in the last hundred years, it is clear that the principle of gauge symmetry is one of the few key principles for electromagnetism, weak and strong interactions. It is also one of few first principles we build the field theory in this and the accompanying articles.

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[1]Here g_s is the strong charge using quark as a reference level; see Remark 4.1.

2. The spontaneous symmetry breaking was another revolution in field theory and in particle physics, going back to [18, 20, 21, 1, 4], followed by the 1964 papers on the introduction to the Higgs field [2, 9, 7]. The success of the classical electroweak theory [26, 3, 24], together with the Gell-Mann's quark theory and Greenberg's introduction of color charge, leads to the standard model; see, among many others [10, 23, 5, 11, 8] and the references therein.

As mentioned earlier, our studies are based on two newly postulated principles, called principle of interaction dynamics (PID) and the principle of representation invariance (PRI). Again, PID takes the variation of the Lagrangian action subject to energy-momentum conservation constraints. This new principle gives an entirely different way of introducing Higgs fields, leading to a natural approach for spontaneous symmetry breaking and mass generation.

3. The introduction of PRI is based purely on mathematical logic, and its validity is unquestionable. Hence any physical theory violating PRI may only be considered as an approximation of laws of Nature.

A crucial consequence of PRI is that for the gauge fields A^a_μ and G^k_μ, corresponding to two different gauge groups $SU(N_1)$ and $SU(N_2)$, the following combinations

$$\alpha A^a_\mu + \beta G^k_\mu$$

are prohibited, since this combination violates PRI. This point of view clearly shows that both the classical electroweak theory and the standard model violates PRI. Of course, this violation does not in any way undermine the importance of the classical electroweak theory and the standard model — as the Einstein theories of relativity do not undermine the Newton laws of physics.

4. There are many attempts to physics beyond standard model. One such attempt is to derive a unified theory based on large gauge group such as $SU(5)$. Unfortunately, the apparent violation of PRI for such theories forces us to take a different route, which has the spirit similar to that of the standard model. Namely, a consistent unified field theory should obey the following requirements:

(1) Its Lagrangian action density must satisfy all known basic symmetry principles, including the principle of general relativity, the principle of Lorentz invariance, the principle of gauge invariance, and the principle of representation invariance (PRI);
(2) The Lagrangian action can be naturally decoupled to Lagrangian actions for individual interactions: the Einstein-Hilbert action for gravity, the $U(1)$ gauge field action for electromagnetism, the $SU(2)$ gauge field action for weak interaction and the $SU(3)$ gauge field action for strong interaction.

In fact, the unified field theory developed by the authors [14, 15] is uniquely determined by these two requirement.

Also, our unified field model appears to match Nambu's vision. In fact, in his Nobel lecture [19], Nambu stated that

Einstein used to express dissatisfaction with his famous equation of gravity

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

His point was that, from an aesthetic point of view, the left hand side of the equation which describes the gravitational field is based on a

> *beautiful geometrical principle, whereas the right hand side, which describes everything else, . . . looks arbitrary and ugly.*
>
> *... [today] Since gauge fields are based on a beautiful geometrical principle, one may shift them to the left hand side of Einsteins equation. What is left on the right are the matter fields which act as the source for the gauge fields ... Can one geometrize the matter fields and shift everything to the left?*

Our understanding of his statement is that the left-hand side of the standard model is based on the gauge symmetry principle, and the right-hand side of the standard model involving the Higgs field is artificial. What Nambu presented here is a general view shared by many physicists that the Nature obeys simple beautiful laws based on a few first principles.

5. In another accompanying article [17], we introduce a weakton model of elementary particles, leading to explanations for sub-atomic decays and the creation/annihilation of matter/antimatter particles, and for the baryon asymmetry problem. Remarkably, in the weakton model, both the spin-1 mediators (the photon, the W and Z vector bosons, and the gluons) and the spin-0 dual mediators introduced in the unified field model have the *same* weakton constituents, differing only by their spin arrangements. The spin arrangements clearly demonstrate that there must be dual mediators with spin-0. This observation clearly supports the unified field model presented in [14, 15]. Conversely, the existence of the dual mediators makes the weakton constituents perfectly fit.

6. Quantum chromodynamics (QCD) was established as an $SU(3)$ quantum gauge field theory for strong interaction; see among many others [10, 5, 23] for the historical accounts.

Three important properties of strong interaction are the quark confinement, asymptotic freedom–discovered in the early 1970s by David Politzer [22] and by Frank Wilczek and David Gross [6]– and the short-range nature. Of course, among many other features of QCD, they are many important studies regarding to these three properties of strong interaction; see the references listed at the end of Chapter 8 of [23].

However due to the complexity of strong interaction, one still does not know how to derive these properties directly from the QCD gauge field theory. One difficulty comes from the fact that there are eight $SU(3)$ gauge fields, representing eight gluons. Strong interaction is taken place under the combined influence of *at least* these eight gauge fields. Fortunately, with PRI, as we mentioned earlier, we are able to find such combined physical quantity S_μ defined by (3.3).

Also, from the derived strong interactions potentials (1.1)–(1.3) above, it is clear that the quark confinement is mainly due to the effect of the dual gluon fields, represented by ϕ^s_k on the right hand side of the $SU(3)$ gauge field equations (3.1).

The paper is organized as follows. Section 2 introduces two basic principles, PID and PRI. Sections 3 and 4 derive the strong interaction potentials. Section 5 explains the quark confinement and asymptotic freedom using the strong interaction potentials derived in Sections 3 and 4. Section explores the connections to the Yukawa potential and explains the short-range nature of the strong interaction.

2. Recapitulation of Two Basic Principles and Unified Field Theory

We introduce in this section the unified field theory based on two basic principles, principle of interaction dynamics (PID) and principle of representation invariance (PRI), postulated recently by the authors [14, 15].

2.1. **Principle of Representation Invariance.**

The $SU(N)$ gauge theory provides an action

$$L = \int \mathcal{L} dx, \tag{2.1}$$

with the action density

$$\mathcal{L} = -\frac{1}{4} G_{ab} F^a_{\mu\nu} F^{b\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi, \tag{2.2}$$

where

$$\begin{aligned}
&F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\lambda^a_{bc} A^b_\mu A^c_\nu, \\
&D_\mu \Psi = (\partial_\mu + igA^a_\mu \tau_a)\Psi, \\
&G_{ab} = \frac{1}{2}\mathrm{Tr}(\tau_a \tau_b^\dagger) = \frac{1}{4N}\lambda^d_{ac}\lambda^c_{db},
\end{aligned} \tag{2.3}$$

and τ_a $(1 \leq a \leq N^2 - 1)$ are the generators of $SU(N)$, λ^c_{ab} are the structure constants with respect to τ_a, $\{G_{ab}\}$ is a Riemannian metric on $SU(N)$ (see [15]), $A^a_\mu = (A^a_0, A^a_1, A^a_2, A^a_3)^T$ $(1 \leq a \leq N^2 - 1)$ are the $SU(N)$ gauge fields, $\Psi = (\Psi_1, \cdots, \Psi_N)^T$ are the N Dirac spinors.

In [15], we have shown that the quantities in (2.2) and (2.3) with indices a, b, c are the $SU(N)$ representation tensors under the transformations

$$\widetilde{\tau}_a = x^b_a \tau_b, \tag{2.4}$$

where (x^b_a) is a K-th order nondegenerate matrix, and $K = N^2 - 1$.

Intuitively speaking, all $SU(N)$ gauge theories should be invariant under transformations (2.4) of different representations of $SU(N)$. This leads us to postulate PRI in [15]:

Principle of Representation Invariance (PRI). *Any $SU(N)$ gauge field theory is independent of the choice of $SU(N)$ generators $\{\tau_a\}$. Namely, the action (2.2) of gauge fields is invariant and the associated gauge field equations are covariant under the transformations (2.4).*

PRI provides a strong restriction on gauge field theories, and we address now some direct consequences of such restrictions, and we refer interested readers to [15] for more details. Here we make a few remarks.

Remark 2.1. The introduction of PRI is based purely on mathematical logic, and its validity is unquestionable. A physical theory violating PRI may only be considered as an approximation of laws of Nature.

Remark 2.2. Based on PRI, for the gauge fields A^a_μ and G^k_μ, corresponding to two different gauge groups $SU(N_1)$ and $SU(N_2)$, the following combinations

$$\alpha A^a_\mu + \beta G^k_\mu \tag{2.5}$$

are prohibited. The reason is that A^a_μ is an $SU(N_1)$-tensor with tensor index a, and G^k_μ is an $SU(N_2)$-tensor with tensor index k. The above combination violates

PRI. This point of view clearly shows that the classical electroweak theory violates PRI. Of course, this remark does not undermine the importance of the classical electroweak theory, in the spirit as the comparison of the Newton mechanics and the Einstein theories of relativity.

2.2. **Principle of Interaction Dynamics.** Let $(M, g_{\mu\nu})$ be a Riemannian manifold of space-time with Minkowski signature, and $A = (A_1, \cdots, A_n)$ be a vector field. We define two differential operators acting on an (r, s)-tensor u as

$$\nabla_A u = \nabla u + u \otimes A, \tag{2.6}$$

$$\text{div}_A u = \text{div} u - A \cdot u. \tag{2.7}$$

For a functional $F(u)$, a tensor u_0 is an extremum point of F under the div_A-free constraint, if u_0 satisfies

$$\frac{d}{d\lambda} F(u_0 + \lambda X)\bigg|_{\lambda=0} = (\delta F(u_0), X) = 0 \qquad \text{for all } X \text{ with } \text{div}_A X = 0.$$

In [15], we have shown that if u_0 is an extremum point of $F(u)$ with the div_A-free constraint, then there exists an $(r-1, s)$ tensor (or $(r, s-1)$ tensor) ϕ such that u_0 is a solution of the following equation

$$\delta F(u) = D_A \phi.$$

We remark that the div_A-free constraint is equivalent to energy-momentum conservation. The original motivation of div_A-free constraints was to take into consideration the presence of dark energy and dark matter in the modified gravitational field equations [16], and then it is natural for us to postulate the following principle of interaction dynamics (PID) [14]:

Principle of Interaction Dynamics (PID). *For all physical interactions there are Lagrangian actions*

$$L(g, A, \psi) = \int_M \mathcal{L}(g_{ij}, A, \psi)\sqrt{-g}dx, \tag{2.8}$$

where $g = \{g_{ij}\}$ is the Riemann metric representing the gravitational potential, A is a set of vector fields representing gauge and mass potentials, and ψ are the wave functions of particles. The action (2.8) satisfy the invariance of general relativity (or Lorentz invariance), the gauge invariance, and PID. Moreover, the states (g, A, ψ) are the extremum points of (2.8) with the div_A*-free constraint.*

A few remarks about PID are now in order.

Remark 2.3. The original motivation for PID was to explain dark energy and dark matter, which cannot be explained in the classical Einstein gravitational field equations. The gravitational field equations based on PID

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - D_\mu \Psi_\nu \tag{2.9}$$

offer a good explanation for both dark matter and dark energy; see [16] for details. Also it is worth mentioning here that the classical Einstein field equations consist of 10 equations solving 6 unknowns [13, 16], and is an overdetermined systems, which will have no solutions under certain physical conditions. The new gravitational field equations (2.9) possess the same number of unknowns and the number of equations are the same, and are mathematically more consistent.

In addition, the new term $D_\mu \Psi_\nu$ in (2.9) cannot be derived 1) from any existing $F(R)$ theories, and 2) from any existing scalar field theory.

Remark 2.4. PID offers a natural spontaneous symmetry breaking and mass generation mechanism.

2.3. **Remarks on Field Theory Based on PID and PRI.** With PRI and PID at our disposal, a unified field theory for interactions in Nature is developed recently in [14, 15].

Our viewpoint to derive the unified field theory is as follows. The topological and geometric structure of the space-time manifold M are determined by a few basic symmetry and dynamics principles: 1) the basic symmetry principles lead to the Riemannian metric $\{g_{\mu\nu}\}$ and the gauge fields $\{G_\mu\}$, 2) the basic dynamic principles determine the field equations for $\{g_{\mu\nu}\}$ and $\{G_\mu\}$, and 3) the field equations determine $\{g_{\mu\nu}\}$ and $\{G_\mu\}$, and consequently the structure of space-time manifold.

Also, a consistent unified field theory should obey the following requirements:

(1) Its Lagrangian action density must satisfy all known basic symmetry principles, including the principle of general relativity, the principle of Lorentz invariance, the principle of gauge invariance, and the principle of representation invariance (PRI);
(2) The Lagrangian action can be naturally decoupled to Lagrangian actions for individual interactions: the Einstein-Hilbert action for gravity, the $U(1)$ gauge field action for electromagnetism, the $SU(2)$ gauge field action for weak interaction and the $SU(3)$ gauge field action for strong interaction.

In fact, the unified field theory developed by the authors [14, 15] is uniquely determined by these two requirement; see [14, 15] for details.

3. Strong Interaction Quark Potential

3.1. **Field equations for strong interaction based on PID and PRI.** Both PID and PRI can be directly applied to strong interaction. In other words, the unified field model can be easily decoupled to study each individual interaction when other interactions are weak. Then the decoupled $SU(3)$ field equations, obeying both PID and PRI, are given as follows [15]:

$$G^s_{kj}\left[\partial^\nu S^j_{\nu\mu} - \frac{g_s}{\hbar c}\lambda^j_{il} g^{\alpha\beta} S^i_{\alpha\mu} S^l_\beta\right] - g_s J^s_{k\mu} = \left[\partial_\mu - \frac{1}{4}\left(\frac{m_\pi c}{\hbar}\right)^2 x_\mu + \frac{g_s \delta}{\hbar c}\alpha_k S^k_\mu\right]\phi^s_k, \tag{3.1}$$

$$i\gamma^\mu\left(\partial_\mu + i\frac{g_s}{\hbar c} S^k_\mu \tau_k\right)\psi - \frac{m_f c}{\hbar}\psi = 0, \tag{3.2}$$

where $\alpha^s_k = (\alpha^s_1, \cdots, \alpha^s_8)$ is the $SU(3)$ constant vector such that $\alpha^s_k \alpha^s_k = 1$, δ is a constant, S^k_μ $(k = 1, \cdots, 8)$ are the $SU(3)$ gauge fields, and

$$S^j_{\mu\nu} = \partial_\mu S^j_\nu - \partial_\nu S^j_\mu + \frac{g_s}{\hbar c}\lambda^j_{kl} S^k_\mu S^l_\nu, \qquad J^s_{k\mu} = \bar\psi \gamma_\mu \tau_k \psi, \qquad \gamma_\mu = \gamma^\mu.$$

In [15], we have shown that if the $SU(3)$ generators λ_k are taken to be the Gell-Mann matrices, then the corresponding metric $G^s_{\mu\nu}$ is Euclidian: $G^s_{\mu\nu} = \delta_{\mu\nu}$.

The above field equations introduce a natural duality:

$$S^k_\mu \longleftrightarrow \phi^k_s = \phi^s_k \qquad \text{for} \quad k = 1, \cdots, 8,$$

which represents the duality between eight gluons and their dual fields.

3.2. **Strong interaction quark potential.** One important point of view in the unified field theory is that each interaction has its own interaction charge, which is the source of the corresponding force. Also, each interaction possesses its own interaction potential and force formulas, given by

$$F = -g\nabla\Phi, \qquad g \text{ is the interaction charge.}$$

The physical quantities representing strong interaction potentials are the eight $SU(3)$ gauge fields S_μ^k. It is clear that the strong interaction potential must be color-independent and given by the following PRI invariant potential

$$S_\mu = \alpha_k S_\mu^k, \tag{3.3}$$

and the corresponding strong interaction potential and force are given by

$$\begin{aligned} &\Phi_s = S_0, && \text{the time component of } S_\mu,\\ &F_s = -g_s\nabla\Phi_s, && g_s \text{ is the strong charge.}\end{aligned} \tag{3.4}$$

We now derive a formula for the quark potential Φ_s from the field equations (3.1) and (3.2). We proceed in several steps as follows.

STEP 1. Using the Gell-Mann matrices as the representation matrices and taking inner product of the field equations with α_k, we derive that

$$\begin{aligned} \partial^\nu S_{\nu\mu} - \frac{g_s}{\hbar c}\lambda_{ij} g^{\alpha\beta} S^i_{\alpha\mu} S^j_\beta - g_s J^s_\mu \\ = \left[\partial_\mu - \frac{1}{4}\left(\frac{mc}{\hbar}\right)^2 x_\mu + \frac{g_s\delta}{\hbar c} S_\mu\right]\phi_s, \end{aligned} \tag{3.5}$$

$$\begin{aligned} \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \triangle\right)\phi_s + \left(\frac{mc}{\hbar}\right)^2\phi_s + \frac{1}{4}\left(\frac{mc}{\hbar}\right)^2 x_\mu\partial^\mu\phi_s \\ = g_s\partial^\mu J^s_\mu + \frac{g_s}{\hbar c}\partial^\mu(\lambda_{ij} g^{\alpha\beta} S^i_{\alpha\mu} S^j_\beta - \delta S_\mu\phi_s), \end{aligned} \tag{3.6}$$

where

$$\begin{aligned} &\lambda_{ij} = \alpha_k\lambda^k_{ij}, && m = m_\pi,\\ &\gamma_\mu = \gamma^\mu, && \tau_k = \tau^k,\\ &J^s_\mu = \alpha_k\bar\psi\gamma_\mu\tau_k\psi, && S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu + \frac{g_s}{\hbar c}\lambda_{ij} S^i_\mu S^j_\nu. \end{aligned} \tag{3.7}$$

Based on the superposition property of the quark potential, $\Phi_s = S_0$ and ϕ_s obey a linear relationship. Namely, we can choose the supplement equations properly, which are due to the introduction of the dual fields, such that the $\mu = 0$ equations of (3.5) and (3.6) are linear. In other words, the supplement equations contain the following two equations:

$$\begin{aligned} &\lambda_{ij}\left[\partial^\nu(S^i_\nu S^j_0) - g^{\alpha\beta} S^i_{\alpha 0} S^j_\beta\right] + \delta S_0\phi_s = 0,\\ &\partial^\mu\left[\lambda_{ij} g^{\alpha\beta} S^i_{\alpha\mu} S^j_\beta - \delta S_\mu\phi_s\right] = 0. \end{aligned} \tag{3.8}$$

Also, physically, we take two more supplement equations

$$\begin{aligned} &x_\mu\partial^\mu\phi_s = 0,\\ &\partial^\mu S_\mu = 0, \end{aligned} \tag{3.9}$$

together with the following static assumption:

$$\frac{\partial S_0}{\partial t} = 0, \qquad \frac{\partial\phi_s}{\partial t} = 0.$$

With the above gauge fixing conditions and static assumption, we derive equations for the quark potential ($\mu = 0$) and the dual potential as follows:

$$-\triangle\Phi_s = g_s Q - \frac{1}{4}k_0^2 c\tau\phi_s, \tag{3.10}$$

$$-\triangle\phi_s + k_0^2\phi_s = g_s\partial^\mu J^s_\mu, \tag{3.11}$$

where $x_0 = -c\tau$ is the wave length of ϕ_s, $k_0 = mc/\hbar$, $Q = -J^s_0$. For quarks, we have

$$k_0 = \frac{mc}{\hbar} \sim 10^{16}\text{cm}^{-1}, \qquad k_0 c\tau \geq 10^5. \tag{3.12}$$

STEP 2. SOLUTION OF (3.11). By definition, we have

$$\partial^\mu J^s_\mu = \alpha_k\partial_\mu\bar{\psi}\gamma^\mu\tau_k\psi + \alpha_k\bar{\psi}\gamma^\mu\tau_k\partial_\mu\psi.$$

In view of the Dirac equation (3.2),

$$\partial_\mu\bar{\psi}\gamma^\mu\tau_k\psi = i\frac{g_s}{\hbar c}S^j_\mu\bar{\psi}\gamma^\mu\tau_j\tau_k\psi + i\frac{m_f c}{\hbar}\bar{\psi}\tau_k\psi,$$

$$\bar{\psi}\gamma^\mu\tau_k\partial_\mu\psi = -i\frac{g_s}{\hbar c}S^j_\mu\bar{q}\gamma^\mu\tau_k\tau_j\psi - i\frac{m_f c}{\hbar}\bar{\psi}\tau_k\psi.$$

Hence we arrive at

$$\partial^\mu J^s_\mu = \frac{ig_s}{\hbar c}\alpha_k S^j_\mu\bar{\psi}\gamma^\mu[\tau_j,\tau_k]\psi = -\frac{2g_s}{\hbar c}\alpha_k S^j_\mu\lambda^i_{jk}J^{s\mu}_i.$$

Since $J^{s\mu}_i = J^{si}_\mu$ is the quark current density, we have

$$\alpha_k\lambda^i_{jk}J^{s\mu}_i = \theta^\mu_j\delta(r),$$

where $\delta(r)$ is the Dirac delta function, θ^μ_j is a constant tensor, inversely proportional to the volume of a quark. Hence

$$\alpha_k S^j_\mu\lambda^i_{jk}J^{s\mu}_i = \bar{S}^j_\mu\theta^\mu_j\delta(r), \tag{3.13}$$

where $\bar{S}^j_\mu \sim \bar{S}^j_\mu(0)$ takes the following average value

$$\bar{S}^j_\mu = \frac{1}{|B_{\rho_0}|}\int_{B_{\rho_0}} S^j_\mu dV.$$

Here ρ_0 is the radius of a quark. Later, we shall see that

$$S^j_\mu \sim \frac{1}{r} \text{ as } r \to \infty.$$

Hence we deduce that

$$\bar{S}^j_\mu = \xi^j_\mu\rho_0^{-1} \qquad (\xi^j_\mu \text{ is a constant tensor}). \tag{3.14}$$

Finally, we arrive at

$$\partial^\mu J^s_\mu = -\frac{\kappa\delta(r)}{\rho_0} \qquad (\rho_0 \text{ is the radius of a quark}). \tag{3.15}$$

where κ is a constant given by

$$\kappa = \frac{2g_s}{\hbar c}\xi^j_\mu\theta^\mu_j \quad \text{inversely proportional to the volume } V_q \text{ of a quark.} \tag{3.16}$$

Therefore, equation (3.11) is rewritten as

$$-\triangle\phi_s + k_0^2\phi_s = -\frac{g_s\kappa\delta(r)}{\rho_0}, \tag{3.17}$$

whose solution is given by

$$\phi_s = -\frac{g_s\kappa}{\rho_0}\frac{1}{r}e^{-k_0 r}. \tag{3.18}$$

STEP 3. SOLUTION OF (3.10). The quantity $g_s Q = -g_s J_0^s$ is the strong charge density of a quark, and without loss of generality, we assume that

$$Q = \beta\delta(r), \tag{3.19}$$

and $\beta > 0$ is a constant, inversely proportional to the quark volume. Hence (3.10) can be rewritten as

$$-\triangle\Phi_s = g_s\beta\delta(r) + \frac{g_s A}{\rho_0}\frac{1}{r}e^{-k_0 r}, \tag{3.20}$$

where A is a constant given by

$$A = \frac{k_0^2 c\tau\kappa}{4} \qquad \text{with physical dimension } \frac{1}{L}. \tag{3.21}$$

Assume $\Phi_s = \Phi_s(r)$ is radially symmetric, then (3.20) can be rewritten as

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)\Phi_s = g_s\beta\delta(r) + \frac{g_s A}{\rho_0}\frac{1}{r}e^{-k_0 r},$$

whose solution takes the form

$$\Phi_s = g_s\left[\frac{\beta}{r} - \frac{A}{\rho_0}\varphi(r)e^{-k_0 r}\right], \tag{3.22}$$

where φ solves

$$\varphi'' + 2\left(\frac{1}{r} - k_0\right)\varphi' - \left(\frac{2k_0}{r} - k_0^2\right)\varphi = \frac{1}{r}. \tag{3.23}$$

Assume that the solution φ of (3.23) is given by

$$\varphi = \sum_{k=0}^{\infty}\alpha_k r^k. \tag{3.24}$$

Inserting φ in (3.23) and comparing the coefficients of r^k, we obtain the relations

$$\begin{aligned} &\alpha_1 = k_0\alpha_0 + \frac{1}{2}, \\ &\alpha_2 = \frac{1}{2}k_0^2\alpha_0 + \frac{1}{3}k_0, \\ &\quad\vdots \\ &\alpha_k = \frac{2k_0}{k+1}\alpha_{k-1} - k_0^2\alpha_{k-2} \qquad \text{for } k \geq 2, \end{aligned} \tag{3.25}$$

where α_0 is a free parameter with dimension L. Hence

$$\varphi(r) = \alpha_0\left(1 + k_0 r + \frac{r}{2\alpha_0} + o(r)\right),$$

and often it is sufficient to take a first-order approximation.

STEP 4. STRONG INTERACTION QUARK POTENTIAL. Formula (3.22) provides an accurate strong interaction quark potential formula:

$$\Phi_q = g_s \beta_q \left[\frac{1}{r} - \frac{A_q}{\rho_0} \varphi(r) e^{-k_0 r} \right], \tag{3.26}$$

where $A_q = \alpha_0 A$ is a dimensionless parameter, $k_0 = 1/r_0$, r_0 is the average radius of hadrons, and $\varphi(r)$ is a dimensionless function, which can approximately be given by

$$\varphi(r) = 1 + \frac{r}{r_0}. \tag{3.27}$$

Namely, there are three constants in (3.26), A_q, β_q, and r_0. Also,

(3.28) β_q is inversely proportional to the quark volume V_q.

For simplicity, we can take $\beta_q = 1, \varphi = 1$, then (3.26) takes the following form

$$\Phi_q = g_s \left[\frac{1}{r} - \frac{A_q}{\rho_0} e^{-k_0 r} \right].$$

4. Layered Formulas of Strong Interaction Potentials

Different from gravity and electromagnetic force, strong interaction is short-ranged with different strengths in different levels. For example, in the quark level, strong interaction confines quarks inside hadrons, in the nucleon level, strong interaction bounds nucleons inside atoms, and in the atom and molecule level, strong interaction almost diminishes. This layered phenomena can be well-explained using the unified field theory based on PID and PRI. We derive in this section strong interaction potentials in different levels.

We start with strong interaction nucleon potential. For strong interaction of nucleons, we still use the $SU(3)$ QCD action

$$\mathcal{L} = -\frac{1}{4} S^k_{\mu\nu} S^{k\mu\nu} + \hbar c \bar{n} \left(i\gamma^\mu D_\mu - \frac{mc}{\hbar} \right) n, \tag{4.1}$$

where S^k_μ are the strong interaction gauge fields,

$$n = (a_1 n_0, a_2 n_0, a_3 n_0), \qquad a_1^2 + a_2^2 + a_3^2 = 1, \tag{4.2}$$

n_0 is the wave function of a nucleon, and

$$D_\mu n = (\hbar c \partial_\mu + \frac{i g_s}{\hbar c} S^k_\mu \tau_k) n. \tag{4.3}$$

The action $\mathcal{L}$ defined by (4.1) is $SU(3)$ gauge invariant. Physically, this means that nucleons are indistinguishable in strong interaction. With PID, the corresponding field equations corresponding to the action $\mathcal{L}$ are

$$\partial^\nu S^k_{\nu\mu} + \frac{g_s}{\hbar c} \lambda^k_{ij} g^{\alpha\beta} S^i_{\alpha\mu} S^j_\beta - g_s Q^k_\mu = \left(\partial_\mu - \frac{k_1^2}{4} x_\mu \right) \phi^k_n, \tag{4.4}$$

$$i\gamma^\mu \left(\partial_\mu + i\frac{g_s}{\hbar c} S^k_\mu \tau_k \right) n - \frac{mc}{\hbar} n = 0, \tag{4.5}$$

where the parameter k_1 is defined by

$$r_1 = \frac{1}{k_1} = 10^{-13} \text{ cm is the Yukawa radius,} \tag{4.6}$$

which is the average radius of nucleons, and

$$Q^k_\mu = \bar{n}\gamma_\mu \tau^k n \qquad (\lambda^k = \lambda_k). \tag{4.7}$$

Let the nucleon strong interaction Φ_n and its dual potential ϕ_n be defined by

$$\Phi_n = \alpha_k S^k_0, \qquad \phi_n = \alpha_k \phi^k_n.$$

In the same spirit as for deriving the quark potential (3.26), we deduce that

$$\begin{aligned} -\triangle \Phi_n &= g_s Q_0 - \frac{1}{4} k_1^2 c\tau_1 \phi_n, \\ -\triangle \phi_n + k_1^2 \phi_n &= g_s \partial^\mu Q_\mu, \end{aligned} \tag{4.8}$$

where $c\tau_1$ is the wave length of ϕ_n, and

$$Q = \alpha_k Q^k_\mu = (Q_0, Q_1, Q_2, Q_3)$$

represents nucleon current density. Similar to (3.15) and (3.19), we have

$$\partial^\mu Q_\mu = -\frac{k_n \delta(r)}{\rho_n}, \qquad Q_0 = \beta_n \delta(r), \tag{4.9}$$

where ρ_n is the radius of a nucleon, β_n and k_n are constants, inversely proportional to the volume of nucleons. Consequently, we derive the following strong nucleon potential:

$$\Phi_n = \beta_n g_s \left[\frac{1}{r} - \frac{A_n}{\rho_n} \varphi(r) e^{-k_1 r} \right], \tag{4.10}$$

where $\varphi(r)$ is as (3.24) and (3.25), and

$$A_n = \frac{\kappa_n k_1^2 c\tau_1}{4\beta_n}.$$

Note that β_q and β_n are inversely proportional to the volumes V_q and V_n, respectively. Hence

$$\frac{\beta_n}{\beta_q} = \frac{N V_q}{V_n} = N \left(\frac{\rho_0}{\rho_n} \right)^3, \qquad N = 3.$$

Here $N = 3$ is the number of strong charges in a nucleon. We then conclude that the strong interaction nucleon potential can be rewritten as (always assuming $\beta_q = 1$):

$$\Phi_n = 3 g_s \left(\frac{\rho_0}{\rho_n} \right)^3 \left[\frac{1}{r} - \frac{A_n}{\rho_n} \varphi(r) e^{-k_1 r} \right].$$

In summary, for a particle with N strong charges and radius ρ, the strong interaction potential can be written as

$$\Phi = N g_s \left(\frac{\rho_0}{\rho} \right)^3 \left[\frac{1}{r} - \frac{A}{\rho} \varphi(r) e^{-kr} \right], \tag{4.11}$$

where ρ_0 is the quark radius, A is a dimensionless constant depending on the particle, $1/k$ is the radius of strong attraction, and $\varphi(r) = 1 + kr$.

More specifically, we, respectively, obtain the strong quark potential Φ_q, the strong nucleon potential Φ_n and the strong atom/molecule potential Φ_a as follows:

$$\begin{aligned}
\Phi_q &= g_s \left[\frac{1}{r} - \frac{A_q}{\rho}(1+k_0 r)e^{-k_0 r}\right], \\
\Phi_n &= 3g_s \left(\frac{\rho_0}{\rho_n}\right)^3 \left[\frac{1}{r} - \frac{A_n}{\rho_n}(1+k_1 r)e^{-k_1 r}\right], \\
\Phi_a &= 3Ng_s \left(\frac{\rho_0}{\rho_a}\right)^3 \left[\frac{1}{r} - \frac{A_a}{\rho_a}(1+k_1 r)e^{-k_1 r}\right],
\end{aligned} \tag{4.12}$$

where N is the number of nucleons in an atom/molecule. These formulas provide strong interaction potentials for particles in different levels.

Remark 4.1. *1. We remark here that the strong charge g_s used in this article is to use quark as a reference level. If we use weaktons as the reference level, the layered strong interaction potentials are given by*

$$\Phi = g_s(\rho)\left[\frac{1}{r} - \frac{A_s}{\rho}\left(1+\frac{r}{R}\right)e^{-r/R}\right], \tag{4.13}$$

where A_s is a constant depending on the particle type, and R is the attracting radius of strong interactions given by

$$R = \begin{cases} 10^{-16}\text{cm} & \text{for } w^* \text{ and quarks,} \\ 10^{-13}\text{cm} & \text{for hadrons.} \end{cases}$$

Also, the strong charge is defined by

$$g_s(\rho) = \left(\frac{\rho_w}{\rho}\right)^3 g_s, \tag{4.14}$$

where g_s is the strong charge using weakton as a reference level. In other words, the g_s in (4.14) is equivalent to

$$\left(\frac{\rho_0}{\rho_w}\right)^3 g_s,$$

where g_s is the strong charge used in (4.12) and throughout this article.

2. The strong interaction potential between two particles with N_1, N_2 charges g_s and radii ρ_1, ρ_2 is given by

$$\Phi_s = N_1 N_2 g_s(\rho_1) g_s(\rho_2)\left[\frac{1}{r} - \frac{\bar{A}_s}{\sqrt{\rho_1\rho_2}}\left(1+\frac{r}{R}\right)e^{-r/R}\right], \tag{4.15}$$

where $g_s(\rho_k)$ $(k=1,2)$ are as in (4.14), and $\bar{A}_s$ is a constant depending on the types of two particle involved.

5. Quark Confinement and Asymptotic Freedom

With the strong interaction potential formulas at our disposal, we are ready to derive quark confinement and asymptotic freedom for strong interactions, two important phenomena in particle physics.

5.1. **Quark confinement.** We start with quark confinement. The strong interaction bounding energy E of two particles is given by

$$E = g_s\Phi(r), \qquad \Phi(r) \text{ is given by (4.11).}$$

Based on the quark potential (3.26), there are two radii: the quark radius ρ_0 and the quark attraction radius $r_0 = 1/k_0$:

$$\rho_0 \sim 10^{-19} \text{ cm}, \qquad r_0 \sim 10^{-16} \text{ cm}. \tag{5.1}$$

Hence we derive from (3.28) the strong force between quarks:

$$F_q = -g_s\frac{d\Phi_q}{dr} = g_s^2\left[\frac{1}{r^2} - \frac{A_1}{\rho_0}\frac{1}{r_0}e^{-k_0 r}\right], \tag{5.2}$$

which implies that there are two radii $\bar{r}_1$ and $\bar{r}_2$ such that $\rho_0 < \bar{r}_1 < r_0 < \bar{r}_2$ and

$$F_q\begin{cases} > 0 & \text{for } 0 < r < \bar{r}_1, \\ < 0 & \text{for } \bar{r}_1 < r < \bar{r}_2. \end{cases} \tag{5.3}$$

It is clear then that in the region where $\bar{r}_1 < r < \bar{r}_2$, strong force between quarks is attractive. In particular, the largest attraction force between quarks is of the following order:

$$F \sim \frac{1}{\rho_0},$$

which indicates that the largest strong attractive force between quarks $F \to \infty$ if $\rho_0 \to 0$. This is the reason for quark confinement.

The quark confinement can also be better explained from the viewpoint of the strong quark bounding energy E_q and the hadronic bounding energy E_n. We have

$$\frac{E_q}{E_n} = \frac{A_q}{\rho_0}e^{(k_1-k_0)r}\Big/\left(3\left(\frac{\rho_0}{\rho_n}\right)^6\frac{A_n}{\rho_n}\right) \sim \left(\frac{\rho_n}{\rho_0}\right)^6 \tag{5.4}$$

Based on physical estimates of $\rho_n = 10^{-15}$cm and $\rho_0 = 10^{-19}$cm, we derive that

$$E_q \sim 10^{24}E_n. \tag{5.5}$$

We know that $E_n \sim 10^{-2}$GeV, then we obtain that

$$E_q \geq 10^{22}\text{GeV}.$$

This clearly shows that there is no observed free quarks and the quark is confined in hadrons.

5.2. **Asymptotic freedom.** The strong interaction potentials provide also a natural explanation of the asymptotic freedom phenomena. By (5.3), we see that

$$F_q \sim 0 \qquad \text{near } r = \bar{r}_1.$$

This indicates that there is a free shell region inside a proton, for example, with radius $\widetilde{r}$, such the three quarks are free in this shell region.

When a low energy electron collides with the proton, the electromagnetic force causes the electron moving away, leading to the elastic scattering

$$e^- + p \to e^- + p.$$

However, when a high speed electron collides with a proton, it can run into the inside of the proton, interacting with one of the quarks. Since the quark was in a free shell region with no force acting upon it, this particular quark will behavior as a free quark. As it moves into the attracting region of the proton, the quark confinement will hold this quark, which, at the same time, will collide with gluons, exchanging quarks, leading to the following inelastic scatterings:

$$e^- + p \to e^- + p + \pi^0,$$
$$e^- + p \to e^- + n + \pi^+.$$

This explains the asymptotic freedom.

6. Yukawa potential and Short-Range Nature of Strong Interaction

One of the mysteries of the strong interaction is the different characteristics exhibited in the quark level and in the nucleon level. In the quark level,

$$\text{quark strong interaction at } r = 10^{-16}\text{cm is infinitely } \textit{attractive}; \tag{6.1}$$

while in the nucleon level,

$$\text{nucleon strong interaction for } 0 < r < 10^{-13}\text{cm is } \textit{repulsive}. \tag{6.2}$$

The different characteristics of the strong interactions demonstrated in (]ref6.1r) and (6.2) can hardly explained by any existing theory. However, the layered strong interaction potentials in (4.12) derived based on PID and PRI lead to a natural explanation of these characteristics, as well as explanations of the quark confinement and asymptotic freedom in the previous section. In this section, we shall explain the Yukawa force between nucleons and the short-range nature of strong interaction.

6.1. **Modified Yukawa potential based on PID and PRI.** Based on the classical strong interaction theory, the potential bounding hadrons for nucleons is the Yukawa potential:

$$\Phi_Y = -\frac{g}{r}e^{-k_1 r}, \qquad r_1 = \frac{1}{k_1} = 10^{-13}\text{cm}, \tag{6.3}$$

where g is the meson charge. The corresponding Yukawa strong force is

$$F_Y = -g\frac{d\Phi_Y}{dr} = -g^2\left(\frac{1}{r^2} + \frac{1}{r_1 r}\right)e^{-k_1 r} < 0 \qquad \text{for } r > 0, \tag{6.4}$$

indicating that the strong nucleon force is always attractive. However, experimentally, we know [27] that

$$F_{\text{experiment}} \begin{cases} > 0 & \text{for } 0 < r < \bar{r} = \frac{1}{2} \times 10^{-13}\text{cm}, \\ < 0 & \text{for } \bar{r} < r. \end{cases} \tag{6.5}$$

The discrepancy between (6.4) and (6.5) shows that the Yukawa potential is valid only in a small attractive range of strong nucleon force.

The derived nucleon potential Φ_n in (4.12) is schematically shown in Figure 6.1, which is consistent with the experimental results; see [27]. Also the corresponding strong nucleon force is given by

$$F_n = -g_s\frac{d\Phi_n}{dr} = 3g_s^2\left(\frac{\rho_0}{\rho_n}\right)^6\left[\frac{1}{r^2} - \frac{A_n}{\rho_n}\frac{r}{r_1^2}e^{-r/r_1}\right], \tag{6.6}$$

where

$$\rho_n = 10^{-15}\text{cm}, \qquad r_1 = 10^{-13}\text{cm}. \tag{6.7}$$

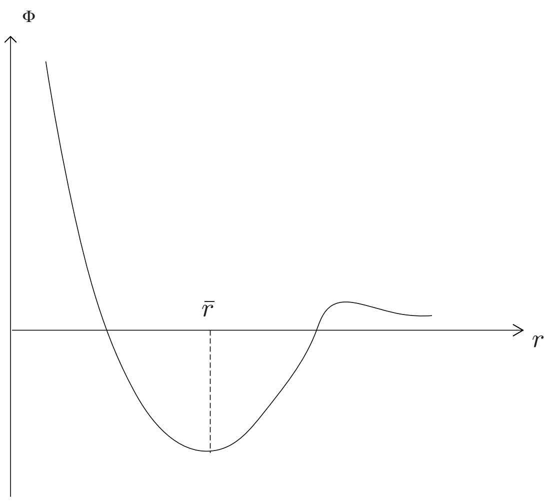

FIGURE 6.1.

By (6.6) and (6.7), we can determine A_n from the experimental value $\bar{r}$ in (6.5) as follows.

Set $F_n = 0$, then we have

$$r^3 e^{-r/r_1} = \frac{\rho_n r_1^2}{A_n}. \tag{6.8}$$

Using $\bar{r} = r_1/2$, this formula shows that

$$A_n = 8e^{1/2} \times 10^{-3}. \tag{6.9}$$

In addition, by (6.8) and (6.9), we derive the two zeros $\bar{r}$ and r_2 of F_n:

$$\bar{r} = \frac{r_1}{2}, \qquad r_2 \sim 9r_1. \tag{6.10}$$

In addition, using the magnitude of the Yukawa force and the magnitude of the attractive force of F_n are given by

$$F_Y \sim -2g^2/r_1^2, \qquad F_n \sim -3g_s^2 \left(\frac{\rho_0}{\rho_n}\right)^6 \frac{A_n}{\rho_n} \frac{r_1}{r_1^2},$$

which lead to

$$3g_s^2 \left(\frac{\rho_0}{\rho_n}\right)^6 \frac{A_n r_1}{\rho_n} = 2g^2 \qquad (g^2 \sim 10\hbar c). \tag{6.11}$$

6.2. **Physical conclusions on nucleon strong force.** The discussion in the previous subsection leads to the following physical conclusions:

(1) The modified Yukawa formula based on PID and PRI is

$$F_n = g^2 \left[\frac{1}{4\sqrt{e}} \frac{1}{r^2} - \frac{2r}{r_1^3} e^{-r/r_1}\right], \tag{6.12}$$

where $g^2 = 10\hbar c$ is the usual nucleon interaction constant, and e is the base of nature logarithm.

(2) The attraction and repelling regions of the strong nucleon force F_n are

$$F_n \begin{cases} > 0 & \text{for } 0 < r < r_1/2, \\ < 0 & \text{for } r_1/2 < r < 9r_1, \\ > 0 & \text{for } 9r_1 < r, \end{cases}$$

where $r_1 = 10^{-13}$cm.

(3) The largest attraction of F_n is achieved at $r \sim 1.5r_1$ with value given by

$$F_{\max} = -\frac{g^2}{\sqrt{e}r_1^2}.$$

(4) By (6.11), the value of the strong charge g_s is given by

$$g_s^2 = \frac{1}{2} \times 10^{25}\hbar c.$$

(5) The strong nucleon interaction constant A_n is given by

$$A_n = 8\sqrt{e} \times 10^{-3}.$$

6.3. **Short-range nature of strong interaction.** By the layered potentials for strong interactions, we see that when the nucleons form an atom, the nucleon potential is no longer valid, and the correct potential becomes the strong interaction potential for atoms given by the third formula in (4.12). The corresponding force formula is given by

$$F_a = 9N^2 g_s^2 \left(\frac{\rho}{\rho_a}\right)^6 \left[\frac{1}{r^2} - \frac{A_a}{\rho_a}\frac{r}{r_1^2} e^{-r/r_1}\right], \tag{6.13}$$

where

$$\text{for atom: } \rho_a = 10^{-8}\text{cm}, \qquad \text{for molecule: } \rho_a = 10^{-7}\text{cm}. \tag{6.14}$$

Also, bounding force between atoms and molecules are electromagnetic force with strength given by

$$\frac{e^2}{\hbar c} = \frac{1}{137}. \tag{6.15}$$

where e is the electric charge. Hence at the atom/molecule scales, the ratio between strong force and electromagnetic attraction force is

$$\frac{F_a}{F_e} = 9N^2 g_s^2 \left(\frac{\rho_0}{\rho_a}\right)^6 \Big/ N^2 e^2 = 9g_s^2 \left(\frac{\rho_0}{\rho_a}\right)^6 \Big/ e^2, \tag{6.16}$$

where in the last term, the first e is the base of the natural logarithm, and the second e is the electric charge. Consequently,

$$\frac{F_a}{F_e} \sim \begin{cases} 10^{-38} & \text{at the atomic level,} \\ 10^{-44} & \text{at the molecular level.} \end{cases} \tag{6.17}$$

This clearly demonstrates the short-range nature of strong interaction.

7. Conclusions

This paper addresses a number of important consequences of the duality induced from the new field equations for strong interaction derived in [15] based on two basic principles PID and PRI. First, one prediction of this duality is the existence of eight dual gluon scalar fields. From the field theoretical point of view, these fields are needed to produce the strong attraction force, leading to quark confinement.

Second, with the duality, we derive three levels of strong interaction potentials: the quark potential S_q, the nucleon/hadron potential S_n and the atom/molecule potential S_a. These potentials clearly demonstrate many features of strong interaction consistent with experimental observations. In particular, these potentials offer a clear mechanism for both quark confinement and asymptotic freedom.

Third, in the nuclear level, the new potential is an improvement of the Yukawa potential. As the distance between two nucleons is increasing, the nuclear force corresponding to the nucleon potential S_n behaves as repelling, then attracting, then repelling again and diminishes. This is exactly the picture that the experimental observations tell us. It is worth mentioning that the classical Yukawa potential can not be derived from classical QCD, while the new potentials are indeed derived directly from the new field equations.

Finally, this paper is part of a research program on unified field theory for interactions in nature. Among other features the unified field model derived in [14, 15] can be easily decoupled to study individual interactions.

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