Law of Gravity, Dark Matter and Dark Energy

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Supported in part by NSF, ONR and Chinese NSF

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PCGM 32: Pacific Coast Gravity Meeting, April 1-2, 2016
Inspired by the Albert Einstein’s vision, our general view of Nature is as follows:

*Nature speaks the language of Mathematics: the laws of Nature are represented by mathematical equations, are dictated by a few fundamental principles, and take the simplest and aesthetic forms.*

We intend to derive experimentally verifiable laws of Nature based only on a few fundamental first principles, *guided by experimental and observation evidences.*
I. Laws of Gravity, Dark Matter and Dark Energy

General relativity (Albert Einstein, 1915):

- **The principle of equivalence** says that the space-time is a 4D Riemannian manifold \((M, g_{\mu\nu})\) with \(g_{\mu\nu}\) being regarded as gravitational potentials.

- **The principle of general relativity** requires that the law of gravity be covariant under general coordinate transformations, and dictates the Einstein-Hilbert action:

\[
L_{EH}(\{g_{\mu\nu}\}) = \int_M \left(R + \frac{8\pi G}{c^4} S\right) \sqrt{-g} d\mathbf{x}.
\]

- **The Einstein field equations** are the Euler-Lagrangian equations of \(L_{EH}\):

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}, \quad \nabla^\mu T_{\mu\nu} = 0
\]
**Dark Matter:** Theoretically for galactic rotating stars, one should have

\[ \frac{v_{\text{theoretical}}^2}{r} = \frac{M_{\text{observed}}}{r^2} G \]

In 1970s, Vera Rubin observed the discrepancy between the observed velocity and theoretical velocity

\[ v_{\text{observed}} > v_{\text{theoretical}}, \]

which implies that

\[ \Delta M = (\text{the mass corresponding to } v_{\text{observed}}) - M_{\text{theoretical}} > 0, \]

which is called **dark matter.**

**Dark Energy** was introduced to explain the observations since the 1990s by indicating that the universe is expanding at an accelerating rate. The 2011 Nobel Prize in Physics was awarded to Saul Perlmutter, Brian P. Schmidt and Adam G. Riess for their observational discovery on the accelerating expansion of the universe.
New Gravitational Field Equations (Ma-Wang, 2012):

The presence of dark matter and dark energy implies that the energy-momentum tensor of visible matter \( T_{\mu\nu} \) may no longer be conserved: \( \nabla^\mu T_{\mu\nu} \neq 0 \).

Consequently, the variation of \( L_{EH} \) must be taken under energy-momentum conservation constraint, leading us to postulate a general principle, called principle of interaction dynamics (PID) (Ma-Wang, 2012):

\[
(3) \quad \left. \frac{d}{d \lambda} \right|_{\lambda=0} L_{EH}(g_{\mu\nu} + \lambda X_{\mu\nu}) = 0 \quad \forall X = \{X_{\mu\nu}\} \text{ with } \nabla^\mu X_{\mu\nu} = 0.
\]

Then we derive a new set of gravitational field equations

\[
(4) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu,
\]

\[
\nabla^\mu \left[ \frac{8\pi G}{c^4} T_{\mu\nu} + \nabla_\mu \Phi_\nu \right] = 0.
\]
• The new term $\nabla_\mu \Phi_\nu$ cannot be derived
  – from any existing $f(R)$ theories, and
  – from any scalar field theories.

In other words, the term $\nabla_\mu \Phi_\nu$ does not correspond to any Lagrangian action density, and is the direct consequence of PID.

• The field equations (4) establish a natural duality:

  spin-2 graviton field $\{g_{\mu\nu}\}$ $\longleftrightarrow$ spin-1 dual vector graviton field $\{\Phi_\mu\}$
Central Gravitational Field

Consider a central gravitational field generated by a ball $B_{r_0}$ with radius $r_0$ and mass $M$. It is known that the metric of the central field for $r > r_0$ can be written in the form

\[
\text{(5)} \quad ds^2 = -e^u c^2 dt^2 + e^v dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad u = u(r), v = v(r).
\]

Then the field equations (4) take the form

\[
\text{(6)} \quad v' + \frac{1}{r} (e^v - 1) = -\frac{r}{2} u' \phi',
\]

\[
u' - \frac{1}{r} (e^v - 1) = r (\phi'' - \frac{1}{2} v' \phi'),
\]

\[
u'' + \left( \frac{1}{2} u' + \frac{1}{r} \right) (u' - v') = -\frac{2}{r} \phi'.
\]
We have derived in (Ma & Wang, 2012) an approximate gravitational force formula:

\begin{equation}
F = \frac{mc^2}{2}e^u \left[ -\frac{1}{r} (e^v - 1) - r\phi'' \right].
\end{equation}

This can be further simplified as

\begin{equation}
F = mM G \left( -\frac{1}{r^2} - \frac{k_0}{r} + k_1 r \right) \quad \text{for } r > r_0,
\end{equation}

\[ k_0 = 4 \times 10^{-18}\text{Km}^{-1}, \quad k_1 = 10^{-57}\text{Km}^{-3}, \]

demonstrating the presence of both dark matter and dark energy. Here the first term represents the Newton gravitation, the attracting second term stands for dark matter and the repelling third term is the dark energy. We note that our modified new formula is derived from first principles.
(Hernandez, Ma & Wang, 2015):

Let \( x(s) \overset{\text{def}}{=} (x_1(s), x_2(s), x_3(s)) = (e^s u'(e^s), e^{v(e^s)} - 1, e^s \phi'(e^s)) \). Then the non-autonomous gravitational field equations (6) become an autonomous system:

\[
x'_1 = -x_2 + 2x_3 - \frac{1}{2}x_1^2 - \frac{1}{2}x_1x_3 - \frac{1}{2}x_1x_2 - \frac{1}{4}x_1x_3,
\]

\[
x'_2 = -x_2 - \frac{1}{2}x_1x_3 - x_2^2 - \frac{1}{2}x_1x_2x_3,
\]

\[
x'_3 = x_1 - x_2 + x_3 - \frac{1}{2}x_2x_3 - \frac{1}{4}x_1x_3^2,
\]

\((x_1, x_2, x_3)(s_0) = (\alpha_1, \alpha_2, \alpha_3) \quad \text{with} \quad r_0 = e^{s_0}.
\]

- The asymptotically flat space-time geometry is represented by \( x = 0 \), which is a fixed point of the system (9).

- There is a two-dimensional stable manifold \( E^s \) near \( x = 0 \), which can be
parameterized by

$$x_3 = h(x_1, x_2) = -\frac{1}{2} x_1 + \frac{1}{2} x_2 + \frac{1}{16} x_1^2 - \frac{1}{16} x_2^2 + O(|x|^3).$$

The field equations (9) are reduced to a two-dimensional dynamical system:

$$x'_1 = -x_1 - \frac{1}{8} x_1^2 - \frac{1}{8} x_2^2 - \frac{3}{4} x_1 x_2 + O(|x|^3),$$

$$x'_2 = -x_2 + \frac{1}{4} x_1^2 - x_2^2 - \frac{1}{4} x_1 x_2 + O(|x|^3),$$

$$(x_1, x_2)(s_0) = (\alpha_1, \alpha_2),$$

with $x_3$ being slaved by (10).
Figure 1: Only the orbits on $\Omega_1$ with $x_1 > 0$ will eventually cross the $x_2$-axis, leading to the sign change of $x_1$, and to a repelling gravitational force.
**Asymptotic Repulsion Theorem (Hernandez-Ma-Wang, 15)** For a central gravitational field, the following assertions hold true:

1) The gravitational force $F$ is given by

$$F = -\frac{mc^2}{2}e^uu',$$

and is asymptotic zero:

(12) \hspace{1cm} F \to 0 \quad \text{if} \quad r \to \infty.

2) If the initial value $\alpha$ in (11) is near the Schwarzschild solution with $0 < \alpha_1 < \alpha_2/2$, then there exists a sufficiently large $r_1$ such that the gravitational force $F$ is repulsive for $r > r_1$:

(13) \hspace{1cm} F > 0 \quad \text{for} \quad r > r_1.
The above theorem is valid provided the initial value $\alpha$ is small. Also, all physically meaningful central fields satisfy the condition (note that a black hole is enclosed by a huge quantity of matter with radius $r_0 \gg 2MG/c^2$).

In fact, the Schwarzschild initial values are as

$$(14) \quad x_1(r_0) = x_2(r_0) = \frac{\delta}{1 - \delta}, \quad \delta = \frac{2MG}{c^2r_0}.$$  

The $\delta$-factors for most galaxies and clusters of galaxies are

$$(15) \quad \text{galaxies } \delta = 10^{-7}, \quad \text{cluster of galaxies } \delta = 10^{-5}.$$  

The dark energy phenomenon is mainly evident between galaxies and between clusters of galaxies, and consequently the above theorem is valid for central gravitational fields generated by both galaxies and clusters of galaxies.

The asymptotic repulsion of gravity (dark energy) plays the role to stabilize the large scale homogeneous structure of the Universe.
Law of gravity (Ma-Wang, 2012)

1. (Einstein’s PE). The space-time is a 4D Riemannian manifold \( \{\mathcal{M}, g_{\mu\nu}\} \), with the metric \( g_{\mu\nu} \) being the gravitational potential;

2. The Einstein PGR dictates the Einstein-Hilbert action (1);

3. The gravitational field equations (4) are derived using PID, and determine gravitational potential \( g_{\mu\nu} \) and its dual vector field \( \Phi_\mu = \nabla_\mu \phi \);

4. Gravity can display both attractive and repulsive effect, caused by the duality between the attracting gravitational field \( g_{\mu\nu} \) and the repulsive dual vector field \( \Phi_\mu \), together with their nonlinear interactions governed by the field equations (4).

5. It is the nonlinear interaction between \( g_{\mu\nu} \) and the dual field \( \Phi_\mu \) that leads to a unified theory of dark matter and dark energy phenomena.
II. New Blackhole Theorem

Schwarzschild metric in the exterior gravitational fields of a centrally symmetric ball with total mass $M$ and with radius $R$:

$$ds^2 = - \left(1 - \frac{R_s}{r}\right) c^2 dt^2 + \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad R_s = \frac{2MG}{c^2}.$$

Tolman-Oppenheimer-Volkoff (TOV) metric in the interior of the centrally symmetric ball of idealized fluid: $T^{\mu\nu} = (\rho + p) u^\mu u^\nu + pg^{\mu\nu}$:

$$ds^2 = - \left[\frac{3}{2} \left(1 - \frac{R_s}{R}\right)^{1/2} - \frac{1}{2} \left(1 - \frac{R_s r^2}{R^3}\right)^{1/2}\right]^2 c^2 dt^2$$

$$+ \left[1 - \frac{R_s r^2}{R^3}\right]^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$
Assume the validity of the Einstein theory of general relativity, then the following assertions hold true:

1. black holes are closed: matters can neither enter nor leave their interiors;

2. black holes are innate: they are neither born to explosion of cosmic objects, nor born to gravitational collapsing; and

3. black holes are filled and incompressible, and if the matter field is non-homogeneously distributed in a black hole, then there must be sub-blackholes in the interior of the black hole.
Key observations:

- The singularity at the Schwarzschild radius $R_s = \frac{2MG}{c^2}$ is physical:

\[
ds^2 = -\left[1 - \frac{R_s}{r}\right] c^2 dt^2 + \left[1 - \frac{R_s}{r}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2.
\]

At $r = R_s$, time freezes, and there is no motion.

The Eddington-Finkelstein coordinate transformation

\[
r^* = r + \frac{2GM}{c^2} \ln \left|\frac{r}{R_s} - 1\right|
\]

is not differentiable, and is not allowed both physically and mathematically.

The same problem occurs for all other coordinate transformations.
• The geometrical realization of a black hole is dictated by the Schwarzschild and the TOV metrics, and shows also that nothing will get in the black hole:

\[ ds^2 = \left[ 1 - \frac{R_s}{r} \right]^{-1} \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad \text{for } r > R_s, \]

\[ ds^2 = \left( 1 - \frac{r^2}{R_s^2} \right)^{-1} \left( dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad \text{for } r < R_s. \]

• Take for example the supermassive black-hole at the center of our galaxy, the Milky Way. By the classical theory, this black-hole would continuously gobble matters nearby. Then the radius \( R_s \) would increase in cubic rate. Then it would not hard to see the black hole will consume the entire Milky Way, and eventually the entire Universe.

• A rigorous proof is also given in terms of energy conservation law by coupling the gravitational field equations and the fluid dynamic equations through a symmetry-breaking mechanism.
Supernovae Explosion and AGN Jets. Consider e.g. an active galactic nucleus (AGN), which occupies a spherical annular shell region $R_s < r < R_1$, where $R_s$ is the Schwarzschild radius of the black hole core of the galaxy.

The relativistic effect is then reflected in the radial force, gives rise to a huge explosive force near $r = R_s$:

$$\frac{\nu}{1 - \frac{R_s}{r}} \frac{R_s^2}{r^4} P_r.$$

The basic mechanism of the formation of AGN jets is that the radial temperature gradient causes vertical Bénard convection cells. Each Bénard convection cell has a vertical exit, where the circulating gas is pushed by the radial force, and then erupts leading to a jet. Each Bénard convection cell is also an entrance, where the external gas is attracted into the nucleus, is cycloaccelerated by the radial force as well, goes down to the interior boundary $r = R_s$, and then is pushed toward to the exit. Thus the circulation cells form jets in their exits and accretions in their entrances.

This mechanism can also be applied to supernovae explosion.
III. Summary

Tian Ma & Shouhong Wang, Mathematical Principles of Theoretical Physics, Science Press, Beijing, 518 pp., August, 2015.

- Inspired by the Albert Einstein’s vision, our general view of Nature is as follows:
  
  *Nature speaks the language of Mathematics: the laws of Nature are represented by mathematical equations, are dictated by a few fundamental principles, and take the simplest and aesthetic forms.*

- We have derived experimentally verifiable laws of Nature based only on a few fundamental first principles, guided by experimental and observation evidences.

- The results are established with solid mathematical foundation, are in agreement with the existing observations and experiments we are aware of, solve a number of longstanding challenges, and offer new predictions.
• We have discovered three fundamental principles:
  - the principle of interaction dynamics (PID), (Ma-Wang, 2012),
  - the principle of representation invariance (PRI) (Ma-Wang, 2012), and

• PID takes the variation of the action under the energy-momentum conservation constraints, and is required by the dark matter and dark energy phenomena (for gravity), by the quark confinements (for strong interaction), and by the Higgs field (for the weak interaction).

For example, the following gravitational field equations (Ma-Wang, 2012) is due to PID:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu. \]
• PRI requires that the gauge theory be independent of the choices of the representation generators. These representation generators play the same role as coordinates, and in this sense, PRI is a coordinate-free invariance/covariance, reminiscent of the Einstein principle of general relativity.

PRI is purely a logic requirement for the gauge theory.

• PSB offers an entirely different route of unification from the Einstein unification route which uses large symmetry group. The three sets of symmetries — the general relativistic invariance, the Lorentz and gauge invariances, as well as the Galileo invariance — are mutually independent and dictate in part the physical laws in different levels of Nature.

For a system coupling different levels of physical laws, part of these symmetries must be broken.
These three new principles have profound physical consequences, and, in particular, provide a new route of unification for the four interactions:

1) the general relativity and the gauge symmetries dictate the Lagrangian;
2) the coupling of the four interactions is achieved through PID and PRI in the field equations, which obey the PGR and PRI, but break spontaneously the gauge symmetry;
3) the unified field model can be easily decoupled to study individual interaction, when the other interactions are negligible; and
4) the unified field model coupling the matter fields using PSB.

The new theory gives rise to solutions and explanations to a number of longstanding mysteries in modern theoretical physics, including e.g. 1) dark matter and dark energy phenomena, 2) the structure of back holes, 3) the structure and origin of our Universe, 4) the unified field theory, 5) quark confinement, and 6) mechanism of supernovae explosion and active galactic nucleus jets.