Field Theory of Four Interactions

Tian Ma, Shouhong Wang
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I. Gravity and Principle of Interaction Dynamics (PID)

II. Principle of Representation Invariance (PRI)

III. Unified Field Theory
I. Gravity and Principle of Interaction Dynamics (PID)

Four fundamental forces/interactions in Nature:
Galileo, Newton and Poincaré: The Newton’s laws of motion are covariant under the Galileo transformation: $x' = x + vt, t' = t$.

Principle of Special Relativity (Einstein 1905): Laws of motion are covariant under Lorentz transformations:

- Minkowski space $M$ with metric $ds^2 = -c^2 dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$.
- Tensors transform under the Lorentz group: $LG^4$.

Energy-momentum relation: \[ E^2 = p^2 c^2 + m^2 c^4 \]

\[ \mathcal{L} = -mc^2 \sqrt{1 - \frac{v^2}{c^2}}, \quad \vec{p} = \frac{\partial \mathcal{L}}{\partial \vec{v}} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad E = \vec{p} \cdot \vec{v} - \mathcal{L} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \]
**General relativity (Einstein, 1915):**

- **principle of general relativity:** Physical laws are equivalent under both inertial and non-inertial frames.

- **principle of equivalence:** a non-inertial frame is equivalent to an inertial system in a gravitational field.

These two principles lead to the following:

- the space-time manifold is a 4D Riemannian manifold \((M, g_{\mu\nu})\)

- tensor fields transform under the transformation group: \(\left( \frac{\partial \varphi^\mu(p)}{\partial x^\nu} \right)_{p\in M} \).
Einstein field equations:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}, \quad \nabla^\mu T_{\mu\nu} = 0.
\]

are the Euler-Lagrange equations of the Einstein-Hilbert functional

\[
L_{EH}(g_{\mu\nu}) = \int_M \left( R + \frac{8\pi G}{c^4} g^{\mu\nu} S_{\mu\nu} \right) \sqrt{-g} dx
\]

Challenges:

• The Einstein field equations do not explain the dark matter and dark energy.

• Non well-posedness: 10 equations solving for 6 unknowns \( g_{\mu\nu} \).
**Principle of Interaction Dynamics (PID) (Ma-Wang, 2012):** Least action with energy-momentum conservation constraints.

With PID, we derive the following gravitational field equations:

\[
L_{EH}(g_{\mu\nu}) = \int_M \left( R + \frac{8\pi G}{c^4} S \right) \sqrt{-g} dx
\]

\[
\delta L_{EH} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{8\pi G}{c^4} T_{\mu\nu}
\]

\[
(\delta L_{EH}(g_{\mu\nu}), X) = 0 \quad \forall \nabla^\mu X_{\mu\nu} = 0 \quad \implies \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu
\]

\[
D^\mu \left( \frac{8\pi G}{c^4} T_{\mu\nu} + \nabla_\mu \Phi_\nu \right) = 0
\]

**Note:** The new term \( \nabla_\mu \Phi_\nu \) cannot be derived 1) from any existing \( f(R) \) theories, and 2) from any scalar field theories.
New gravitational vector bosonic field:

- The new vector particle field $\Phi_\mu$ is a spin-1 massless bosonic particle:

$$\Box \Phi_\nu = \frac{e}{\hbar c} A_\mu \nabla^\mu \Phi_\nu + \frac{8\pi G}{c^4} \nabla^\mu T_{\mu \nu}$$

- The nonlinear interaction between this particle field $\Phi_\mu$ and the graviton leads to a unified theory of dark matter and dark energy and explains the acceleration of expanding universe.

- Consider a spherically symmetric central matter field with mass $M$ and radius $r_0$. The force exerted on an object with mass $m$ is given by

$$F = mMG \left[ -\frac{1}{r^2} - \frac{c^2}{2GM} \left( 2 + \frac{2GM}{c^2} \frac{1}{r} \right) \frac{d\varphi}{dr} + \frac{c^2}{2GM} \Phi \right], \text{ for } r > r_0.$$
The first term is the classical Newton gravitation, the 2nd and 3rd terms are the coupling interaction between matter and the scalar potential $\varphi$.

This formula can be further simplified to derive the following approximate formula for $r_0 < r < r_1 \approx 10^{21} - 10^{22} km$:

$$F = mMG \left[ -\frac{1}{r^2} - \frac{k_0}{r} + k_1 r \right],$$

(1)

where $k_0 = 4 \times 10^{-18} km^{-1}$ and $k_1 = 10^{-57} km^{-3}$, which are estimated using rotation curves of galactic motion.

\[ L\left(\{g_{\mu\nu}\}, A, \psi\right) = \int_M L\left(\{g_{\mu\nu}\}, A, \psi\right) \sqrt{-g} dx \]

2. \( L \) obeys the principle of general relativity, the Lorentz and gauge invariances.

3. The interaction fields \((g_{\mu\nu}, A, \psi)\) are extrema of \( L \) with \( \text{div}_A \)-free constraints:

\[ \frac{\delta}{\delta g^{\mu\nu}} L\left(\{g_{ij}\}, A, \psi\right) = \left(\nabla_\mu + \alpha_b A^b_\mu\right) \Phi_\nu, \]

\[ \frac{\delta}{\delta A^a_\mu} L\left(\{g_{ij}\}, A, \psi\right) = \left(\nabla_\mu + \beta^{(a)}_b A^b_\mu\right) \phi^a, \]

\[ \frac{\delta}{\delta \psi} L\left(\{g_{ij}\}, A, \psi\right) = 0. \]
II. Principle of Representation Invariance (PRI)

Fundamentals of Quantum Physics:

- Max Planck 1900 (blackbody radiation): the energy of microscopic systems is discrete: \( E = n\hbar \omega, \quad n = 0, 1, 2, \cdots \) \((\omega \) is the frequency of the EM wave)

- Albert Einstein 1905 (Light quanta (photon \( \gamma \))): postulated that for a photon \( \gamma \):
  \[
  E = \hbar \omega, \quad \vec{p} = \hbar \vec{k}, \quad \frac{m_e v^2}{2} = \hbar \omega - W.
  \]

- Louis de Broglie matter wave, 1924: Other matter particle such as an electron possesses similar wave behavior, and the motion of free particles can be described by a plane wave function:

  \[
  \psi(x, t) = Ae^{-\frac{i}{\hbar} (Et - \vec{p} \cdot \vec{x})} \quad \Rightarrow \quad E = \hbar c \frac{\partial}{\partial t}, \quad \vec{p} = -\hbar c \nabla
  \]
• Based on classical $E = \frac{1}{2m}p^2 + V(x)$, Schrödinger (1925) derived then the following Schrödinger equation:

$$\hbar i \psi_t = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi.$$

Then we have

$$\frac{d}{dt} \int_M |\psi(x, t)|^2 dx = 0$$

Hence

$$|\psi(x, t)|^2$$ stands for the probability of finding the particle at $x$ and $t$.

**Note:** Schrödinger equation is NOT Lorentz invariant.
• Using Einstein’s energy-momentum relation: \( E^2 = p^2 c^2 + m^2 c^4 \), Klein-Gordon introduced the KG equations:

\[
\left[ \frac{\partial^2}{c^2 \partial t^2} - \nabla^2 \right] \psi + \left( \frac{mc}{\hbar} \right)^2 \psi = 0,
\]

which is Lorentz invariant! Hence it is a relativistic wave equation.

**Note:**

1. \( \int_M |\psi(x, t)|^2 dx \) is not conserved, and \( |\psi(x, t)|^2 \) does not represent probability density.
2. KG does not describe the particle motion, instead it is a field equation for bosonic particles
Dirac equations, 1928: In view of \( E^2 = p^2 c^2 + m^2 c^4 \), writing \( E = c\alpha_k p_k + \alpha_0 mc^2 \) and using again \( E = \hbar i \frac{\partial}{\partial t} \), \( \vec{p} = -\hbar i \nabla \), Dirac derived the Dirac equations:

\[
\begin{bmatrix}
i \gamma^\mu \partial_\mu - \frac{mc}{\hbar}
\end{bmatrix} \psi = 0
\]

\[
\psi = \begin{pmatrix}
\psi_1 \\
\vdots \\
\psi_4
\end{pmatrix}
\]

\[
\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3
\]

Dirac matrices

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

Pauli matrices

Remark: Dirac equations are Lorentz invariant:

\[
\tilde{x}^\mu = a^\mu_\nu x^\nu, \quad A = (a^\mu_\nu) \in LG^4,
\]

\[
\tilde{\psi} = U_A \psi, \quad U_A \in SU(4), \quad U_A^{-1} \gamma^\mu U_A = a^\mu_\nu \gamma^\nu.
\]
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**Principle of gauge invariance**, J. C. Maxwell (1861), H. Weyl (1919), O. Klein (1938), C. N. Yang & R. L. Mills (1954): Certain physical properties of fermionic particles $\Psi$ are not distinguishable under the $SU(N)$ gauge transformations:

- **gauge fields**: $A^a_\mu (a = 1, \cdots, N^2 - 1)$
- **Dirac fields**: $\Psi = (\psi^1, \cdots, \psi^N)^t$
- **connection**: $D^a_\mu = \partial^a_\mu + igA^a_\mu \tau_a$
- **curvature**: $F^{a}_{\mu\nu} = \frac{i}{g}[D^a_\mu, D^a_\nu] = (\partial^a_\mu A^a_\nu - \partial^a_\nu A^a_\mu + g\lambda^a_{bc}A^b_\mu A^c_\nu)\tau_a$

**gauge transformation**: $U(x) = e^{i\theta^a(x)\tau_a} \in SU(N)$,

$$\tilde{\Psi}(x) = U(x)\Psi(x), \quad \tilde{A}^a_\mu \tau_a = \frac{i}{g}(\partial^a_\mu U)\Psi + U A^a_\mu \tau_a U^{-1}$$

where $\theta^a = \theta^a(x) (1 \leq a \leq N^2 - 1)$ are real parameters, and the traceless Hermitian matrices $\tau_a$ are generators of $SU(N)$ with $[\tau_a, \tau_b] = \tau_a \tau_b - \tau_b \tau_a = i\lambda^c_{ab} \tau_c$. 
Principle of Representation Invariance (PRI) (Ma-Wang, 2012): Physical laws for an $SU(N)$ gauge theory are independent of representations of $SU(N)$:

- Transformation of the generators $\tau_a = \{\tau_1, \cdots, \tau_K\}$: $\tilde{\tau}_a = x^b_ax^b_b$, $X = (x^b_a)$.

- $\theta^a$, $A^a_\mu$, and $\lambda^c_{ab}$ are $SU(N)$-tensors under this transformation.

- $G_{ab} = \frac{1}{4N} \lambda^c_{ad} \lambda^d_{cb} = \frac{1}{2} \text{Tr}(\tau_a \tau_b^\dagger)$ is a symmetric positive definite 2nd-order covariant $SU(N)$-tensor, which can be regarded as a Riemannian metric on $SU(N)$.

- The representation invariant action and gauge field equations are

  $$ L = \int_M \left[ -\frac{1}{4} G_{ab} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu}^a F_{\alpha\beta}^b + \bar{\Psi} \left[ i\gamma^\mu (\partial_\mu + igA^a_\mu \tau_a) - m \right] \Psi \right], $$

  $$ \begin{align*}
  G_{ab} \left[ \partial^\nu F^b_{\nu\mu} - g\lambda^b_{cd} g^{\alpha\beta} F^c_{\alpha\mu} A^d_{\beta} \right] - g\bar{\Psi} \gamma_\mu \tau_a \Psi = (\partial_\mu + \alpha_b A^b_\mu)\phi_a, \\
  (i\gamma^\mu D_\mu - m)\Psi = 0 \quad \text{Dirac eqs for fermions}
  \end{align*} $$
Evidence for PID # 2: Spontaneous symmetry breaking and Higgs particle

- The Yang-Mills action $-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}^{a}F_{\alpha\beta}^{b}$ does not contain mass term $m^{2}g^{\mu\nu}A_{\mu}^{a}A_{\nu}^{a}$. The field equations do not have mass, inconsistent with physics.

- Y. Nambu 1960, Y. Nambu & G. Jona-Lasinio 1961 proposed that one needs to introduce a physically natural mechanism to destroy the gauge symmetry to generate mass. The mechanism is called spontaneous symmetry breaking (BSS).

- Brout-Englert, Higgs, and Guralnik-Hagen-Kibble 1964 proposed to add action of a particle field $\phi$ into the Yang-Mills: $\mathcal{L}_{YM} + \mathcal{L}_{H}(\phi)$ such that a mass term is generated at the vacuum state $\phi = 1$ of $\phi$:

$$\mathcal{L}_{YM} + \mathcal{L}_{H}(\phi' + 1) = \mathcal{L}_{YM} + m^{2}A_{\mu}^{a}A^{\mu a} + \text{other terms in } \mathcal{L}_{H}$$

- As Nambu pointed out, the introduction of the Higgs field $\phi$ is very artificial, and the electroweak theory based on Higgs mechanism violates PRI.
Mass generation with PID (Ma-Wang, 2012):

- PID fields equations:

$$\partial^\mu F^a_{\mu\nu} - g\lambda^a_{bc}g^{\alpha\beta}F^b_{\alpha\nu}A^c_{\beta} - gJ^a_{\nu} = \left[\partial_{\nu} - \frac{m^2x_{\nu}}{4} + \alpha_bA^b_{\nu}\right]\phi^a$$

where $\phi^a$ are scalar fields, $m^2x_{\nu}/4$ is the mass potential of the scalar field $\phi^a$.

- Let $\phi_0$ be the vacuum state of $\phi^a$, and transform $\phi^a \rightarrow \phi^a + \phi_0$ and $A^a_{\mu} \rightarrow A^a_{\mu}$:

$$\partial^\mu F^a_{\mu\nu} - m^2aA^a_{\nu} - g\lambda^a_{bc}g^{\alpha\beta}F^b_{\alpha\nu}A^c_{\beta} - gJ^a_{\nu} = \left[\partial_{\nu} - \frac{m^2x_{\nu}}{4} + \alpha_bA^b_{\nu}\right]\phi^a - \frac{m^2x_{\nu}\phi_0}{4}$$

$$\partial^{\nu}\partial_{\nu}\phi^a + m^2\phi_a = \alpha_b\partial^{\nu}(A^b_{\nu}\phi^a) - g\partial^{\nu}\left[\lambda^a_{bc}g^{\alpha\beta}F^b_{\alpha\nu}A^c_{\beta} - J^a_{\nu}\right]$$

where $m^2a = \alpha_a\phi_0$. 
III. Unified Field Theory

The unified field model is derived based on the following principles:

- principles of general relativity and Lorentz invariance Einstein (1905, 1915)
- principle of interaction dynamics (PID), M.-W. (2012)
- principle of representation invariance (PRI), M.-W. (2012)

Lagrangian action functional is the natural combination of the Einstein-Hilbert functional, the standard $U(1)$-QED, SU(2)-weak and SU(3)-strong interaction actions.
Unified field model (Ma-Wang, 2012):

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{8\pi G}{c^4} T_{\mu\nu} = \left[ \nabla_\mu - \frac{e \alpha^E}{\hbar c} A_\mu - \frac{g_w \alpha^w_a}{\hbar c} W^a_\mu - \frac{g_s \alpha^s_s k}{\hbar c} S^k_\mu \right] \Phi_\nu, \]

\[ \partial^\nu F_{\nu\mu} - e \bar{\psi} \gamma_\mu \psi = \left[ \nabla_\mu - \frac{e \alpha^E}{\hbar c} A_\mu - \frac{g_w \alpha^w_a}{\hbar c} W^a_\mu - \frac{g_s \alpha^s_s k}{\hbar c} S^k_\mu \right] \phi^E, \]

\[ G^{ww}_{ab} \left[ \partial^\nu W^b_\nu - \frac{g_w}{\hbar c} \lambda_c^b g^{\alpha\beta} W^c_\alpha W^d_\beta \right] - g_w \bar{L} \gamma_\mu \sigma_a L = \left[ \nabla_\mu + \frac{1}{4} \left( \frac{m_{HC}}{\hbar} \right)^2 x_\mu - \frac{e \alpha^E}{\hbar c} A_\mu - \frac{g_w \alpha^w_a}{\hbar c} W^a_\mu - \frac{g_s \alpha^s_s k}{\hbar c} S^k_\mu \right] \phi^w, \]

\[ G^{ss}_{kj} \left[ \partial^\nu S^j_\nu - \frac{g_s}{\hbar c} \Lambda^j_c d g^{\alpha\beta} S^c_\alpha S^d_\beta \right] - g_s \bar{q} \gamma_\mu \tau_k q = \left[ \nabla_\mu + \frac{1}{4} \left( \frac{m_{\pi C}}{\hbar} \right)^2 x_\mu - \frac{e \alpha^E}{\hbar c} A_\mu - \frac{g_w \alpha^w_a}{\hbar c} W^a_\mu - \frac{g_s \alpha^s_s j}{\hbar c} S^j_\mu \right] \phi^s, \]

\[ (i \gamma^\mu \tilde{D}_\mu - \tilde{m}) \Psi = 0. \]
Conclusions and Predictions of the Unified Field Model

1. **Duality:** The unified field model induces a natural duality:

\[
\begin{align*}
\{ g_{\mu\nu} \} \text{ (massless graviton)} & \quad \leftrightarrow \quad \Phi_\mu, \\
A_\mu \text{ (photon)} & \quad \leftrightarrow \quad \phi^E, \\
W^a_\mu \text{ (massive bosons } W^\pm \text{ & } Z) & \quad \leftrightarrow \quad \phi^w_a \quad \text{for } a = 1, 2, 3, \\
S^k_\mu \text{ (massless gluons)} & \quad \leftrightarrow \quad \phi^s_k \quad \text{for } k = 1, \cdots, 8.
\end{align*}
\]

2. **Decoupling and Unification:** An important characteristics is that the unified model can be easily decoupled. Namely, Both PID and PRI can be applied directly to individual interactions.

For gravity alone, we have derived modified Einstein equations, leading to a unified theory for dark matter and dark energy.
3. **Spontaneous symmetry breaking and mass generation mechanism:** We obtained a much simpler mechanism for mass generation and energy creation, completely different from the classical Higgs mechanism. This new mechanism offers new insights on the origin of mass.

4. The two $SU(2)$ and $SU(3)$ constant vectors $\{\alpha^w_a\}$ and $\{\alpha^s_k\}$, containing 11 parameters, represent the portions distributed to the gauge potentials by the weak charge $g_w$ and strong charge $g_s$. We define, e.g., the total potential $S_\mu$:

$$S_\mu = \alpha^s_k S^k_\mu = \{S_0, S_1, S_2, S_3\}$$

=\{strong charge potential, strong rotational potential\}. 
5. Strong interaction

• Quantum Chromodynamics based on an $SU(3)$ gauge theory (Yukawa, Yang-Mills, Gell-Mann, O. Greenberg, ...)

• Layered strong interactions potentials (Ma-Wang, 2012 & 13): based on two new principles: principle of interaction dynamics (PID), and principle of representation invariance (PRI):

\[
S_0 = g_s(\rho) \left[ \frac{1}{r} - \frac{A_s}{\rho} \left( 1 + \frac{r}{R} \right) e^{-r/R} \right], \quad g_s(\rho) = g_s \left( \frac{\rho_w}{\rho} \right)^3,
\]

where $A_s$ is a constant depending on the particle type, and $R$ is the attracting radius of strong interactions given by

\[
R = \begin{cases} 
10^{-16} \text{cm} & \text{for } w^* \text{ and quarks}, \\
10^{-13} \text{cm} & \text{for hadrons}.
\end{cases}
\]
These potentials match very well with experimental data.

Again, strong interaction demonstrates both attraction and repelling behaviors.
Quark confinement: With these potentials, the binding energy of quarks can be estimated as

\[ E_q \sim \left( \frac{\rho_n}{\rho_0} \right)^6 E_n \sim 10^{24} E_n \sim 10^{22} \text{GeV}, \]

where \( E_n \sim 10^{-2} \text{GeV} \) is the binding energy of nucleons.

This clearly explains the quark confinement.
Short-range nature of strong interaction:

With the strong interaction potentials, at the atom/molecule scales, the ratio between strong force and electromagnetic attraction force is

\[
\frac{F_a}{F_e} = 9g_s^2 \left( \frac{\rho_0}{\rho_a} \right)^6 / e^2 \sim \begin{cases} 10^{-38} & \text{at the atomic level,} \\ 10^{-44} & \text{at the molecular level.} \end{cases}
\]

This demonstrates the short-range nature of strong interaction.
6. Weak interactions

- An $SU(2)$ gauge theory

- Ma-Wang, 2012 & 2013: Layered gauge weak interaction potentials:

$$W_0 = g_w(\rho)e^{-r/r_0} \left[ \frac{1}{r} - \frac{A_w}{\rho} \left( 1 + \frac{2r}{r_0} \right) e^{-r/r_0} \right], \quad g_w(\rho) = g_w \left( \frac{\rho_w}{\rho} \right)^3$$

where $\rho_w$ and $\rho$ are the radii of the constituent weakton and the particle, $A_w$ is a constant depending on the types of particles, and $r_0 \simeq 10^{-16} \text{cm}$ is the radius of weak interaction.

- The potentials show that the weak interaction is also short-ranged

- The weak interaction/force demonstrates both attraction and repelling behaviors.
7. Stability of matters

The attraction and repelling nature of all four forces leads to the stability of the matter in the universe.