combination of right hand (positive) aspect of PC 1 and bottom (negative) aspect of PC 2
Tips for data manipulation in *Mathematica*

*Flatten[ ]* - Takes a nested list and flattens it into a single list.

\[
\begin{align*}
\text{In[171]} &= \text{Flatten}[\{\{1, 2\}, \{3, 4\}\}] \\
\text{Out[171]} &= \{1, 2, 3, 4\}
\end{align*}
\]

*Partition[ ]* - Opposite of flatten: groups items into a nested list.

\[
\begin{align*}
\text{In[172]} &= \text{Partition}[\{1, 2, 3, 4\}, 2] \\
\text{Out[172]} &= \{\{1, 2\}, \{3, 4\}\}
\end{align*}
\]

*Transpose[ ]* - Switches columns to rows and rows to columns.

\[
\begin{align*}
\text{In[168]:= data} &= \{\{1, 2\}, \{3, 4\}\}; \\
data \text{ // MatrixForm} \\
\text{Transpose[data] \text{ // MatrixForm}}
\end{align*}
\]

\[
\begin{align*}
\text{Out[169]:= MatrixForm}[&
\begin{pmatrix}
1 & 2 \\
3 & 4
\end{pmatrix}]
\end{align*}
\]

\[
\begin{align*}
\text{Out[170]:= MatrixForm}[&
\begin{pmatrix}
1 & 3 \\
2 & 4
\end{pmatrix}]
\end{align*}
\]
Step 3: Principal Components Analysis

Principal Components Analysis (PCA) ordnates the objects in your analysis by arranging them in a shape space. Similarities and differences can easily be seen in a PCA plot.

The axes of a PCA plot are Principal Components (PCs). The first PC of any analysis is, by definition, the one that shows the largest axis of variation in shape. The second PC shows the next largest axis of variation that is uncorrelated with the first, the third PC shows the third largest axis of variation, and so on.

Each point on a PCA plot represents the shape of a single object from your analysis. The closer two objects are, the more similar they are in shape.
Step 3: PCA (cont.)

A PCA plot is often called a *morphospace* in GMM because each point on the plot represents a different shape or, more specifically, a different configuration of landmarks. An important part of understanding PCA results is to explore how shape varies in the PCA plot.

The thin-plate spline grids in this PCA plot show how shape varies along PC1 and PC2 for a data set of osteostracan fish head shields (shown at right). For example, at the left of the plot landmark 1 is located far in front of 8 and 11, at the right it is very close to 8 and 11. The species at the left of the plot on the previous slide have shapes like the grids on the left in this plot. Exploring morphospace with these grids can help understand the meaning of the PCA.
Step 3: PCA (cont.)

You can also explore the distribution of shape by referring back to your original photographs.

Compare these shapes to the grids on the previous slide.
Step 3: PCA (cont.)

To do a PCA of shape in Mathematica:

```
PrincipalComponentsOfShape[proc, {1,2}, labels]
```

where `proc` is matrix of Procrustes superimposed coordinates from Step 2, the list `{1,2}` tells the function to plot the first and second principal components, and `labels` is a list of text labels for each object in `proc`.

This function provides the following output:

1. A PCA plot showing the objects with labels
2. Text output explaining how much of the variation in shape is explained by each PC
3. A graphic representation of the mean shape in your data set, where each landmark is indicated by its number and a convex hull has been placed around the landmarks
4. A morphospace model showing thin-plate spline snapshots of shape variation in the PCA plot.
Step 3: PCA (cont.)