Shape modelling from PCA

In R
<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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<tbody>
<tr>
<td>9:30 - 10:30 am</td>
<td>Finish tarsal exercise</td>
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<tr>
<td>10:30 - 11:00 am</td>
<td>Coffee Break</td>
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<tr>
<td>11:00 am - 12:30 pm</td>
<td>Shape modeling</td>
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<tr>
<td>12:30 - 1:30 pm</td>
<td>Lunch Break</td>
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<td>1:30 - 3:00 pm</td>
<td>Quantitative evolution</td>
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<td>3:00 - 3:30 pm</td>
<td>Tea break</td>
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<tr>
<td>4:30 - 5:00 pm</td>
<td>Regression, multivariate, shape modeling</td>
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Shape modelling...
How to construct models of shapes in morphospace

Ingredients:
1. mean shape (consensus)
2. eigenvectors
3. the score (address) of the point to be modelled

Model = consensus + \( \sum \) (scores * eigenvectors)
Shape modelling: note scores of the models
Shape models

Basic equation for 2D shape space:

\[ \text{model} = \text{score PC1} \times \text{vector PC1} + \text{score PC2} \times \text{vector PC2} + \text{consensus} \]

Model at center of the space:

\[ \text{model} = 0 \times \text{vector PC1} + 0 \times \text{vector PC2} + \text{consensus} \]
\[ = \text{consensus} \]

Model at center right:

\[ \text{model} = 0.12 \times \text{vector PC1} + 0 \times \text{vector PC2} + \text{consensus} \]
\[ = 0.12 \times \text{vector PC1} + \text{consensus} \]

Model at upper right:

\[ \text{model} = 0.12 \times \text{vector PC1} + 0.14 \times \text{vector PC2} + \text{consensus} \]
PCA in R

1. Obtain Procrustes coordinates
   \[ \text{proc} \leftarrow \text{gpagen(lands)} \]

2. Convert coordinates to two-dimensional matrix
   \[ \text{coords2d} \leftarrow \text{two.d.array(proc$coords)} \]

3. Calculate consensus and flatten to single vectors
   \[ \text{consensus} \leftarrow \text{apply(proc$coords, c(1,2), mean)} \]
   \[ \text{consensusvec} \leftarrow \text{apply(coords2d, 2, mean)} \]

4. Calculate Procrustes residuals (Procrustes coordinates - consensus)
   \[ \text{resids} \leftarrow \text{t(t(coords2d)-consensusvec)} \]

5. Calculate covariance matrix
   \[ \text{P} \leftarrow \text{cov(resids)} \]

6. Calculate eigenvector and eigenvalues with SVD
   \[ \text{pca.stuff} \leftarrow \text{svd(P)} \]

7. Calculate PCA scores
   \[ \text{scores} \leftarrow \text{resids%*%pca.stuff$u} \]
Create your own modelling function

The following function creates a shape model for a point in the first two dimensions of shape space:

```r
model.space <- function(PC1,PC2) {
    model <- matrix(PC1*eigenvectors[,1]+PC2*eigenvectors[,2]+consensusvec,
                     nrow=9,ncol=2,byrow=T)
    plotRefToTarget(consensus,model,sub=paste("PC1=" ,PC1," PC2=" ,PC2," " ))
}
model.space(0.1, -0.05)
```

The following code creates a series of eight models across PC 1 (holding PC 2 and all other dimensions at 0.0)

```r
for(i in seq(from=-0.1, to=0.05, length.out=8)) {model.space(i,0)}
```

You can flip from one to the next with (CNTL-arrow) in PC or (Command-arrow) in Mac.
Recreate Procrustes coordinates

Recreate the original Procrustes landmark coordinates using all eigenvector dimensions:

```r
coords2d.svd <- t(t(scores%*%solve(eigenvectors))+consensusvec)
```

Coords from geomorph

![Coords from geomorph](image1)

Coords reconstructed from SVD

![Coords reconstructed from SVD](image2)
Dimensionality and shape modeling

Remember that if you create a model with only one (or two or three) eigenvectors, its position on other eigenvectors is 0

In other words

$1 \times \text{PC 1 eigenvectors} + \text{consensus is model at point } (1, 0, 0, 0, 0, 0, 0, 0...)$

in shape space