Multivariate regression and MANOVA

Extending and modeling
Univariate Regression

Create a variable

> my.rand.var <- rnorm(28)

Regress PC1 onto the variable

> my.regr <- lm(scores[,1]~my.rand.var)

summary(my.regr)

Call:
  lm(formula = scores[, 1] ~ my.rand.var)

Residuals:
  Min      1Q  Median      3Q     Max
-0.087875 -0.027512  0.001516  0.034111  0.055853

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.001486   0.008099   0.183    0.856
my.rand.var -0.007462   0.008540  -0.874    0.390

Residual standard error: 0.0419 on 26 degrees of freedom
Multiple R-squared:  0.02852, Adjusted R-squared:  -0.008842
F-statistic: 0.7633 on 1 and 26 DF,  p-value: 0.390
Univariate regression cont.

> anova(my.regr)

Analysis of Variance Table

Response: scores[, 1]

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>my.rand.var</td>
<td>1</td>
<td>0.001340</td>
<td>0.0013402</td>
<td>0.7633</td>
<td>0.3903</td>
</tr>
<tr>
<td>Residuals</td>
<td>26</td>
<td>0.045647</td>
<td>0.0017557</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multivariate Regression

Regress PC1-PC14 onto the variable

```r
> my.regr <- lm(scores[,1:14]~my.rand.var)

summary(my.regr)

Response Y1 :

Call:  lm(formula = Y1 ~ my.rand.var)

Residuals:
   Min     1Q  Median     3Q    Max
-0.087875 -0.027512  0.001516  0.034111  0.055853

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.001486   0.008099   0.183    0.856
my.rand.var -0.007462   0.008540  -0.874    0.390

Residual standard error: 0.0419 on 26 degrees of freedom
Multiple R-squared:  0.02852, Adjusted R-squared:  -0.008842
F-statistic: 0.7633 on 1 and 26 DF,  p-value: 0.390
```
Multivariate Regression cont.

```r
> anova(my.regr)
Analysis of Variance Table

             Df Pillai approx F num Df den Df Pr(>F)
(Intercept)  1 0.0000  0.00000     14     13 1.0000
my.rand.var  1 0.5157  0.98878     14     13 0.5108
Residuals    26
```

Compared to Univariate results

```r
> anova(my.regr)
Analysis of Variance Table

Response: scores[, 1]

             Df Sum Sq Mean Sq F value Pr(>F)
my.rand.var  1 0.001340 0.0013402  0.7633 0.3903
Residuals    26 0.045647 0.0017557
```
Modeling results of regression or MANOVA

General principles

1. slopes and intercepts describe lines

2. regression is shape variables onto independent variable, therefore slopes and intercepts describe lines in shape space (=PC space)

3. every point in shape space corresponds to a configuration of landmarks

4. you can model the predicted values of a linear regression by using slopes and intercepts to predict scores and then convert the scores to landmark coordinates using eigenvectors
Model the results

Regress all the PCs onto the variable

```r
> my.regr <- lm(scores~my.rand.var)

my.regr$coefficients

Model for my.rand.var == -1

model <- my.regr$coefficients[2,]*-1+my.regr$coefficients[1,] + consensusvec
```
Repeat model with regularly spaced values along variable

my.rand.var = -1.00
Multivariate ANOVA (MANOVA)

> my.anova <- aov(scores[,1:14]~photo.groups)

> anova(my.anova)

Analysis of Variance Table

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Pillai</th>
<th>approx F</th>
<th>num Df</th>
<th>den Df</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1</td>
<td>0.00000</td>
<td>0.0000</td>
<td>14</td>
<td>13</td>
<td>1.0000</td>
</tr>
<tr>
<td>photo.groups</td>
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<td>0.67647</td>
<td>1.9416</td>
<td>14</td>
<td>13</td>
<td>0.1202</td>
</tr>
<tr>
<td>Residuals</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No statistical difference between Andrew’s and Nadia’s photography
Modeling MANOVA

First calculate group means (mean of PC scores for each group)

```r
> group1.mean <- apply(scores[as.vector(photo.groups)==1,],2,mean
> group2.mean <- apply(scores[as.vector(photo.groups)==2,],2,mean)
```

These means represent a position in the multivariate shape space. A model of that point is a model of the mean shape for that group:

```r
> model <- t(t(group1.mean%*%solve(eigenvectors)) + consensusvec)
> plotRefToTarget(consensus,matrix(model,nrow=9,ncol=2,byrow=T))
```

Average face as photographed by Andrew

Average face as photographed by Nadia
Simpler method for modeling ANOVA

plotRefToTarget(consensus, apply(proc $coords[, as.vector(photo.groups)==1], c(1, 2), mean))

plotRefToTarget(consensus, apply(proc $coords[, as.vector(photo.groups)==2], c(1, 2), mean))

Average face as photographed by Andrew

Average face as photographed by Nadia