1. You are given a video of a nuclear explosion and want to estimate the yield of the bomb, i.e., the energy $E$ released by the explosion. From the video, you can determine the blast radius $R$ at a time $t$ shortly after detonation. Assume you also know the density $\rho$ of the surrounding air. What is an approximate expression for $E$?

A. $R^3 \rho^2 / t$
B. $R \rho t$
C. $\rho t^2 / R^2$
D. $R^5 \rho / t^2$
E. $R^2 / t^3$

**Solution:** The correct answer is D. You could determine the correct answer simply by finding the units of each answer choice; only answer choice D has the correct units of energy. We encourage you, however, to take a more systematic approach similar to the following. In the SI system, $R$ has units of meters (m), $\rho$ has units of kg/m$^3$, $t$ has units of seconds (s), and $E$ has units of newtons, i.e., kg·m$^2$/s$^2$. Assuming $E$ is a function of powers of $R$, $\rho$, and $t$ alone, we write $E = R^a \rho^b t^c$ for some numbers $a$, $b$, and $c$; this has units of m$^a$·(kg·m$^{-3}$)·s = m$^{a-3}$·kg$^b$·s$^c$. Therefore, for the dimensions to work out, we need to have $a - 3 = 2$, $b = 1$, and $c = -2$. Note that this expression for $E$ is only approximate; a more accurate expression is $E = CR^5 \rho / t^2$ for some dimensionless constant $C$ (which we cannot find using dimensional analysis).

Dimensional analysis and scaling arguments can be quite helpful for what could otherwise be very complicated calculations; these techniques are used often by working physicists. Those who are interested in learning more might want to look up the Buckingham $\pi$ theorem.

British physicist G.I. Taylor used this method to determine that the yield of the bomb used in the Trinity test was approximately 22 kilotons of TNT, which was within 10% of the U.S. government’s classified official estimate of 20 kilotons. He published this estimate in the *Proceedings of the Royal Society of London A*, to the displeasure of the U.S. government!

2. You are sitting in a boat on a pond, and throw an anchor (which has density about ten times that of water) overboard. What happens to the water level of the pond?

A. It goes up.
B. It goes down.

A. It goes up.
C. It stays the same.
D. There’s not enough information given to determine this.
E. The water level goes down just after the anchor hits the water, but goes up as the anchor sinks.

**Solution:** The correct answer is B. When the anchor is in the boat, it is acting to displace its *weight* in water (since the boat is floating), but when it is in the pond, it is displacing its *volume* in water. Since the anchor is denser than water, the water level goes down when it is thrown in the pond.

Problems 3 and 4 concern the following physical situation:

A thin, uniform, rigid rod of mass $m$ and length $L$ is hung from its end at a frictionless pivot attached to the ceiling. You hit the rod horizontally with a hammer at a distance $d$ from the pivot, imparting an impulse $F\Delta t$, where $F$ is a constant force and $\Delta t$ is a few milliseconds.

3. What are the magnitudes of the velocity $v$ of the center of mass of the rod and the angular velocity $\omega$ of the rod about its center of mass just after the impact? The moment of inertia of the rod about its center of mass is $mL^2/12$.

   A. $v = \frac{F\Delta t}{m}$ and $\omega = \frac{3F\Delta t(2d-L)}{2mL^2}$
   B. $v = \frac{F\Delta t}{2m}$ and $\omega = \frac{6F\Delta t(d-L)}{mL}$
   C. $v = \frac{F\Delta t}{m}$ and $\omega = \frac{6F\Delta t(d-L)}{mL^2}$
   D. $v = \frac{F\Delta t}{m}$ and $\omega = \frac{6F\Delta t(2d-L)}{mL^2}$
   E. $v = \frac{F\Delta t}{2m}$ and $\omega = \frac{6F\Delta t(2d-L)}{mL}$

**Solution:** The correct answer is C. Newton’s second law says that $F = ma$, and treating the force as constant as indicated in the problem gives $v = \frac{F\Delta t}{m}$. Another way of looking at this to remember that the impulse imparted to an object is equal to the resulting change in momentum, which gives $F\Delta t = mv$.

The magnitude of the torque on the rod about its center of mass is $F(d-L/2)$, since the rod is uniform and therefore its center of mass is located a distance $L/2$ from the pivot. Therefore the rotational form of Newton’s second law, $\tau = I\alpha$, here reads $F(d-L/2) = (mL^2/12)(\omega\Delta t)$, and thus $\omega = \frac{6F\Delta t(2d-L)}{mL^2}$.

4. What value of $d$ results in no net force on the rod at the pivot just after the impact?

   A. $L/2$   B. $L/3$   C. $5L/6$   D. $L$   E. $2L/3$
Solution: The correct answer is \( E \). Were the rod free, the speed of the end of the rod where the pivot was would be 

\[
v - \frac{L}{2} \omega = \frac{F \Delta t}{m} \left[ 1 - \frac{6F \Delta t(2d-L)}{mL^2} \cdot \frac{L}{2} \right] = \frac{F \Delta t}{m} \left( 4 - \frac{6d}{L} \right).
\]

In order for this speed to be zero (and therefore for the net force on the rod at the pivot to be zero just after impact) we need 

\[ 4 - \frac{6d}{L} = 0, \]

that is, 

\[ d = \frac{2L}{3}. \]

This point on the rod is called the center of percussion. It is related to the “sweet spot” on a baseball bat. The length \( d \) is called the radius of gyration of the rod; it is the length of a simple pendulum that has the same mass and oscillation period as the rod. We encourage you to do some research and find out more!

5. Twelve 1 \( \Omega \) resistors are connected to make the edges of a cube. What is the equivalent resistance between opposite corners of the cube?

A. \( \frac{5}{6} \)  B. 1  C. \( \frac{1}{2} \)  D. \( \frac{3}{2} \)  E. \( \frac{4}{3} \)

Solution: The correct answer is \( A \). There are a number of ways to arrive at the correct answer. One could use Kirchoff’s loop and junction rules, writing a system of linear equations and solving. While this is a systematic way of dealing with a general circuit, here there is a lot of symmetry we can exploit to do the problem much more quickly.

One way is to introduce a current of 1 A going into vertex A and coming out of vertex H; see the figure above. By symmetry, upon getting to vertex A the current splits into three \( \frac{1}{3} \) A currents. Similarly, by the symmetry of the cube the currents going into vertex H must each be \( \frac{1}{3} \) A. The current going through each of the six remaining edges is \( \frac{1}{6} \) A. For example, the current through edge AB splits...
into 1/6 A through edge BG and 1/6 A through edge BC. Now that we know the currents, we can find the voltage drop along some path from A to H in a direction of current flow by using Ohm’s law. For example, along path ABGH the voltage drop is (1/3 A)(1 Ω) + (1/6 A)(1 Ω) + (1/3 A)(1 Ω) = 5/6 V. Therefore, the equivalent resistance is (5/6 V)/(1 A) = 5/6 Ω.

Another way of quickly solving the problem is to notice that, by symmetry, the voltage is the same at vertices A, B, and D, and similarly vertices E, G, and C lie at the same potential. Therefore, an equivalent circuit is given by three resistors in parallel from vertex A, in series with six resistors in parallel (two connected to each of the resistors coming from vertex A), in series with three resistors in parallel. This circuit is, in turn, equivalent to a 1/3 Ω resistor, 1/6 Ω resistor, and 1/3 Ω resistor in series, which gives an equivalent resistance of 1/3 Ω + 1/6 Ω + 1/3 Ω = 5/6 Ω.

6. Who won the 2014 Nobel Prize in Physics? What did they win it for?
   A. François Englert and Peter Higgs for their prediction of the Higgs boson
   B. François Englert and Peter Higgs for leading experiments at the LHC that resulted in the discovery of the Higgs Boson
   C. Brian Greene and Edward Witten for their numerous contributions to string theory
   D. Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura for their invention of efficient blue LEDs
   E. Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura for their prediction of topological insulators

Solution: The correct answer is D. We’ll leave you to look this one up, if you didn’t already know it. For a more personal account of Nakamura’s journey to the invention, you might enjoy this article, currently available freely online: http://www.scientificamerican.com/article/blue-chip-2000-07-05/. (This article, “Blue Chip” by Glenn Zorpette, was printed in Scientific American on July 5, 2000 and is available freely online.)