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Teacher – Vigo County School Corporation
Gifted and Talented programs for middle school students for 25 years.
Which fractions can be reduced? (mark them out) Which ones are left?

When a fraction cannot be reduced we can say the numerator and denominator are relatively prime. To be relatively prime, two numbers can only have a common factor of 1.

Which of the following pairs of numbers are relatively prime?

12 and 17, 8 and 9, 15 and 22, 36 and 49, 21 and 48, 20 and 27, 14 and 63

Now consider the natural numbers 1-20 inclusive. How many of these numbers are relatively prime to 20?

Consider how we can expand this concept to natural numbers from 1-50, 1-100, 1-366, 1-????

GOTTA BE A BETTER WAY!!!!!!!!!!!!!
Let's go back to our 1-10 problem!

Which fractions did we “mark out”? What was the common factor?

Since the common factors were 2’s and/or 5’s we could consider another approach to finding how many numbers are relatively prime to a particular number.

Consider the prime factorization of 10- $2 \cdot 5$. This means every 2nd number can be divided by 2 and every 5th number can be divided by 5.

So consider taking $10\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 4$. There are 4 natural numbers less than 10 that are relatively prime to 10.

Try this logic for the number of natural numbers less than or equal to 20 that are relatively prime to 20.

Try 50 Try 100 Try 350

Try 256 You pick the number!!!!

Follow up problem- Of 8000 apples harvested, every third apple was too small, every fourth apple was too green and every tenth apples was bruised. The remaining apples were perfect! How many perfect apples were harvested? (Adapted from Mathematics Teaching in Middle School)
Consider the following problem:

At Math City High School there are 12 players on the basketball team. All of the players are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 of the players are taking Spanish and 5 of the players are taking both languages. How many players are taking French?
Consider the problem-
There are 27 cats at the pound. 14 of them are short-haired. 11 of them are kittens. 5 of them are long-haired adult cats (not kittens). How many of them are short-haired kittens?

Where do we start?
How about three overlapping groups!

Consider the problem- At the Goooddog Obedience School, dogs can learn to do three tricks: sit, stay, and roll over. Of the dogs at the school:

- 50 dogs can sit
- 29 dogs can stay
- 34 dogs can roll over
- 9 dogs can do all three
- 17 dogs can sit and stay
- 12 dogs can stay and roll over
- 18 dogs can sit and roll over
- 9 dogs can do none

How many dogs are in the school?

How many dogs can do exactly 2 tricks?
Alternate method of doing problems like this!!!

Two overlapping events

Three overlapping events
EXAMPLE OF DAILY BOARD PROBLEMS

Side ONE - Concept Averages

1. What is the average of 6, 15, and 45?
2. What is the average of \( \frac{1}{2} \) and \( \frac{3}{4} \)?
3. Find the average of the even numbers greater than 1 and less than 15.
4. If the average of five numbers 4.9, 3.1, 7.7, \( k \), and 9.3 is 5.1, determine the value of \( k \).
5. In a set of five numbers, the average of the first two numbers is 10, and the average of the last three is 15. What is the average of all five numbers?
6. A class of thirty students had an average score of 68 on a mathematics test. Another class of 25 students had an average score of 70 on the same test. What was the average score for all the students?
7. The average of two numbers is 7. When a third number is included, the average of the three numbers in 8. What is the third number?
8. The average of -3 and a second number is 2. What is the second number?
9. The average of a set of 10 numbers is 20. If one of the numbers is removed from the set, the average of the remaining numbers is 19. What number was removed?
10. A teacher recorded the test scores for a class of 25 students. Using these recorded scores, the average was calculated to be 72. Sam’s score of 86 was incorrectly recorded as 36. What was the correct average?
11. In a competition, the average score of Pat’s first four games was 6.5; the average of her next five games was 6.4. If she scored 9 on her tenth game, what was her overall average?
12. Dullard Academy served 2068 dull lunches from Monday to Thursday inclusive. Given that the daily average for the five day week was 516 lunches, how many dull lunches were served on Friday?

Side One - Concept - Averages

1. 22 2. 5/8 3. 8 4. 0.5 5. 13
6. 68 10/11 7. 10 8. 7 9. 29 10. 74 11. 6.7
12. 512
Side TWO (Harder)

1. The angles of a triangle are in the ratio of 2:3:4. What is the size of the largest angle?

2. If \( x = (n + 1)(n + 2)(n + 3) \) where \( n \) is a positive integer then \( x \) is NOT always divisible by
   A) 1  B) 2  C) 3  D) 5  E) 6

3. If the result of \( 10^{25} - 25 \) is written in decimal notation, the sum of the digits of that number is?

4. A shopkeeper receives the following bill for 22 model X DVD players: \$29.3\# with the first and last digits smudged beyond recognition. The shopkeeper knows that each DVD player costs over \$25.00. The cost of each DVD player is between
   A) 25 and 28  B) 28 and 32  C) 32 and 35  D) 35 and 40  E) 40 and 50

5. Two candles of equal length start burning at the same time. One of the candles will burn down in 4 hours, the other in 5 hours. How many hours will they have to burn before one candle is 3 times the length of the other?

6. A father in his will left all his money to his children in the following manner: \$1000 to the first born and 1/10 of what then remains, then \$2000 to the second born and 1/10 of what then remains, then \$3000 to the third born and 1/10 of what then remains and so on. When this was done each child had the same amount of money. How many children were there?

7. If \( a + b = 1 \) and \( a^2 + b^2 = 2 \), then \( a^4 + b^4 = ? \)

8. What is the last digit of the number \((7^5)^3\)?

9. A motor car manufacturer makes \( m \) cars per week. The production is increased by \( n\% \). The number of cars made per week is now
   A) \( m + n \)  B) \( m + \frac{n}{100} \)  C) \( \frac{mn}{100} \)  D) \( m(1 + \frac{n}{100}) \)  E) \( 1 + \frac{mn}{100} \)

SOLUTIONS

Side TWO (Harder)

1. 80 degrees  2. D  3. 444  4. C

5. 40/11  6. 9  7. 3 \( \frac{1}{2} \)  8. 3  9. D