The talk began with a list of the five interconnected ideas which proved the focus of the presentation: reflections on elementary school curricula, some random pedagogical insights, some contemporary mathematics, secondary school curriculum development, and computational mathematics. Each part provided thoughts for take-home reflection and use.

**Reflections on Elementary School Curricula**

Indiana University received a grant to study the revised *Investigations in Number, Data, and Space* curriculum. That study was complicated by the timing: it coincided with the implementation of the No Child Left Behind Act. While some think ideal research in classrooms would be “scientific” research, actual research in classrooms is quite complex. Teachers and researchers are torn between fidelity to a curriculum and fidelity to each unique child learning mathematics alongside many other subjects and in the context of his or her own complex life.

**Attending to Pedagogical Nuance**

Three instances of children’s understandings, or misunderstandings, in mathematics were presented. In the first example, a child seemed to understand the commutative property of multiplication for a story context with four rocks in six bags versus six rocks in four bags, but when presented with a context using much larger numbers, 78 and 324, she or he did not appear to understand that the property was still applicable. This suggests that understanding of the commutative property of multiplication might develop in a magnitude-dependent way.

In another instance, students were asked to make up a story for the number sentence $27 \div 6 = 4 \quad R3$. One response was “There are 6 bridges and 27 cars. If the same amount of cars are on each bridge, *and the drivers all want to be on the bridges*, how many cars are
on each bridge? How many cars aren't able to get on?” The italicized phrase suggests that there are fine nuances in children's understanding of operations that are vital for a full understanding of the concepts.

In the third case, the importance of language was highlighted through examples from transcripts of tutoring sessions. In each case, the tutor was saying no without saying “no.” For example, a student says, “Let's see $2(x + 1)$ is $2x + 1$,” and the tutor responds, “Good but $2(x + 1)$ is $2x + 2.$” The result is that the tutor, or teacher, sends a mixed message.

Vertical Articulation with Some Contemporary Mathematics
Next, audience members were asked to work on a mathematical task. Using four, five, and then six points to represent four, five, and six people and two different colors, say red and blue, to represent friends or strangers, the task was to see if the points could be connected (each pair of pants connected by either a red line or a blue line) without creating a monochromatic triangle. After working on the task, participants could see that no matter how one colored the edges of a graph on six (or more) vertices, there will be at least one monochromatic triangle. The study of the emergence of unavoidable structure is called Ramsey Theory. Very few Ramsey Numbers are known.

In some circumstances, even students in elementary school can understand a problem that no one has yet solved. If more students were given an authentic understanding of mathematics, we would stand a better chance of solving more problems. Problems that confront industry and research are problems that don't have easy solutions. This problem is one of the hardest known problems, despite decades of study, we simply don't know. As a result of challenges like this one and increasing computational resources, computational mathematics is growing by leaps and bounds.

Secondary School Curriculum and Interdisciplinary Learning
Elementary and middle schools are much better at integrating subjects than high schools. Yet there are a wide variety of topics that could be used to bridge disciplines, for example modules that could be used in either biology classes or mathematics classes. An example of how minimum inversion numbers are used in genetics and the construction of phylogenetic trees was presented.

Computational Mathematics
What will math education look like in ten years? What if we continue to lose scientific and technical expertise to other countries? What if we let people in other nations lead, innovate, and take risks? The software program *Mathematica* 6 was described and offered
as an example of where the practice and teaching of mathematics are headed. Three online resources were presented:

- http://www.wolframresearch.com/
- http://demonstrations.wolfram.com/
  (where the program Mathematica Player can be downloaded for free).

Five Take-Home Messages

The presentation concluded with five key ideas to take home:

1. Teach beyond the test and the text: apply fidelity to the curriculum, to the mathematics, and to each child’s learning.
2. Be alert for nuance.
3. Vertical articulation shouldn’t end with high school.
4. Interdisciplinarity shouldn’t end in elementary school.
5. The future is here — computational mathematics.