

Implementation of a Standards-based High School Mathematics Program

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In *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001), mathematical proficiency is broken down into five strands:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.” (p. 116)

Of these strands, traditional mathematics education has put an almost exclusive emphasis on procedural fluency. One of the goals of standards-based mathematics programs has been to bring a more balanced approach to mathematics education, producing mathematics learners who go beyond procedural fluency to include conceptual understanding as well as strategic competence, adaptive reasoning, and a productive disposition.

My association with the Indiana Math Initiative (IMI) began with my presenting workshops on graphing calculators for IMI districts during their previous LSC (Local Systemic Change) grant, which dealt with standards-based reform mathematics in the middle school grades.

As the BCSC District Coordinator for IMI in this IU/NSF Math Science Partnership grant, I have worked with Coordinators from the other districts since the inception of our grant in 2002. As math department chairman for Columbus North High School and Northside Middle School, I have responsibility for mathematics textbook adoption. I also teach a class on Mathematical Modeling at Indiana University Bloomington. This assignment at IU came about after I acted as one of the instructors for the Mathematical Modeling summer workshops for secondary math and science teachers that were a component of the IU-NSF MSP grant. These workshops were held at Indiana University Bloomington over the summers of 2004, 2005, and 2006.

In all of these experiences, I have seen the positive results in student achievement when a standards-based math program is properly implemented. My goal has been to see such a program implemented in the secondary schools of our districts, as an appropriate follow-up to the

standards-based math programs adopted at our elementary schools in the districts. However, of the eight districts participating in the IU/NSF MSP, Bartholomew Consolidated School Corporation was the only district to adopt non-traditional high school mathematics texts.

Those texts included *College Preparatory Mathematics* for Algebra 1 and Algebra 2, *Discovering Geometry* from Key Curriculum Press and *Connected Geometry* from the Educational Development Center (Published by Glencoe) for geometry, and *Functions Modeling Change* and *Single Variable Calculus* from the Harvard Consortium for Calculus Reform (published by Wiley Press) for precalculus and calculus.

After four years of using these reformed-minded textbooks, it is apparent that adopting a reform-minded text book and establishing reform mathematics classrooms are two different things. There is a wide range of implementation levels among teachers teaching the reform-minded mathematics programs. Also there is a wide range of student achievement among classes and teachers.

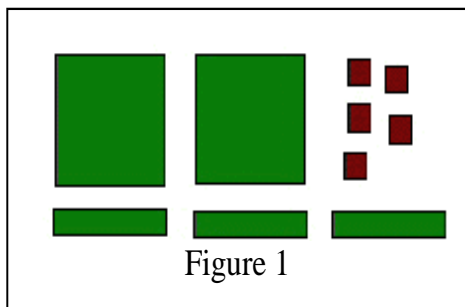
In Indiana, all students take an end-of-course exam (ECA) upon completion of Algebra 1. At both BCSC high schools, there has been a consistent gap in student performance in different teachers' classes. To put that range of scores into perspective, it is necessary to understand some background data on the test itself.

Statewide, the cut-score was set so that passing rates average between 25% and 29% for all students (including 6th, 7th, and 8th graders) taking the Algebra ECA. In BCSC, no 6th graders take algebra, approximately 5%-8% of the students take algebra in seventh grade, 35%-40% take algebra in eighth grade and the remainder take it in ninth grade or begin to study it in ninth grade as part of a two-year program. The passing rate for BCSC 7th graders is 100% and the passing rate for BCSC 8th graders is 85%. The remaining 52% to 60% of the students take the Core 401 in high school in fairly heterogeneous classes. The two-year students are actually mixed in with the one-year students during the second year of their program. The passing rate for these high school students has been around 25%, very close to the state average for all students. Among the high school classes taking the Algebra ECA, the passing rates have varied from 4% to 50% passing. This difference could be attributed to a between classes difference in student ability, except that each teacher teaches three or four algebra classes and the passing rates are fairly consistent for each teacher across classes and from year to year.

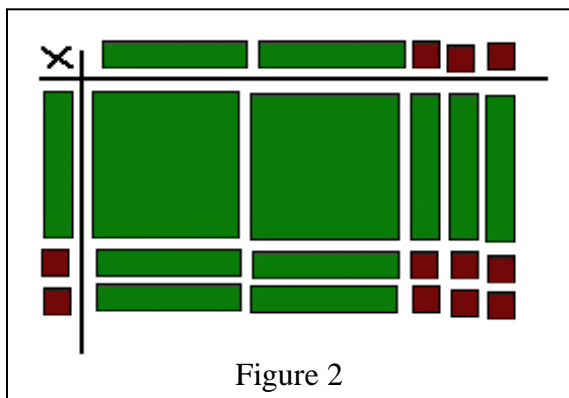
What then causes the wide difference in results? Teacher observation, while not a scientific study, has yielded evidence of differences in teacher implementation of the reform mathematics programs. It does appear that there is a positive correlation between the extent of implementation of the standards-based program and student achievement. The purpose of this paper is to discuss those instructional attributes and behaviors that seem to correlate with higher student achievement and with the spirit of implementing a reform mathematics program.

Components of the College Preparatory Mathematics program are aimed at moving students' mathematical understanding beyond the procedural fluency strand. The CPM program attempts to achieve conceptual understanding in several ways. One of these ways is that topics are often presented first from a conceptual perspective.

¹ The Indiana General Assembly has made completion of Core 40 a graduation requirement for all students beginning with those entering high school in the fall of 2007. The legislation also makes Core 40 a minimum college admission requirement for the state's public four-year universities beginning in the fall of 2011. The Core 40 requirements for mathematics include Algebra I, Algebra II, and Geometry, or completion of an Integrated Math Series I, II, and III.



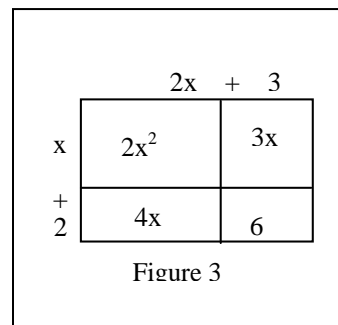
As an example, multiplying binomials is introduced via a concrete modality with algebra tiles. An area model is used in which rectangles are formed to model certain dimensions and the area is found by finding the area of all of the tiles. Three types of tiles are used: unit tiles, the small squares, have dimensions 1 by 1; X tiles, the long rectangles, have dimensions 1 by X, and X-squared tiles, the large squares, have dimensions X by X. The picture in figure 1, for example, shows $2x^2 + 3x + 5$ using algebra tiles.



To multiply $(x+2)$ by $(2x+3)$ a rectangle is formed with dimensions $(x+2)$ and $(2x+3)$. Figure 2 shows the dimensions using algebra tiles along the top and the left side of figure 2. The rectangle with those dimensions is represented by the algebra tiles in the lower right of figure 2. The area of that rectangle can be expressed in two ways; length times width would be the product of $(x+2)$ and $(2x+3)$, and the area can be found by adding the algebra tiles in the lower right section where there are two x-squared tiles, seven x-tiles, and six unit tiles representing $2x^2 + 7x + 6$. In this

model, the x-tiles are separated into two sections with 3 x's in one section and 4 x's in the other. This corresponds to the values obtained when finding the product using the traditional "FOIL" multiplication from the "outside" and "inside" partial products.

Students progress from a concrete to a representational model as they begin to draw the algebra tiles and eventually draw "generic" rectangles. For $(x+2)(2x+3)$, the generic rectangle is shown in figure 3. It can be seen that the area of this rectangle is $2x^2 + 7x + 6$. The intent of the program is that presenting binomial multiplication through an area model with algebra tiles will yield more conceptual understanding than the traditional "FOIL" multiplication method.



In observing teachers teach this topic, it became apparent that not all teachers viewed this model as a conceptual one. Teachers sometimes cut short the work with the algebra tiles, thus bypassing the concrete model, going directly to the generic rectangle representation. That model itself became more procedural and less conceptual as some teachers presented the lesson without ever mentioning areas and dimensions. The rectangle actually just became a "bookkeeping" method to make sure all parts got included in the multiplication procedure. The rectangle became

another mnemonic tool to help manage the procedure much like “FOIL” is used to keep track of “First two,” “Outside two,” “Inside two,” and “Last two.” There was also no connection made to the previous work students had done with the distributive property which is illustrated both in the area model and in other methods for multiplying binomials.

The CPM curriculum made a strong attempt to connect different representations in establishing a firm conceptual understanding, but teachers, either because they did not see the conceptual connection themselves or because they saw their role as primarily teaching procedures, left that connection out of their lessons.

Other topics where similar teacher observations were made included solving equations. The curricular program’s intended conceptual approach is using algebra tiles and a “balance” method, but some teachers were observed reverting back to symbol manipulation procedures. Also the intent of the program was to approach solving proportions by treating them as any other equation rather than approaching them with the special process of cross-multiplying. Some teachers approached them as a special case and had students cross-multiply from the initial introduction of the topic.

All teachers had received a total of eight days of professional development during the first year of implementing the program. That training, however, emphasized the processes and procedures that teachers were to use in the classroom to teach the material, more than the rationale behind those procedures. This does not mean the teacher training is an ineffective approach to a standards-based program. It means that the teacher training is vitally important and should include a strong emphasis on the underlying learning theory. The training should also include reflection and discussion around conceptual understanding and what that means for each topic covered in a course. That stronger emphasis might have convinced teachers to incorporate the approaches that incorporate conceptual understanding into the teaching and learning of algebra. Teachers were often simply told to “trust the program,” when in actuality a stronger emphasis on rationale and research during the training would go a long way in establishing that trust.

The CPM program is also based on the premise that students learn more when they are given tasks that require higher cognitive demand and when the teacher works at maintaining that high cognitive demand throughout the lesson. To achieve this goal, students are given some problems to work on individually and in their study teams before the teacher instructs them as to how to do those particular problems. This is followed by a whole class discussion on the problems as well as the concepts and procedures involved in doing them. From this discussion, the teacher is to make sure that the students have learned the intended mathematics.

This approach appeared to be undermined in several ways. (1) Some teachers did a mini-lesson first, explaining the mathematical procedures prior to the students working and brought down the level of cognitive demand placed on the students. The lesson actually reverted back to the typical teacher demonstrate-student practice model. (2) As the students began to work on the problems, individual students or groups of students asked questions of the teacher who responded by showing them or explaining to them how to complete the problem. (3) Some teachers told the students that this math program required that the students figure out how to do the math on their own without the teacher’s help. This often resulted in phone calls from parents that the teacher “was not teaching.” (4) Because of the class schedule (45-minute periods), teachers often found themselves leaving out or rushing through the class discussion component of the instructional model, so that students had little or no time to make sure they understood the important mathematics that was to be included in the lesson.

A key to overcome these pitfalls is for teachers to meet regularly and have discussions specifically addressing these issues. They can share ways to be aware of the pitfalls and to avoid or correct them. The original training did involve some classroom video clips showing what should happen and what should not happen in a lesson, but during the course of implementation, those topics need to be revisited on a regular basis.

Student engagement is related to the idea of higher demand cognitive tasks, but it is worth considering on its own merit. One of the premises of the CPM program is that students learn best when they are actively working and discussing the mathematics with their classmates. The intent of the program is that this is accomplished through their participation during classroom time spent working with study teams.

Teachers had a varying degree of success in meeting this goal of student-to-student communication on study teams and balancing it with individual accountability. One difference that was noted very early in the implementation is the different approach that teachers took in moving students into those teams. Teachers used a variety of approaches to assigning students to study teams: random assignment, teacher choice, or student choice. The greatest difference, however, came in the physical implementation of those teams on a daily basis.

The most ineffective method appeared to be where students chose their own groups on a daily basis; some groups may have had two people, others up to six, and some students worked individually. Little or no conversation centered on the mathematics beyond an occasional comparing of answers. The “teams” were more of a means of social interaction for students during math class.

Most of the mathematics classrooms have student desks rather than tables. For students to work on teams they would need to move their desks into a group. The desks may have been moved together or the students may have remained in their rows and just turned around periodically to talk or compare answers. Some teachers actually had predetermined teams, but the students were not required to move their desks together. In this scenario, there did tend to be slightly more discussion about mathematics and fewer students were working completely on their own, but again, most, if not all, of the mathematical conversation centered on comparing answers. In both of these approaches, students were allowed to freely discuss things with people outside of their own study team. The result was that if a student wanted to discuss any non-mathematical topic with someone else in the room, he or she could turn to them or get up and go to them and spend class time doing just that. At times, teachers using either of these first two methods for work on study teams complained that the students did not really work well in teams.

Finally, some teachers had predetermined teams and established the routine that the students must turn their desks so that all team members were facing each other and were sitting close enough to see each other’s work. This approach resulted in more student conversation and conversations that went beyond comparing answers as students looked at each other’s work as they discussed it.

As an experiment, I would ask the teacher to require the students to move their desks into a team formation. On one occasion we actually made that change in the middle of a class session. The change in the amount and the quality of the student-to-student communication was immediate and significant.

In implementing any standards-based mathematics program, students will be expected to work together and discuss with their classmates the mathematics they are working on. It is vitally important that teachers are trained in how to work with students in a cooperative learning format and have time throughout the implementation to revisit this topic and to discuss effective

approaches with other teachers. It cannot be assumed that the skills needed to work together on common math problems are inherent to students or that the facilitation skills needed to manage those groups is inherent to teachers.

Communication and individual accountability are also important aspects of students working together to solve math problems. As part of the original training for the CPM curriculum, teachers were shown a number of “study team strategies.” The purpose of these strategies is to require students to discuss the mathematics with other students. This helps the student-speaker evaluate his/her own understanding and clarify his/her thinking. It also helps the student-listener and encourages all students to use and understand correct mathematics vocabulary.

Some of the strategies are as simple as requiring students to put their pencils down as they begin work on a problem to require them to discuss the problem and devise a plan before they begin to write. This strategy is called “Teammates Consult.” Another strategy, called “Swap Meet,” has one or two students in each group move to another group to share what each group has accomplished in solving the problem. Another study team strategy, called “Hot Potato,” has one student in the team do one step of a problem as he/she explains it to the rest of the team. That student then passes the paper to the next team member who completes and explains the next step. This continues until the problem is solved. Each strategy is designed to increase student-to-student communication and/or individual accountability.

During the first two years of implementation, teachers were so focused on the new curriculum itself and the new approaches to teaching some of the algebra topics that the study team strategies were overlooked. Before year three of implementation, summer workshops were held reviewing and practicing those study team strategies. In general, the inclusion of study team strategies increased as did student achievement as measured by the end of course exam. There still existed a wide variation in the degree of teacher implementation of the strategies and the degree of implementation correlated positively with student achievement.

Adopting and implementing a standard-based mathematics program is a much larger task than merely choosing which series to adopt. It must involve significant professional development before implementation and on-going professional development throughout the implementation. The quality of the professional development is also important. If we expect more than procedural proficiency from our students, then it is important that our teachers are more than procedurally proficient at teaching mathematics. They need to know what they are teaching from their own conceptual understanding. They must know how to teach each topic with understanding, with higher cognitive demand tasks, and with student engagement. Finally, they must understand the theory and research basis that underlies what they are doing.

As teachers work together to implement standards-based mathematics, they can work towards a broader mathematical fluency from their students, going beyond conceptual understanding and procedural fluency to include strategic competence, adaptive reasoning, and a productive disposition, the personal view that mathematics is a useful endeavor, and the belief in one’s own capability of doing and understanding.

References

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