ADDING *Some*

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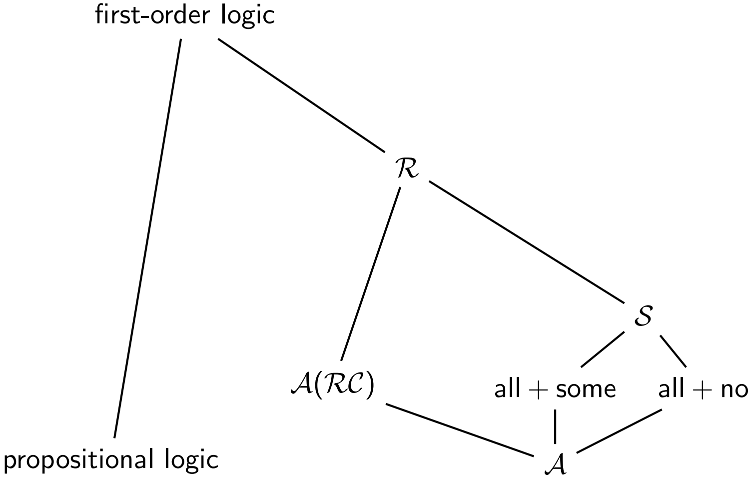
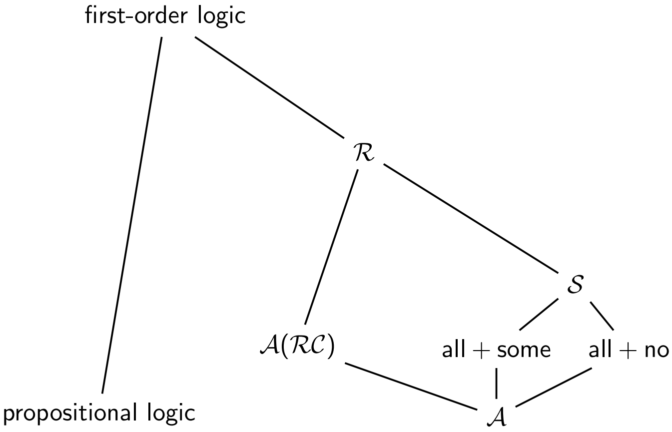
We started with the very small logic $\mathcal{A}$ of All x are y.

And most recently we added verbs and relative clauses.

At this point, we go back to $\mathcal{A}$ and add
Some x are y.

Tomorrow, we'll want to add *some* to the language of verbs, and then negation.


first-order logic
$\mathcal{R}$
$\mathcal{S}$
$\mathcal{A}(\mathcal{RC})$
all + some
all + no
propositional logic
$\mathcal{A}$



My plan is to do a pretty quick treatment of the logic
all + some

We start with a set **P** of unary atoms,
also called nouns.

The sentences today are All p are q and Some p are q, where p and q are atoms.

For the semantics, we use models $\mathcal{M}$ that consist of a set M with interpretations $[\![p]\!]$ of the atoms.

Then we define

$$\begin{array}{lll} \mathcal{M} \models \text{All } p \text{ are } q & \text{iff} & [\![p]\!] \subseteq [\![q]\!] \\ \mathcal{M} \models \text{Some } p \text{ are } q & \text{iff} & [\![p]\!] \cap [\![q]\!] \neq \emptyset \end{array}$$

$M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$[\![p]\!] = \{1, 3, 4\}$
$[\![q]\!] = \{4\}$
$[\![n]\!] = \{1, 7\}$.
$[\![x]\!] = \emptyset$

Then we have the following facts:

$$\begin{aligned}
\mathcal{M} &\models \text{Some } n \text{ are } p \\
\mathcal{M} &\models \text{Some } p \text{ are } n \\
\mathcal{M} &\models \text{Some } p \text{ are } p \\
\mathcal{M} &\not\models \text{Some } x \text{ are } x
\end{aligned}$$

What should $\Gamma \models \varphi$ mean?

All p are v, All v are q, Some v are w $\not\models$ Some p are p.

Can you find a counter-model for this?
That is, can you find a model where the assumptions are true and the conclusion is false?

$$\frac{}{\text{All } p \text{ are } p}\ \textsc{axiom} \qquad \frac{\text{All } p \text{ are } n \quad \text{All } n \text{ are } q}{\text{All } p \text{ are } q}\ \textsc{barbara}$$

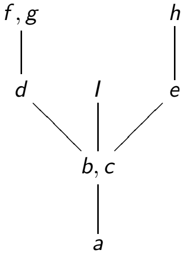
$$\frac{\text{Some } p \text{ are } q}{\text{Some } p \text{ are } p}\ \textsc{some}_1 \qquad \frac{\text{Some } p \text{ are } q}{\text{Some } q \text{ are } p}\ \textsc{some}_2$$

$$\frac{\text{All } q \text{ are } n \quad \text{Some } p \text{ are } q}{\text{Some } p \text{ are } n}\ \textsc{darii}$$

$$\dfrac{\text{All } n \text{ are } q \qquad \dfrac{\dfrac{\text{All } n \text{ are } p \quad \text{Some } n \text{ are } n}{\text{Some } n \text{ are } p}\ \textsc{darii}}{\text{Some } p \text{ are } n}\ \textsc{some}_1}{\text{Some } p \text{ are } q}\ \textsc{darii}$$

In words:
if there is an *n*, and if all *n*s are *p*s and also *q*s, then some *p* is a *q*.

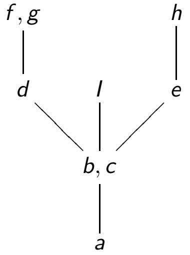
$$\Gamma = \left\{ \begin{array}{l} \text{All a are b,} \\ \text{All a are c,} \\ \text{All b are c,} \\ \text{All c are b,} \\ \text{All c are d,} \\ \text{All b are e,} \\ \text{All d are g,} \\ \text{All f are g,} \\ \text{All g are f,} \\ \text{All e are h,} \\ \text{All b are l,} \\ \text{Some d are e,} \\ \text{Some l are g} \end{array} \right\}$$



Does $\Gamma \models$ *Some f are h*?

Does $\Gamma \models$ *Some d are l*?

$$\Gamma = \left\{ \begin{array}{l} \text{All a are b,} \\ \text{All a are c,} \\ \text{All b are c,} \\ \text{All c are b,} \\ \text{All c are d,} \\ \text{All b are e,} \\ \text{All d are g,} \\ \text{All f are g,} \\ \text{All g are f,} \\ \text{All e are h,} \\ \text{All b are l,} \\ \varphi\text{: Some d are e} \\ \psi\text{: Some l are g} \end{array} \right\}$$



$[\![a]\!] = \emptyset$
$[\![b]\!] = \emptyset$
$[\![c]\!] = \emptyset$
$[\![d]\!] = \{\varphi\}$
$[\![e]\!] = \{\varphi\}$
$[\![f]\!] = \{\varphi, \psi\}$
$[\![g]\!] = \{\varphi, \psi\}$
$[\![h]\!] = \{\varphi\}$
$[\![l]\!] = \{\psi\}$

Here is a model which shows $\Gamma \not\models$ *Some d are l*.
$M = \{\varphi, \psi\}$. The points of the model are sentences!
For all x, put φ in $[\![x]\!]$ provided $d \leq x$ or $e \leq x$.
For all x, put ψ in $[\![x]\!]$ provided $l \leq x$ or $g \leq x$.

Completeness: if $\Gamma \models \varphi$, then $\Gamma \vdash \varphi$

When φ is an All-sentence, it's just like what we saw in Tuesday's homework.

Let's do this when φ is a Some-sentence, say Some y are z.

So we assume that $\Gamma \models$ Some y are z,
and we show using a
canonical model that we devise based on our example above
that $\Gamma \vdash$ Some y are z.

Completeness: if $\Gamma \models$ Some y are z, then $\Gamma \vdash$ Some y are z

At this point, I want to remind you of some notation from earlier in the course.

If we have a set Γ in mind,
we sometimes write

$$x \leq y$$

to mean

$$\Gamma \vdash \text{All } x \text{ are } y$$

We do this to save a little space.

Completeness: if $\Gamma \models$ Some y are z, then $\Gamma \vdash$ Some y are z

Let M be the set of all Some-sentences in Γ.

We will use letters like φ and ψ for these,
and we speak of the first and second atoms in these sentences.

Here is how we interpret nouns in our model:

$$[\![u]\!] \quad = \quad \{\varphi : \underline{\hspace{6cm}}\}$$

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$$
\begin{aligned}
[\![u]\!] &= \{\varphi \in M : \text{one of the two atoms in } \varphi \text{ is } \leq_\Gamma u\} \\
&= \{\text{“Some } x \text{ are } y\text{” in } \Gamma : \text{either } \Gamma \vdash \text{All } x \text{ are } u \text{ or } \Gamma \vdash \text{All } y \text{ are } u\}
\end{aligned}
$$

LEMMA

$\mathcal{M} \models \Gamma$.

Once this is done, we have $\mathcal{M} \models$ Some y are z.
Why?
And we use this last fact to show that $\Gamma \vdash$ Some y are z.

Completeness: if $\Gamma \models$ Some y are z, then $\Gamma \vdash$ Some y are z

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Lemma

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COMPLETENESS: IF $\Gamma \models$ Some y are z, THEN $\Gamma \vdash$ Some y are z

LEMMA

$\mathcal{M} \models \Gamma$.

PROOF.

(first half)
Take an All-sentence in Γ, say All a are b.

Let $\varphi \in [\![a]\!]$. We show that $\varphi \in [\![b]\!]$.

Suppose that φ is Some m are n.

Then either $m \leq a$, or else $n \leq a$.
Since $a \leq b$, we see that either $m \leq b$, or else $n \leq b$.

And this means that $\varphi \in [\![b]\!]$.

That is, $\mathcal{M} \models$ All a are b. □

LEMMA

$\mathcal{M} \models \Gamma$.

PROOF.

(second half)

Take a Some-sentence in Γ, say Some a are b.

This sentence itself belongs to $[\![a]\!] \cap [\![b]\!]$.

So $\mathcal{M} \models$ Some a are b.

□

Recall that we are assuming that $\Gamma \models$ Some y are z, and then proving that $\Gamma \vdash$ Some y are z.

We have a model $\mathcal{M}$, and we showed that $\mathcal{M} \models \Gamma$.

So $\mathcal{M} \models$ Some y are z.

Thus, in our model $[\![y]\!] \cap [\![z]\!] \neq \emptyset$.

Let $\varphi \in \Gamma$ belong to $[\![y]\!] \cap [\![z]\!]$.

Let's write φ as Some a are b.

Since $\varphi \in [\![y]\!]$, either $a \leq y$ or $b \leq y$.

Since $\varphi \in [\![z]\!]$, either $a \leq z$ or $b \leq z$.

Recall that we are assuming that $\Gamma \models$ Some y are z,
and then proving that $\Gamma \vdash$ Some y are z.
Γ contains the sentence Some a are b.

CASE 1: $a \leq y$ AND $a \leq z$

CASE 2: $a \leq y$ AND $b \leq z$

CASE 3: $b \leq y$ AND $a \leq z$

CASE 4: $b \leq y$ AND $b \leq z$

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$$\dfrac{\begin{array}{c}\vdots\\ \text{All } a \text{ are } z\end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\vdots\\ \text{All } a \text{ are } y\end{array} \quad \dfrac{\text{Some } a \text{ are } b}{\text{Some } a \text{ are } a}\,\mathrm{SOME}_1}{\text{Some } a \text{ are } y}\,\mathrm{DARII}}{\text{Some } y \text{ are } a}\,\mathrm{SOME}_2}{\text{Some } y \text{ are } z}\,\mathrm{DARII}$$

CASE 2: $a \leq y$ AND $b \leq z$

CASE 3: $b \leq y$ AND $a \leq z$

CASE 4: $b \leq y$ AND $b \leq z$

Recall that we are assuming that $\Gamma \models$ Some y are z,
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Case 4: $b \leq y$ and $b \leq z$

$$\dfrac{\begin{matrix}\vdots\\ \text{All } b \text{ are } z\end{matrix} \qquad \dfrac{\dfrac{\begin{matrix}\vdots\\ \text{All } b \text{ are } y\end{matrix} \quad \dfrac{\dfrac{\text{Some } a \text{ are } b}{\text{Some } b \text{ are } a}\,\mathrm{SOME}_2}{\text{Some } b \text{ are } b}\,\mathrm{SOME}_1}{\text{Some } b \text{ are } y}\,\mathrm{DARII}}{\text{Some } y \text{ are } b}\,\mathrm{SOME}_2}{\text{Some } y \text{ are } z}\,\mathrm{DARII}$$

Recall that we are assuming that $\Gamma \models$ Some y are z,
and then proving that $\Gamma \vdash$ Some y are z.

CASE 1: $a \leq y$ AND $a \leq z$

CASE 2: $a \leq y$ AND $b \leq z$

You try this one.

CASE 3: $a \leq z$ AND $b \leq y$

You try this one.

CASE 4: $a \leq z$ AND $b \leq z$

TO SEE IF $\Gamma \vdash$ *Some a are b*

- Construct the *All*-graph of Γ.
- Look at All the *Some* sentences in Γ one at a time.
  Let's say that one of them is *Some u are v*.
  Ask if one of the following 4 conditions hold:

  - We can get from u to a, and from u to b.
  - We can get from u to a, and from v to b.
  - We can get from v to a, and from u to b.
  - We can get from v to a, and from v to b.

  If this happens for any sentence *Some u are v* in Γ_{some},
  then we know that $\Gamma \vdash$ *Some a are b*.

- If not, then we want to make a counter-model, say $\mathcal{M}$.

  We take the universe M to be the set Γ_{some}.
  Some u are v $\in [\![W]\!]$ iff either $u \leq W$ or $v \leq W$.

Recall that $\mathcal{A}(\mathcal{RC})$ is our language with

- term formers r all t
- sentence former All t u.

We now want to enlarge this by adding

- sentence former Some t u.

(Of course, we will then want to add term formers r all t.
But I'm not going to go there today.)

$$\frac{}{\textsf{All } p \textsf{ are } p}\ \textsc{axiom} \qquad \frac{\textsf{All } p \textsf{ are } n \quad \textsf{All } n \textsf{ are } q}{\textsf{All } p \textsf{ are } q}\ \textsc{barbara}$$

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$$\frac{\textsf{All } q \textsf{ are } n \quad \textsf{Some } p \textsf{ are } q}{\textsf{Some } p \textsf{ are } n}\ \textsc{darii}$$

$$\frac{\textsf{All } x\ (r \textsf{ all } y) \quad \textsf{All } z\ y}{\textsf{All } x\ (r \textsf{ all } z)}\ \textsc{down}$$

Theorem

There is no finite sound and complete set of rules of the kind we have been studying for this language.

But this doesn't work!

Theorem

There is no finite sound and complete set of rules of the kind we have been studying for this language.

Here is a hint of why this is the case.

$$\text{some } c\ d, \text{all } a\ x, \text{all } a\ y, \text{all } (r \text{ all } a)\ x, \text{all } (r \text{ all } a)\ y \vdash \text{some } x\ y.$$

Let Γ be the set of assumptions on the left.

Claim

$\Gamma \models \text{some } x\ y$

Can you think about this?

Hint

Do it by cases depending on whether $[\![a]\!] =$ or not.

We adopt a new rule, (cases).

$$\dfrac{\dfrac{[\text{some } x\ x]}{\varphi} \quad \dfrac{[\text{all } z_1\ (r_1 \text{ all } x)] \quad \cdots \quad [\text{all } z_k\ (r_k \text{ all } x)]}{\varphi}}{\varphi}\ \text{CASES}$$

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In words, to prove φ from Γ, we may

- Take any variable (=noun) x.
- Add some x are x to Γ, and prove φ.
- Add any set of sentences all z (r all x) to Γ, and again prove φ.

Proof by Cases: An example

We show that

1. $\Gamma \cup \{\text{some } a\ a\} \vdash \text{some } x\ y$
2. $\Gamma \cup \{\text{all } c\ (r \text{ all } a)\} \vdash \text{some } x\ y$

The first is easy from all a x and all a y.

The second comes from

$$\frac{\dfrac{\text{all } c\ (r \text{ all } a) \quad \text{all } (r \text{ all } a)\ y}{\text{all } c\ y} \quad \dfrac{\dfrac{\text{all } c\ (r \text{ all } a) \quad \text{all } (r \text{ all } a)\ x}{\text{all } c\ x} \quad \text{some } c\ d}{\text{some } c\ x}}{\text{some } x\ y}$$

The resulting system is complete.

However, adding proof features like
proof by cases and reductio ad absurdum
complicates the proof search algorithm.

Indeed, in a strictly syllogistic system, the relation $\Gamma \vdash \varphi$ is
in polynomial time,
and adding the extra features could give us proof systems for
which this relation is
co-NP-complete.

In other words, if we have a complete logic for which the problem
$\Gamma \vdash \varphi$
is co-NP-complete.
then (assuming $P \neq NP$), III we cannot hope to find a finite,
purely syllogistic proof system.

For the particular logic that we are studying,

- term formers r all t
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the relation $\Gamma \vdash \varphi$ happens to be in polynomial time.

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the relation $\Gamma \vdash \varphi$ happens to be in polynomial time.

Even more strangely,
it's possible to extend the language and then indeed get a purely syllogistic system!

We add to our current language a new four-place sentence former

$$a \Vdash_{xy} b$$

with semantics

$$\mathcal{M} \models a \Vdash_{xy} b \quad \text{iff} \quad [\![x]\!] \cap [\![y]\!] = \emptyset \text{ implies } [\![a]\!] \subseteq [\![b]\!]$$
$$\text{i.e.,} \quad (\text{Some } x\ y) \vee (\text{All } a\ b)$$

When $x = y$, we write $a \Vdash_{x} b$.

A finite, complete set of rules

Alex Kruckman & LM (2017)

$$\frac{}{\text{All } p \text{ are } p}\ \textsc{axiom} \qquad \frac{\text{All } p \text{ are } n \quad \text{All } n \text{ are } q}{\text{All } p \text{ are } q}\ \textsc{barbara}$$

$$\frac{\text{Some } p \text{ are } q}{\text{Some } p \text{ are } p}\ \textsc{some}_1 \qquad \frac{\text{Some } p \text{ are } q}{\text{Some } q \text{ are } p}\ \textsc{some}_2$$

$$\frac{\text{All } q \text{ are } n \quad \text{Some } p \text{ are } q}{\text{Some } p \text{ are } n}\ \textsc{darii} \qquad \frac{\text{All } x \ (r \text{ all } y) \quad \text{All } z \ y}{\text{All } x \ (r \text{ all } z)}\ \textsc{down}$$

$$\frac{}{a \Vdash_{xy} a}\ \mathrm{R}_0 \qquad \frac{a \Vdash_{yx} b}{a \Vdash_{xy} b}\ \mathrm{R}_1 \qquad \frac{a \Vdash_{xy} b}{a \Vdash_{x} b}\ \mathrm{R}_2 \qquad \frac{a \Vdash_{xy} b \quad b \Vdash_{xy} c}{a \Vdash_{xy} c}\ \mathrm{R}_3 \qquad \frac{\text{all } a \ b}{a \Vdash_{xy} b}\ \mathrm{R}_4$$

$$\frac{a \Vdash_{xy} b}{r \text{ all } b \Vdash_{xy} r \text{ all } a}\ \mathrm{R}_5 \qquad \frac{a \Vdash_{xy} x \quad a \Vdash_{xy} y}{b \Vdash_{xy} r \text{ all } a}\ \mathrm{R}_6 \qquad \frac{a \Vdash_{xy} x \quad a \Vdash_{xy} y}{a \Vdash_{xy} b}\ \mathrm{R}_7$$

$$\frac{a \Vdash_{uv} b \quad u \Vdash_{xy} x \quad v \Vdash_{xy} y}{a \Vdash_{xy} b}\ \mathrm{R}_8 \qquad \frac{\text{some } a \ b \quad a \Vdash_{xy} x \quad b \Vdash_{xy} y}{\text{some } x \ y}\ \mathrm{R}_9 \qquad \frac{\text{some } x \ y}{a \Vdash_{xy} b}\ \mathrm{R}_{10}$$

We add a complement symbol to our nouns and verbs.
We always understand $\overline{\overline{x}}$ to be the same as x.

No embedded relative clauses,
and subjects noun phrases must not contain relative clauses.

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We always understand $\overline{\overline{x}}$ to be the same as x.

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and subjects noun phrases must not contain relative clauses.

formal syntax	read in English
all(p, q)	all p are q
some(p, q)	some p are q
all$(p, r$ all $q)$	all p r all q
all$(p, r$ some $q)$	all p r some q
some$(p, r$ all $q)$	some p r all q
some$(p, r$ some $q)$	some p r some q
some$(p, \overline{q})$	some p aren't q
all$(p, \overline{q})$	no p are q
some$(p, \overline{r}$ some $q)$	some p don't-r some q
some$(p, \overline{r}$ all $q)$	some p don't-r any q
all$(p, \overline{r}$ some $q)$	all p don't-r some q
all$(p, \overline{r}$ all $q)$	all p don't-r any q

A model $\mathcal{M}$ for $\mathcal{R}$ is a set M,
together with interpretation functions

$$\llbracket\ \rrbracket : \mathbf{P} \to \mathcal{P}(M)$$
$$\llbracket\ \rrbracket : \mathbf{R} \to \mathcal{P}(M \times M)$$

This means that for each unary atom $p \in \mathbf{P}$, $\llbracket p \rrbracket \subseteq M$, and for each binary atom r, $\llbracket r \rrbracket \subseteq M \times M$. We interpret literals $\overline{p}$ and $\overline{r}$ using set complements:

$$\llbracket \overline{p} \rrbracket = M \setminus \llbracket p \rrbracket \qquad \llbracket \overline{r} \rrbracket = M^2 \setminus \llbracket r \rrbracket.$$

We then interpret terms by subsets of M in the following way

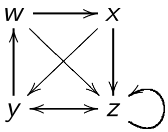
$$\begin{array}{lcl} \llbracket s \text{ all } p \rrbracket & = & \{m \in M : \text{for all } n \in \llbracket p \rrbracket, (m,n) \in \llbracket s \rrbracket\} \\ \llbracket s \text{ some } p \rrbracket & = & \{m \in M : \text{for some } n \in \llbracket p \rrbracket, (m,n) \in \llbracket s \rrbracket\} \end{array}$$

Finally,

$$\begin{array}{lcl} \mathcal{M} \models \text{all}(p,c) & \text{iff} & \llbracket p \rrbracket \subseteq \llbracket c \rrbracket \\ \mathcal{M} \models \text{some}(p,c) & \text{iff} & \llbracket p \rrbracket \cap \llbracket c \rrbracket \neq \emptyset \end{array}$$

One unary atom p, and one binary atom s.

A model $\mathcal{M}$ might have $M = \{w, x, y, z\}$, $[\![p]\!] = \{w, x, y\}$ and s as shown below:



In this model,

$$[\![\overline{p}]\!] = \{z\}$$
$$[\![s \text{ all } p]\!] = \emptyset$$
$$[\![s \text{ some } \overline{p}]\!] = M$$
$$[\![\overline{s} \text{ some } p]\!] = M$$

Here are two $\mathcal{R}$-sentences true in $\mathcal{M}$: some(p, p), and also all$(p, s \text{ some } \overline{p})$.

$$\frac{\mathsf{some}(p,q) \quad \mathsf{all}(q,c)}{\mathsf{some}(p,c)}\ (\mathsf{D1}) \qquad \frac{\mathsf{all}(p,q) \quad \mathsf{all}(q,c)}{\mathsf{all}(p,c)}\ (\mathsf{B})$$

$$\frac{\mathsf{all}(p,q) \quad \mathsf{some}(p,c)}{\mathsf{some}(q,c)}\ (\mathsf{D2}) \qquad \frac{}{\mathsf{all}(p,p)}\ (\mathsf{T}) \qquad \frac{\mathsf{some}(p,c)}{\mathsf{some}(p,p)}\ (\mathsf{I})$$

$$\frac{\mathsf{all}(q,\bar{c}) \quad \mathsf{some}(p,c)}{\mathsf{some}(p,\bar{q})}\ (\mathsf{D3}) \qquad \frac{\mathsf{all}(p,\bar{p})}{\mathsf{all}(p,c)}\ \mathsf{zero} \qquad \frac{\mathsf{some}(p, t\ \mathsf{some}\ q)}{\mathsf{some}(q,q)}\ (\mathsf{II})$$

$$\frac{\mathsf{all}(p, t\ \mathsf{all}\ n) \quad \mathsf{some}(q,n)}{\mathsf{all}(p, t\ \mathsf{some}\ q)}\ (\mathsf{all}\exists) \qquad \frac{\mathsf{some}(p, t\ \mathsf{some}\ q) \quad \mathsf{all}(q,n)}{\mathsf{some}(p, t\ \mathsf{some}\ n)}\ (\exists\exists)$$

$$\frac{\mathsf{all}(p, t\ \mathsf{some}\ q) \quad \mathsf{all}(q,n)}{\mathsf{all}(p, t\ \mathsf{some}\ n)}\ (\mathsf{allall}) \qquad \begin{array}{c}[\overline{\varphi}] \\ \vdots \\ \bot \\ \hline \varphi \end{array}\ \mathsf{RAA}$$