Natural Logic
Homework 1

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PROBLEM 1

The semantics of *All X are Y* in a model is that $[\![X]\!] \subseteq [\![Y]\!]$.

What should the semantics be for the following:

All X which are Y are Z.

What should the semantics be for the following:

Of the X, the only ones which are Y are Z.

It might help to put in concrete plural nouns for X, Y, and Z.

Recall that in any model $\mathcal{M}$ where $[\![x]\!] = \emptyset$, then $\mathcal{M} \models \mathsf{All}\ x\ y$, no matter what y is. The reason for this is that the empty set is a subset of every set, so automatically $[\![x]\!] \subseteq [\![y]\!]$ in $\mathcal{M}$.

Sometimes people don't like this. So let's define a new word All*, and make a logical language using the sentences $\mathsf{All}^*\ x\ y$ for all unary atoms x and y. Let's say that we want to do logic with this language. So in this problem, the only sentences which we consider are those of the form $\mathsf{All}^*\ x\ y$ for all unary atoms x and y. We define

$$\mathcal{M} \models \mathsf{All}^*\ x\ y \quad \text{iff} \quad [\![x]\!] \neq \emptyset \text{ and } [\![x]\!] \subseteq [\![y]\!]$$

Notice that if we have a model $\mathcal{M}$ and an atom x where $[\![x]\!] = \emptyset$, then automatically $\mathcal{M} \not\models \mathsf{All}^*\ x\ y$, no matter what y is.

Problem 2

1. True or false? $\models \mathsf{All}^*\ x\ x$. Why or why not? (Saying that $\models \mathsf{All}^*\ x\ x$ is the same thing as saying that $\emptyset \models \mathsf{All}^*\ x\ x$.)
2. Show that $\mathsf{All}^*\ x\ y \models \mathsf{All}^*\ x\ x$.
3. Show that $\mathsf{All}^*\ x\ y \models \mathsf{All}^*\ y\ y$.
4. Show that $\mathsf{All}^*\ x\ y, \mathsf{All}^*\ y\ z \models \mathsf{All}^*\ x\ z$.

This is a continuation of the last problem. Let's make a logical system for our logic using the following rules:

$$\frac{\mathsf{All}^*\ x\ y}{\mathsf{All}^*\ x\ x}\ \text{R} \qquad \frac{\mathsf{All}^*\ x\ y}{\mathsf{All}^*\ y\ y}\ \text{S} \qquad \frac{\mathsf{All}^*\ x\ y \quad \mathsf{All}^*\ y\ z}{\mathsf{All}^*\ x\ z}\ \text{B}$$

Note that there is no (AXIOM) rule. Using what you just did, the logic is easily sound. Your job in this problem is to show the completeness of the system. The work is strongly based on the completeness of the system A for the All-logic.

Fix a set Γ. Define the canonical model of Γ to be the model $\mathcal{M}$, where M is the set of all atoms, and with the interpretation function given as follows: For all atoms w,

$$[\![w]\!] \;=\; \{x : \Gamma \vdash \mathsf{All}^*\ x\ w\}.$$

PROBLEM 3

1. Show that $\mathcal{M} \models \Gamma$. That is, if the sentence $\mathsf{All}^*\ u\ v$ belongs to Γ, show that it is true in $\mathcal{M}$. [Be sure that you are using the semantics of All^*.]
2. Show that every sentence true in $\mathcal{M}$ can be proved from Γ. That is, if $\mathcal{M} \models \mathsf{All}^*\ a\ b$, then $\Gamma \vdash \mathsf{All}^*\ a\ b$.
3. Use the last two parts to prove that our logic is complete. [This should be quite familiar.]