NATURAL LOGIC
HOMEWORK 2: NAMES AND THE LOGIC OF *All* p *are* q

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August 7-11, 2017

Homework 2: Syntax and Semantics

Here is another logical system. In addition to a set **P** of unary atoms (=nouns) denoting subsets of a given set, our syntax in this homework will also include a set **N** of names. Then we take as sentences the following expressions:

- All p are q
- a isa p
- a is b

In this list and onwards, we'll use a and b for names, and p, q, ..., for nouns. For the semantics, we start with a set M and interpret a name a by an element $[\![a]\!] \in M$.

Note

We might well have different names $n \neq m$ such that $[\![n]\!] = [\![m]\!]$.

This gives us the definition of a model $\mathcal{M}$.
Now for the interpretation of sentences, we go as follows:

$$
\begin{array}{lll}
\mathcal{M} \models \text{All } p \text{ are } q & \text{iff} & [\![p]\!] \subseteq [\![q]\!] \\
\mathcal{M} \models a \text{ isa } p & \text{iff} & [\![a]\!] \in [\![p]\!] \\
\mathcal{M} \models a \text{ is } b & \text{iff} & [\![a]\!] = [\![b]\!]
\end{array}
$$

Homework 2: First case of completeness

Here is a set of rules for our logic:

$$\frac{}{\text{All } p \text{ are } p}\ \text{AX} \qquad \frac{\text{All } p \text{ are } q \quad \text{All } q \text{ are } n}{\text{All } p \text{ are } n}\ \text{BARB}$$

$$\frac{}{a \text{ is } a}\ \text{R} \qquad \frac{b \text{ is } a}{a \text{ is } b}\ \text{S} \qquad \frac{a \text{ is } b \quad b \text{ is } c}{a \text{ is } c}\ \text{T}$$

$$\frac{a \text{ is } b \quad b \text{ isa } p}{a \text{ isa } p}\ \text{U} \qquad \frac{a \text{ isa } p \quad \text{All } p \text{ are } q}{a \text{ isa } q}\ \text{V}$$

We are going to show the completeness of this system.

Problem 1

Suppose that Γ is a set of sentences in this language, and that

$$\Gamma \models \text{All } x \text{ are } y.$$

Let Δ be the set of sentences in Γ that are of the form All p are q. Show that $\Delta \models$ All x are y. (So by the completeness of the All logic, $\Delta \vdash$ All x are y.)

Hint

Use a model-theoretic argument. Take a model of Δ and turn it into a model of Γ. The construction is pretty easy, so don't work too hard.

Homework 2: Remaining cases of completeness

Problem 2

Suppose that Γ is a set of sentences in this language, and that

$$\Gamma \models a \text{ isa } p.$$

Let $\mathcal{M}$ be the set of equivalence classes of names, using the relation

$$b \equiv c \quad \text{iff} \quad \Gamma \vdash b \text{ is } c$$

1. Figure out how to interpret the nouns and names in this model.
2. Show that $\mathcal{M} \models \Gamma$.
3. By the last part, $\mathcal{M} \models a$ isa p.
   Use this to show that $\Gamma \vdash a$ isa p.

"Problem" 3

Everything you did in Problem 2 also holds when

$$\Gamma \models a \text{ is } b.$$

So there's nothing to do here.

Problem 4

Is the following valid or not? And why?

Everyone likes everyone who likes Pat
Pat likes every clarinetist

Everyone likes everyone who likes everyone who likes every clarinetist (1)